CHAPTER 12

Corporate Securities

Firms issue different types of securities to finance the assets of the firm—common stock preferred stock, discount bonds, coupon bonds, convertible bonds, warrants, convertible bonds, and so on. Some are issued to the public and are actively traded in the secondary markets. Others are placed publicly, but trade infrequently. Yet others are privately placed, and trade seldom if at all. The purpose of this chapter is to show how all of the firm's securities outstanding can be valued using only information regarding the firm's common stock price and volatility rate. This is possible because all of the firm's securities have the same source of uncertainty—the overall market value of the firm's assets. Consequently, all of the firm's securities have price movements that are perfectly correlated with one another over short periods of time. With such being the case, all corporate securities can be valued using information about the price and volatility rate of any *one* of the firm's outstanding securities. We choose to use the common stock of the firm because, of all the firm's securities, it has the deepest and most active secondary market. In this sense, all corporate securities may be considered common stock derivatives.

To develop the corporate security valuation framework, we rely on the BSM option valuation results from Chapter 7. The underlying source of uncertainty is the firm's overall market value, which we assume is log-normally distributed in the future. We also assume that a risk-free hedge may be formed between each of the firm's securities and the firm's overall value. As a practical matter, the firm's overall value (i.e., the sum of the market values of all of the firm's constituent securities) does not trade as a single asset, however, small changes in the value of the firm are perfectly correlated with the changes in the value of its stock. This means that, as long as the firm's common stock is actively traded, we can apply risk-neutral valuation with no loss in generality.

The chapter proceeds as follows. First, we address the valuation of corporate bonds assuming that the firm that has two securities outstanding—zero-coupon bonds and common stock. Second, we extend the framework to include multiple bond issues with varying degrees of seniority. Third, we value rights and warrants. Rights and warrants are call option-like securities written by the firm on its own stock. But because the firm issues these securities, option exercise implies that the value of existing shareholder equity is diluted, and the effects of dilution can have significant value. The same is true of convertible bonds, which are a hybrid security with bond and warrant-like features. Convertible bonds are the focus of the last section of the chapter.

VALUING CORPORATE BONDS[1]

The first security that we consider is a corporate bond. Unlike bonds issued by the U.S. Treasury, corporate bonds have default risk. There is always some possibility, however remote, that a firm will be unable to make a promised payment to bondholders. The effect of default risk on corporate bond valuation is shown in three different ways. First, the bond value is modeled as the value of the firm less the value of the stock, where the stock is modeled as a call option on the value of the firm. Next we use put-call parity to reformulate the value of a corporate bond as the difference between the value of a risk-free bond and the value of a put option that allows the managers of the firm to put the firm's assets to bondholders if the firm value is less than the bond's face value when they mature. Finally, we show that the value of a corporate bond is equivalent to the value of a portfolio that consists of a long position in the risk-free bonds and a long position in the firm's stock. As was noted in the introduction, all corporate securities may be formulated in this way.

Stock as Call Option on Firm

The bond valuation model assumes that the firm has two securities outstanding—a zero-coupon bond and stock. The bond's current value is denoted B, its face value is F, and its term to maturity is T. The market value of the firm's stock is denoted S, and the market value of the firm is $V \equiv S + B$. Since the bond has no coupons, bond default can be triggered only at bond maturity, when the value of the firm's assets is less than the face value of the bond.[2]

Under the above assumptions, the value of the firm's bond equals the total value of the firm less the value of the firm's stock, that is,

$$B = V - S \tag{12.1}$$

The firm's stock, in turn, can be thought of as a call option on the value of the firm. In the event that the market value of the firm is greater than the face value of the bond at the bond's expiration, the shareholders receive the value of the firm less the payment of the face value to bondholders; otherwise, they receive nothing, that is,

$$\tilde{S}_T = \max(\tilde{V}_T - F, 0) \tag{12.2}$$

Assuming the firm's value is log-normally distributed at the end of the bond's life, the current value of the firm's stock is given by the BSM call option valuation formula,

[1] The model developed in this section is frequently referred to as the "Merton model." Merton (1974) was the first to use the BSM framework to value corporate securities.

[2] With coupon-bearing bonds, default may also occur when the firm cannot meet a coupon interest payment.

$$S = VN(d_1) - Fe^{-rT}N(d_2) \tag{12.3}$$

where

$$d_1 = \frac{\ln(Ve^{rT}/F) + 0.5\sigma_V^2 T}{\sigma_V\sqrt{T}},\ d_2 = d_1 - \sigma_V\sqrt{T}$$

and the volatility rate is the volatility rate of the firm rather than the stock. From (12.1) and (12.3), the value of the risky bond may be written

$$B = V - [VN(d_1) + Fe^{-rT}N(d_2)] \tag{12.4}$$

The stock valuation equation (12.3) and the bond valuation equation (12.4) are useful in developing intuition regarding the relative values of the claims held by shareholders and bondholders. Suppose that the firm experiences an unexpected labor strike and now believes its earnings will be significantly below normal during the next few quarters. Naturally, the firm's value drops immediately. At first blush, one might think the shareholders of the firm are the only security holders to suffer since they are the residual claimants of the firm. The valuation equations (12.3) and (12.4) tell us otherwise, however. The stock's delta, that is, the change in stock value with respect to a change in firm value, may be derived from (12.3)[3] and has the form,

$$\Delta_S = N(d_1) \tag{12.5}$$

The bond's delta may be derived from (12.4) and is

$$\Delta_B = 1 - N(d_1) = N(-d_1) \tag{12.6}$$

Thus an unexpected drop in the firm value is split between the shareholders and the bondholders. The shareholders absorb Δ_S per dollar of firm value change, and the bondholders absorb Δ_B. Naturally, the sum of the changes in value is one, that is, $\Delta_S + \Delta_B = 1$. The only instance in which the shareholders absorb the full amount of the change is when the value of the firm is considerably greater than the face value of the bonds, making the bonds are essentially default-free.

ILLUSTRATION 12.1 Value corporate bond as firm value less call option.

Assume that the firm has a current value of 25, and its annual volatility rate is 20%. The firm has two securities outstanding—a zero-coupon bond and common stock. The bond matures in five years and has a face value of 20. The stock pays no dividends, and the risk-free rate of interest is 5%. Compute the values and volatility rates of the firm's stock and bonds.

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[3] The partial derivatives of the BSM call option formula with respect to changes in the formula's determinants are given in Chapter 7, Appendix 7E.

To compute the value of the stock, we use the BSM call option valuation formula (12.3), that is,

$$S = 25N(d_1) - 20e^{-0.05(5)}N(d_2)$$

where

$$d_1 = \frac{\ln(25e^{0.05(5)}/20) + 0.5(0.20^2)5}{0.20\sqrt{5}} = 1.2816 \text{ and } d_2 = 1.2816 - 0.20\sqrt{5} = 0.8344$$

The risk-neutral probabilities are $N(d_1) = 0.9000$ and $N(d_2) = 0.7980$, and the call option value is 10.071.

The OPTVAL function library contains a number of valuation routines for corporate securities. They all have the prefix OV_CORP_. The valuation function for the firm's stock given the value of the firm is

OV_CORP_STOCK_FIRM(*firm, face, t, r, vf, vind*)

where *firm* is the value of the firm, *face* is the face value of the firm's zero-coupon bonds, *t* is the term to maturity of the bond's in years, *r* is the risk-free interest rate, and *vf* is the volatility rate of the firm. The term, *vind*, is an indicator variable whose value is set equal to 1 if the function is to return the stock's value and 2 if the function is to return the volatility rate. For the illustration at hand,

S = OV_CORP_STOCK_FIRM(25, 20, 5, 0.05, 0.20, 1) = 10.071

To compute the stock's rate of return volatility, we use the elasticity (eta) of the stock value with respect to the firm value. Recall from the early discussion on dynamic strategies in Chapter 10 that, because the option's (stock's) rate of return is perfectly correlated with the asset's (firm's) rate of return, the stock's volatility rate may be written as a function of the firm's volatility rate, that is,

$$\sigma_S = \eta_S \sigma_V$$

where η_S is the elasticity (eta) of the value with respect to the firm value. The eta, in turn, is

$$\eta_S = \Delta_S\left(\frac{V}{S}\right)$$

where Δ_S is the stock's delta. From the above results, we know $\Delta_S = N(d_1) = 0.9000$, so the stock's rate of return volatility is

$$\sigma_S = N(d_1)\left(\frac{V}{S}\right)\sigma_V = 0.9000\left(\frac{25}{10.071}\right)0.20 = 44.68\%$$

This value can be verified using the OPTVAL function

OV_CORP_STOCK_FIRM(*firm, face, t, r, vf, vind*)

The function value is

σ_S = OV_CORP_STOCK_FIRM(25, 20, 5, 0.05, 0.20, 2) = 44.68%

With the stock value known, the bond value can be computed using (12.4), that is,

$$B = 25 - 10.071 = 14.929$$

Since the stock's delta is 0.9000, the bond's delta is 0.1000 (i.e., the delta of the firm is 1), the bond's eta is

$$\eta_B = \Delta_B\left(\frac{V}{B}\right) = 0.1000\left(\frac{25}{14.929}\right) = 16.74\%$$

and the bond's rate of return volatility is $\sigma_B = 0.1674(0.20) = 3.35\%$. These values can be verified using the functions

$$B = \text{OV\_CORP\_BOND\_FIRM}(25, 20, 5, 0.05, 0.20, 1) = 14.929$$

and

$$\sigma_S = \text{OV\_CORP\_BOND\_FIRM}(25, 20, 5, 0.05, 0.20, 2) = 3.35\%$$

Note that since both the rate of return of the bond and the rate of return of the stock are perfectly correlated, the volatility rate of the firm is a market value-weighted average of the volatility rates of the bond and the stock, that is,

$$\sigma_V = 0.0335\left(\frac{14.929}{25}\right) + 0.4468\left(\frac{10.071}{25}\right) = 20.00\%$$

Under the assumptions of the model, all corporate securities can be written is this way, albeit with different weights.

Figure 12.1 helps us develop the intuition for corporate bond valuation. It illustrates the tradeoff between shareholder and bondholder values as the value of the overall firm changes. All other parameters in the valuation equations are held constant. At very low levels of firm value, the probability of default is extremely high. Since the shareholders are unlikely to receive anything after bondholders are paid, the value of the bonds is simply the firm value. As firm value rises, the probability of default falls. Bond value rises, but at a decreasing rate, since the bondholders never receive more than the face value of their bonds. Share value, however, increases at an increasing rate. Once the bondholders are paid off, all of the firm's value goes to shareholders.

FIGURE 12.1 Values of bond and stock as a function of the value of the firm.

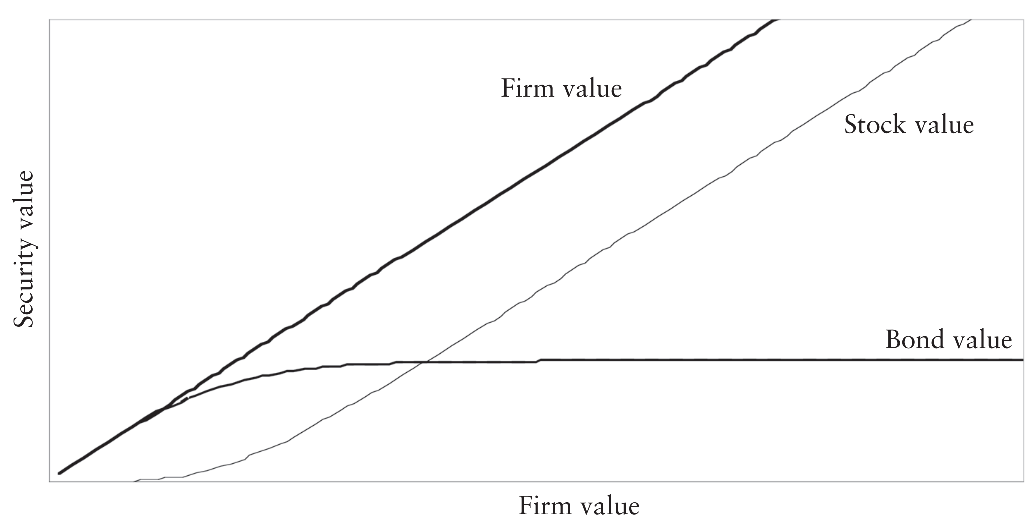
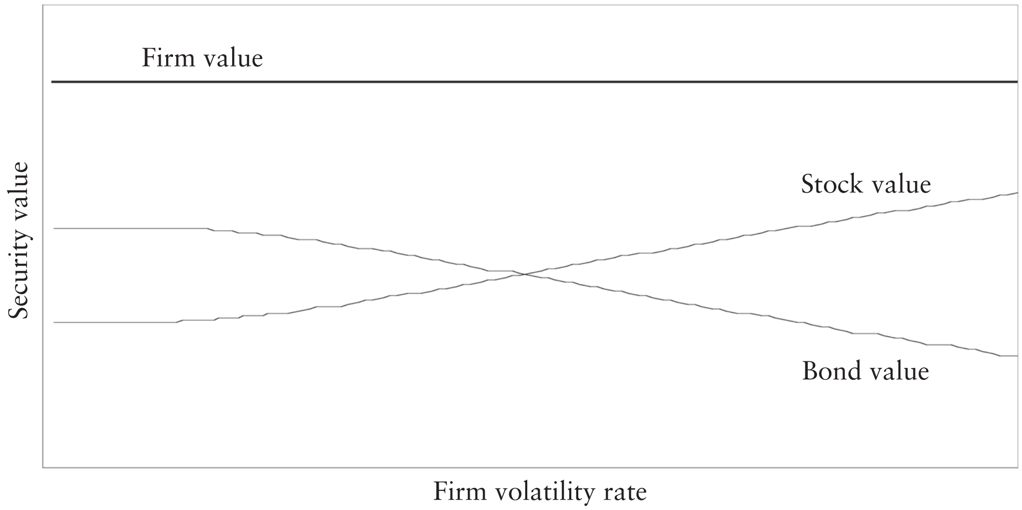


FIGURE 12.2 Values of bond and stock as a function of the firm's volatility rate.



Another interesting feature of (12.3) is that the value of the firm's stock increases with an increase in the firm's volatility rate, as is shown in Figure 12.2. The intuition for this result is that, the higher the volatility, the greater the chance that the firm's value exceeds the face value of the bonds by a large amount at the bond's maturity. Higher volatility also increases the chance that the firm's value is below the bond's face value by a large amount. Since the shareholders do not face any liability from this shortfall, however, the value of stock is unaffected.

The bondholder, on the other hand, is affected by the shortfall. Recall that, in the bond valuation formula (12.4), the bondholder owns the firm but is short a call option. With an increase in the firm's volatility rate, the value of the call rises. Holding other factors constant, the bondholders suffer. Figure 12.2 also shows the zero-sum nature of the effects of volatility rate changes on the values of the bonds and the stock of the firm. As firm's volatility rate rises, the value of the bonds falls and the value of stock rises.

Risky Bond Equals Risk-Free Bond Less Present Value of Expected Loss

Valuing a corporate bond as the difference between the firm value and the value of a call option provides several useful economic insights. But, this is only one possible formulation. By applying put-call parity to (12.4), we can derive an alternate specification that further enhances our understanding of bond valuation. In the context of valuing corporate securities, put-call parity[4] may be written

$$V - c = Fe^{-rT} - p \tag{12.7}$$

where c is the value of a call option written on the firm, that is, $c \equiv VN(d_1) - Fe^{-rT}N(d_2)$, and p is the value of the corresponding put option, that is, $p \equiv Fe^{-rT}N(-d_2) - VN(-d_1)$. Thus, the corporate bond value may also be written

[4] The European-style put-call parity was developed in Chapter 6.

$$B = Fe^{-rT} - [Fe^{-rT}N(-d_2) - VN(-d_1)] \tag{12.8}$$

Equation (12.8) says that the value of a corporate bond equals the difference between the value of a risk-free zero-coupon bond with face value F and the value of a put that allows the managers of the firm to put the firm's assets to the bondholders if firm value falls below the bonds' face value at maturity. To understand the economic intuition underlying the put, recall that in Chapter 7 we show that the value of a put option may be written

$$e^{-rT}\left[F - Ve^{rT}\frac{N(-d_1)}{N(-d_2)}\right]N(-d_2) \tag{12.9}$$

In (12.9), the term,

$$Ve^{rT}\frac{N(-d_1)}{N(-d_2)}$$

is the expected firm value at time T conditional on the value of the firm being less than the face value of the bonds, that is, $E(\tilde{V}_T | V_T < F)$. From a corporate bond perspective, this is called the bond's *expected recovery value*—what bondholders expect to receive in the event of default. If we subtract the expected recovery value conditional upon default from the bond's face value, we get the *expected loss* of the bond at time T conditional upon default, that is, $F - E(\tilde{V}_T | V_T < F)$, which may be calculated using the term in squared brackets of (12.9). The full expression (12.9) is, therefore, the *present value of the expected loss* on the bond conditional on the value of the firm being less than the bond's face value at time T times the *probability of default*, $\Pr(V_T < F) = N(-d_2)$. Hence, equation (12.9) provides another perspective on why bond value falls as the volatility rate rises (see Figure 12.2). As volatility rises, the expected loss conditional on default rises as does the probability of default.

ILLUSTRATION 12.2 Compute present value of expected loss on corporate bond.

Assume that the firm has a current value of 25, and its annual volatility rate is 20%. The firm has two securities outstanding—zero-coupon bonds and common stock. The bonds mature in five years and have a face value of 20. The stock pays no dividends, and the risk-free rate of interest is 5%. Compute the risk-neutral probability of default, the present value of the expected loss conditional upon default, the value of the firm's bonds, and the value of the firm's stock.

The value of a risk-free bond with five years remaining to maturity is

$$Fe^{-0.05(5)} = 20e^{-0.05(5)} = 15.576$$

The risk-neutral probability of default is

$$N(-d_2) = 1 - N(d_2) = 1 - 0.7980 = 0.2020$$

which can be verified using the OPTVAL function, OV_CORP_PROB_DEFAULT(*firm, face, t, alpha, vf*)

where *firm* is the value of the firm, *face* is the face value of the firm's zero-coupon bonds, *t* is the term to maturity of the bond's in years, *alpha* is the expected rate of appreciation in the value of the firm, and *vf* is the volatility rate of the firm. In a risk-neutral world, *alpha* is set equal to the risk-free rate of interest:

$$\Pr(V_T < F) = \text{OV\_CORP\_PROB\_DEFAULT}(25, 20, 5, 0.05, 0.20) = 0.2020$$

The expected recovery value conditional upon default is

$$25e^{0.05(5)}\left(\frac{0.1000}{0.2020}\right) = 15.888$$

and may be computed using

OV_CORP_RECOVERY_VALUE(*firm, face, t, alpha, vf*)

where all of the function arguments are as defined above. Substituting the problem parameters, we find

OV_CORP_RECOVERY_VALUE (25, 20, 5, 0.05, 0.20) = 15.888

The expected loss conditional upon default is. Alternatively, we can use the function

OV_CORP_EXPECTED_LOSS (25, 20, 5, 0.05, 0.20) = 4.112

The present value of the expected loss conditional upon default times the probability default is

$$e^{-0.05(5)}(4.112)(0.2020) = 0.647$$[5]

The value of the corporate bond is therefore

$$B = 15.576 - 0.647 = 14.929$$

and the value of the common stock is

$$S = V - B = 25 - 14.929 = 10.071$$

Note that the bond and stock values are consistent with Illustration 12.1.

Risky Bond as Portfolio of Risk-Free Bond and Stock

The bond valuation (12.8), in turn, can be rearranged to provide further economic insight. To see this, first gather terms on Fe^{-rT} and substitute $S + B$ for V, that is,

$$B = Fe^{-rT}N(d_2) + (S + B)N(-d_1) \tag{12.10}$$

Next rearrange terms to isolate B and then divide through by $N(d_1)$. The resulting equation for the corporate bond value is

$$B = Fe^{-rT}\left(\frac{N(d_2)}{N(d_1)}\right) + S\left(\frac{N(-d_1)}{N(d_1)}\right) \tag{12.11}$$

[5] To check this computation, compute the value of the put option on the right-hand side of (12.8) using OV_OPTION_VALUE(25, 20, 5, 0.05, 0.0, 0.20, “p”, “e”) = 0.647.

Equation (12.11) says that any corporate bond may be written as a portfolio consisting of a long position in risk-free bonds and a long position in the firm's stock. The number of units of the risk-free bond is

$$\frac{N(d_2)}{N(d_1)}$$

and the number of units of the firm's stock is

$$\frac{N(-d_1)}{N(d_1)}$$[6]

ILLUSTRATION 12.3 Write firm's bond value as portfolio consisting of risk-free bonds and stock.

Assume that the firm has a current value of 25, and its annual volatility rate is 20%. The firm has two securities outstanding—a zero-coupon bond and common stock. The bond matures in five years and has a face value of 20. The stock pays no dividends, and the risk-free rate of interest is 5%. Write the value of the bond as a portfolio consisting of a risk-free bond and the firm's stock.

From Illustration 12.1, we know that the stock's value is 10.071, $N(d_1) = 0.9000$, and $N(d_2) = 0.7980$. From Illustration 12.2, we know that the value of the risk-free bond is $20e^{-0.02(5)} = 15.576$, and $N(-d_1) = 0.1000$. The value of the firm's bond is therefore

$$\begin{aligned} B &= Fe^{-rT}\left(\frac{N(d_2)}{N(d_1)}\right) + S\left(\frac{N(-d_1)}{N(d_1)}\right) \\ &= 20e^{-0.05(5)}\left(\frac{0.7980}{0.9000}\right) + 10.071\left(\frac{0.1000}{0.9000}\right) = 14.929 \end{aligned}$$

The number of units of risk-free bonds and stock may also be computed using the OPTVAL function

OV_CORP_RFBOND_STOCK_SPLIT(*firm, face, t, r, vf, sind*)

where *sind* is an indicator variable whose value is set equal to 1 to find the number of units of risk-free bonds, and 2 to find the number of units of stock. All other function arguments are as defined above. For example,

OV_CORP_RFBOND_STOCK_SPLIT(25, 20, 5, 0.05, 0.20, 1) = 0.887

and

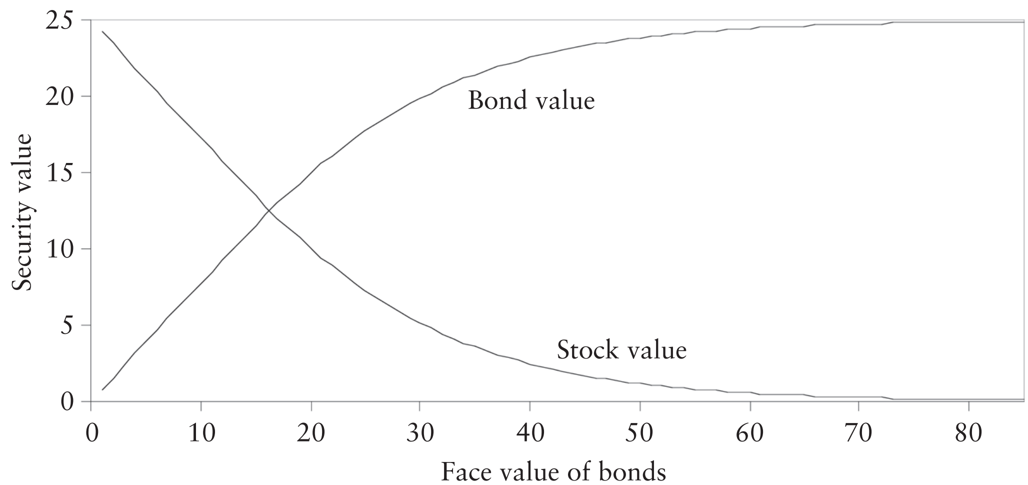
OV_CORP_RFBOND_STOCK_SPLIT(25, 20, 5, 0.05, 0.20, 2) = 0.111.

Figures 12.3A and 12.3B illustrate the dynamics of increased leverage on the values of the firm's bonds and stock as well as on the numbers of units of risk-free bonds and stock to hold in order to synthetically create a corporate bond.

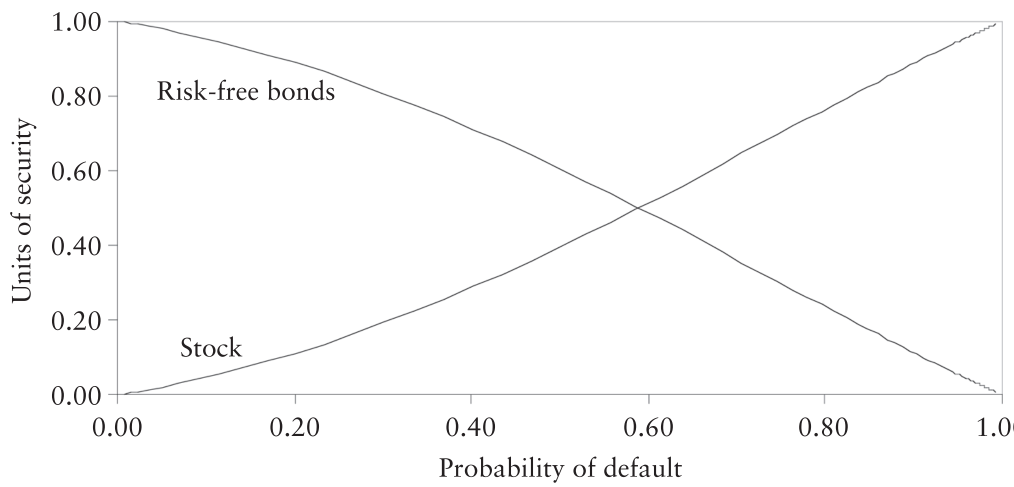
[6] Note that the weights do not sum to one. There is no reason that they should.

FIGURE 12.3 Replication of corporate bond value using risk-free bonds and common stock. Parameters: $V = 25$, $T = 5$, $r = 0.05$, and $\sigma_V = 0.20$. The face value of the corporate bonds (F) varies from 1 to 85.

Panel A: Security value as a function of face value of bonds



Panel B: Units of risk-free bonds and stock as a function of probability of default



All of the parameters are the same as in the previous illustrations except that the face value of the bonds (F) is allowed to vary from 1 to 85. In Panel A, the bond and stock values are plotted. As the face value of the bonds increases, the value of the bonds rises and the value of the stock falls. This stands to reason. As the face value of the bonds increases, the present value of the face amount (i.e., the first term in (12.8)) increases proportionately, however, the value of the put option (i.e., the second term in (12.8)) increases at an increasing rate. Even though the face value of the value of the bonds is allowed to rise beyond the value of the firm (i.e., $V = 25$), the market value of the bonds converges to the value of the firm since the bondholders' claim cannot exceed the value of the firm's assets. In Panel B, the horizontal axis is the probability of default, which can be computed using $N(-d_2)$. The vertical axis is the numbers of units of risk-free bonds and stock that are necessary to replicate the bond value. At a default probability of zero, the portfolio consists almost entirely of risk-free bonds. This stands to reason since the firm's bonds are essentially risk-free. As the default probability increases, less units of risk-free bonds are held and more money is invested in stock. At high levels of default, the number of units of stock is high, however, the value of the stock is negligible.

Estimating Bond Value Using Stock Price Information

The bond valuation formulas, (12.4), (12.8), and (12.10), are useful in developing economic intuition about the relation between bond value and the value of the firm. From a practical standpoint, however, these bond valuation formulas are not particularly useful. One reason is that we do not know the value of the firm. For most U.S. corporations, the only corporate security that is actively traded is the common stock. Corporate bonds trade largely in the over-the-counter market.[7] Because such trades are private negotiations, there is no mandated reporting of trade prices and quantities. Without knowing the price of the bonds, we cannot compute the value of the firm.[8] Another reason is that we have no means of estimating the firm's expected future volatility rate from historical data. Corporate bonds trade relatively infrequently, and the time between trades may vary from minutes apart to months apart. Indeed, it not uncommon for newly-issued corporate bonds to trade only in the first few days following issuance and never again. Thus, even if the current price of the bond can be observed, the lack of historical bond price data undermines our ability to compute historical firm prices and, hence, the firm's historical volatility rate. In sum, the bond valuation formulas, (12.4), (12.8), and (12.10), are difficult to implement because we cannot reliably identify the firm's current value, V, or its expected future volatility rate, σ_V.

As it turns out, both of these problems can be circumvented. To understand how, consider the bond valuation equation (12.4), and assume, for the moment, that we know the firm's volatility rate σ_V. Since all parameters on the right hand-side of the valuation equation (12.4) are known, except for V, we can solve for V iteratively using a numerical search procedure such as Microsoft Excel's SOLVER function. With the firm value known, we compute the bond value as $B = V - S$.

Unfortunately, we do not know σ_V, so we are left with one equation and two unknowns. One way to solve for this somewhat perplexing problem is to find a second equation that is also a function of σ_V and V, and then to solve for σ_V and V simultaneously. Since stock return volatility can be estimated using an available stock price history, a reasonable starting point is to look for some relation between stock return volatility σ_S and firm return volatility σ_V. In Illustration

[7] This assumes, of course, that the bonds were publicly issued. Many bond issues are private placements.

[8] One possibility for estimating the value of the firm is to use the value of the firm's assets. The book value of assets, however, is seldom a good proxy for the firm's market value.

12.1, we relied upon the mechanics of Chapter 10 to demonstrate that, since the stock return and the firm return are perfectly correlated, the relation between stock return volatility and firm return volatility of the firm may be expressed as $\sigma_S = \eta_S \sigma_V$, where η_S is the elasticity of the stock value with respect to the firm value. The elasticity measure, in turn, equals the stock's delta times the ratio of the firm's value to the value of the stock, that is,

$$\sigma_S = N(d_1)\frac{V}{S}\sigma_V \tag{12.12}$$

Isolating the known from the unknown parameters, we get

$$S\sigma_S = N(d_1)V\sigma_V \tag{12.13}$$

Since we can observe the value of the stock S and can estimate the stock return volatility σ_S from historical price data, we have two equations, (12.3) and (12.13), and two unknowns—the value of the firm, V, and the volatility rate of the firm, σ_V. Thus we can solve for the unknown parameters uniquely. With V identified, the value of the corporate bond may be computed as $B = V - S$.

ILLUSTRATION 12.4 Value corporate bond form stock price information.

Assume that the firm's stock pays no dividends, has a value of 8, and has a volatility rate of 50%. Also assume the firm has a single zero-coupon bond. The bond promises to be redeemed at its face value of 20 at the end of five years. Finally, the five-year, zero-coupon, risk-free interest rate is 5%. Compute the value of the bond.

To compute the value of the bond, insert the problem information into equations (12.3) and (12.13), that is,

$$8 = VN(d_1) - 20e^{-0.05(5)}N(d_2)$$

and

$$8(0.50) = 4 = N(d_1)V\sigma_V$$

where

$$d_1 = \frac{\ln(Ve^{0.05(5)}/20) + 0.5\sigma_V^2(5)}{\sigma_V\sqrt{5}} \text{ and } d_2 = d_1 - \sigma_V\sqrt{5}$$

With two equations and two unknowns, we can solve uniquely for V and σ_V using Excel's SOLVER function. The firm value is 22.503, and the firm's volatility rate is 21.02%. The bond value is $B = 22.503 - 8 = 14.503$.

The OPTVAL library contains a function that solves for the firm's value and volatility rate given information about the stock, that is,

OV_CORP_FIRM_STOCK(*stock, face, t, r, vs, vind*)

where *stock* is the value of the stock, *face* is the face value of the bonds, *t* is the bonds' term to maturity in years, and *vs* is the volatility rate of the stock. The argument *vind* is an indicator variable whose value is set equal to 1 if the function is to return the firm's value and 2 if the function is to return the firm's volatility rate. Applying the function, the value of the firm is

$$V = \text{OV\_CORP\_FIRM\_STOCK}(8, 20, 5, 0.05, 0.50, 1) = 22.503$$

and the firm's volatility rate is

$$\sigma_V = \text{OV\_CORP\_FIRM\_STOCK}(8, 20, 5, 0.05, 0.50, 2) = 21.02\%$$

Estimating Bond Value Using Stock and Stock Option Price Information

Estimating the stock's volatility rate using historical price data presents a subtle theoretical inconsistency. If we assume the firm's volatility rate is constant, the stock's volatility rate is not and will change as the firm's value moves and time passes. This means that estimating the stock return volatility from a time-series of stock price data is error-prone. An alternative means of estimating stock return volatility is to compute implied volatility based on exchange-traded option prices. We cannot use the BSM model to compute implied volatility of the stock return, however, since the BSM model assumes that stock price (not the firm's value) is log-normal.

Again, these problems can be circumvented. An exchange-traded call option written on the firm's stock is a call on a call since the firm's stock is a call on the firm's value. Similarly, an exchange-traded put option written on the firm's stock is a put on a call. In other words, we can now apply the compound option valuation mechanics from Chapter 8 to the corporate security valuation problem at hand. Specifically, we use exchange-traded option prices to infer the value of the firm and its volatility rate. First, we model the value of a call option, and then we turn to the put option value.

To value a call on the firm's stock, first recall the valuation equation that we developed for the equity of the firm (12.3), that is,

$$S = VN(d_1) - Fe^{-rT}N(d_2) \tag{12.3}$$

where

$$d_1 = \frac{\ln(Ve^{rT}/F) + 0.5\sigma_V^2 T}{\sigma_V\sqrt{T}}, \text{ and } d_2 = d_1 - \sigma_V\sqrt{T}$$

The exchange-traded call on the firm's stock is assumed to have an exercise price of X and a time to expiration of t, where $t < T$. In this corporate finance context, a European-style call on a call formula may be written

$$c = VN_2(d_1,a_1;\rho) - Fe^{-rT}N_2(d_2,a_1;\rho) - Xe^{-rt}N_1(a_2) \tag{12.14}$$

where

$$a_1 = \frac{\ln(Ve^{rt}/V^*) + 0.5\sigma_V^2 T}{\sigma\sqrt{t}}, \; a_2 = a_1 - \sigma_V\sqrt{t}, \text{ and } \rho = \sqrt{\frac{t}{T}}$$

Like in Chapter 8, we adopt the subscript "1" to indicate the univariate normal probability $N_1(d)$ and "2" to indicate the bivariate normal probability, $N_2(a,b;\rho)$.

The first step in valuing the call using (12.14) is to determine the critical value of the firm at time t above which the call will be exercised. It can be determined by iteratively searching for the firm value V^* that makes the value of the stock (i.e., the underlying call) equal to the exercise price of the call, that is,

$$V^* N_1(d_1^*) - Fe^{-r(T-t)} N_1(d_2^*) = X \tag{12.15}$$

With V^* known, the call option value can be computed using (12.14).

Notice the similarity between the structure of (12.14) and the structure of (12.3). The first two terms of the right-hand side of (12.14) correspond to the formula (12.3). Upon exercising the call, we receive the stock, which can be valued using the BSM formula. The last term on the right-side is the present value of the call's exercise price, $e^{-rt}X$, times the risk-neutral probability that the firm value will exceed the critical firm value at time t, $N_1(a_2)$. This is the expected cost of exercising the compound call conditional upon it being in the money at time t. The term, $N_2(d_2,a_2;\rho)$, is the risk-neutral compound probability that the asset price exceeds V_t^* at time t and exceeds the face value of the bonds F at time T. In other words, the firm value must jump both hurdles for the stock to be in the money at time T. The sign of the correlation coefficient reflects whether the firm value should move in the same or opposite direction in the interval between time 0 and time t as in the interval between time t and time T in order for the stock to be in-the-money at time T. For a call on a call, the sign is positive because the firm value must increase in both intervals. For a put on a call, the sign is negative because the firm value must be low enough for the compound option to be exercised at time t and yet high enough to exceed the face value of the bonds at time T.

A European-style put option on the shares of the firm has a similar valuation equation. The value of the put is

$$p = Fe^{-rT} N_2(d_2,-a_1;-\rho) - VN_2(d_1,-a_1;-\rho) + Xe^{-rt} N_1(-a_2) \tag{12.16}$$

where all notation is defined above. In (12.16), $N_1(-a_2)$ is the risk-neutral probability that the asset price is below the critical firm value at time t, V^*. In this region, the compound option is exercised. The stock (i.e., underlying call) value, however, increases with the value of the firm. The correlation in the compound probability that is negative and the term, $N_2(d_2,-a_2;-\rho)$, is the risk-neutral compound probability that the firm value is below V^* at time t and exceeds the face value of the bonds F at time T.

ILLUSTRATION 12.5 Value corporate bond using stock and stock option price information.

Assume that the firm's stock pays no dividends, has a price of 8, and has a volatility rate of 50%. Assume also there exists a six-month call option written on the stock, with an exercise price of 10 and a market price of 0.5312. The firm has a single issue of zero-coupon bonds outstanding. The face value of the bonds is 20, and their term to maturity is five years. Finally, the five-year zero-coupon interest rate on risk-free debt is 5%. Compute the value of the bonds.

To compute the value of the bonds, we need to identify the value of the firm and the firm's volatility rate. To do so, we use information regarding the stock's price and its volatility rate with valuation equation (10.3) and the call's price with valuation equation (10.14). Our two equations are

$$8 = VN(d_1) - 20e^{-0.05(5)}N(d_2)$$

where

$$d_1 = \frac{\ln(Ve^{0.05(5)}/20) + 0.5\sigma_V^2 5}{\sigma_V\sqrt{5}} \text{ and } d_2 = d_1 - \sigma_V\sqrt{5}$$

and

$$0.5312 = VN_2(d_1, a_1; \rho) - 20e^{-0.05(5)}N_2(d_2, a_1; \rho) - 10e^{-0.05(5)}N_1(a_2)$$

where

$$a_1 = \frac{\ln(Ve^{0.05(5)}/V^*) + 0.5\sigma_V^2(0.5)}{\sigma\sqrt{0.5}},\ a_2 = a_1 - \sigma_V\sqrt{0.5}\text{, and } \rho = \sqrt{\frac{0.5}{5}}$$

Rather than compute these formula values by hand, we will use the OPTVAL functions

OV_CORP_STOCK_FIRM(*firm, face, t, r, vf, vind*)

which solves for the value of the firm's stock, and

OV_CORP_OPTION_FIRM(*firm, face, t, r, vf, cp, x, topt*)

which solves for the value of an option written on the firm's stock. The parameters of the functions are as before: *firm* is the value of the firm, *face* is the face value of the firm's zero-coupon bonds, *t* is the term to maturity of the bond's in years, *r* is the risk-free interest rate, and *vf* is the volatility rate of the firm. The term, *vind*, is an indicator variable whose value is set equal to 1 if the function is to return the stock's value and 2 if the function is to return the volatility rate. The additional parameters of the option valuation function are as follows: *cp* is a call/put indicator variable ("c" or "p"), *x* is option's exercise price, and *topt* is the time to expiration of the option. Using the problem information, the appropriate function calls are

8 = OV_CORP_STOCK_FIRM(*firm*, 20, 5, 0.05, *vf*,1)

and

0.5312 = OV_CORP_OPTION_FIRM(*firm*, 20, 5, 0.05, *vf*, "c",10, 0.5)

The Excel function SOLVER can be used to identify the values of *firm* and *vf* that allow the above expressions to hold exactly. The solution values for the value of the firm and its volatility rate are 22.503 and 21.02%. The value of the bonds is therefore 22.503 – 8.00 or 14.503.

Computing Expected Returns on Corporate Securities

The mechanics used to value corporate securities can also be used to determine the relation between the expected rate of return and risk of different corporate securities. From Chapter 10, we know that if a risk-free hedge can be formed between each of the securities of the firm and the underlying value of the firm, the expected rate of return of security *i* may be expressed as

$$E_i = r + (E_V - r)\eta_i \tag{12.17}$$

where E_i is the expected rate of return of security *i*, E_V is the expected rate of return of the firm, *r* is the risk-free rate of interest, and η_i is the security *i*'s eta (i.e., its price elasticity with respect to the firm value). As noted earlier in the chapter, the eta may be expressed as a function of delta, that is,

$$E_i = r + (E_V - r)\Delta_i\left(\frac{V}{V_i}\right) \tag{12.18}$$

where V_i is the value of security *i*, and

$$\sum_{i=1}^{n} V_i = V$$

where *n* is the number of securities of the firm. In the above illustrations, $n = 2$ since the firm has only two types of corporate securities—a zero-coupon bond and common stock.

ILLUSTRATION 12.6 Compute expected returns for corporate bond and stock.

Assume that the firm has a current value of 25, an expected rate of return of 12%, and a volatility rate of 20%. It has two securities outstanding—a zero-coupon bond and common stock. The bond matures in five years and has a face value of 20. The stock pays no dividends, and the risk-free rate of interest is 5%. Compute the expected rates of return and the volatility rates of the firm's bond and stock, and plot them in a figure showing expected return on the vertical axis and return volatility on the horizontal axis.

The volatility rates of the bond and the stock may be computed using the OPTVAL functions

σ_B = OV_CORP_BOND_FIRM(25, 20, 5, 0.05, 0.20, 2) = 3.35%

and

σ_S = OV_CORP_STOCK_FIRM(25, 20, 5, 0.05, 0.20, 2) = 44.68%

which were introduced earlier in the chapter. The OPTVAL Function Library also contains functions for computing the deltas and etas of the bond and the stock. The syntax of the functions are

OV_CORP_BONDDELTA_FIRM(*firm, face, t, r, vf, gind*)

and

OV_CORP_STOCKDELTA_FIRM(*firm, face, t, r, vf, gind*),

where *firm* is the value of the firm, *face* is the face value of the firm's zero-coupon bonds, *t* is the term to maturity of the bond's in years, *r* is the risk-free interest rate, and *vf* is the volatility rate of the firm. The term, *gind*, is an indicator variable whose value is set equal to "d" or "D" if the function is to return the delta and "e" or "E" if the function is to return the volatility rate. For the illustration at hand,

$$\eta_B = \text{OV\_CORP\_BONDDELTA\_FIRM}(25, 20, 5, 0.05, 0.20, \text{"e"}) = 16.74\%$$

and

$$\eta_S = \text{OV\_CORP\_STOCKDELTA\_FIRM}(25, 20, 5, 0.05, 0.20, \text{"e"}) = 223.41\%$$

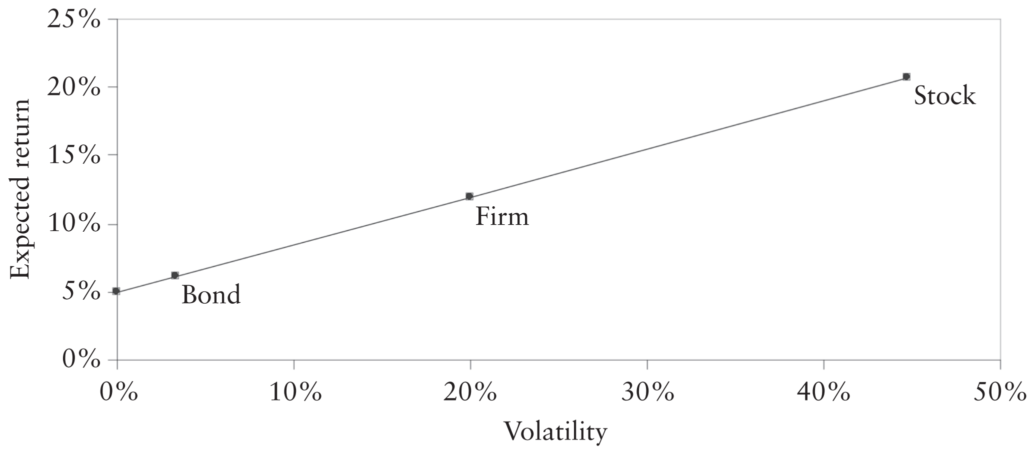
The expected returns for the bond and the stock are, therefore,

$$E_B = 0.05 + (0.12 - 0.05)0.1674 = 6.17\%$$

and

$$E_S = 0.05 + (0.12 - 0.05)2.2341 = 44.68\%$$

For the problem parameters, the expected return/risk attributes of the firm's securities fall on a straight line emanating from the risk-free interest rate:



VALUING SUBORDINATED DEBT

Subordinated debt refers to bonds of different seniority. Interest payments to bondholders follow a pecking order. The most senior bondholders are paid first, followed by the next most senior, and so on. In the event of bankruptcy, some tranches may be paid while others may not.

To value subordinated bond issues, we again begin with a framework in which we know the firm's value and volatility rate. Three bond issues are considered. They are labeled 1 through 3 in descending order of seniority. Their market values are denoted B_1, B_2, and B_3, and their respective face values are F_1, F_2, and F_3. All bonds are zero-coupon bonds and mature at time T. The notation $c(V,F)$ denotes the BSM call option formula value where the underlying asset price is V and the option has an exercise price of F. To value each of the bond issues, we apply the valuation-by-replication technique.

The value of the most senior bond issue can be obtained using (12.4), that is,

$$B_1 = V - c(V,F_1) \tag{12.19}$$

In spirit, the most senior bondholders hold a portfolio in which they are long the value of the firm and short a call option whose exercise price equals the face value of the bonds. In other words, the senior bondholders own the firm but are not entitled to any firm value that goes beyond the face value of their bonds. The residual value goes to the remaining stakeholders of the firm.

Now, consider the bondholders with intermediate seniority. Again, bond valuation equation (12.4) applies in spirit. The intermediate bondholders are the firm value net of the value of the senior bonds, $V - B_1$, and are short a call option whose exercise price equals the sum of the face values of the senior and intermediate claims, $c(V, F_1 + F_2)$. The value of the intermediate bonds is therefore

$$B_2 = V - B_1 - c(V, F_1 + F_2) \tag{12.20}$$

The call option value in (12.20) represents the aggregate value of the remaining stakeholders of the firm—junior bondholders as well as the stockholders.

Finally, consider the most junior bondholders. The junior bondholders are long the firm value net of the senior and intermediate bondholder values, $V - B_1 - B_2$, and are short a call option whose exercise price equals the sum of the face amounts of all bond issues, $F_1 + F_2 + F_3$. This call is the value of the shareholders' claim since they receive the value of the firm's assets at the bonds' maturity net of the bondholder claims. The value of the junior bonds may be written

$$B_3 = V - B_1 - B_2 - c(V, F_1 + F_2 + F_3) \tag{12.21}$$

ILLUSTRATION 12.7 Value subordinated bonds.

Assume that the firm's value is 90, its expected return is 12%, and its volatility rate is 30%. Assume also that there are three issues of five-year, zero-coupon bonds outstanding. In decreasing order of seniority, they have face values of 50, 30, and 20. The risk-free interest rate is 4%. Compute the value of each bond issue, its expected rate of return, and its volatility rate.

To value the bond issues, we begin with the most senior bonds and apply the valuation-by-replication principle. Holding the senior bonds is like being long the firm and short a call option with an exercise price of 50 and a time to expiration of five years. Using the BSM call option valuation formula, the call value is

OV_CORP_STOCK_FIRM(90, 50, 5, 0.04, 0.30, 1) = 51.382

Applying (12.19), the value of the most senior bond issue is 90 – 51.382 = 38.618. As noted earlier in the chapter, the expected rate of return of security i, is

$$E_i = r + (E_V - r)\Delta_i\left(\frac{V}{V_i}\right)$$

where V_i is the value of security i, and

$$\sum_{i=1}^{n} V_i = V$$

where n is the number of securities of the firm. In this illustration, $n = 4$ since there are three bonds issues plus the common stock. The delta value for the most senior bonds equals one less the delta of a call on the firm with an exercise price of 50. The delta of the call may be computed using

OV_CORP_STOCKDELTA_FIRM(90, 50, 5, 0.04, 0.30, "D") = 0.9344

The delta of the senior bonds is, therefore, 1 – 0.9344 = 0.0656, and the expected rate of return of the senior bonds is

$$E_{B_1} = r + (E_V - r)\Delta_{B_1}\left(\frac{V}{B_1}\right)$$

$$= 0.04 + (0.12 - 0.04)(0.0656)\left(\frac{90}{36.618}\right) = 5.222\%$$

Finally, since the senior bond's eta equals

$$\eta_{B_1} = 0.0656\left(\frac{90}{36.618}\right) = 15.277\%$$

the volatility rate of the senior bond is $\sigma_{B_1} = \eta_{B_1}\sigma_V = 0.15277(0.30) = 4.583\%$.

To value the second most senior tranche, we apply the valuation-by-replication technique yet once again. Holding the intermediate bond is like being long the residual value of the firm after the most senior bondholders have been paid and being short a call option with an exercise price of 80 (i.e., the sum of the face values of the senior and intermediate bonds). Again, the BSM call option valuation formula can be applied. The call option value is

OV_CORP_STOCK_FIRM(90, 80, 5, 0.04, 0.30, 1) = 34.818

Thus the value of the intermediate bonds is 51.382 – 34.818 or 16.563. The combined delta of the senior and intermediate bonds equals one less the delta of a call on the firm with an exercise price of 80. The delta of the call may be computed using

OV_CORP_STOCKDELTA_FIRM(90, 80, 5, 0.04, 0.30, "D") = 0.7908

The delta of the intermediate bonds is therefore 1 – 0.7908 – 0.0636 = 0.1437. Consequently, its expected rate of return is

$$E_{B_2} = r + (E_V - r)\Delta_{B_2}\left(\frac{V}{B_2}\right)$$

$$= 0.04 + (0.12 - 0.04)(0.1437)\left(\frac{90}{16.563}\right) = 10.245\%$$

Finally, since the intermediate bond's eta equals

$$\eta_{B_1} = 0.0656\left(\frac{90}{36.618}\right) = 15.277\%$$

the volatility rate of the bond is $\sigma_{B_2} = 0.78065(0.30) = 23.419\%$.

Finally, holding the most junior bonds is like being long the residual value of the firm after the senior and intermediate bondholders have been paid and being short a call option with an exercise price of 100 and a time to expiration of 100. The BSM value of the call is

OV_CORP_STOCK_FIRM(90, 100, 5, 0.04, 0.30, 1) = 26.853

With only the junior bonds included with the stock, the value of the stock is 26.853. The value of the junior bonds, therefore, is the residual value less the stock value, 34.818 – 26.853, or 7.965. Similarly, the stock's delta is

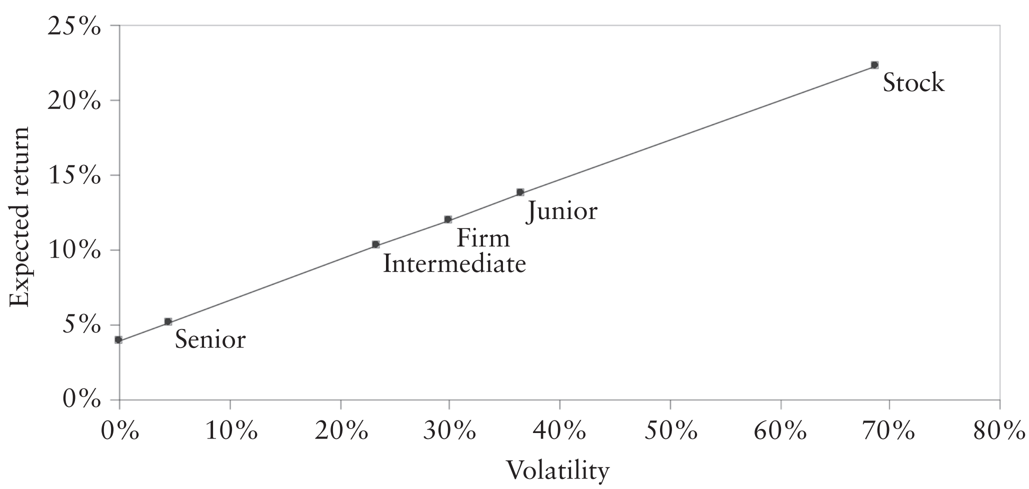
OV_CORP_STOCKDELTA_FIRM(90, 100, 5, 0.04, 0.30, "D") = 0.6831

so the junior bond's delta must be 1 – 0.0656 – 0.1437 – 0.6831 = 0.1076. The junior bond's eta is 121.631%, its expected return is 13.371%, and its volatility rate is 36.489%.

As noted above, holding the stock of a firm is like holding a call option on the firm's value with the exercise price being equal to the sum of the face values of all bond issues. In this illustration, the stock value is 26.853, its delta is 0.6831, and its eta is 228.955%. Its expected rate of return is 22.316%, and its volatility rate is 68.687%. The table below summarizes all of the computations:

Bond Issues	Face Value	Firm Value Before Claim	Call Value	Bond Value	Market Weight	Delta	Eta	Expected Return	Volatility Rate
Senior	50	90.000	51.382	38.618	0.4291	0.0656	15.277%	5.222%	4.583%
Intermediate	30	51.382	34.818	16.563	0.1840	0.1437	78.065%	10.245%	23.419%
Junior	20	34.818	26.853	7.965	0.0885	0.1076	121.631%	13.731%	36.489%
Total bonds				63.147	0.7016	0.3169			
Stock		26.853		26.853	0.2984	0.6831	228.955%	22.316%	68.687%
Total				90.000	1.000	1.000			
Market value weighted average								12.000%	30.000%

Note that a market-value, weighted averages of the expected rates of return and volatility rates of the individual corporate securities equals the assumed expected rate of return of the firm, 12%, and the assumed volatility rate of the firm, 30%. The following figure summarizes the expected return/risk relation for the bonds and the stock in this illustration.



As was the case for the corporate bond discussed in the first section, the values of the securities other than the common stock are usually unknown. Consequently, the value of the firm, V, is unknown. In addition, since the price histories of the bonds are generally not available, it is impossible to develop a historical estimate the volatility rate of the firm, σ_V. Assuming the value of the stock and its volatility are known, we can solve for V and σ_V based on S and σ_S using the same iterative procedure as we used earlier. Assuming the value of the stock is observed in the marketplace is 26.583 and the stock's historical volatility rate is (coincidently)

$$0.6831\left(\frac{90}{26.583}\right)(0.30) = 68.69\%$$

(i.e., the assumed parameters in Illustration 12.7), the value of the firm is

OV_CORP_FIRM_STOCK(26.583, 100, 5, 0.04, 0.6869, 1) = 90

and the firm's volatility rate is

OV_CORP_FIRM_STOCK(26.583, 100, 5, 0.04, 0.6869, 2) = 0.30

Note that the exercise price in the above computations is the sum of the face values of all three bond issues in Illustration 12.7 (i.e., we treat all three bond issues as if they were one issue). Thus, we can perform the valuation of all three bond issues in Illustration 12.7 based on only on the knowledge of the current stock value and its volatility rate.

VALUING WARRANTS

Rights and *warrants* are option-like securities issued by the firm. Usually they are attached to bond or preferred stock offerings by the firm in order to entice the buyers of the securities to accept lower coupon interest or dividend payments. Rights tend to be short-term and at-the-money when they are issued, and warrants tend to be long-term and out-of-the-money. Since there is little distinction between rights and warrants from a valuation standpoint, only the term, "warrants" is used in the remainder of this section.

Like call options, warrants provide holders with the right to buy the underlying stock at a predetermined price within a specified period of time. Unlike call options, however, warrants are issued by the firm. Since the exercise of the warrants creates more shareholders and the firm has a fixed amount of assets, exercising an in-the-money warrant dilutes the value of existing shares. This section focuses on warrant valuation in a manner that explicitly considers the effects of dilution induced by the prospect of warrant exercise.[9]

[9] The approach used here is based on Smith (1976).

To understand the effects of dilution on warrant (and stock) valuation, the BSM option valuation framework developed in Chapter 7 is again applied. The notation is as follows. The aggregate market value of the shares of the common stock currently outstanding is denoted S, and n_S is the number of shares outstanding. The current share price is therefore S/n_S. Similarly, W is the aggregate market value of the warrants currently outstanding, n_W is the number of shares of stock sold to warrant holders if the warrants are exercised (for simplicity, one warrant is assumed to provide the right to buy one share), and W/n_W is the current warrant price per share. The firm is assumed to have only two sources of financing, stock and warrants, so the aggregate market value of the firm is $V = S + B$. The total market value of the firm is assumed to be log-normally distributed at the warrants' expiration. The rate of return of the firm, $\ln(\tilde{V}_T/V)$, is therefore normally distributed and its variance is denoted σ_V^2. The stock is assumed to pay no dividends during the warrant's life. Finally, the warrant contract parameters are T, the time to expiration of warrants, and X, the aggregate exercise price of the warrants. The exercise price per share of stock is X/n_W. The proportion of the firm owned by warrant holders if they exercise their warrants is called the *dilution factor* and is denoted

$$\gamma = \frac{n_W}{n_S + n_W}$$

As usual, r is the risk-free rate of interest.

To understand how to value warrants, first consider their value at expiration. At time T, the value of the firm's existing assets is $\tilde{V}_T$. If the warrants are in-the-money, the warrant holders will exercise, paying the firm X is cash and driving the firm value to $\tilde{V}_T + X$. In return for paying X, the warrant holders receive proportion γ of the value of the overall firm, that is, $\gamma(\tilde{V}_T + X)$. Thus the terminal value of the warrants may be written

$$\tilde{W}_T = \max[\gamma(\tilde{V}_T + X) - X, 0] \tag{12.22}$$

Separating known from unknown values in (12.22),

$$\tilde{W}_T = \max[\gamma\tilde{V}_T - (1-\gamma)X, 0] \tag{12.23}$$

Note the structure of the warrant value at expiration (12.23) is similar to the terminal value of a European-style call. The underlying asset price at expiration is $\gamma\tilde{V}_T$, which is log-normally distributed by assumption, and the exercise price is $(1 - \gamma)X$. It follows, therefore, that the current value of the warrants is

$$W = \gamma V N(d_1) - e^{-rT}(1-\gamma)XN(d_2) \tag{12.24}$$

where

$$d_1 = \frac{\ln\left[\frac{\gamma V e^{rT}}{(1-\gamma)X}\right] + 0.5\sigma_V^2 T}{\sigma\sqrt{T}} \text{ and } d_2 = d_1 - \sigma\sqrt{T}$$

The *market value per warrant* is simply W from (12.24) divided by n_W.

Like in the case of corporate bond valuation, the warrant valuation equation (12.24) seems somewhat circular in the sense that the warrant value, W, appears on both sides of the equation—directly on the left-hand side of (12.24) and indirectly through V (i.e., W is embedded in V) on the right-hand side. This does not undermine the use of the formula, however. With all of the other valuation parameters known, we can find the value of W that satisfies the equation using a numerical search procedure such as SOLVER in Excel. A pre-programmed function for valuing warrants, OV_CORP_WARRANT_STOCK, is also provided in the OPTVAL Function Library.

ILLUSTRATION 12.8 Value warrant.

Suppose that you have been hired by an internet firm to determine the worth of warrants written on the firm. In an initial public offering a few months ago, the firm sold 5 million shares of stock in the marketplace, while giving the employees of the firm 7 million shares with seven-year European-style warrants with an exercise price of $35 per share on 7 million additional shares.[10] Compute the value of each warrant assuming the current stock price is $40 per share, and the stock pays no dividends. The stock and the warrants are the firm's only two sources of financing. One warrant entitles its holder to one share of common stock. Assume that the risk-free rate of interest is 4%, and that the standard deviation of the rate of return of the firm is 30%. In the interest of completeness, compute the expected returns and volatility rates of the firm's stock and warrants assuming that the firm's expected return is 12%.

In the event the warrants are exercised, the dilution factor is

$$\gamma = \frac{7{,}000{,}000}{12{,}000{,}000 + 7{,}000{,}000} = 36.84\%$$

That is, the warrant holders receive 36.84% of the firm value. The remaining shareholders are the investment public, with 26.32%, and the employees, with 36.84%. Prior to the exercise of the warrants, the investment public held 41.67% and the employees held 58.33%. The aggregate exercise proceeds from the exercise of the warrants are

$$X = 7{,}000{,}000 \times 35 = 245{,}000{,}000$$

and the current market value of the firm is

$$V = 12{,}000{,}000 \times 40 + W = 480{,}000{,}000 + W$$

The aggregate market value of the warrants is computed by solving

[10] Warrants are sometimes issued to provide incentives. For example, warrants may be issued to employees as an incentive to work hard. In doing so, they gather a greater share of the firm if it becomes successful.

$$W = 0.3684VN(d_1) - e^{-0.04(7)}0.6316(245{,}000{,}000)N(d_2)$$

where

$$d_1 = \frac{\ln\left[\dfrac{0.3684Ve^{0.04(7)}}{0.6316(245{,}000{,}000)}\right] + 0.5(0.30^2)(7)}{0.30\sqrt{7}} \text{ and } d_2 = d_1 - 0.30\sqrt{7}$$

The solution to this problem can be obtained using SOLVER in Excel. The aggregate warrant value is \$118,066,271 or \$16.867 per share. The intermediate computations for the final solution value are: $d_1 = 1.1949$, $d_2 = 0.4012$, $N(1.1949) = 0.8839$, and $N(0.4012) = 0.6559$. Alternatively, the warrant value can be computed using the OPTVAL function

OV_CORP_WARRANT_STOCK(*s, ns, nw, x, t, r, v, vfs, vind*)

where *s* is the current stock price, *ns* is the number of shares outstanding, *nw* is the number of warrants outstanding, *x* is the exercise price per share, *t* is the warrant's time to expiration in years, r is the risk-free interest rate, *v* is the volatility rate, *vfs* is either "V" or "S," depending upon whether the volatility rate is the volatility rate of the firm or of the stock, respectively, and *vind* is an indicator variable whose value is set equal to 1 if the function is to return the firm's value and 2 if the function is to return the firm's volatility rate. For the problem information at hand, the warrant value per share is

OV_CORP_WARRANT_STOCK(40, 12000000, 7000000, 35, 7, 0.04, 0.30, "V", 1)
= 16.867

and its volatility rate is

OV_CORP_WARRANT_STOCK(40, 12000000, 7000000, 35, 7, 0.04, 0.30, "V", 2)
= 0.4949

In order to compute the warrant's expected return, we need its eta. The warrant's delta is

OV_CORP_WARRANTDELTA_STOCK(40, 12000000, 7000000, 35, 7, 0.04, 0.30, "V", 1)
= 0.326

which means its eta is

$$0.326\left(\frac{598{,}066{,}271}{118{,}066{,}271}\right) = 164.97\%$$

and its expected return is

$$E_W = 0.04 + (0.12 - 0.04)1.6497 = 17.20\%$$

Since the warrant's delta is 0.326, the stock's delta is 0.674. The stock's eta is therefore,

$$0.674\left(\frac{598{,}066{,}271}{480{,}000{,}000}\right) = 0.8402$$

its expected return is

$$E_S = 0.04 + (0.12 - 0.04)0.8402 = 10.72\%$$

and its volatility rate is

$$\sigma_S = 0.8402(0.30) = 25.21\%$$

The expected return/volatility rate for each security is plotted in the figure below. The figure is unusual to the extent that the stock's return/risk parameters are below that of the firm. This arises because the firm has no securities more junior than stock.

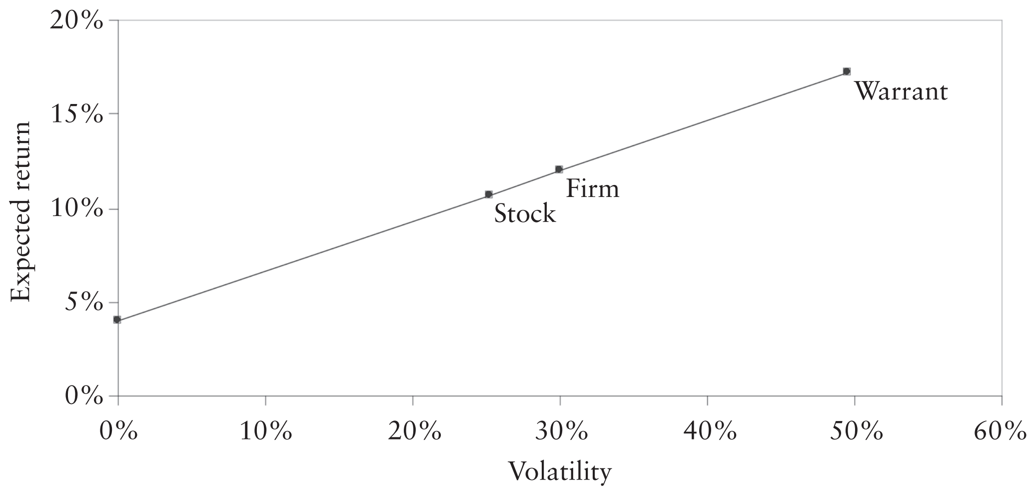


Illustration 12.8 assumes we can estimate the volatility rate of the firm, σ_V. Like corporate bonds, warrants are not actively traded. Without historical price series for both the stock and the warrant, we cannot obtain a history of firm values from which to estimate the historical rate of return volatility, σ_V. Fortunately, if we know the current value of the stock and the stock's volatility rate, we can solve for V and σ_V based on S and σ_S using the same iterative procedure as we used earlier. In Illustration 12.8, we know the current value of the stock is \$40 per share. Suppose we collect a history of stock prices and find that $\sigma_S = 25.21\%$. From discussions earlier in the chapter, we know that the return volatility of the stock may be expressed as a function of the return volatility of the firm, that is,

$$\sigma_S = N(d_1)\left(\frac{V}{S}\right)\sigma_V \tag{12.25}$$

Equation (12.25), together with (12.24), can be used to solve for V and σ_V uniquely. The OPTVAL function that performs this computation is

OV_CORP_WARRANT_STOCK(40, 12000000, 7000000, 35, 7, 0.04, 0.2521, "S", 1) = 16.867

Again, like in the case of corporate bonds, we can value warrants in a manner so as to incorporate the effects of dilution based on only on the knowledge of the current stock value and its volatility rate.

Before proceeding with the valuation of convertible bonds, it is worthwhile to assess the approximate magnitude of the effects of dilution. As noted earlier, it is not uncommon for individuals to value warrants using the BSM call option valuation formula with no adjustment for the effects of dilution. This practice overstates the warrant. The degree of bias is related to a number of the war-

TABLE 12.1 Assessing the effects of potential dilution on warrant valuation for at-the-money warrants.

Market/Warrant Parameters		Dilution Factor	Warrant Value	Call Option Value	Percent Difference
Stock					
Price	40.00	5.00%	23.028	23.065	0.161%
No. of shares	10,000,000	10.00%	22.987	23.065	0.338%
Volatility type	*S*	15.00%	22.943	23.065	0.533%
Volatility rate	40.00%	20.00%	22.894	23.065	0.748%
		25.00%	22.840	23.065	0.986%
Warrant		30.00%	22.780	23.065	1.251%
Exercise price	40.00	35.00%	22.713	23.065	1.548%
Years to expiration	10	40.00%	22.639	23.065	1.882%
		45.00%	22.555	23.065	2.263%
Market		50.00%	22.461	23.065	2.691%
Interest rate (r)	4.00%	55.00%	22.352	23.065	3.188%
		60.00%	22.227	23.065	3.768%
		65.00%	22.082	23.065	4.454%
		70.00%	21.908	23.065	5.281%
		75.00%	21.697	23.065	6.304%

rant's underlying parameters, particularly the dilution factor. For Illustration 12.8, the BSM value is

OV_OPTION_VALUE(40, 35, 7, 0.04, 0.00, 0.2521, "c", "e") = 16.983

which means that using the BSM formula overstates value by 11.6 cents. Table 12.1 shows the effects of the dilution factor on warrant valuation in more detail. For the assumed warrant valuation parameters, the degree of bias is only 0.161% when the dilution factor is 5%, however, the degree of bias is more than 2.5% when the dilution factor is 50%.

VALUING CONVERTIBLE BONDS

A *convertible bond* is a hybrid security with bond-like and option-like features. Like a corporate bond, it promises to make periodic coupon interest payments throughout its life and then to repay the principal at some fixed maturity date. Also, like a corporate bond, there is a risk of default if the firm fails to make an interest payment or repay the principal at maturity. Aside from the bond features, however, a convertible bond allows its holder to exchange the bond for shares of the firm's stock. On first appearance, it may seem to be the case that a convertible bond may be valued by replication as the sum of the values of a corporate bond and a warrant, both of which we have already valued. Unfortunately, this is not the case because the bond must be forfeit in order to receive the shares. In this sense, it is like an exchange option of one risky asset for another.[11]

To value convertible bonds, we use the same approach that we used for corporate bonds and warrants. We assume that the firm has two sources of financing—common stock and convertible bonds. The market value of all shares outstanding is S, and the market value of the convertible bonds is CV. Thus, the value of the firm is $V = S + CV$. For convenience, the convertible bonds are assumed to be discount bonds. The market value of the overall firm is assumed to be log-normally distributed when the bond's mature. The number of shares outstanding is n_S, the number of shares underlying the convertible bonds n_{CV}, and F is the face value of the bonds. The firm's volatility rate is σ_V, and r is the risk-free rate of interest.

The bondholder's decision to convert at the bond's expiration depends on whether the per share market value of the stock exceeds the implicit exercise price of the embedded option, that is,

$$\frac{\tilde{V}_T}{n_S + n_{CV}} - \frac{F}{n_{CV}} > 0 \qquad (12.26)$$

The first term on the left hand-side is the per share value of the stock if the convertibility option is exercised. The second term on the left-hand side is the exercise price per share being paid by the bond holder if he chooses to exercise. This is not a cash exercise price; the bond holder merely forfeits the face value (principal repayment) of the bond in return for shares of higher value.

To express things at an aggregate level, multiply the left-hand side of (12.26) by n_{CV} and substitute the dilution factor,

$$\gamma = \frac{n_{CV}}{n_S + n_{CV}}$$

Like in the case of warrants, the exercise of in-the-money convertible bonds dilutes the value of the existing shareholders' equity. The convertibility option has a terminal value of $\max(\gamma V_T - X, 0)$ at time T, that is, the convertible bondholder will elect to exchange his bond for stock if it is profitable to do so. Applying the BSM call option valuation formula, we find that the value of the convertibility option is

$$c(\gamma V, F) = \gamma V N(d_1) - Fe^{-rT}N(d_2) \qquad (12.27)$$

where

[11] Recall that exchange options were valued in Chapter 8. That framework is not directly applicable here since there is only one source of uncertainty (i.e., the firm's value) and dilution must be considered.

$$d_1 = \frac{\ln[\gamma V e^{rT}/F] + 0.5\sigma_V^2 T}{\sigma_V\sqrt{T}} \text{ and } d_2 = d_1 - \sigma_V\sqrt{T}$$

Applying the corporate bond valuation equation (12.8), the current value of the convertible bonds is therefore

$$CV = Fe^{-rT} - p(V,F) + c(\gamma V,F) \tag{12.28}$$

where $p(V,F)$ is the BSM put option value for a put written on V with exercise price F. In other words, the value of the convertible bonds equals the value of risk-free bond with the same face amount and maturity date as the convertible bond less the value of the put option providing the firm with the right to put the assets of the firm to the bondholders in the event of default plus the value of the option to convert the bond into shares if it profitable to do so.

ILLUSTRATION 12.9 Value convertible bonds.

Assume that the firm's value is 12,000 and its volatility rate is 30%. Assume also that the stock pays no dividends, and there are currently 400 shares outstanding. The firm's convertible bond has a face value of 4,000, has five years to maturity, and may be exchanged into 100 shares of stock. The risk-free rate of interest is 4%. Compute the convertible bond value, the share price, and the aggregate value of stocks and bonds outstanding. Also, compute and plot the expected rates of return and volatility rates of all securities.

First, compute the value of the risk-free bonds, that is,

$$Fe^{-rT} = 4{,}000e^{-0.04(5)} = 3{,}274.92$$

Next compute the value of the firm's put option to default. Since the face value of the bonds is 4,000 and the value of the firm is 12,000, the bonds are close to risk-free and the put option is worth only 40.52, and the value of the corporate bond without the convertibility feature is 3,274.92 – 40.52 = 3,234.41. This computation can be performed using the OPTVAL function

OV_CORP_BOND_FIRM(12000, 4000, 5, 0.04, 0.30, 1) = 3,234.41

Finally, we compute the value of the option to convert. The dilution factor is

$$\gamma = \frac{100}{400 + 100} = 20\%$$

The aggregate value of the call option to convert is 382.85; therefore the current value of the convertible bond is

$$3{,}274.92 - 40.52 + 382.85 = 3{,}617.26$$

A convertible bond valuation function is included in the OPTVAL library. The function is

OV_CORP_CVBOND_FIRM(*firm, ns, ncb, face, t, r, vf, vind*)

where *firm* is the firm value, *ns* is the number of shares of stock outstanding, *ncb* is the number of shares underlying the convertible bonds, *face* is the face value of the convertible bonds, *t* is the term to maturity of the bonds in years, *r* is the risk-free interest rate, and *vf* is the volatility rate of the firm. The term *vind* is an indicator variable. A value of 1 returns the convertible bond value and 2 returns the convertible bond volatility rate. For the problem at hand,

OV_CORP_CVBOND_FIRM(12000, 400, 100, 4000, 5, 0.04, 0.30, 1) = 3,617.26

With the convertible bond value being 3,617.26, the value of the firm's common stock is

$$S = V - B = 12{,}000 - 3{,}617.26 = 8{,}382.74$$

and the firm's share price is

$$\frac{8{,}382.74}{400} = 20.96$$

The expected rate of return and volatility rate for each of the firm's securities can be computed in the same manner as previous illustrations. For convenience, the expected return and volatility rate of a straight bond with the same face value and maturity as the convertible bond are also computed. The results are as follows:

Security	Expected Return	Volatility
Risk-free	4.00%	0.00%
Bond	4.34%	1.29%
Stock	10.18%	23.16%
Firm	12.00%	30.00%
Convertible bond	16.23%	45.85%

The figure below shows that all securities fall on a line emanating from the risk-free rate of interest. The straight bond is nearly risk-free and lies at the extreme left of the figure. The convertible bond, on the other hand, is more risky than the stock given its embedded option. For the parameters of this problem, the expected return/risk characteristics of the stock are below those of the firm.

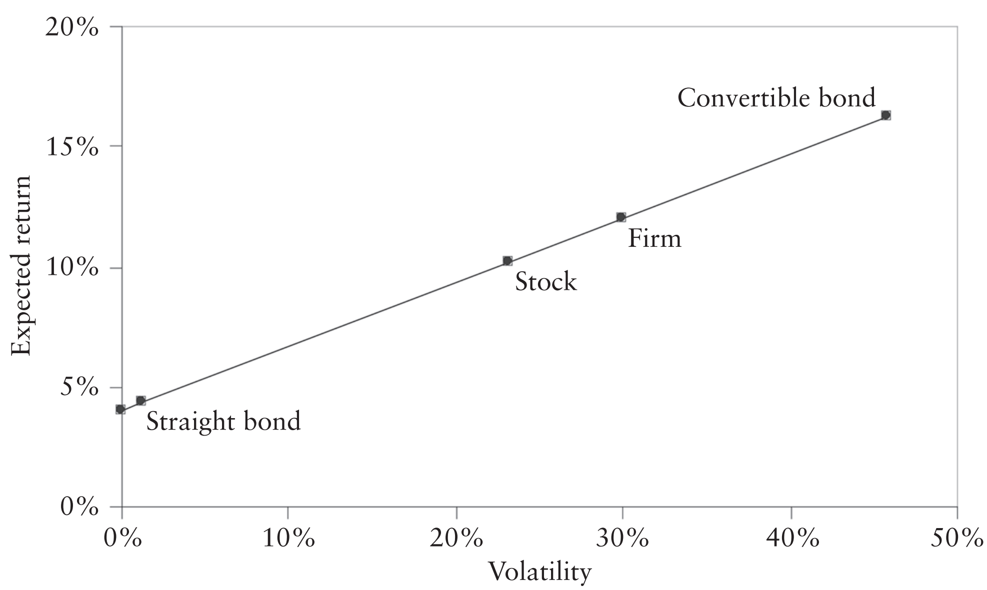


Illustration 12.9 is intended only to illustrate the convertible bond valuation mechanics. Like in the previous valuation problems of this chapter, it is generally the case that the value of the firm, V, and the volatility rate of the firm, σ_V, are not known. But again we can circumvent the problem by using the value of the stock, S, and the volatility rate of the stock, σ_S. To understand how to do this, note that equation (12.28) is incomplete. If we know the value of the stock, we can write the value of the firm as the sum of the values of the stock and the convertible bonds, that is,

$$V = S + Fe^{-rT} - p(V,F) + c(\gamma V,F) \tag{12.29}$$

We also know that the relation between the volatility of the stock and the volatility of the firm is

$$\sigma_S = \eta_S \sigma_V \tag{12.30}$$

Assuming the value of the stock S and its volatility σ_S are known, we can solve uniquely for V and σ_V. For the sake of illustration, assume that the current value of the common shares of the firm is 8,382.74. Also, assume that a history of stock prices was collected and that the estimate of the historical stock return volatility is 23.16% (which just happens to equal the

$$0.5393\left(\frac{12{,}000}{8{,}382.74}\right)(0.30) = 0.2316$$

in Illustration 12.9. With two equations, (12.29) and (12.30), and two unknowns, we can solve uniquely for V and σ_V using Excel's SOLVER function. Alternatively, OPTVAL includes the function

OV_CORP_CV_FIRM_STOCK(*stock, ns, ncb, face, t, r, vs, vind*)

where *stock* is the value of the stock, *ns* is the number of shares of stock, *ncb* is the number of shares underlying the convertible bonds, *face*, is the face value of the bonds, *t* is the term to maturity of the bonds in years, *r* is the risk-free interest rate, and *vs* is the volatility rate of the stock. The argument *vind* is an indicator telling the function to return the firm value 1 or the firm volatility rate 2. Applying the function

OV_CORP_CV_FIRM_STOCK(8382.74, 400, 100, 4000, 5, 0.04, 0.2316, 1) = 12,000

and

OV_CORP_CV_FIRM_STOCK(8382.74, 400, 100, 4000, 5, 0.04, 0.2316, 2) = 0.50

We can then value the convertible bonds of the firm using market information for only the common shares, that is, the current stock value and its volatility rate.

SUMMARY

In this chapter, we show how to value an array of different types of corporate securities using only information about the firm's common stock price and the stock's volatility rate. This is possible because we assume that there is a single source of uncertainty—the firm's value. With a single source of uncertainty, all of the firm's securities have price movements that are perfectly correlated with one another over short periods of time. Thus, under the BSM assumptions, all corporate securities can be valued using information about the price and volatility rate of any *one* of the firm's outstanding securities. We use the firm's common stock because, of all the firm's securities, it has the deepest and most active secondary market.

In the model's development, we simplify the firm's securities in order to focus on the economic intuition underlying valuation. Initially, we assume that the firm has two sources of financing—common stock and bonds. The common stock pays no dividends, and bonds pay no coupons. Under these assumptions, the firm's common stock is a European-style call option written on the value of the firm, where the exercise price and time to expiration of the option equal the face value and term to maturity of the firm's zero-coupon bonds, respectively. The firm's bond value, in turn, equals the firm value less the value of the stock. Due to European-style put-call parity, it also equals the value of a risk-free bond less the value of a European-style put option that allows the managers of the firm to put the firm's assets to the bondholders if asset value falls below the face value of the bonds. While the model becomes more complicated when bonds pay coupons and stocks pay dividends and valuation requires using the lattice-based procedures such as those described in Chapter 9, the economic intuition remains intact.

We then extend the model to include other types of corporate securities such as warrants and convertible bonds. For expositional convenience, we assume that the (embedded) options in these securities are European-style. In this way, we can apply the BSM formula directly and use it to develop the economic intuition regarding the valuation problem. In practice, both warrants and convertible bonds may be exercised at any time during the option's life, and convertible bonds are often callable by the firm. These valuation considerations can be handled using lattice-based procedures (as opposed to analytical formulas).

Finally, while it is beyond the scope of the chapter, the model can be generalized to handle multiple sources of uncertainty. Corporate bond prices, for example, may change for reasons other than firm value changes (e.g., changes in the level of interest rates). Again, numerical methods are used to address such valuation problems. The lattice-based procedures (e.g., the binomial and trinomial methods) and Monte Carlo simulation techniques described in Chapter 9 can be modified to handle multiple sources of underlying risk.

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