CHAPTER 15

Stock Index Products: Strategy Based

Many stock index products are inextricably linked to particular index derivative trading strategies. This chapter focuses on such products. The first is portfolio insurance. Portfolio insurance is a means of protecting a stock portfolio against the prospect of declining prices. Like any insurance policy, the face amount of the insurance is prespecified as is the life of the policy. The insurance is purchased by buying a put, either directly or synthetically, with an exercise price equal to the face amount of the insurance and a time to expiration equal to the term of the policy. Buying the put directly is called *passive portfolio insurance*; creating it synthetically, *dynamic portfolio insurance*. The first section describes a variety of portfolio insurance trading strategies.

The second group of products are funds based on an index/option trading strategy. The first such product to appear in the marketplace was based on the CBOE's Buy-Write Index (BXM). The BXM buy-write strategy involves buying the S&P 500 index portfolio and selling one-month, at-the-money call options. While such a strategy should theoretically perform the same as the S&P 500 portfolio on a risk-adjusted basis (as we demonstrated in Chapter 10), it has performed better over the last 16 years. The reason is that index options appear to have been overpriced (i.e., their implied volatility has been too high relative to realized volatility) and converge to their correct values over time. The second section describes the BXM trading strategy in detail and shows its historical performance.

The final group of index products that we discuss is market volatility derivatives. Essentially two types exist—contracts on realized volatility and contracts on volatility implied by index option prices. In the third section, we describe different volatility contract specifications and show how the CBOE's Market Volatility Index (VIX) can be constructed from a portfolio of S&P 500 index options. We then illustrate how volatility derivatives can be used as an alternative investment in an asset allocation framework.

INSURING STOCK PORTFOLIOS

Portfolio insurance is a means of protecting your portfolio against the prospect of declining prices. Like any insurance policy, the face amount of the insurance as well as the term of the policy are specified. The insurance is created by buying a put, either directly or synthetically. The put's exercise price is the face amount of the policy and its time to expiration is the term. Buying the put directly is called *passive portfolio insurance*; creating it synthetically, *dynamic portfolio insurance*.

The history of portfolio insurance in the United States is an interesting story in financial innovation.[1] It began in the mid-1970s when Hayne Leland, a Berkeley finance professor, dreamed up the concept of dynamic portfolio insurance. The easiest way to create portfolio insurance is to buy a put option written on the stocks in the portfolio, but, at the time, neither put options in general nor index put options in particular were traded. Leland's idea, further refined with Mark Rubinstein, was to mimic the payoffs of an insured portfolio by continuously rebalancing a portfolio of stock and T-bills or a portfolio of stocks. As the market rose, risk-free bonds would be liquidated and more stocks purchased. As the market fell, stocks would be sold and risk-free bonds purchased. The two academics enlisted the help of a professional marketer named John O'Brien, formed an advisory firm called "Leland-O'Brien-Rubinstein" (LOR) and began marketing portfolio insurance. Their service was to provide clients with instructions on how to rebalance their portfolios as the market moved. They landed their first client in the fall of 1980.

An early problem in implementing the strategy was that it was difficult and costly for many clients to buy and sell simultaneously the stocks in their portfolio. Program trading was in its infancy. Another problem was that active portfolio managers did not take kindly to outsiders giving them orders to buy or sell stocks in their portfolio with little or no warning. Consequently, the birth of S&P 500 index futures in March 1982 was a godsend. Index futures allowed managers to tailor their market risk exposures quickly and inexpensively, without touching the stocks in their portfolios. The market for portfolio insurance flourished. By 1987, more than $60 billion in stock portfolios were covered by dynamic portfolio insurance.

The end came with the market crash on Monday, October 19, 1987. On Friday, October 16, 1987, there was a nervousness in the market. The S&P 500 index fell by more than 5% during the trading day. Figure 15.1 shows that the December 1987 futures price was at a discount relative to the index several times during the day including at the close. The nervousness grew over the weekend, and, by Monday morning, there was outright panic. The December 1987 futures price opened about 19 points lower than its Friday close and at an 18 point discount to the index. See Figure 15.2. With the decline in the market, dynamic portfolio insurances triggers were hit, and futures contracts were sold. But, the success of LOR dynamic portfolio insurance depends on the futures price being at its theoretical level, and the futures contract stays at its theoretical level only when index arbitrageurs are at work. On the morning of October 19, the trading of stocks on the NYSE was hopelessly congested. And, without

[1] See Bernstein (1996, pp. 316–320).

FIGURE 15.1 Intraday prices for the December 1987 S&P 500 futures and S&P 500 index on Friday, October 16, 1987.

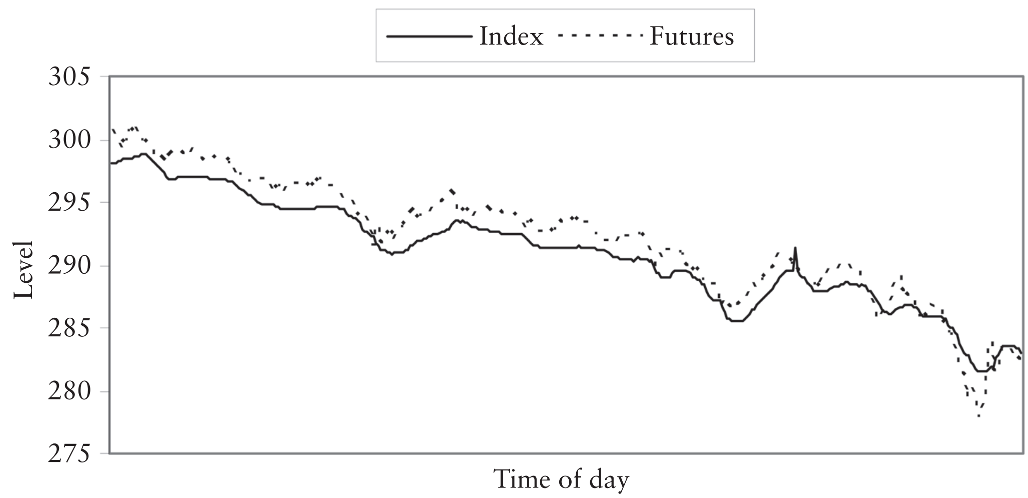
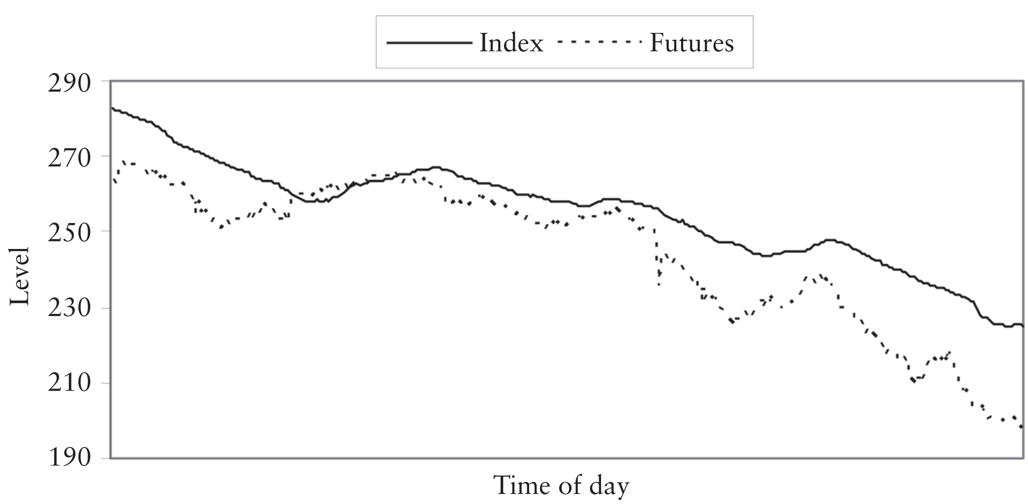


FIGURE 15.2 Intraday prices for the December 1987 S&P 500 futures and S&P 500 index on Monday, October 19, 1987.



the ability to sell shares, index arbitrageurs were unwilling to buy the futures. Consequently, the futures price went into a freefall, reaching levels as high as 30 points below the reported index level.

The failure of dynamic portfolio insurance during the October 1987 crash spurred interest in passive portfolio insurance. While index put options had been launched by the CBOE in March 1983, their trading volume was modest. After the crash, trading volume exploded. With such strong demand, the OTC markets took notice. Since exchange-traded products are standardized, portfolio managers have limited degrees of freedom in setting floor values and insurance horizons. In addition, OTC option dealers have the flexibility to write puts on any basket of stocks that the customer wants as opposed to the handful of indexes offered by the exchanges.[2]

This section focuses first on the passive portfolio insurance created by buying index puts. It then uses the mechanics of a passively insured portfolio and the Black-Scholes/Merton (BSM) option valuation formula to develop many of the dynamic portfolio insurance trading strategies that have been used in the marketplace.

Passive Portfolio Insurance

Passive portfolio insurance involves buying an index put option. To start, assume that the underlying stock portfolio is an index like the S&P 500 and that the put is written directly on the index. The level of the S&P 500 index is denoted S, δ is its dividend yield rate,[3] and σ is its volatility rate. The price of a European-style index put with exercise price X and time to expiration T is p. To insure the value of the index portfolio, we buy one put for each $e^{-\delta T}$ units in the index portfolio.[4] Since the continuous dividend yield assumption involves immediately reinvesting all dividends in the index portfolio, $e^{-\delta T}$ units grow to exactly 1 unit at expiration. The initial and terminal values of this portfolio are shown in Table 15.1. If the index level at expiration closes below the floor value of the insurance, gains on the put will offset exactly any losses on the stocks. If the index level closes above the floor value, the investor earns the gains.

TABLE 15.1 Insuring an index portfolio using an index put.

Position	Initial Value	Terminal Value (T)	
		$S_T \leq X$	$S_T > X$
Buy index portfolio	$-Se^{-\delta T}$	$\tilde{S}_T$	$\tilde{S}_T$
Buy index put	$-p$	$X - \tilde{S}_T$	0
Net portfolio value	$Se^{-\delta T} + p$	X	$\tilde{S}_T$

ILLUSTRATION 15.1 Create static portfolio insurance by buying index puts.

Suppose you are an index fund manager with a $50 million position in the S&P 500 portfolio, and want to buy insurance that the portfolio value will not fall below a level of $50 million by the end of next year. The current level of the S&P 500 is 1,500, its dividend yield rate is 1.5%, and its volatility rate is 20%. The current risk-free interest rate is 6%.

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[2] Responding to customer demand for more flexible contracts, the CBOE introduced FLEX options in which the customer is allowed to choose the exercise price, expiration date, the style of option, and the means of settlement. The underlying index, however, must be the same as the index options already trading at the CBOE.

[3] The constant dividend yield rate assumption is the more difficult case. Adjusting for discrete dividend payments means only subtracting the present value of the promised portfolio dividends from the value of the portfolio being insured.

[4] If the stock portfolio is not the index but rather a stock portfolio with risk level β_P relative to the index, we scale the number of puts by a factor of β_P.

Find the appropriate number (and exercise price) of the index puts needed to provide for the \$50 million floor value in one year. Show the initial and terminal values of the insured portfolio for a range of index levels between 500 and 2,500 in increments of 100. The denomination of the S&P 500 index option is 100 times the index level.

Compute number of puts and exercise price

To find the number of index puts to buy today, you need to find the number of units of the index portfolio that we will have in one year. With \$50 million in the index portfolio and each unit worth 1,500, you currently have

$$n = \frac{50{,}000{,}000}{1{,}500(100)} = 333.333$$

units, where the 100 in the denominator is the contract multiplier of the S&P 500 index option contract. As a result of the S&P 500 index paying dividends, however, the number of index units will grow to $n = 333.33e^{0.015(1)} = 338.371$ by the end of one year. The required number of index puts is therefore 338.371. With the number of index puts computed, you now must compute the exercise price of each put. With the floor value of the portfolio set at \$50 million in one year, the exercise price of each put should be

$$X = \frac{50{,}000{,}000}{338.371(100)} = 1{,}477.67$$

Compute terminal values

At an exercise price of 1,477.67, each put costs \$76.3363 (according to the BSM formula), that is,

OV_OPTION_VALUE(1500,1477.67,1,0.06,0.015,0.20,“P”,“E”) = 76.3363

The total cost of the portfolio insurance is therefore

$$\$76.3363 \times 100 \times 338.371 = \$2{,}583{,}000$$

With this level of insurance, the values of the insured portfolio in one year for index levels ranging between 500 and 2,500 are as follows:

Terminal Values

Index Level	Stock Portfolio Value	Value of Puts	Insured Portfolio Value
500	16,918,551	33,081,449	50,000,000
600	20,302,261	29,697,739	50,000,000
700	23,685,972	26,314,028	50,000,000
800	27,069,682	22,930,318	50,000,000
900	30,453,392	19,546,608	50,000,000
1,000	33,837,102	16,162,898	50,000,000
1,100	37,220,812	12,779,188	50,000,000
1,200	40,604,523	9,395,477	50,000,000
1,300	43,988,233	6,011,767	50,000,000
1,400	47,371,943	2,628,057	50,000,000
1,500	50,755,653	0	50,755,653

Terminal Values			
Index Level	Stock Portfolio Value	Value of Puts	Insured Portfolio Value
1,600	54,139,363	0	54,139,363
1,700	57,523,074	0	57,523,074
1,800	60,906,784	0	60,906,784
1,900	64,290,494	0	64,290,494
2,000	67,674,204	0	67,674,204
2,100	71,057,915	0	71,057,915
2,200	74,441,625	0	74,441,625
2,300	77,825,335	0	77,825,335
2,400	81,209,045	0	81,209,045
2,500	84,592,755	0	84,592,755

Compute initial values

The tables that follow show the initial values of the portfolio assuming you purchased the required number of puts and the index level immediately changes to a different level. Note that, before the put's expiration, the portfolio value may be substantially less than the floor value of $50 million. At an index level of 500, for example, the insured portfolio value is only $47,088,238, well short of the $50 million required. The reason is, of course, that you bought European-style puts, that is, you bought an insurance policy to guarantee at least $50 million in one year. The portfolio value, $47,088,238, is simply the present value of the $50 million, that is, $47,088,238 = $50,000,000$e^{-0.06(1)}$. In one year, if the index level remains at 500, the stock portfolio value will have grown to $16,918,551 due to the re-investment of dividends (i.e., $16,666,667$e^{0.015(1)}$ = $16,918,551) and the terminal value of the puts is their exercise value, $30,081,449 (i.e., 338.371(100)(1,477.67 – 500)).

Portfolio Values with One Year to Expiration			
Index Level	Stock Portfolio Value	Value of Puts	Insured Portfolio Value
500	16,666,667	30,421,560	47,088,227
600	20,000,000	27,088,239	47,088,239
700	23,333,333	23,755,263	47,088,596
800	26,666,667	20,426,203	47,092,869
900	30,000,000	17,119,553	47,119,553
1,000	33,333,333	13,889,916	47,223,250
1,100	36,666,667	10,839,846	47,506,513
1,200	40,000,000	8,100,014	48,100,014
1,300	43,333,333	5,783,980	49,117,313
1,400	46,666,667	3,948,601	50,615,267
1,500	50,000,000	2,583,000	52,583,000
1,600	53,333,333	1,624,612	54,957,946
1,700	56,666,667	986,331	57,652,998

Portfolio Values with One Year to Expiration			
Index Level	Stock Portfolio Value	Value of Puts	Insured Portfolio Value
1,800	60,000,000	580,381	60,580,381
1,900	63,333,333	332,300	63,665,633
2,000	66,666,667	185,807	66,852,474
2,100	70,000,000	101,801	70,101,801
2,200	73,333,333	54,814	73,388,147
2,300	76,666,667	29,082	76,695,749
2,400	80,000,000	15,240	80,015,240
2,500	83,333,333	7,904	83,341,237

Dynamic Insurance Using Stocks and Risk-Free Bonds

Dynamic portfolio insurance does the same thing as the passive portfolio insurance, except that a put option is not purchased directly. Instead the fund manager dynamically rebalances a portfolio consisting of stocks and risk-free bonds (or a portfolio of stocks and stock index futures or a portfolio of stocks, index futures and risk-free bonds) in such a way that the payoff contingencies of the portfolio exactly match the payoff contingencies of the passively insured portfolio. Dynamic portfolio insurance using a mix of the stock portfolio and risk-free bonds is discussed first, followed by dynamic insurance using the stock portfolio and stock index futures and then by the stock portfolio together with index futures and risk-free bonds. Recall that this is the order that LOR followed in providing their portfolio insurance advisory service.

To create a dynamic portfolio insurance portfolio, you need to create a portfolio of stocks and risk-free bonds in such a way that the portfolio has (1) the same value as the passive portfolio insurance portfolio; and (2) the same change in value as the passive portfolio insurance portfolio with respect to a change in the level of the index S. The *value constraint* is

$$V = Se^{-\delta T} + p = w_S S e^{-\delta T} + w_B X e^{-rT} \tag{15.1}$$

where w_S is the number of units of the index portfolio, w_B is the number of risk-free bonds, and X is the floor value of the portfolio insurance at time T (i.e., the exercise price of the dynamically created put option). To identify the *delta constraint*, first substitute the European-style put option valuation equation from Table 14.13 in Chapter 14 into equation (15.1) and then take the partial derivative of (15.1) with respect to a change in the index level S, that is,

$$\frac{\partial V}{\partial S} = e^{-\delta T} - e^{-\delta T}N(-d_1) = w_S e^{-\delta T} + w_B \frac{\partial X e^{-rT}}{\partial S} = w_S e^{-\delta T} \tag{15.2}$$

Note that since the value of the risk-free bonds with respect to a change in the index level is 0, the appropriate number of units of the stock portfolio to buy can be identified by factoring $e^{-\delta T}$ from (15.2), that is,

$$w_S = 1 - N(-d_1) = N(d_1) \tag{15.3}$$

Substituting (15.3) back into the value constraint (15.1), the appropriate number of bonds can be identified by solving

$$Se^{-\delta T} + Xe^{-rT}N(-d_2) - Se^{-\delta T}N(-d_1) = Se^{-\delta T}N(d_1) + w_B Xe^{-rT} \tag{15.4}$$

Since $Se^{-\delta T} = Se^{-\delta T}N(-d_1) + Se^{-\delta T}N(d_1)$, the appropriate number of bonds to sell is

$$w_B = N(-d_2) \tag{15.5}$$

Asset-or-Nothing and Cash-or-Nothing Options Before returning to the illustration, it is worthwhile to note that you can identify the appropriate number of index portfolio units and bond portfolio units by using the values of two more primitive options. The payoffs of portfolio insurance can be replicated by buying an asset-or-nothing call and a cash-or-nothing put with the same exercise price. The asset-or-nothing call provides the upside. If the index portfolio value rises, you will exercise the call and take delivery of the stock portfolio. If the index portfolio value falls, you will exercise the put and take delivery of the cash. Summing the option values, you get $Se^{-\delta T}N(d_1) + Xe^{-rT}N(-d_2)$. Note how the probability terms in this expression correspond to the values of w_S and w_B, respectively.

ILLUSTRATION 15.2 Create dynamic portfolio insurance using stock portfolio and risk-free bonds.

Assume that you face the same insurance situation as in Illustration 15.1, except that you want to use a dynamic portfolio insurance strategy with stocks and risk-free bonds. Find the appropriate weights of the index portfolio and risk-free bonds. Show the initial values of the insured portfolio for a range of index levels between 500 and 2,500 in increments of 100.

Compute portfolio weights

The values of w_S and w_B are obtained using (15.3) and (15.5). At the current index level of 1,500, the weights are 0.6554 and 0.4207, respectively. Multiplying each weight by the value of the stock and bond portfolios and then summing, you get \$52,583,000, exactly the figure you started with using passive portfolio insurance.

Compute initial values

The table below shows the portfolio weights for different levels of the index. As the index level rises, you sell bonds and buy more stocks according to the new values of w_S and w_B. Conversely, as the index falls, you sell stocks and buy bonds. The insured portfolio behaves exactly like the passively insured portfolio described earlier. Assuming the index level falls to 500, for example, you will be entirely in bonds. If the index level stays at that level for the entire year, the terminal value of the bonds will be \$50 million (i.e., the current value plus risk-free interest, \$47,088,238$e^{0.06(1)}$), exactly the desired result.

Portfolio Values with One Year to Expiration

Index Level	Stock Portfolio Value	Value Bonds	w_S	w_B	Insured Portfolio Value
500	16,666,667	47,088,227	0.0000	1.0000	47,088,227
600	20,000,000	47,088,227	0.0000	1.0000	47,088,239
700	23,333,333	47,088,227	0.0003	0.9998	47,088,596
800	26,666,667	47,088,227	0.0030	0.9984	47,092,869
900	30,000,000	47,088,227	0.0156	0.9907	47,119,553
1,000	33,333,333	47,088,227	0.0518	0.9662	47,223,250
1,100	36,666,667	47,088,227	0.1249	0.9116	47,506,513
1,200	40,000,000	47,088,227	0.2371	0.8201	48,100,014
1,300	43,333,333	47,088,227	0.3762	0.6969	49,117,313
1,400	46,666,667	47,088,227	0.5219	0.5576	50,615,267
1,500	50,000,000	47,088,227	0.6554	0.4207	52,583,000
1,600	53,333,333	47,088,227	0.7651	0.3006	54,957,946
1,700	56,666,667	47,088,227	0.8475	0.2045	57,652,998
1,800	60,000,000	47,088,227	0.9052	0.1332	60,580,381
1,900	63,333,333	47,088,227	0.9432	0.0835	63,665,633
2,000	66,666,667	47,088,227	0.9670	0.0507	66,852,474
2,100	70,000,000	47,088,227	0.9813	0.0299	70,101,801
2,200	73,333,333	47,088,227	0.9897	0.0172	73,388,147
2,300	76,666,667	47,088,227	0.9944	0.0097	76,695,749
2,400	80,000,000	47,088,227	0.9970	0.0054	80,015,240
2,500	83,333,333	47,088,227	0.9984	0.0029	83,341,237

Dynamic Insurance Using Stocks and Index Futures

Rebalancing the portfolio that consists of stocks and risk-free bonds is not the only means of dynamically insuring a stock portfolio. In practice, dynamic portfolio insurance can also be created with a trading strategy that involves a portfolio of stocks and stock index futures. To identify the appropriate number of units of the stock portfolio and the index futures to use in creating an insured portfolio, use the value and delta constraints. Starting with the value constraint,

$$V = Se^{-\delta T} + p = w_S Se^{-\delta T} + w_F F = w_S Se^{-\delta T} \tag{15.6}$$

Note that since the futures involves no initial outlay, the number of units of the stock portfolio to buy is simply

$$w_S = 1 + \frac{p}{Se^{-\delta T}} \tag{15.7}$$

Substituting the European-style put option valuation equation from Table 14.13 in Chapter 14 into equation (15.6) and then taking the partial derivative of (15.6) with respect to a change in the index level S, the delta constraint is

$$\frac{\partial V}{\partial S} = e^{-\delta T} - e^{-\delta T}N(-d_1) = w_S e^{-\delta T} + w_F \frac{\partial F}{\partial S} \tag{15.8}$$

The cost of carry relation is $F = Se^{(r-\delta)T}$, which implies

$$\frac{\partial F}{\partial S} = e^{(r-\delta)T}$$

Substituting into (15.8),

$$\frac{\partial V}{\partial S} = e^{-\delta T} - e^{-\delta T}N(-d_1) = w_S e^{-\delta T} + w_F e^{(r-\delta)T} \tag{15.9}$$

Factoring $e^{-\delta T}$ and rearranging,

$$w_F = e^{-rT}[N(d_1) - w_S] \tag{15.10}$$

ILLUSTRATION 15.3 Create dynamic portfolio insurance using stock portfolio and stock index futures.

Assume that you face the same insurance that you did in Illustration 15.1, except that you want to use a dynamic portfolio insurance strategy with stocks and index futures. Find the appropriate weights of the index portfolio and index futures. Show the initial values of the insured portfolio for a range of index levels between 500 and 2,500 in increments of 100.

Compute portfolio weights

The values of w_S and w_F are obtained using (15.7) and (15.10). At the current index level of 1,500, the weights are 1.0517 and –0.3732, respectively. Multiplying w_S by the value of the stock portfolio (i.e., the futures position has no value), you get $52,583,000, exactly the figure you started with using passive portfolio insurance. Note that you have more units of the stock than before since you took the money needed to buy the put under the passive insurance scheme and invested it in stocks.

Portfolio Values with One Year to Expiration					
Index Level	Futures Price	Stock Portfolio Value	w_S	Insured Portfolio Value	w_F
500	523.01	16,666,667	2.8253	47,088,227	–2.6608
600	627.62	20,000,000	2.3544	47,088,239	–2.2173
700	732.22	23,333,333	2.0181	47,088,596	–1.9003
800	836.82	26,666,667	1.7660	47,092,869	–1.6603
900	941.43	30,000,000	1.5707	47,119,553	–1.4645
1,000	1046.03	33,333,333	1.4167	47,223,250	–1.2854
1,100	1150.63	36,666,667	1.2956	47,506,513	–1.1025
1,200	1255.23	40,000,000	1.2025	48,100,014	–0.9092
1,300	1359.84	43,333,333	1.1335	49,117,313	–0.7132
1,400	1464.44	46,666,667	1.0846	50,615,267	–0.5299
1,500	1569.04	50,000,000	1.0517	52,583,000	–0.3732
1,600	1673.64	53,333,333	1.0305	54,957,946	–0.2499
1,700	1778.25	56,666,667	1.0174	57,652,998	–0.1600
1,800	1882.85	60,000,000	1.0097	60,580,381	–0.0984
1,900	1987.45	63,333,333	1.0052	63,665,633	–0.0585
2,000	2092.06	66,666,667	1.0028	66,852,474	–0.0337
2,100	2196.66	70,000,000	1.0015	70,101,801	–0.0189
2,200	2301.26	73,333,333	1.0007	73,388,147	–0.0104
2,300	2405.86	76,666,667	1.0004	76,695,749	–0.0056
2,400	2510.47	80,000,000	1.0002	80,015,240	–0.0030
2,500	2615.07	83,333,333	1.0001	83,341,237	–0.0016

Dynamic Insurance Using Stock Portfolio and Dynamic Adjustment of Index Futures and Risk-Free Bonds

As was noted earlier, stock portfolio managers may be reluctant to change the composition of their stock holdings as the market moves. In addition, transaction costs in the stock market are generally higher than in the index futures and risk-free bond markets. Consequently, we now focus on a dynamic portfolio insurance strategy that allows the stock portfolio manager to leave his equity holdings untouched.

Again start with the portfolio value constraint. In this case, it is written

$$V = Se^{-\delta T} + p = Se^{-\delta T} + w_F F + w_B X e^{-rT} \tag{15.11}$$

Since the futures involves no initial outlay, the number of units of risk-free bonds equals

$$w_B = \frac{p}{Xe^{-rT}} \tag{15.12}$$

Substituting the European-style put option valuation equation from Table 14.13 in Chapter 14 into equation (15.11) and then taking the partial derivative of (15.11) with respect to a change in the index level S, the delta constraint is

$$\frac{\partial V}{\partial S} = e^{-\delta T} - e^{-\delta T}N(-d_1) = e^{-\delta T} + w_F\frac{\partial F}{\partial S} \quad (15.13)$$

The cost of carry relation is $F = Se^{(r-\delta)T}$, which implies

$$\frac{\partial F}{\partial S} = e^{(r-\delta)T}$$

Substituting into (15.13),

$$\frac{\partial V}{\partial S} = -e^{-\delta T}N(-d_1) = w_F e^{(r-\delta)T} \quad (15.14)$$

Factoring $e^{-\delta T}$ and rearranging,

$$w_F = e^{-rT}N(-d_1) \quad (15.15)$$

ILLUSTRATION 15.4 Create dynamic portfolio insurance using stock portfolio and dynamic adjustment of stock index futures and risk-free bonds.

Assume that you want the same insurance as in Illustration 15.1. In this instance, however, leave the number of units in the stock portfolio untouched and dynamically insure your portfolio using stock index futures and risk-free bonds. Determine the appropriate number of risk-free bonds and index futures to execute this strategy. Show the initial values of the insured portfolio for a range of index levels between 500 and 2,500 in increments of 100.

Compute portfolio weights

The values of w_B and w_F are obtained using (15.12) and (15.13). At the current index level of 1,500, the number of risk-free bonds is 0.0549 and the number of index futures is –0.3245. Multiplying w_B by the value of the risk-free bond, and adding the value of the stock portfolio the stock portfolio, you get $52,583,000, exactly the figure you started with using passive portfolio insurance (recall the futures position has no initial value).

Note that, under this scheme, the dynamic adjustment has to do with risk-free bonds and index futures. As the market falls, the short futures position generates cash, which is used, in turn, to buy more units of risk-free bonds. As the market rises, the sale of risk-free bonds is used to cover the losses on the short position in the index futures. All the while, the number of units invested in the stock portfolio remains intact, and the insured portfolio values are the same as under the previous alternatives.

Portfolio Values with One Year to Expiration						
Index Level	Futures Price	Stock Portfolio Value	Bond Value	w_B	w_F	Insured Portfolio Value
500	523.01	16,666,667	47,088,227	0.6461	–0.9418	47,088,227
600	627.62	20,000,000	47,088,227	0.5753	–0.9418	47,088,239
700	732.22	23,333,333	47,088,227	0.5045	–0.9415	47,088,596
800	836.82	26,666,667	47,088,227	0.4338	–0.9389	47,092,869
900	941.43	30,000,000	47,088,227	0.3636	–0.9271	47,119,553
1,000	1046.03	33,333,333	47,088,227	0.2950	–0.8929	47,223,250
1,100	1150.63	36,666,667	47,088,227	0.2302	–0.8241	47,506,513
1,200	1255.23	40,000,000	47,088,227	0.1720	–0.7185	48,100,014
1,300	1359.84	43,333,333	47,088,227	0.1228	–0.5875	49,117,313
1,400	1464.44	46,666,667	47,088,227	0.0839	–0.4502	50,615,267
1,500	1569.04	50,000,000	47,088,227	0.0549	–0.3245	52,583,000
1,600	1673.64	53,333,333	47,088,227	0.0345	–0.2213	54,957,946
1,700	1778.25	56,666,667	47,088,227	0.0209	–0.1436	57,652,998
1,800	1882.85	60,000,000	47,088,227	0.0123	–0.0893	60,580,381
1,900	1987.45	63,333,333	47,088,227	0.0071	–0.0535	63,665,633
2,000	2092.06	66,666,667	47,088,227	0.0039	–0.0311	66,852,474
2,100	2196.66	70,000,000	47,088,227	0.0022	–0.0176	70,101,801
2,200	2301.26	73,333,333	47,088,227	0.0012	–0.0097	73,388,147
2,300	2405.86	76,666,667	47,088,227	0.0006	–0.0053	76,695,749
2,400	2510.47	80,000,000	47,088,227	0.0003	–0.0028	80,015,240
2,500	2615.07	83,333,333	47,088,227	0.0002	–0.0015	83,341,237

Practical Considerations in Choosing Between Passive and Dynamic Portfolio Insurance

The mechanics of the above illustrations show that dynamic portfolio insurance must be more expensive than passive portfolio insurance. As the market moves and time passes, the portfolio manager is left readjusting his portfolio weights, incurring transaction costs with each adjustment. For this reason, portfolio managers often set trigger levels whereby portfolio adjustments are not made until the market moves by, say, 5%. The effect of not making continuous and instantaneous adjustments is that the insurance scheme will have a lower floor value and less upside potential. One may question the reason for the existence of dynamic portfolio. The reason, as alluded to earlier, is that index put options did not exist when portfolio insurance was first introduced into the marketplace.

Another distinction between passive and dynamic portfolio insurance is worthy of note. When a portfolio manager buys an index put, he locks in the amount he will pay for expected future volatility (i.e., the price he pays for the put implies the level of volatility). If subsequently realized volatility is lower than expected, the passive portfolio insurer will have overpaid for insurance.

Under the dynamic scheme, this would not have been the case. On the other hand, if subsequently realized volatility is higher than expected, the dynamic portfolio insurer will pay more for insurance, akin to buying fire insurance on your home after the fire has started.

INDEX OPTION BUY-WRITE STRATEGIES

Option trading strategies are becoming an increasingly important part of the investment landscape. Indeed, since mid-2004, more than 42 new buy-write investment products (closed-end funds or structured products) alone have been launched with more than $18 billion in assets. Many of these are index products, and, currently, the most popular buy-write index is the Chicago Board Options Exchange's Buy-Write Index (BXM).

The BXM is based on a buy-write trading strategy using the S&P 500 index portfolio and index call options. In Chapter 10, we defined a buy-write strategy as buying an asset and selling a call option against it. Such a strategy contributes incremental return over and above the asset return conditional on the underlying asset price staying with in a tight range during the life of the call. Unconditionally, however, the buy-write strategy is risk-reducing (relative to holding the asset alone) and, hence, should lead to lower returns. In this section, we describe the BXM trading strategy, and examine and discuss its historical performance.

Buy-Write Return Distributions and the Central Limit Theorem

Evaluating the performance of trading strategies involving options can be difficult because the nonlinear payoff structure of an option can dramatically affect the skewness of the return distibution. Recall, from Chapter 3, commonly used portfolio performance evaluation techniques assume the portfolio's return distibution is normal or, at least, symmetric. To analyze this problem, we focus on the BXM index.

The BXM index is a total return index based on writing the nearby, just out-of-the-money S&P 500 call option against the S&P 500 index portfolio each month on the day the previous nearby contract expires, which is usually the third Friday of the month.[5] Assuming for the moment that the S&P 500 portfolio pays no dividends, its continuously compounded return over the one-month holding period is

$$R_S = \ln\left(\frac{S_T}{S_0}\right) \tag{15.16}$$

[5] Since expirations occur monthly and there are 52 weeks in the calendar year, some "one-month" options have 28 days to expiration at the time they are written and others have 35 days.

where S_0 and S_T are the index levels on adjacent option expiration days. Over the same period, the continuously compounded return on the BXM buy-write strategy over the month is

$$R_{BXM} = \ln\left(\frac{S_T - \max(S_T - X, 0)}{S_0 - C_0}\right) \tag{15.17}$$

where C_0 is price of the call when it is sold and $\max(S_T - X,0)$ is the price of the call when it expires. Note that the the buy-write return over the month has a truncated distribution, that is,

$$R_{BXM} = \begin{cases} \ln\left(\dfrac{X}{S_0 - C_0}\right) & \text{if } S_T > X \\ \ln\left(\dfrac{S_T}{S_0 - C_0}\right) & \text{if } S_T \leq X \end{cases} \tag{15.18}$$

In other words, when the call expires in the money, the buy-write return is capped at

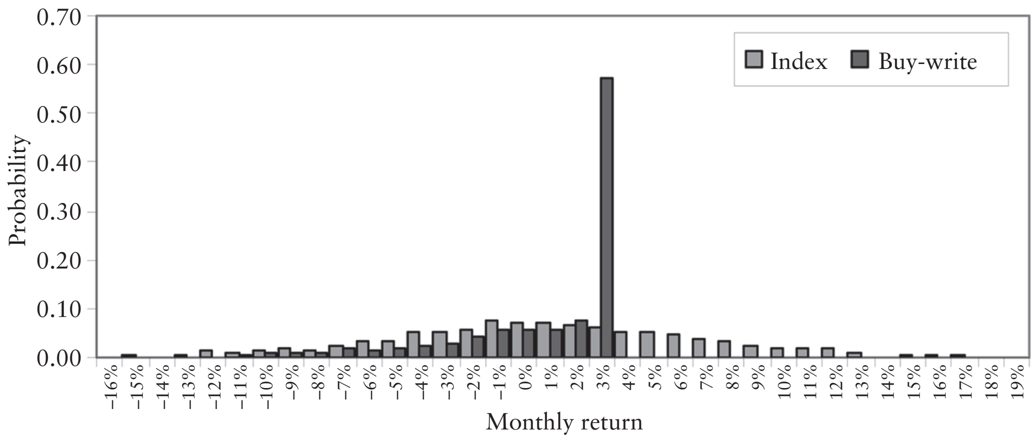
$$\ln\left(\frac{X}{S_0 - C_0}\right)$$

When the call finishes out of the money, the buy-write return is higher than index return as a result of the sales proceeds of the call.

To illustrate the implications of (15.18) with respect to the shape of the buy-write return distibution, we perform a Monte Carlo simulation.[6] In the simulation, the continuously compounded index returns are assumed to be normally distributed with a mean (μ) of 12% and a volatility rate (σ) of 20% annually. In each simulation run, a single monthly index return is generated. Based on this index return, the monthly return of a buy-write strategy is computed using (15.18). The price of the one-month, at-the-money call at the beginning of the month is set equal to its BSM value. The assumed risk-free rate of interest is 6%. The call's price at the end of the month is set equal to its exercise proceeds. The simulation is repeated 1,000 times. Figure 15.3 shows a histogram the results. In the figure, the lighter bars represent index returns. Not surprisingly, the returns appear symmetrically distributed around the mean monthly return of 1%. This merely tells us that the simulation procedure is working. The darker bars represent the reutrns of the buy-write index. Note the large spike at about 3%.[7] More than 50% of the time, the call option finishes in the-money and the buy-write strategy realizes its maximum monthly return. This makes sense

[6] The Monte Carlo simulation procedure is described at length in Chapter 9.

FIGURE 15.3 Return distributions from Monte Carlo simulation of index portfolio versus at-the-money buy-write strategy over one-month holding period. (Assumed parameters: μ = 0.12 and σ = 0.20. Number of trials is 1,000.)



because the index is expected to appreciate in value (i.e., $\mu > 0$), and a call option that is currently at the money is therefore expected to be in the money at expiration. Note that if the call option finishes out of the money, the probability of a particular negative buy-write return is less then the probability of the index return. This is merely another way of saying that, if the option finishes out of the money, the buy-write return will exceed the index return. The negative skewness of the buy-write strategy implies that the portfolio performance measures in Chapter 3.

The histogram in Figure 15.3 represents the return distibution of the buy-write strategy if it is used only once. A buy-write strategy program, however, involves selling call options again and again over a long period of time, say, 10 or 20 years. In Appendix A: Elementary Statistics, we discussed the Central Limit Theorem. In the context of the buy-write strategy, the Central Limit Theorem says that even though the monthly return distribution is highly negatively skewed, the distribution of the mean of monthly returns over time becomes approximately normal with mean μ and variance σ^2/n, where n is the number of months over which the buy-write strategy is repeated. Figures 15.4 and 15.5 show the shape of the return distibutions if the monthly buy-write strategy is repeated over a 10-year and 20-year horizons, respectively. For the 10-year horizon the buy-write return distribution remains negatively skewed, however, the degress of skewness is trivial in relation to Figure 15.3. For the 20-year horizon, the buy-write strategy has approximately a normal distribution. Indeed, if we apply the Jarque-Bera test of normality (see Appendix A at the end of the book), the null hypothesis that the buy-write return distibution is normal is not rejected. In other words, depending on the trading strategy and the number of times the strategy is repeated in the trading program, it may or may not be

[7] Given the simulation parameters, the maximum monthly return of the buy-write strategy is 2.59%.

FIGURE 15.4 Return distributions from Monte Carlo simulation of index portfolio versus at-the-money buy-write strategy over 120-month holding period. (Assumed parameters: μ = 0.12 and σ = 0.20. Number of trials is 1,000.)

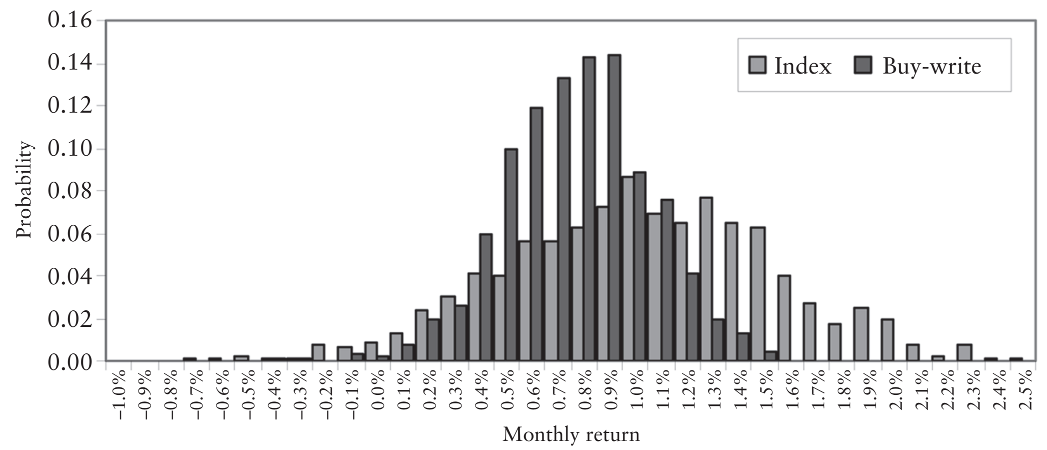
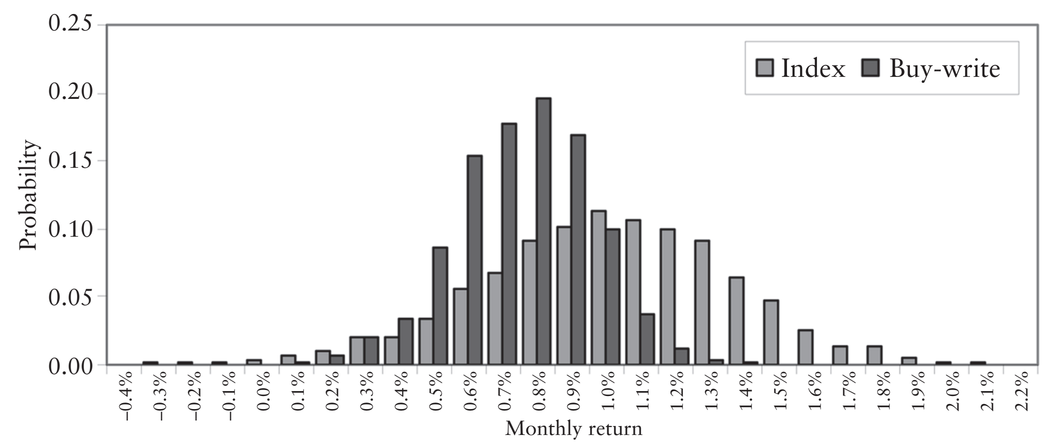


FIGURE 15.5 Return distributions from Monte Carlo simulation of index portfolio versus at-the-money buy-write strategy over 240-month holding period. (Assumed parameters: μ = 0.12 and σ = 0.20. Number of trials is 1,000.)



appropriate to apply the mean-variance portfolio perfromance measures from Chapter 3. Before applying any of these techniques, it is a good idea to examine the return distibution, compute its skewness, and, perhaps, perform a test for normality. In the event that skewness is a problem, applying performance measures based on mean/semivariance may provide a more accurate assessment.[8]

[8] While using semivariance as a risk measure is intuitively appealing, it is somewhat *ad hoc* in nature. Stutzer (2000) offers an alternative approach that is more rigorous from a theoretical standpoint.

Historical Performance of BXM

In this section, we examine the hsitorical performance of the BXM over the 211-month period June 1988 through December 2005. The data history is available on the CBOE's website[9] and is contained in the Excel file **BXM history.xls**. The first two series in the file are the total return index levels of the BXM and the S&P 500 (SPTR), and the second two are continuously compounded monthly returns. The monthly money market rates in the final column are the continuously compounded rates of return of a 30-day Eurodollar time deposit whose number of days to maturity matches the number of days in the month. The Eurodollar rates were downloaded from Datastream.

Table 15.2 shows that the average monthly return of the one-month money market instruments over the 211-month period was 0.398%. Over the same period, the S&P 500 index portfolio generated an average monthly return of 0.920%, while the BXM generated an average monthly return of 0.926%. Surprisingly, the monthly average monthly return of the BXM was 0.6 basis points *higher* than the S&P 500 even though the BXM risk, as measured by the standard deviation of return, was substantially lower. For the BXM, the standard deviation of monthly returns was 2.747%, while, for the S&P 500, the standard deviation was 4.071%. In other words, BXM produced a monthly return approximately equal to the S&P 500 index portfolio, but at about 67% of the S&P 500's risk (i.e., 2.747% versus 4.071%), where risk is measured in the usual way.

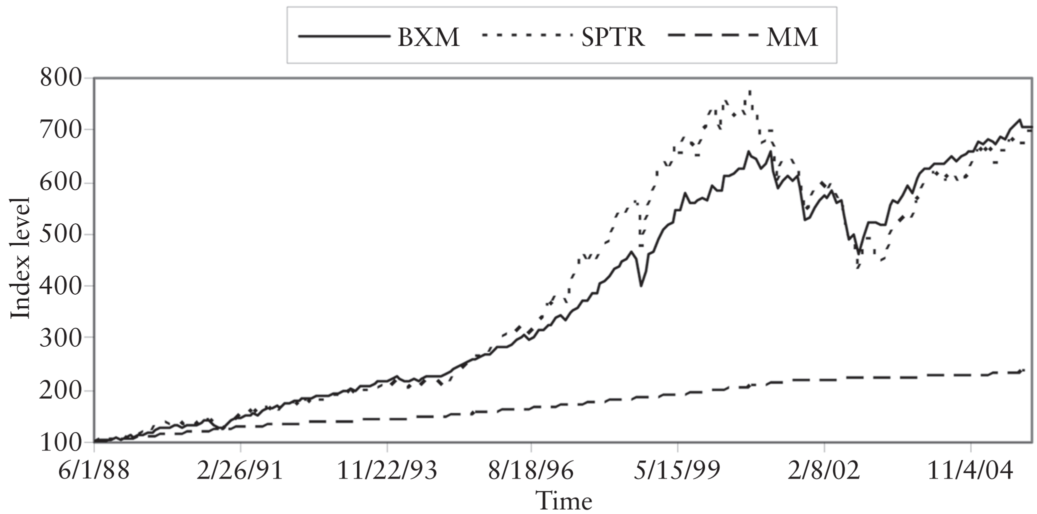
The realized returns and risks of the BXM, the S&P 500, and the 30-day money market instrument are summarized in Figure 15.6. For purposes of comparison, we assume a 100 investment in each instrument on June 1, 1988, and then watch how the investment value moves through time. As the figure shows, the BXM tracked the S&P 500 index closely at the outset. Then, starting in 1992, the BXM began to rise faster than the S&P 500, but, by mid-1995, the level of the S&P 500 total return index surpassed the BXM. Beginning in 1997, the S&P 500 index charged upward in a fast but volatile fashion. The BXM

TABLE 15.2 Summary statistics for monthly returns of CBOE's BXM index, the S&P 500 index, and money market deposits during the period June 1988 through December 2005.

	BXM	SPTR	MM
No. of months	211	211	211
Mean	0.926%	0.920%	0.407%
Median	1.236%	1.280%	0.443%
Standard deviation	2.747%	4.071%	0.191%
Skewness	–1.420	–0.597	0.133
Excess kurtosis	5.006	1.120	–0.576
Jarque-Bera test statistic	291.187	23.583	3.543
Probability or normal	0.000	0.000	0.170

[9] The CBOE's BXM webpage is at http://www.cboe.com/micro/bxm/introduction.aspx.

FIGURE 15.6 Month-end total return indexes for BXM index, S&P 500 total return index (SPTR), and 30-day money market index (MM) for the period June 1988 through December 2005.



lagged behind, as should be expected. When the market reversed in mid-2000, the BXM again moved ahead of the S&P 500. The steadier path taken by the BXM reflects the fact that it has lower risk than the S&P 500.

Table 15.2 also reports the skewness and excess kurtosis of the monthly return distributions as well as the Jarque-Bera statistic for testing the hypothesis that the return distribution is normal. It is interesting to note that both the S&P 500 portfolio and the BXM have negative skewness. For the BXM, negative skewness should not be surprising in the sense that a buy-write strategy truncates the upper end of the index return distribution. But, the Jarque-Bera statistic rejects the hypothesis that returns are normal for the BXM and S&P 500 but not for the money market rates. The negative skewness for the BXM and S&P 500 does not appear to be severe, however. Figure 15.5 shows the standardized monthly returns of the S&P 500 and BXM in relation to the normal distribution. The S&P 500 and BXM return distributions appear more negatively skewed than the normal, but only slightly. What stands out in the figure is that both the S&P 500 and the BXM return distributions have greater kurtosis than the normal distribution. This is reassuring in the sense that most portfolio performance measures work well for symmetric distributions but not asymmetric ones.

To evaluate the historical performance of the BXM, we use the Sharpe ratio and *M*-squared performance measures from Chapter 3. Risk is measured using the standard deviation and the semi-standard deviation of portfolio returns. To the extent that BXM returns are skewed, the measures derived from the two different models will differ. Since the standardized BXM return distribution show slight negative skewness, the performance measures based on semi-standard deviation should be less than their standard deviation counterparts, but not by much. The portfolio performance results over the period June 1988 through December are reported in Table 15.3.

TABLE 15.3 Estimated performance measures based on monthly returns of CBOE's BXM index and the S&P 500 index during the period June 1988 through December 2005.

Performance Measure	Total Risk Measure	BXM	SPTR
Sharpe ratio	Standard deviation	0.18901	0.12596
	Semistandard deviation	0.26144	0.18203
M-squared	Standard deviation	0.257%	
	Semistandard deviation	0.224%	

The results of Table 15.3 support two main conclusions. First, the BXM has outperformed the S&P 500 index on a risk-adjusted basis over the BXM's history. Both the Sharpe ratio and *M*-squared performance measures support this conclusion, independent of whether total risk is measured using standard deviation or semistandard deviation. The outperformance using standard deviation as the total risk measure, for example, is 25.7 basis points per month. Second, the performance measures using mean/semistandard deviation are slightly lower than their counterparts using mean/standard deviation. The reason is, of course, that the BXM returns are negatively skewed, as was indicated in Table 15.2 and Figure 15.6. The effect of skewness is impounded through the risk measure. The skewness "penalty" is about 3.3 basis points per month.

In an efficiently functioning capital market, the risk-adjusted return of a buy-write strategy using S&P 500 index options should be no different than the S&P 500 portfolio. Yet, the BXM has provided an abnormally high return relative to the S&P 500 index portfolio over the period June 1988 through December 2005. What could cause such an aberration? One possible explanation, suggested by Stux and Fanelli (1990), Schneeweis and Spurgin (2001), and others, is that the volatilities implied by option prices are too high relative to realized volatility. Indeed, Bollen and Whaley (2004) argue that there is excess buying pressure on S&P 500 index puts by portfolio insurers. Since there are no natural counterparties to these trades, market makers must step in to absorb the imbalance. As the market maker's inventory becomes large, implied volatility will rise relative to actual return volatility, with the difference being the market maker's compensation for hedging costs and/or exposure to volatility risk.[10] The implied volatilities of the corresponding calls also rise from the reverse conversion arbitrage supporting put-call parity.

To examine whether this explanation is consistent with the observed performance of the BXM, Whaley (2004) compares the average implied volatility[11] of the calls written in the BXM strategy to the average realized volatility over the call's life. Figure 15.8 shows that the difference has not been constant through time, perhaps indicating variation in the demand for portfolio insurance. The

[10] Bollen and Whaley (2004) also show that the same phenomenon does not exist for options on high market capitalization stocks whose empirical return distributions are shaped similarly to the S&P 500 returns.

[11] The implied volatility was computed by setting the observed call price equal to the Black-Scholes (1973)/Merton (1973) call option formula.

difference is persistently positive, however, with the mean (median) difference between the ATM call implied volatility and realized volatility being about 167 (234) basis points on average.[12]

FIGURE 15.7 Distribution of standardized monthly returns for the BXM index and S&P 500 total return index (SPTR) indexes during the period June 1988 through December 2005. Normally distributed standard returns are also included.

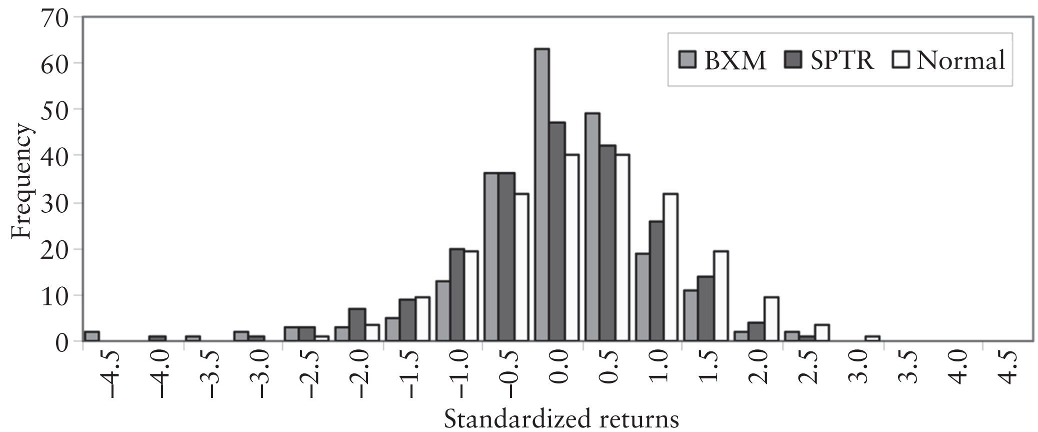
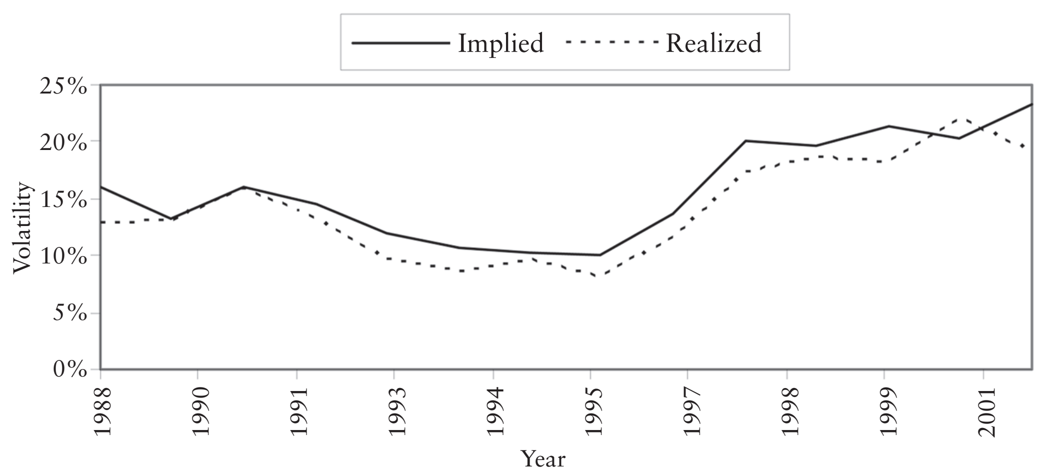


FIGURE 15.8 Average implied and realized volatility for S&P 500 index options in each year 1988 through 2001.

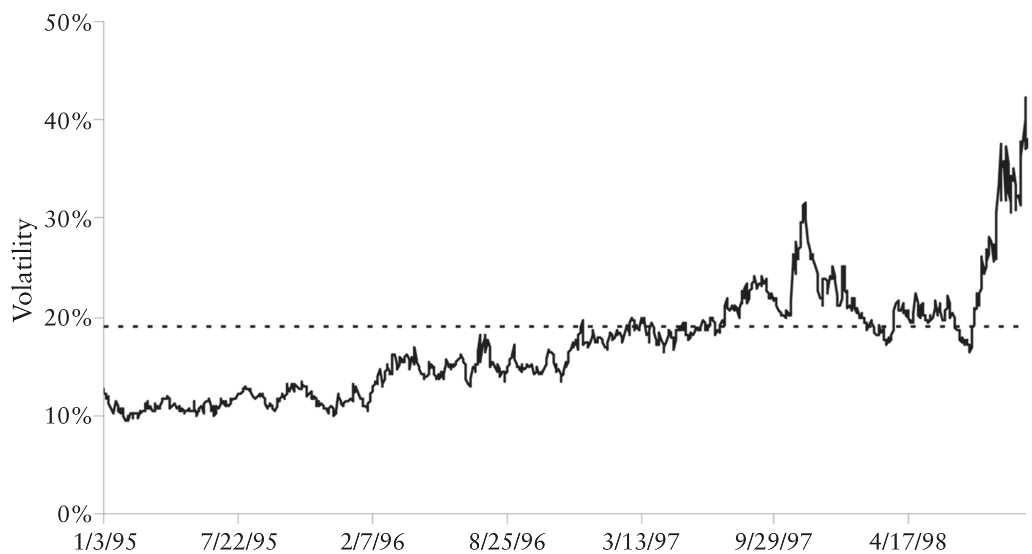


[12] Indeed, Whaley (2004) recreates the BXM where the one-month calls are traded at their BSM values using the standard deviation of the returns actually earned over the option's life rather that at their bid/ask quotes for the calls observed in the marketplace as the volatility parameter, and finds that the buy-write strategy performs no differently than the S&P 500 index portfolio on a risk-adjusted basis.

VOLATILITY DERIVATIVES

Another relatively new stock index product are *volatility derivatives*. Although such instruments had been contemplated since the early 1990s,[13] it was not until the Long-Term Capital Management (LTCM) fiasco in late 1998 that the market finally began to recognize the value of trading stock market volatility as a separate asset class. LTCM lost $1.3 billion from being short implied stock index volatility through the sale of long-term index call and put options on stock market indexes in the United States and Europe.[14] Assuming the aggregate delta values of the calls and puts were approximately equal, the overall position would be relatively immune to movements in the underlying indexes. Unfortunately, however, being short index options implies having significant vega-risk. Indeed, LTCM had written so many index options with volatilities in the range of 19% that their net vega exposure was –$40 million in the U.S. alone. Market volatility rose significantly during the last months before LTCM's collapse—to a level of nearly 45% by mid-September 1998 as Figure 15.9 shows. With these option positions being marked-to-market on a daily basis, the cash drain was enormous. It is exactly this type of risk exposure that volatility derivatives are intended to address.

FIGURE 15.9 Implied volatility of S&P 500 index options prior to the collapse of Long-Term Capital Management.



[13] The Chicago Board Options Exchange (CBOE) contemplated launching trading volatility options as early as 1993. See Whaley (1993). On January 19, 1998, the Deutsche Terminborse (DTB) became the first exchange in the world to list volatility futures. The CBOE launched trading of VIX futures on its CBOE Futures Exchange on March 26, 2004, with contracts on three-month realized variance being launched on May 18, 2004. The CBOE launched VIX options on February 24, 2006.

[14] Lowenstein (2000) provides a brief account of LTCM's trading strategies. The sale of stock index options accounted for nearly 30% of the $4.5 billion in firm losses.

Today volatility derivative contracts are written not only on stock indexes, but also interest rates, currencies, and commodities like crude oil. Prior to the advent of volatility derivatives, stock market volatility risk was managed using options written on the underlying index. The problem with doing so is that it is expensive. Options have two sources of price risk—risk associated with movements in the underlying index level and risk associated with movements in the market's perception of expected future volatility rate. The only way to isolate the volatility exposure is by trading the options and delta-hedging using the underlying index, index futures, and other index options.

This section describes volatility derivative contracts and their uses. We focus primarily on stock market volatility since stock market volatility contracts are the most actively traded. The discussion has two parts. First, we discuss realized volatility contracts and their applications, and then we turn to implied volatility contracts.

Realized Volatility Derivative Contracts

At the outset, we need to correct a misnomer. Industry has come to refer to realized volatility contracts as volatility swaps. A *volatility swap* is not a swap; it is a forward contract. They have traded in OTC markets for more than five years, and are now also exchange-traded. A volatility forward (or swap) is written on the *realized future volatility* of an asset (say, the S&P 500 index). At expiration, its payoff is based on the statistical formula for the annualized standard deviation of index return, that is,

$$\sigma_{\text{realized}} = \sqrt{\frac{\sum_{t=2}^{n_T}\left[\ln\left(\frac{S_t}{S_{t-1}}\right) - \text{mean}\right]^2}{n_T - 2}} \times \sqrt{\text{no. of time intervals in a year}} \tag{15.19}$$

where n_T is the number of price observations used in the computation,[15] and S_t is the index level. Volatility forwards are usually based on daily closing prices, however, since they are traded primarily in the OTC market, any frequency (e.g., hourly, weekly) is possible. The contract also specifies the source from which for the prices will be obtained. Volatility forwards are sometimes based on squared returns, and sometimes on squared deviations. Formula (15.19) shows squared deviations. The formula for squared returns is the special case where the mean term in the squared brackets of (15.19) is set equal to zero and the adjustment in the numerator is increased to $n_T - 1$. Finally, the volatility is annualized. For daily prices, the last term on the right-hand side is usually $\sqrt{252}$, that is, the square root of the typical number of business days in a year. For weekly prices, the last term is $\sqrt{52}$.

The value represented by formula (15.19) is the price of the asset underlying the forward contract at expiration. The only difference is the underlying asset is

[15] Note that n_T prices produce $n_T - 1$ returns. Since we lose one degree of freedom from estimating the mean (see Appendix A, "Elementary Statistics," of this book), the appropriate divisor is $n_T - 2$.

not tradable; it is simply a computation of realized volatility. At inception, the buyer and seller agree to a fixed delivery price (quoted as an annualized volatility), σ_X, on the expiration date, T. As expiration approaches, the forward's settlement price becomes more and more certain because some of the prices used in (15.19) have been realized already. On the last day before expiration, only the index level on expiration day remains unknown. Upon settlement, the buyer receives

$$\text{Notional} \times (\sigma_{\text{realized}} - \sigma_X) \tag{15.20}$$

that is, the notional amount of the swap times the difference between the realized and contracted volatility. The seller receives the opposite amount. Sometimes the volatility derivatives are written on the square of volatility, or variance. The buyer of a *variance swap* receives the payoff,

$$\text{Notional} \times (\sigma^2_{\text{realized}} - \sigma^2_X) \tag{15.21}$$

ILLUSTRATION 15.5 Compute settlement price of realized volatility swap.

Suppose that on Friday, August 1, 2003, you bought a 13-week volatility forward from an OTC derivatives dealer. Its price was 0.12, and its notional amount was $100 million. Compute the settlement price and the settlement proceeds using squared weekly returns. Recompute the values using squared deviations. Comment on the difference. The Friday closing index levels over the period were as follows:

Friday Close	S&P 500 Index
20030801	980.15
20030808	977.59
20030815	990.67
20030822	993.06
20030829	1008.01
20030905	1021.39
20030912	1018.63
20030919	1036.30
20030926	996.85
20031003	1029.85
20031010	1038.06
20031017	1039.32
20031024	1028.91
20031031	1050.71

The first step is to compute the weekly returns. Next compute the mean weekly return, and the squared returns and deviations. Compute the sum of squares and the annualized volatility. To annualize weekly returns, use the factor, $\sqrt{52}$. The cash settlement proceeds are $1.27 million for the squared returns contract and $.65 million for squared deviations. The difference is unusually large because the S&P 500 index level rose abnormally during this 13-week period, at least relative to historical standards. The rate of return of the S&P 500 index was about 7.2%—nearly 30% on an annualized basis.

Friday Close	S&P 500 Index	S&P 500 Return	Squared Returns	Squared Deviations
20030801	980.15			
20030808	977.59	–0.00262	0.00000684	0.00006340
20030815	990.67	0.01329	0.00017665	0.00006310
20030822	993.06	0.00241	0.00000581	0.00000863
20030829	1008.01	0.01494	0.00022327	0.00009206
20030905	1021.39	0.01319	0.00017388	0.00006145
20030912	1018.63	–0.00271	0.00000732	0.00006485
20030919	1036.30	0.01720	0.00029577	0.00014044
20030926	996.85	–0.03881	0.00150634	0.00195002
20031003	1029.85	0.03257	0.00106068	0.00074097
20031010	1038.06	0.00794	0.00006305	0.00000672
20031017	1039.32	0.00121	0.00000147	0.00001709
20031024	1028.91	–0.01007	0.00010134	0.00023759
20031031	1050.71	0.02097	0.00043958	0.00024395
	Mean	0.00535		
	Total		0.00033850	0.00030752
	Annualized volatility		0.13267	0.12646
	Notional amount		100,000,000	100,000,000
	Forward price		0.120	0.120
	Cash settlement value		1,267,275	645,649

The CBOE Futures Exchange (CFE) launched its three-month realized volatility futures contract on May 18, 2004. The CFE is an all-electronic exchange that was created by the Chicago Board Options Exchange (CBOE) in March 2004. The CFE's realized volatility contract is based on S&P 500 return variance rather than return standard deviation, and its product specifications are provided in Table 15.4. The contract denomination is $50 per variance point. A price quotation of 633.50, for example, means the contract value is $31,675. Up to four contracts may trade simultaneously. The contracts are on the March quarterly expiration cycle (March, June, September, December). The final settlement date is the third Friday of the contract month. Trading stops at the close on the preceding business day.

The final settlement price is a variance number and assumes the mean return is zero. Hence, the realized volatility formula (15.19) becomes

$$\sigma^2_{\text{realized}} = \frac{\sum_{t=1}^{n_a} \left[\ln\left(\frac{S_t}{S_{t-1}} \right) \right]^2}{n_e - 1} \times 252 \qquad (15.22)$$

TABLE 15.4 Selected terms of S&P 500 three-month variance futures contract.

Exchange	CBOE Futures Exchange (CFE)
Ticker symbol	VT
Contract unit	$50 per variance point
Tick size	0.5 of one variance point
Tick value	$25
Trading hours	8:30 AM to 3:15 PM CST
Contract months	Up to four contract months on the March cycle (Mar., Jun., Sep., Dec.)
Last day of trading	Close of trading on business day before final settlement date.
Final settlement date	Third Friday of contract month.
Final settlement price	Final settlement price is based on the standardized calculation of the realized variance of the S&P 500. This calculation uses continuously compounded daily returns for a three-month period assuming a mean daily return of zero. The calculated variance is then annualized assuming 252 business days per year. The final settlement price is this annualized, calculated variance multiplied by 10,000.

where n_a is the actual number of trading days in the three-month interval, and n_e is the expected number of days in the three-month interval. Normally, n_a and n_e are equal. In the event of a market disruption during the contract's life, however, n_a will be less than n_e. Generally speaking, a "market disruption event," as determined by the CFE, occurs when trading on the primary exchanges of a significant number of S&P 500 stocks is suspended or limited in some way or when the primary exchange on which index stocks unexpectedly closes early (or does not open) on a particular day. For each market disruption event, the value of n_a is reduced by one.

Volatility versus Variance Contracts Industry has come to define volatility as the standard deviation of the natural logarithm of the price ratios.[16] If the forward is defined in terms of variance (i.e., volatility squared) rather than volatility, the payoff structure is quite different. Consider Figures 15.10 and 15.11, which plot the payoffs of a volatility forward contract versus a variance forward contract for long and short positions. Since the horizontal axis is defined in terms of volatility, its terminal payoffs are a linear function of volatility. The variance forward, on the other hand, is nonlinear. The long variance position (the dotted line in Figure 15.10) has convexity. As volatility falls, the terminal payoff of the long variance position decreases, but at a decreasing rate. At the same time, as volatility increases, the terminal payoff of the long variance forward increases at an increasing rate. Indeed, the variance payoffs loosely resemble a long call position, while the variance payoffs of the short variance futures resemble a short call position.

[16] Recall that this is consistent with the BSM model's use of continuously compounded returns.

FIGURE 15.10 Payoff structure of volatility and variance forward contracts: Long positions.

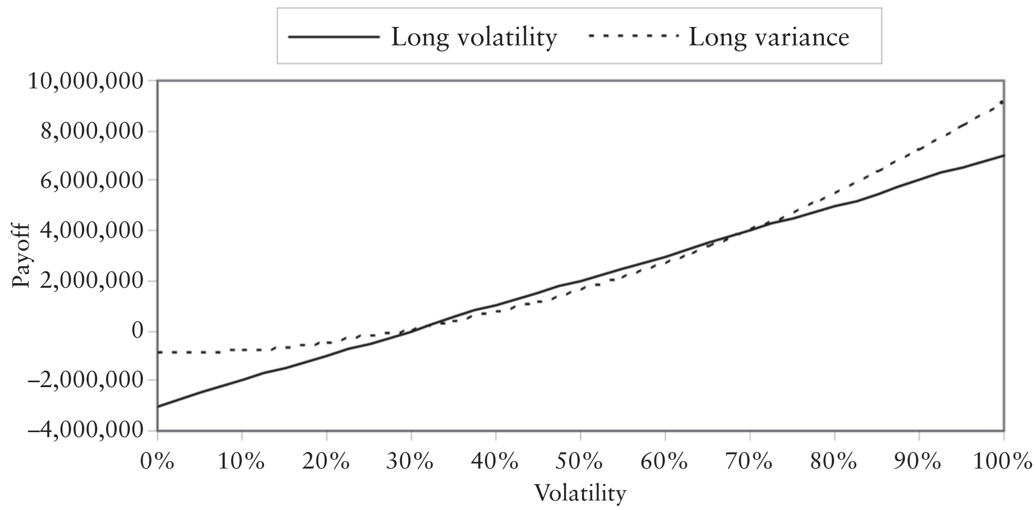
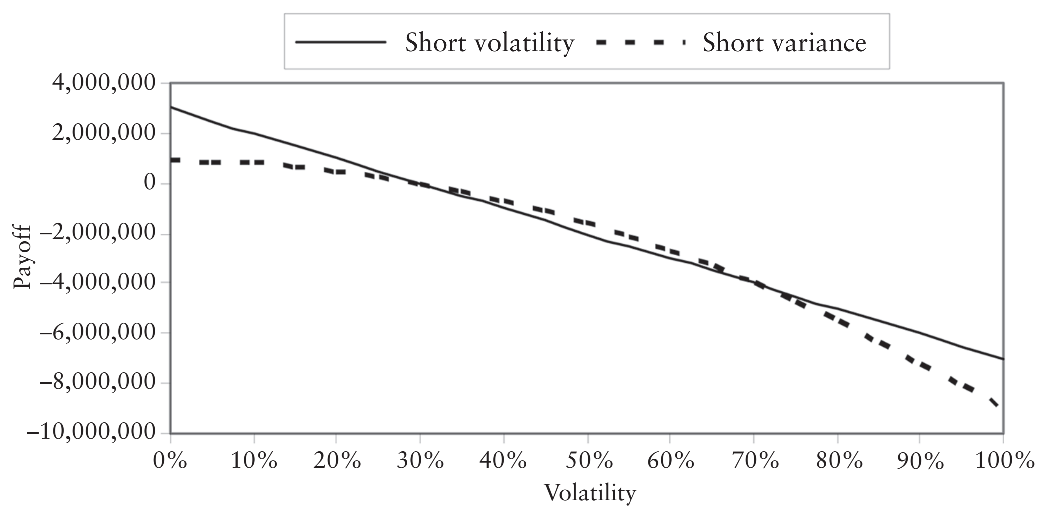


FIGURE 15.11 Payoff structure of volatility and variance forward contracts: Short positions.



Expected Return/Risk Management Applications At first blush, the volatility forward contract seems to be purely a speculative instrument. Traders who believe future volatility will be high relative to the forward price will go long the swap, and those who believe that the market will be very calm will go short. But, the hedging possibilities using realized volatility forwards are many. In the normal course of operation, for example, some market participants become inherently short volatility. Consider LTCM's ill-fated index option strategy. Because index option implied volatilities were as high as they had been anytime since the October 1987 market crash, LTCM sold both index calls and puts with the belief that implied volatility would return to normal levels. Unfortunately, a problem arose when implied volatility continued to rise and their positions were marked-to-market. The cash drain was enormous. Buying realized volatility forwards would have hedged this exposure, at least in part. The same is true for index option market makers who are short market volatility as a result of selling index puts to portfolio insurers.[17]

Another hedging possibility is for risk arbitrageurs. Immediately after a merger is announced, risk arbitrageurs step in and buy shares of a target firm and sell the shares of bidder. Because the probability that the merger will be successful is not known, the prices of the target and the bidder will not fully reflect the terms of the offer. If the merger is successful, the spread between the prices will converge. Before the deal is consummated, however, market volatility may increase, making the merger less likely, thereby causing the spread to widen. Buying a realized volatility forward contract can hedge this type of risk exposure.

Yet another application is for individuals or portfolio managers who attempt to track some sort of benchmark index. During periods of high volatility, the portfolio may require more frequent rebalancing and greater transaction cost expenses. Again, buying a realized forward contract on volatility can hedge this exposure.

Implied Volatility Derivatives Contracts

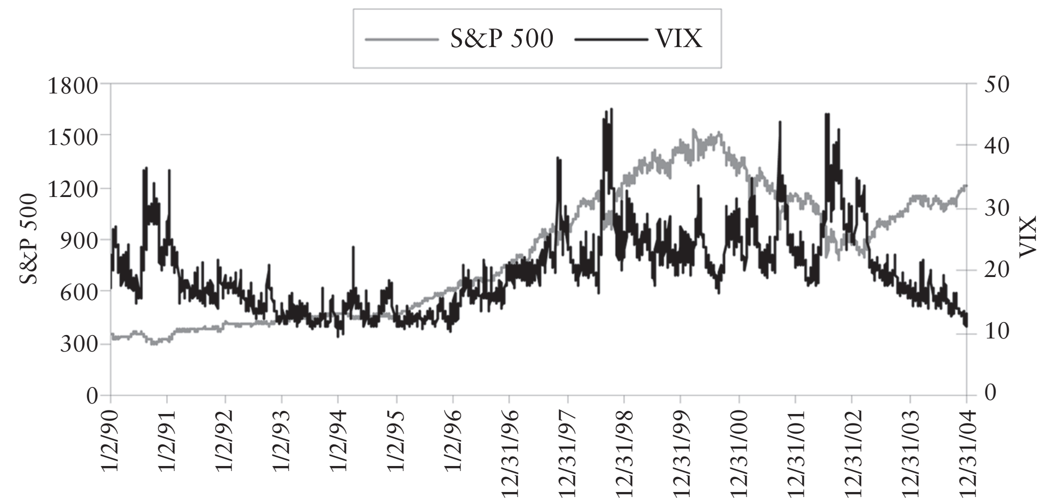
The CFE also lists a futures contract written on the implied return volatility of the S&P 500 index. The CBOE Market Volatility Index or VIX is constructed in such a way that it represents the implied volatility of an at-the-money S&P 500 index option with exactly thirty calendar days to expiration. It is sometimes called the "investor fear gauge" because it is set by *investors* and expresses their consensus view about *expected future stock market volatility*. The specific details of its construction are contained in Appendix 15A of this chapter. What is interesting about its construction is that the index can be created using a static portfolio of SPX options. This is important since arbitrage between the VIX futures and the underlying VIX index promotes liquidity in both markets.

The relation between the movements of the VIX and the movements of the S&P 500 index are important to understand. Figure 15.12 shows the daily levels of the S&P 500 index and the VIX during the period January 1990 through December 2004. A number of interesting patterns appear. First, note that the VIX level (i.e., the dark line) is more jagged than the S&P 500 index level. What this means is that the volatility of the volatility of the S&P 500 index is greater than the volatility of the index itself.[18] Second, there tends to be an inverse rela-

[17] On a typical day, S&P 500 put option volume (and open interest) is nearly double that of S&P 500 calls.

[18] Time-series variation in the expected volatility of stock indexes has been documented in a number of studies. Day and Lewis (1992), for example, demonstrate that the expected variance of the S&P 100 index follows a mean-reverting process. They also show that implied volatilities from S&P 100 index options (OEX) explain a significant amount of the changes in expected variance. In a related paper, Fleming (1998) finds that OEX implied volatilities are good (but not perfect) forecasts of future volatility.

FIGURE 15.12 Daily levels of the S&P 500 index and the VIX during the period January 1990 through December 2004.



tion between the level of VIX and the level of the S&P 500 index—as the stock market goes up, volatility tends to fall. During 2003 and 2004, for example, the S&P 500 is systematically increasing while the level of VIX falls. Third, the inverse correlation is not perfect. During 1996 and 1997, for example, the level of market volatility is increasing while the stock market is also increasing. All of these phenomenon contribute to making futures contracts on the VIX a potentially new and useful expected return/risk management tool, as we will see in the illustration that follows.

The CFE VIX futures contract has, as its underlying, the VIX. The futures contract specifications are given in Table 15.5. Its denomination is \$100 times the increased-value VIX. The "increased-value VIX" (ticker symbol VBI) is simply the level observed in the marketplace times ten ($VBI = VIX \times 10$). The tick size of the contract is 0.1 of one VBI point or \$10. The available contract months include the two near-term contract months plus two contract months on the February quarterly cycle (February, May, August, and November). The expiration day is the third Friday of the contract month, although trading stops on the preceding Tuesday. The contract is cash-settled on the Wednesday preceding the third Friday, at a special opening quotation (SOQ).

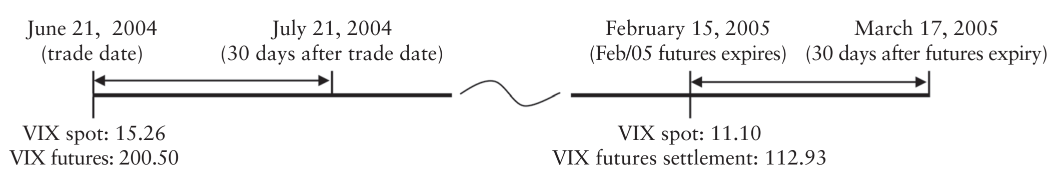
To understand the distinction between the VIX and the VIX futures, consider Figure 15.13. The figure assumes that we traded the February 2005 VIX futures on June 21, 2004. At the close on June 21, the VIX level was 15.26, and the Feb/05 VIX was at 200.50. Recall that the futures is scaled by 10, so the futures price represents a volatility rate of 20.05%. As the figure illustrates, the level of VIX reflects the market's expected future volatility over the next thirty calendar days (from June 21 to July 21, 2004), while the VIX futures reflects the expected future market volatility during a 30-calendar day period beginning when the Feb/05 futures contract expires and ending thirty calendar days later (February 15 to March 17, 2005). In other words, the VIX futures is a one-month forward volatil-

TABLE 15.5 Selected terms of Market Volatility Index (VIX) futures contract.

Exchange	CBOE Futures Exchange (CFE)
Ticker symbol	VX
Contract unit	$100 times Increased-Value VIX[a]
Tick size	0.1 of one VBI point
Tick value	$10
Trading hours	8:30 AM to 3:15 PM CST
Contract months	Two near-term contract months plus two contract months on the February quarterly cycle (Feb., May, Aug., and Nov.)
Expiration day	Third Friday of the contract month.
Last day of trading	Tuesday prior to the third Friday of the expiring month.
Final settlement date	Wednesday prior to the third Friday of the expiring month.
Final settlement price	Cash settled. Final settlement price for VIX futures shall be 10 times a Special Opening Quotation (SOQ) of VIX calculated from the options used to calculate the index on the settlement date. The opening price for any series in which there is no trade shall be the average of that option's bid price and ask price as determined at the opening of trading. The final settlement price will be rounded to the nearest 0.10.

[a] Increased-Value VIX (VBI) is 10 times the VIX index level.

FIGURE 15.13 VIX index and February 2005 VIX futures assuming futures was traded on June 21, 2004.



ity rate that begins some time in the future. As it turns out, on February 15, 2005, the Feb/05 VIX was cash settled in the morning at ten times the spot level of VIX, 112.93. By the end of the day, the level of VIX had fallen to 11.10.

The convergence of the Feb/05 VIX futures to the VIX index over the period June 21, 2004 through February 15, 2005 is shown in Figure 15.14. The VIX is multiplied by 10 to put it on the same scale as the futures price. Where the two prices were about 50 points apart in June 2004, they slowly and steadily converged to the same level at expiration. Figure 15.15 shows the open interest of the Feb/05 VIX futures contract. In June 2004, the Feb/05 futures was a distant contract maturity and did not have much open interest. Through time, as the shorter contract maturities expired, the open interest in the Feb/05 contract rose, reaching a peak above 6,000 contracts in January 2005. Like most cash-settled futures, open interest remained high until contract settlement.[19]

[19] Recall that, in Chapter 1, we discussed the fact that futures contracts with physical delivery are generally unwound before contract maturity to avoid the costs of transportation. With cash settlement, no such costs exist.

FIGURE 15.14 Convergence of February 2005 VIX futures price to VIX spot price (10 times observed VIX) over the period June 21, 2004 through February 16, 2005.

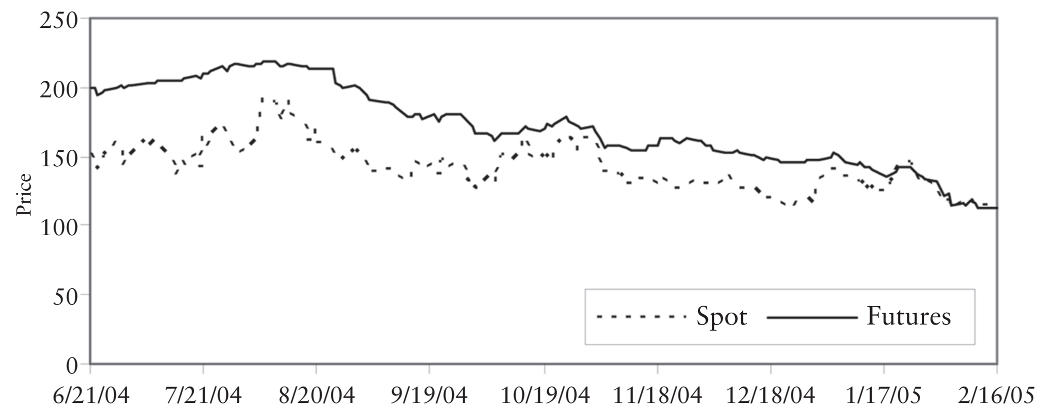
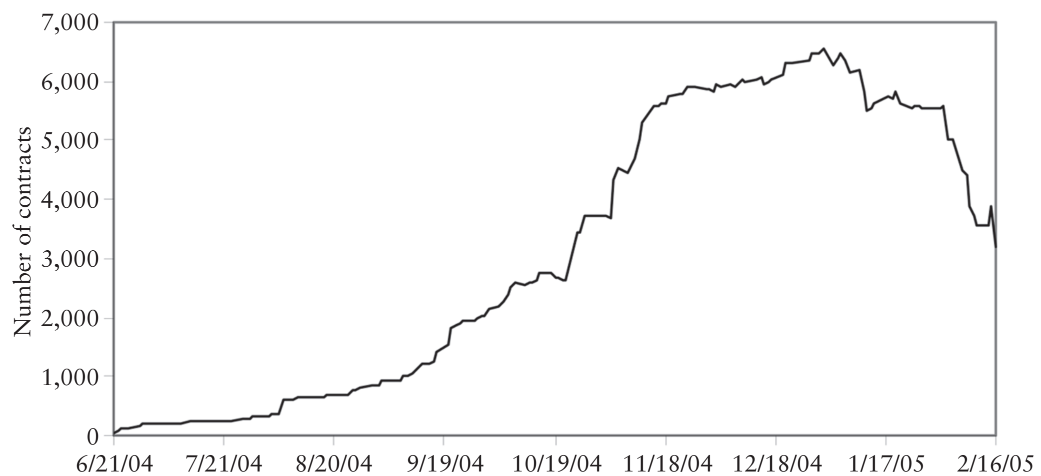


FIGURE 15.15 Open interest of February 2005 VIX futures over its life (June 21, 2004 through February 16, 2005).



To get a sense for how VIX futures contracts are priced, let us assume that we are considering the variance of S&P 500 index returns over the next 60 calendar days (i.e., two months). If the returns of the index are independent through time, we can write

$$\bar{\sigma}^2_{1\text{-}60}\left(\frac{60}{365}\right) = \bar{\sigma}^2_{1\text{-}30}\left(\frac{30}{365}\right) + \bar{\sigma}^2_{31\text{-}60}\left(\frac{60-30}{365}\right) \tag{15.23}$$

In (15.23), $\bar{\sigma}^2_{1\text{-}60}$ and $\bar{\sigma}^2_{1\text{-}60}$ can be considered spot rates of variance, that is, the expected variance rates over the next 30 calendar days and 60 calendar days, respectively. The term,

$$\bar{\sigma}^2_{31\text{-}60}\left(\frac{30}{365}\right)$$

however, is a forward variance, that is, the average variance rate that we can expect to observe over a 30-day period beginning 30 days from now. To determine the forward volatility rate, we can rearrange (5.23) to yield

$$\bar{\sigma}_{31\text{-}60} = \sqrt{\frac{\bar{\sigma}^2_{1\text{-}60}\left(\frac{60}{365}\right) - \bar{\sigma}^2_{1\text{-}30}\left(\frac{30}{365}\right)}{\frac{60-30}{365}}} \tag{15.24}$$

Equation (15.24) provides us with the insight we need in understanding how to value the VIX futures. The rate on the left-hand side of (15.24) can be thought of as the VIX futures price. In order to estimate its value, we need to know the two variance rates in the numerator on the right-hand side. One way to get these values is to request quotes on 30-day and a 60-day variance forwards from an OTC swap dealer. Another is to use S&P 500 index options to imply the variance rates of 30- and 60-day intervals.[20] Note that, in this particular instance, the rate $\bar{\sigma}_{1\text{-}30}$ is also the current level of the VIX because the forward period begins in exactly 30 calendar days. Whether the forward price exceeds the current spot price, as it did for the Feb/05 VIX futures, depends upon whether the term structure of realized variance swaps is upward- or downward-sloping. In an upward-sloping environment, the forward price will exceed the spot price, and vice versa. Given that volatility tends to follow a mean-reverting process, the forward rate will be equal to the spot rate on average.

ILLUSTRATION 15.6 Estimate VIX futures price.

Suppose that you are given the assignment of determining the fair value of the VIX futures where the contract expires in exactly 15 days. You have contacted an OTC derivatives dealer, and he quoted you rates of 400 and 420 on 15-day and 45-day realized variance swaps.

The quoted realized variance swap rates straddle the forward period corresponding to the VIX futures. Hence, the fair value of the VIX futures can be determined by

$$VIX \text{ futures} = \sqrt{\frac{420\left(\frac{45}{365}\right) - 400\left(\frac{15}{365}\right)}{\frac{45-15}{365}}} = 20.74$$

expressed in VIX points, or 207.40 expressed in VBI points.

[20] The procedure in Appendix 15A of this chapter can be adapted to handle this exercise.

Expected Return/Risk Management Applications Exchange-traded futures on volatility also offer a number of new expected return/risk management strategies. In the illustration below, we show that VIX futures can be regarded as a new asset class and can potentially improve the expected return/risk opportunity set. Indeed, because the returns of the S&P 500 portfolio and the returns of the VIX are inversely correlated, the diversification effects can well surpass other strategies such as diversifying across countries.[21] VIX futures can also be used to manage individual stock volatility. Individual stock volatility can be thought of as the sum of two components: stock market volatility and firm-specific volatility. Market volatility products allow investors to hedge the stock market volatility component to develop selected exposures in the idiosyncratic risk of individual stocks.[22]

One caveat is necessary, however. Many stock market volatility hedging needs are long-term. The VIX futures contract, on the other hand, is on the stock market volatility rate in a thirty-day forward period. Consequently, in order to effectively hedge a short volatility position over a long period of time, it may be necessary to buy a strip of VIX futures so that the volatility rate over the entire hedge interval may be captured.

ILLUSTRATION 15.7 Using VIX futures as alternative investment.

Suppose that you are a pension fund manager and have just finished your stock portfolio allocation decisions for the year. The expected return of the stock portfolio is 12%, and its standard deviation of return is 16%. The risk-free interest rate is 4%. Since the pension fund has a stated risk tolerance level of 14%, you place 12.5% of the portfolio's funds in risk-free bonds and 87.5% in the stock portfolio. The expected overall portfolio return is therefore

$$E_P = 0.125(0.04) + 0.875(0.12) = 0.04 + (0.12 - 0.04)(0.14/0.16) = 11\%^{23}$$

Now suppose that you have just become aware of VIX futures contract. Since stock market volatility tends to follow a mean-reverting process, you believe that the expected rate of price appreciation in the VIX futures is 0%. After some statistical analysis, you assess the volatility of the rate of price appreciation in the VIX futures to be 80%, and the correlation between the VIX futures return and your stock portfolio return to be –0.6. Can you benefit by buying or selling VIX futures?

To answer the question, you need to recall from Chapter 5 that the expected return and risk of a portfolio that consists of a long position in the asset and n_F futures contracts may be written

$$E_P = E_S + n_F E_F$$

and

[21] Stock returns in different countries tend to be positively correlated. A major economic shock in one market is usually felt across markets.

[22] Whaley (1993) demonstrates that, for large market capitalization firms, nearly 50% of movement in individual stock volatility rate is explained by movements in the market volatility rate.

[23] The expected return/risk mechanics is drawn directly from Chapter 3. Risk tolerance is the maximum return volatility (expressed in standard deviation of return) that the portfolio is willing to sustain.

$$\sigma_P = \sqrt{\sigma_S^2 + n_F^2\sigma_F^2 + 2n_F\rho_{SF}\sigma_S\sigma_F}$$

where E is expected rate of return, σ is the standard deviation of return, and ρ is the correlation between rates of return. With the current allocation, the expected excess return-to-risk ratio (i.e., the Sharpe ratio) is

$$\text{Sharpe ratio} = \frac{E_S - r}{\sigma_S} = \frac{0.12 - 0.04}{0.16} = 0.5$$

Can you arrive at a higher Sharpe ratio by buying/selling VIX futures?

To answer this question, you can use Excel's SOLVER to find the value of n_F that maximizes

$$\frac{E_P - r}{\sigma_P} = \frac{0.12 - 0.04}{\sqrt{0.16^2 + n_F^2(0.80^2) + 2n_F(-0.6)(0.16)(0.80)}}$$

where, because the expected return on the VIX futures is zero, it does not appear in the numerator of the portfolio's excess return-to-risk ratio. For the problem information at hand, the optimal value of n_F is 0.12. At $n_F = 0.12$, the expected portfolio return (E_P) stays at 12% (since the expected return on the VIX futures is 0), however, the standard deviation of portfolio return (σ_P) is only 12.8% and the expected excess return/risk ratio is 0.625. If the pension fund does permit borrowing, the final portfolio should consist of only the stock portfolio and a long position the VIX futures, and no money in risk-free bonds. If the pension fund allows for borrowing and wants to maintain its stated risk tolerance of 14%, it must lever up the portfolio by 14/12.8 – 1 = 0.09375. Thus, the optimal allocation is to borrow 9.375% of the portfolio's value, invest 109.375% of the portfolio's value in the stock portfolio, and buy 1.09375(0.12) VIX futures. The expected return of the overall portfolio is now 12.75% at a 14% risk tolerance, well above the 11% expected when the VIX futures are not considered. The figure shown below summarizes the results of this illustration. Without the VIX futures, the pension fund is expected to reside at point A with an expected return of 11%. With the VIX futures as part of the portfolio, the fund has a higher expected return, 12.75%, at the same level of risk and resides at point B.

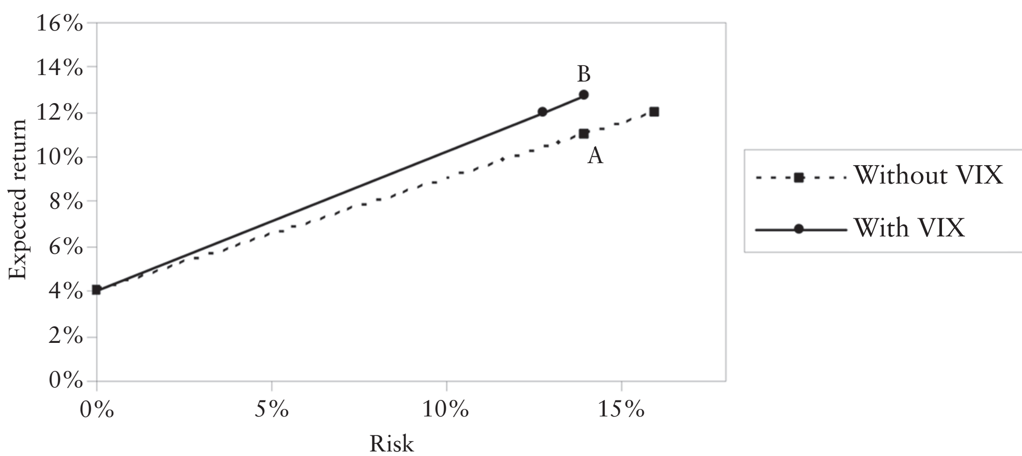


TABLE 15.6 Selected terms of Market Volatility Index (VIX) option contract.

Exchange	Chicago Board Options Exchange (CBOE)
Ticker symbol	VIX
Contract unit	100 times CBOE Market Volatility index
Exercise price increments	2-1/2 point increments
Exercise style	European
Tick size	0.05 point up to $3 premiums; .10 point over $3
Tick value	$5; $10
Trading hours	8:30 AM to 15:15 PM CST
Contract months	Two near-term contract months plus two contract months on the February quaterly cycle (Feb., May, Aug., and Nov.)
Expiration day	Wednesday that is 30 days prior to the third Friday of the calendar month immediately following the expiring month.
Last day of trading	Tuesday prior to expiration date each month.
Final settlement price	Cash settled. Exercise settlement value shall be a Special Opening Quotation (SOQ) of VIX calculated from the sequence of opening prices of options used to calculate the index on the settlement date. The opening price for any series in which there is no trade shall be the average of that option's bid price and ask price as determined at the opening of trading. Exercise will result in the delivery of cash on the business day following expiration. The exercise settlement amount is equal to the difference between the exercise-settlement value and the exercise price of the option times $100.

The Chicago Board Options Exchange (CBOE) launched VIX option contracts on Friday, February 24, 2006. Like the VIX futures, the CBOE's VIX options contract has, as its underlying, the VIX. The option contract specifications are given in Table 15.6. Its ticker symbol is "VIX," and its denomination is $100 times the level of the CBOE's Market Volatility index. The tick size (value) of the contract is 0.05 ($5) for option premiums below $3.00 ($300), and 0.10 ($100) for premiums greater than $3 ($300). The available contract months include the two near-term contract months plus two contract months on the February quarterly cycle (February, May, August, and November). The expiration day is the Wednesday that is 30 days before the third Friday of the calendar month following the expiring month. Trading stops on the Tuesday before the expiration day. The contract is cash-settled on the day after the expiration at at a special opening quotation (SOQ). The exercise settlement amount equals the difference between the exercise-settlement value and the exercise price times $100.

SUMMARY

Many stock index products are inextricably linked to particular index derivative trading strategies. This chapter focuses on such products. The first is portfolio insurance. After examining a brief history on how the strategy evolved, a detailed analysis of passive and dynamic portfolio insurance schemes is provided. Passive insurance means buying an appropriate number of index puts. Dynamic insurance implies that the portfolio consists of either stocks and risk-free bonds or stocks and index futures and is rebalanced continuously through time and as the market moves in such a way that the portfolio payoffs mimic the payoffs of an insured portfolio. The second group of products are funds based on an index/option trading strategy. Including options in an investment portfolio can dramatically affect the shape of the portfolio's rate of return distribution, undermining the usefulness of commonly applied portfolio performance evaluation techniques. We examine this problem using the history of buy-write returns for the CBOE's Buy-Write Index (BXM). The final group of index products that we discuss is market volatility derivatives. Essentially two types exist—contracts on realized volatility and contracts on volatility implied by index option prices. We describe different volatility contract specifications and show how the CBOE's Market Volatility Index (VIX) can be constructed from a portfolio of S&P 500 index options. We also illustrate how volatility derivatives can be used as an alternative investment in an asset allocation framework.

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APPENDIX 15A: CONSTRUCTION OF THE CBOE'S MARKET VOLATILITY INDEX (VIX)

The purpose of this appendix is to describe the algorithm with which the CBOE's Market Volatility Index (VIX) is computed.[24] The VIX is the expected future volatility of the S&P 500 index over the next thirty days. It is an *implied* volatility in that it is based on S&P 500 index option prices. Unlike the implied volatilities from the BSM option valuation model, however, the VIX does not depend on a particular return distribution.[25]

To compute the VIX, an eight-step procedure is used.

Step 1: Collect relevant information. The information needed to compute the VIX is (1) the bid/ask price quotes of all nearby and second nearby call and put options

[24] The procedure for calculating VIX is described in CBOE (2003). The theory underlying the procedure is based on the Breeden and Litzenberger (1978) result that the probability density function of asset price can be inferred from the prices of options written on that asset, where the options have a common expiration date and continuum of exercise prices. Demeterfi, Derman, Kamal, and Zou (1999) apply this result in a discretized form to arrive at an equation for the volatility of asset price.
[25] Recall that the BSM model assumes a log-normal asset price distribution at the option's expiration.

traded on the S&P 500 index; and (2) the risk-free interest rate corresponding to each expiration date. For each option series, the bid/ask midpoint is computed. The difference between the call midpoint and put midpoint at each exercise price is computed.

Step 2: Compute the time to expiration in minutes and then years from the current time until option expiration. The time to expiration in minutes is the sum of three components.[26] First, we must compute the number of minutes from the current time until midnight on the same day. We next compute the number of minutes from midnight today until midnight on the day before expiration. Finally, we must compute the number of minutes from midnight on the day before expiration until cash settlement at the open on expiration day. The last number is, of course, a constant. The time of cash settlement is at 8:30 AM on expiration day. The number of minutes from midnight on the day before expiration until the time of expiration is therefore

$$8.5 \text{ hours} \times 60 \text{ minutes per hour} = 510 \text{ minutes}$$

The first and second components depend upon the time of day and the number of days to expiration, respectively.

To illustrate, assume that we are computing the level of VIX at 8:38 AM (CST) on October 6, 2003. The number of minutes to midnight on October 6 is

$$22 \text{ minutes} + 15 \text{ hours} \times 60 \text{ minutes per hour} = 922 \text{ minutes}$$

On October 6, 2003, the nearby and second expirations of the S&P 500 index options are the October 17, 2003 and November 21, 2003, respectively, and the number of days to expiration are 12 and 47 days inclusive of the current date and the expiration date. The current date and expiration date are already incorporated, however. The number of minutes until midnight on the current date is 922, and the number of minutes from midnight on the day before expiration until time of expiration on the expiration day is 510. Thus we reduce the number of days to expiration for the nearby and second nearby expirations to 10 and 45 and compute the number of minutes. With 1,440 minutes in each 24-hour day, the number of minutes for the second component of the nearby contract is

$$10 \text{ days} \times 1{,}440 \text{ minutes per day} = 14{,}400$$

and the number of minutes for the second component of the second nearby contract is

$$45 \text{ days} \times 1{,}440 \text{ minutes per day} = 64{,}800$$

The total numbers of minutes for the two contract expirations are therefore

$$\text{Nearby contract: } 922 + 14{,}400 + 510 = 15{,}832$$

and

[26] Time to expiration is computed in minutes to conform to industry practice.

Second nearby contract: 922 + 64,800 + 510 = 66,232

The times to expiration in years are then computed as

$$T_1 = 15{,}832/525{,}600 = 0.0301217656$$

and

$$T_2 = 66{,}232/525{,}600 = 0.1260121766$$

where 525,600 in the number of minutes in a calendar year (i.e., 1,440 minutes per day times 365 days).

Step 3: Compute the interest accumulation factor for each option expiration. The interest accumulation factor is defined as the terminal amount that $1 will accumulate to by the option's expiration if invested at the risk-free rate of interest. On October 6, 2003, the risk-free rate corresponding to the nearby expiration was 0.920% on an annualized basis, and the risk-free rate corresponding to the second nearby expiration was 0.850%.[27] The accumulation factors for the nearby and second nearby contracts were

$$e^{r_1 T_1} = e^{0.00920(0.03012177)} = 1.0002772$$

and

$$e^{r_2 T_2} = e^{0.00850(0.12601218)} = 1.0010717$$

respectively.

Step 4: Identify the at-the-money options for each option expiration. To identify the at-the-money options for each expiration, we must first compute the bid/ask midpoints for all calls and puts with the nearby and second nearby contract expirations. This is shown in Tables 15A.1 and 15A.2. For each exercise price for which a call price and put price are available, compute the absolute difference between the call price and put price. Note that the calls and puts with zero bid prices are excluded for consideration. Such options appear in bold face. The exercise price with the lowest absolute difference is defined as the at-the-money option. On October 6, 2003, the nearby at-the-money exercise price is 1030 (as is shown in Table 15A.1), and the second nearby exercise price is 1035 (as is shown in Table 15A.2).

Step 5: Compute the forward index level for each contract expiration. With the identity of the at-the-money options known, we compute the implied forward index level using the forward value version of put-call parity, that is,

$$F_i = X_i + e^{r_i T_i}(C_i - P_i)$$

[27] On this particular day, the yield curve of the risk-free rate was inverted at short maturities.

TABLE 15A.1 Nearby S&P 500 index option prices used in the computation of the VIX on October 6, 2003 at 8:38 AM (CST).

Nearby Contract Expiration: 20031017

Exercise Price	Call Price Quotes			Put Price Quotes			Absolute Difference
	Bid	Ask	Midpoint	Bid	Ask	Midpoint	
725	304.10	307.10	305.600	**0.00**	**0.50**		
750	279.10	282.10	280.600	**0.00**	**0.50**		
775	254.10	257.10	255.600	**0.00**	**0.50**		
800	229.10	232.10	230.600	**0.00**	**0.40**		
825	204.10	207.10	205.600	**0.00**	**0.25**		
850	179.10	182.10	180.600	0.05	0.20	0.125	180.475
875	154.20	157.20	155.700	0.10	0.20	0.150	155.550
890	139.20	142.20	140.700	**0.00**	**0.50**		
900	129.30	132.30	130.800	0.20	0.40	0.300	130.500
910	119.40	122.40	120.900	**0.00**	**0.50**		
915	114.40	117.40	115.900	0.05	0.50	0.275	115.625
925	104.50	107.50	106.000	0.25	0.60	0.425	105.575
930	100.00	102.60	101.300	0.30	0.70	0.500	100.800
935	95.10	97.10	96.100	0.50	0.60	0.550	95.550
940	90.20	92.20	91.200	0.45	0.90	0.675	90.525
945	85.30	87.30	86.300	0.40	0.90	0.650	85.650
950	80.40	82.40	81.400	0.65	1.00	0.825	80.575
955	75.80	77.80	76.800	0.75	1.10	0.925	75.875
960	70.90	72.90	71.900	0.80	1.30	1.050	70.850
970	61.30	63.30	62.300	1.10	1.60	1.350	60.950
975	56.50	58.50	57.500	1.50	1.90	1.700	55.800
980	51.80	53.80	52.800	1.70	2.20	1.950	50.850
985	47.20	49.20	48.200	2.00	2.50	2.250	45.950
990	42.60	44.60	43.600	2.30	3.10	2.700	40.900
995	38.20	40.20	39.200	3.00	3.70	3.350	35.850
1005	29.50	31.50	30.500	4.40	5.20	4.800	25.700
1010	25.50	27.50	26.500	5.40	6.40	5.900	20.600
1015	21.80	23.80	22.800	6.60	7.60	7.100	15.700
1020	18.50	19.50	19.000	8.00	9.00	8.500	10.500
1025	16.00	16.90	16.450	9.90	10.90	10.400	6.050
1030	13.00	14.00	13.500	11.60	13.20	12.400	1.100
1035	10.10	11.50	10.800	14.00	15.60	14.800	4.000
1040	8.00	9.00	8.500	16.80	18.40	17.600	9.100
1045	6.10	7.00	6.550	19.90	21.50	20.700	14.150
1050	4.70	5.50	5.100	23.20	25.20	24.200	19.100
1055	3.40	4.20	3.800	26.90	28.90	27.900	24.100
1060	2.50	3.30	2.900	30.90	32.90	31.900	29.000
1065	1.90	2.40	2.150	35.20	37.20	36.200	34.050

TABLE 15A.1 (Continued)

Exercise Price	Call Price Quotes			Put Price Quotes			Absolute Difference
	Bid	Ask	Midpoint	Bid	Ask	Midpoint	
1070	1.30	1.80	1.550	39.60	41.60	40.600	39.050
1075	0.90	1.40	1.150	44.20	46.20	45.200	44.050
1100	0.10	0.20	0.150	68.60	70.60	69.600	69.450
1115	**0.00**	**0.50**		83.40	85.40	84.400	84.400
1125	**0.00**	**0.15**		93.40	95.40	94.400	94.400
1135	**0.00**	**0.50**		102.90	105.90	104.400	104.400
1150	**0.00**	**0.10**		117.80	120.80	119.300	119.300
1175	**0.00**	**0.50**		142.80	145.80	144.300	144.300
1200	**0.00**	**0.50**		167.80	170.80	169.300	169.300
1225	**0.00**	**0.50**		192.80	195.80	194.300	194.300
1250	**0.00**	**0.50**		217.80	220.80	219.300	219.300
1275	**0.00**	**0.50**		242.80	245.80	244.300	244.300
1300	**0.00**	**0.50**		267.80	270.80	269.300	269.300
1325	**0.00**	**0.50**		292.80	295.80	294.300	294.300
1350	**0.00**	**0.50**		317.70	320.70	319.200	319.200
1375	**0.00**	**0.50**		342.70	345.70	344.200	344.200

TABLE 15A.2 Second nearby S&P 500 index option prices used in the computation of the VIX on October 6, 2003 at 8:38 AM (CST).

Second Nearby Contract Expiration: **20031121**

Exercise Price	Call Price Quotes			Put Price Quotes			Absolute Difference
	Bid	Ask	Midpoint	Bid	Ask	Midpoint	
600	427.70	430.70	429.200	**0.00**	**0.30**		
625	402.70	405.70	404.200	**0.00**	**0.50**		
650	377.80	380.80	379.300	**0.00**	**0.50**		
675	352.80	355.80	354.300	**0.00**	**0.50**		
700	327.90	330.90	329.400	**0.00**	**0.50**		
725	303.00	306.00	304.500	**0.00**	**0.50**		
750	278.10	281.10	279.600	**0.00**	**0.50**		
775	253.30	256.30	254.800	0.10	0.60	0.350	254.450
800	228.50	231.50	230.000	0.30	0.80	0.550	229.450
825	203.90	206.90	205.400	0.60	1.10	0.850	204.550
850	179.40	182.40	180.900	1.10	1.60	1.350	179.550
875	155.00	158.00	156.500	1.70	2.20	1.950	154.550
895	135.80	138.80	137.300	2.30	3.10	2.700	134.600
900	131.20	134.20	132.700	2.60	3.30	2.950	129.750
925	107.70	110.70	109.200	3.90	4.70	4.300	104.900
950	85.40	87.40	86.400	6.00	7.00	6.500	79.900
975	64.00	66.00	65.000	9.50	10.50	10.000	55.000
980	60.00	62.00	61.000	10.20	11.80	11.000	50.000

TABLE 15A.1 (Continued)

Second Nearby Contract Expiration:			**20031121**				
Exercise Price	**Call Price Quotes**			**Put Price Quotes**			**Absolute Difference**
	Bid	**Ask**	**Midpoint**	**Bid**	**Ask**	**Midpoint**	
985	56.00	58.00	57.000	11.20	12.80	12.000	45.000
990	52.10	54.10	53.100	12.30	13.90	13.100	40.000
995	48.30	50.30	49.300	13.50	15.10	14.300	35.000
1005	41.20	43.20	42.200	16.80	17.90	17.350	24.850
1010	37.80	39.80	38.800	17.90	19.50	18.700	20.100
1015	34.50	36.50	35.500	19.70	21.30	20.500	15.000
1020	31.40	33.40	32.400	21.30	23.30	22.300	10.100
1025	28.40	30.40	29.400	23.30	25.30	24.300	5.100
1035	22.90	24.90	23.900	27.90	29.90	28.900	5.000
1050	16.20	17.80	17.000	35.90	37.90	36.900	19.900
1060	12.40	14.00	13.200	42.10	44.10	43.100	29.900
1065	10.70	12.30	11.500	45.40	47.40	46.400	34.900
1070	9.50	10.00	9.750	48.90	50.90	49.900	40.150
1075	8.20	9.20	8.700	52.50	54.50	53.500	44.800
1080	7.00	8.00	7.500	56.30	58.30	57.300	49.800
1100	3.50	4.30	3.900	73.00	75.00	74.000	70.100
1125	1.40	1.90	1.650	95.70	97.70	96.700	95.050
1150	0.60	0.90	0.750	119.20	122.20	120.700	119.950
1175	**0.00**	**0.50**		143.80	146.80	145.300	145.300
1200	**0.00**	**0.50**		168.60	171.60	170.100	170.100
1225	**0.00**	**0.50**		193.50	196.50	195.000	195.000
1250	**0.00**	**0.50**		218.40	221.40	219.900	219.900
1275	**0.00**	**0.50**		243.40	246.40	244.900	244.900

For the nearby at-the-money options, the forward price is

$$F_1 = 1030 + 1.0002771586449(13.500 - 12.400) = 1031.10$$

For the second nearby at-the-money options, the forward price is

$$F_2 = 1025 + 1.0010716773370(29.400 - 26.600) = 1029.99$$

Step 6: Identify the option series used in the computation of the VIX. In computing the VIX, only the prices of out-of-the-money calls and puts are used. To distinguish between in-the-money and out-of-the-money options, the exercise price just below the implied forward price ($X_{i,0}$) is used. The out-of-the-money calls are those with exercise prices greater than or equal $X_{i,0}$, and the out-of-the-money puts are those with exercise prices less than or equal to $X_{i,0}$. If any of these option series have a bid price equal to zero, they are eliminated from consideration.[28] For the nearby and second nearby option series in the illustration, the exercise prices just below the forward index levels are $X_{1,0} = 1030$ and $X_{2,0}$

= 1035. Since this procedure identifies two options (a call and a put) at exercise price $X_{i,0}$, the arithmetic average of the call and put prices is used.

Step 7: Compute the implied variance for each contract expiration. The formula for computing the implied variance for the nearby contract is

$$\sigma_1^2 = \frac{2}{T_1}\sum_{i=1}^{n_1}\frac{\Delta X_{1,i}}{X_{1,i}^2}e^{r_1T_1}O(X_{1,i}) - \frac{1}{T_1}\left(\frac{F_1}{X_{1,0}}-1\right)^2 \qquad (15A.1)$$

where T_1 is the nearby contract month's time to expiration expressed in years, n_1 is the number of out-of-the-money option series for the nearby contract month, $X_{1,i}$ is the exercise price of the i-th option, r_1 is the interest rate corresponding to option's expiration date, F_1 is the forward index level implied by the at-the-money call and put prices, $O(X_{1,i})$ is the bid/ask price midpoint of the nearby option with an exercise price of $X_{1,i}$, and $X_{1,0}$ is the exercise price just below the implied nearby forward price. The summation term also includes the at-the-money options. For the at-the-money options, the average of the call and put midpoints is used as $O(X_{1,i})$. Finally, the term $\Delta X_{1,i}$ is the average of the exercise prices that straddle option i's exercise price. At the highest and lowest exercise prices, $\Delta X_{1,i}$, is the absolute difference between option i's exercise price and the adjacent exercise price. The last term on the right-hand side is called the displacement factor.

The same procedure is used to compute the second nearby implied variance,

$$\sigma_2^2 = \frac{2}{T_2}\sum_{i=1}^{n_1}\frac{\Delta X_{2,i}}{X_{2,i}^2}e^{r_2T_2}O(X_{2,i}) - \frac{1}{T_2}\left(\frac{F_2}{X_{2,0}}-1\right)^2 \qquad (15A.2)$$

To illustrate the mechanics of these computations, first compute the values of the last term on the right-hand side (i.e., the displacement factors) of the nearby and second nearby contracts. For the nearby contract,

$$\frac{1}{T_1}\left(\frac{F_1}{X_{1,0}}-1\right)^2 = \frac{1}{0.03012177}\left(\frac{1031.10}{1030}-1\right)^2 = 0.3789\times10^{-4}$$

and, for the second nearby contract,

$$\frac{1}{T_2}\left(\frac{F_2}{X_{2,0}}-1\right)^2 = \frac{1}{0.12601218}\left(\frac{1027.80}{1035}-1\right)^2 = 0.00018843$$

[28] In the event that the bid prices of two calls (puts) at adjacent exercise prices are equal to zero, all call (put) option series with higher (lower) exercise prices are eliminated even though they may have nonzero bid prices.

Next take the sum in the first term on the right-hand side. Table 15A.3 shows the values of each of the n_1 terms for the nearby contract, and Table 15A.4 shows the values of each of the n_2 terms of the second nearby contract. The first term in the nearby contract's summation is

$$\frac{\Delta X_{1,1}}{X_{1,1}^2} e^{r_1 T_1} O(X_{1,1}) = \frac{25}{850^2} \times 1.0002772 \times 0.125 = 0.433 \times 10^{-5}$$

as is shown in Table 15A.3. Note that the option price used in the expression is the forward price (i.e., the current price carried forward until the end of the contract's life). The sum of the weighted average of the forward option prices is 0.0005943786 for the nearby contract and 0.0025376773 for the second nearby contract. The variance of the nearby contract is therefore

$$\sigma_1^2 = \frac{2}{0.03012177} \times 0.00059438 - 0.3789 \times 10^{-4} = 0.03942717$$

and the variance of the second nearby contract is

$$\sigma_2^2 = \frac{2}{0.12601218} \times 0.00253768 - 0.00018843 = 0.04008827$$

Step 8: Compute the annualized volatility over the next 30 calendar days. The variances of the nearby and second nearby contracts correspond to times to expiration of T_1 years and T_2 years, respectively. VIX, however, maintains a constant time to expiration of 30 days or 30/365 = 0.0821917808 years. To find the variance over the 30 calendar-day interval, we must interpolate between the variances of the nearby and second nearby contracts, that is,

$$\sigma_{30\text{-day}}^2 = \left(\frac{T_2 - T_{30\text{-day}}}{T_2 - T_1}\right)\sigma_1^2 T_1 + \left(\frac{T_{30\text{-day}} - T_1}{T_2 - T_1}\right)\sigma_2^2 T_2$$
$$= 0.00328583$$

To compute the level of VIX, we annualize the 30-day variance and take the square root, that is,

$$VIX = \sqrt{\sigma_{30\text{-day}}^2\left(\frac{1}{T_{30\text{-day}}}\right)} = \sqrt{0.03997755\left(\frac{1}{0.08219178}\right)} = 19.99\%$$

This is precisely the level of VIX reported by the CBOE at 8:38 AM (CST) on October 6, 2003. The Excel file, **VIX computation.xls**, contains the background computations used in this illustration.

TABLE 15A.3 Nearby S&P 500 index option prices contribution to the computation of the VIX on October 6, 2003 at 8:38 AM (CST).

Nearby Contract Expiration: 10/17/2003

C/P	Exercise Price	Price Midpoint	ΔX_i	Weight	Weight Times Forward Option Price
P	850	0.125	25	0.0000346021	0.0000043265
P	875	0.150	25	0.0000326531	0.0000048993
P	900	0.300	20	0.0000246914	0.0000074095
P	915	0.275	12.5	0.0000149303	0.0000041070
P	925	0.425	7.5	0.0000087655	0.0000037264
P	930	0.500	5	0.0000057810	0.0000028913
P	935	0.550	5	0.0000057194	0.0000031465
P	940	0.675	5	0.0000056587	0.0000038207
P	945	0.650	5	0.0000055989	0.0000036403
P	950	0.825	5	0.0000055402	0.0000045719
P	955	0.925	5	0.0000054823	0.0000050725
P	960	1.050	7.5	0.0000081380	0.0000085473
P	970	1.350	7.5	0.0000079711	0.0000107640
P	975	1.700	5	0.0000052597	0.0000089440
P	980	1.950	5	0.0000052062	0.0000101548
P	985	2.250	5	0.0000051534	0.0000115985
P	990	2.700	5	0.0000051015	0.0000137779
P	995	3.350	7.5	0.0000075756	0.0000253852
P	1005	4.800	7.5	0.0000074256	0.0000356526
P	1010	5.900	5	0.0000049015	0.0000289267
P	1015	7.100	5	0.0000048533	0.0000344680
P	1020	8.500	5	0.0000048058	0.0000408610
P	1025	10.400	5	0.0000047591	0.0000495081
X_0	1030	12.950	5	0.0000047130	0.0000610500
C	1035	10.800	5	0.0000046676	0.0000504235
C	1040	8.500	5	0.0000046228	0.0000393045
C	1045	6.550	5	0.0000045786	0.0000299985
C	1050	5.100	5	0.0000045351	0.0000231357
C	1055	3.800	5	0.0000044923	0.0000170753
C	1060	2.900	5	0.0000044500	0.0000129085
C	1065	2.150	5	0.0000044083	0.0000094805
C	1070	1.550	5	0.0000043672	0.0000067710
C	1075	1.150	15	0.0000129800	0.0000149311
C	1100	0.150	25	0.0000206612	0.0000031000
				Sum	0.0005943786

TABLE 15A.4 Second nearby S&P 500 index option prices contribution to the computation of the VIX on October 6, 2003 at 8:38 AM (CST).

Second Nearby Contract Expiration:		11/21/2003			
C/P	Exercise Price	Price Midpoint	ΔX_i	Weight	Weight Times Forward Option Price
P	775	0.350	25	0.0000416233	0.0000145838
P	800	0.550	25	0.0000390625	0.0000215074
P	825	0.850	25	0.0000367309	0.0000312548
P	850	1.350	25	0.0000346021	0.0000467629
P	875	1.950	22.5	0.0000293878	0.0000573675
P	895	2.700	12.5	0.0000156050	0.0000421787
P	900	2.950	15	0.0000185185	0.0000546882
P	925	4.300	25	0.0000292184	0.0001257738
P	950	6.500	25	0.0000277008	0.0001802484
P	975	10.000	15	0.0000157791	0.0001579600
P	980	11.000	5	0.0000052062	0.0000573292
P	985	12.000	5	0.0000051534	0.0000619076
P	990	13.100	5	0.0000051015	0.0000669015
P	995	14.300	7.5	0.0000075756	0.0001084467
P	1005	17.350	7.5	0.0000074256	0.0001289715
P	1010	18.700	5	0.0000049015	0.0000917559
P	1015	20.500	5	0.0000048533	0.0000995995
P	1020	22.300	5	0.0000048058	0.0001072852
X_0	1025	26.850	7.5	0.0000071386	0.0001918770
C	1035	23.900	12.5	0.0000116689	0.0002791852
C	1050	17.000	12.5	0.0000113379	0.0001929503
C	1060	13.200	7.5	0.0000066750	0.0000882041
C	1065	11.500	5	0.0000044083	0.0000507497
C	1070	9.750	5	0.0000043672	0.0000426258
C	1075	8.700	5	0.0000043267	0.0000376823
C	1080	7.500	12.5	0.0000107167	0.0000804617
C	1100	3.900	22.5	0.0000185950	0.0000725984
C	1125	1.650	25	0.0000197531	0.0000326275
C	1150	0.750	25	0.0000189036	0.0000141929
				Sum	0.0025376773