

Currency Products

Futures on *foreign exchange* (FX) rates were the first financial futures contract introduced by an exchange. On May 16, 1972, the Chicago Mercantile Exchange launched trading futures on three currencies—the British pound, the Deutsche-mark, and the Japanese yen. Before that time there was little need for derivatives markets on currencies. Exchange rates were essentially fixed as a result of the Bretton Woods Agreement, which required each country to fix the price of its currency in relation to gold. With the failure of the Bretton Woods Agreement and the removal of the gold standard in 1971, exchange rates began to fluctuate more freely, motivating a need for exchange rate risk management tools. FX options and futures options did not appear until about 10 years later, being introduced by the Philadelphia Stock Exchange and the Chicago Mercantile Exchange in 1982. While exchange-traded derivatives are not nearly as active as stock index and interest rate derivatives, currency derivatives today account for about 12% of the notional amount of all OTC derivatives trading worldwide.¹

This chapter has four sections. In the first section, exchange-traded and OTC FX derivative markets are discussed. In the second section, arbitrage relations and valuation methods for FX forward, futures, option, and swap contracts are provided. For currencies, the continuous net cost of carry no-arbitrage relations and valuation methods apply. The third section illustrates a number of important currency risk management strategies. Among them are using a currency swap or a strip of currency forwards to redenominate fixed rate debt in one currency into another, using forward/options to manage the price risks of single and multiple transactions, and using forward/options to manage balance sheet risk.

MARKETS

By far the largest market in currencies is the *interbank* market. Major banks around the world trade both spot and forward currencies on a 24-hour basis. Spot transactions call for delivery and payment within two days. Forward trans-

¹ *Bank for International Settlements* (June 2004).

actions call for delivery and payment at the time specified in the forward contract. Table 16.1 contains quoted spot and forward currency rates on Monday, March 27, 2006. These rates are indicative of what might be charged by major New York banks on large purchases/sales (greater than USD 1 million) of the various currencies. According to the table, buying U.S. dollars with Canadian dollars in the spot market costs CAD 1.1696/USD. On the other hand, selling U.S. dollars for Canadian dollars generates CAD 1.1693/USD. It is also worth noting that an important inverse relation exists between the purchases and sales of different currencies. Buying U.S. dollars with Canadian dollars is the same as selling Canadian dollars for U.S. dollars. For each Canadian dollar sold, we generate $1/1.1696$ or 0.85499 U.S. dollars. Similarly, selling U.S. dollars for Canadian dollars is the same as buying Canadian dollars with U.S. dollars. The exchange rate in this case is $1/1.1693$ or USD 0.85521/CAD. This inverse relation will prove useful throughout the remaining pages of the chapter.

The forward exchange rates quoted in Table 16.1 have times to expiration up to five years. These particular standard maturities are reported to give a sense for the relation between forward exchange rates and their terms to maturity, that is, the *term structure of forward exchange rates*. Banks are generally willing to quote a forward rate on any maturity that a customer requests. Shorter-term contracts are generally more active and competitively-traded, as is reflected by the fact that the spread between the quoted bid and ask rates is narrower for short maturities than long maturities. Since the CAD/USD forward rates are lower than the spot rate, the U.S. dollar is selling at a *forward discount* (relative to the Canadian dollar), or, alternatively, the Canadian dollar is said to be selling at a *forward premium* (relative to the U.S. dollar).

TABLE 16.1 Bid and ask spot and forward exchange rates drawn from Bloomberg on Monday, March 27, 2006.

| Term | USD/GBP | | CHF/USD | | CAD/USD | | USD/EUR | | JPY/USD | |
|---------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| | Bid Rate | Ask Rate | Bid Rate | Ask Rate | Bid Rate | Ask Rate | Bid Rate | Ask Rate | Bid Rate | Ask Rate |
| Spot | 1.7475 | 1.7478 | 1.3095 | 1.3097 | 1.1693 | 1.1696 | 1.2009 | 1.2011 | 116.67 | 116.69 |
| 1 week | 1.7476 | 1.7479 | 1.3086 | 1.3088 | 1.1690 | 1.1694 | 1.2014 | 1.2016 | 116.56 | 116.58 |
| 1 month | 1.7480 | 1.7483 | 1.3055 | 1.3057 | 1.1683 | 1.1686 | 1.2031 | 1.2033 | 116.21 | 116.23 |
| 2 month | 1.7487 | 1.7490 | 1.3011 | 1.3014 | 1.1673 | 1.1676 | 1.2054 | 1.2056 | 115.71 | 115.73 |
| 3 month | 1.7494 | 1.7497 | 1.2971 | 1.2973 | 1.1662 | 1.1667 | 1.2076 | 1.2078 | 115.24 | 115.26 |
| 4 month | 1.7503 | 1.7506 | 1.2928 | 1.2931 | 1.1653 | 1.1657 | 1.2099 | 1.2101 | 114.74 | 114.76 |
| 5 month | 1.7510 | 1.7514 | 1.2891 | 1.2895 | 1.1643 | 1.1647 | 1.2118 | 1.2121 | 114.29 | 114.32 |
| 6 month | 1.7519 | 1.7522 | 1.2853 | 1.2856 | 1.1633 | 1.1637 | 1.2139 | 1.2142 | 113.82 | 113.84 |
| 9 month | 1.7543 | 1.7547 | 1.2746 | 1.2750 | 1.1605 | 1.1610 | 1.2197 | 1.2200 | 112.48 | 112.51 |
| 1 year | 1.7562 | 1.7567 | 1.2648 | 1.2654 | 1.1582 | 1.1587 | 1.2249 | 1.2252 | 111.23 | 111.26 |
| 2 year | 1.7602 | 1.7615 | 1.2297 | 1.2309 | 1.1495 | 1.1503 | 1.2438 | 1.2444 | 106.78 | 106.84 |
| 3 year | 1.7645 | 1.7688 | 1.1981 | 1.2013 | 1.1413 | 1.1436 | 1.2617 | 1.2629 | 103.10 | 103.21 |
| 4 year | 1.7685 | 1.7763 | 1.1685 | 1.1737 | 1.1315 | 1.1348 | 1.2785 | 1.2807 | 99.87 | 100.04 |
| 5 year | 1.7760 | 1.7853 | 1.1395 | 1.1620 | 1.1272 | 1.1333 | 1.2958 | 1.2989 | 96.78 | 97.05 |

TABLE 16.2 Conversion rates between euro and national currencies when irrevocably fixed on December 31, 1998.

| Country | Currency | 1 euro = |
|------------|----------|----------|
| Austria | ATS | 13.7603 |
| Belgium | BEF | 40.3399 |
| Finland | FIM | 5.94573 |
| France | FRF | 6.55957 |
| Germany | DEM | 1.95583 |
| Ireland | IEP | 0.787564 |
| Italy | ITL | 1936.27 |
| Luxembourg | LUF | 40.3399 |
| NLG | NLG | 2.20371 |
| Portugal | PTE | 200.482 |
| Spain | ESP | 166.386 |

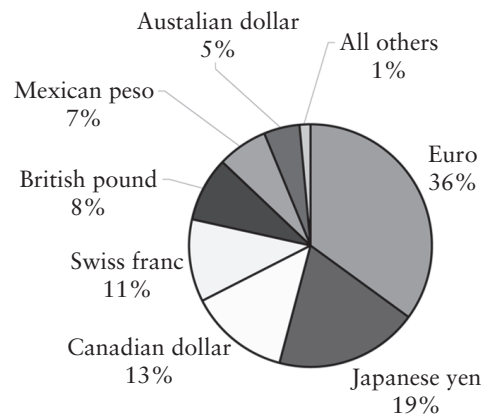
Exchange-traded FX futures and options markets are also active worldwide. In recent years, contract volume has waned. One reason is that, on December 31, 1998, 11 European countries irrevocably fixed their currencies to the Euro. The countries who are members of the European Union (EU), their former currencies, and the fixed exchange rates are reported in Table 16.2. With all of these countries adopting the euro as the common currency, the need to hedge currency risk across EU countries is eliminated. A second reason is that the OTC currency derivatives market has usurped some of the trading volume from the derivatives exchanges. The OTC market is more well suited to tailor FX derivatives contracts to meet customer risk management needs.

Futures

In the United States, the most active FX futures contracts are traded on the Chicago Mercantile Exchange's International Monetary Market division. Figure 16.1 shows the breakdown of the CME's FX futures by number of contracts traded during the calendar year 2003. The total contract volume during this period was 31,873,938 contracts. The euro futures contract is the most active, with about 36% of the total contract volume. The Japanese yen futures was second at 19%, followed by the Canadian dollar futures and the Swiss franc futures with 13% and 11% of contract volume, respectively. All of the aforementioned contracts are USD denominated (quoted in USD per unit of the underlying currency). The CME also lists cross-rate futures contracts, however, the trading volume is slight and is included with less active USD denominated contracts with the category heading "All others."

Each of the exchange's contracts has preset terms. Contract specifications of the CME's euro futures are listed in Table 16.3. The contract requires EUR 125,000 to be delivered on the third Wednesday of the contract month. The price of the euro futures contract is quoted in USD/EUR. The minimum price

FIGURE 16.1 Relative trading volumes of FX futures contracts traded on the Chicago Mercantile Exchange during the calendar year 2003. Total contract volume was 31,873,938 contracts.



Source: Data compiled from www.cme.com.

TABLE 16.3 Selected terms of euro futures contract.

| | |
|---------------------|--|
| Exchange | Chicago Mercantile Exchange (International Monetary Market Division) |
| Contract unit | 125,000 euro |
| Tick size | \$0.0001 per euro |
| Tick value | \$12.50 per contract |
| Trading hours | 7:20 AM to 2:00 PM (CST) GLOBEX: Monday through Thursday, 5 PM to 4 PM; Sundays and holidays, 5 PM to 4 PM. |
| Contract months | Six months in March quarterly expiration cycle (Mar., Jun., Sep., Dec.). |
| Last day of trading | 9:16 AM on second business day immediately preceding third Wednesday of contract month. |
| Final settlement | Physical delivery on third Wednesday of contract month. |

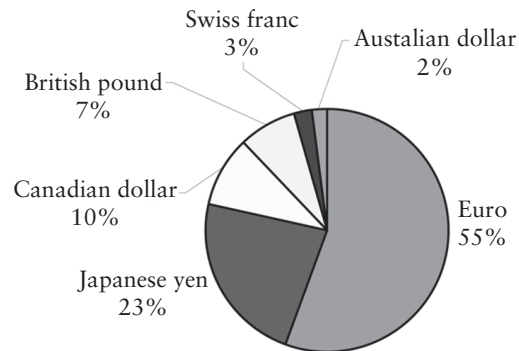
Source: www.cme.com.

movement (i.e., tick size) is USD 0.0001. Such a movement implies a change in contract value of USD 12.50. Six contract months on the March quarterly expiration cycle (March, June, September, December) are available on any given time. The contracts trade virtually 24 hours a day—from 7:20 AM to 2:00 PM (CST) in an open outcry format in the trading pits of Chicago but from 5:00 PM to 4:00 PM the following afternoon on GLOBEX.

Options

In the United States, FX options take two forms: options on FX futures and options on FX spot currencies. The CME's futures options are the most active,

FIGURE 16.2 Relative trading volumes of FX futures option contracts traded on the Chicago Mercantile Exchange during the calendar year 2003. Total contract volume was 2,142,684 contracts.



Source: Data compiled from www.cme.com.

followed by the Philadelphia Stock Exchange's (PHLX) spot currency options. Figure 16.2 shows the breakdown of trading volume for the CME's FX futures option contracts by underlying currency during the calendar year 2003. The total contract volume was 2,142,684, approximately 6.7% of the FX futures volume. Euro FX futures option contracts were the most active, with 55% of the total contract volume. The Japanese yen and Canadian dollar contracts followed, with 23% and 10% of the total volume, respectively. Like in the case of currency futures, the USD denominated option contracts are the most active. Cross-rate futures account for little contract volume.

The CME's futures options and the PHLX's spot currency options are very similar in nature, however, there are some minor distinctions. The contract specifications of the CME's EUR futures option contract and the PHLX's EUR option contract are provided in Tables 16.4 and 16.5, respectively. The EUR futures options require the delivery of the underlying futures; so they have a contract denomination of EUR 125,000. The tick size and tick value are the same as the underlying futures, as are the trading hours. The CME's most active futures options are American-style, although they also offer European-style contracts. The American-style contracts can be exercised at any time to gain a position in the underlying futures. When a call is exercised, the option holder receives a long position in the underlying futures and is marked-to-market at the difference between the futures price and the exercise price of the option. Available contract months include the next four months in the March quarterly expiration cycle plus two that are not. Standing on March 30, 2005, this means that June, September, and December 2005 as well as the March 2006 contracts are traded as well as April and May 2005. Because the underlying futures are on a quarterly expiration cycle, the April and May futures options are written on the June 2005 futures. The futures options expire at the close of trading on the second Thursday preceding the third Wednesday of the contract month.

The currency options traded on the PHLX are half the size of the CME's futures option, EUR 62,500. The tick size is the same, and a one tick movement is worth \$6.25. The PHLX offers both American-style and European-style

TABLE 16.4 Selected terms of euro futures option contract.

| | |
|---------------------|---|
| Exchange | Chicago Mercantile Exchange (International Monetary Market Division) |
| Contract unit | One euro futures contract |
| Tick size | \$0.0001 per euro |
| Tick value | \$12.50 per contract |
| Trading hours | 7:20 AM to 2:00 PM (CST) GLOBEX: Monday through Thursday, 5 PM to 4 PM; Sundays and holidays, 5 PM to 4 PM. |
| Exercise style | American |
| Contract months | Four contract months in March quarterly cycle and two serial months, not in the March quarterly cycle, plus four weekly expirations. |
| Last day of trading | Quarterly and serial options: Close of trading on second Thursday preceding third Wednesday of contract month. Weekly options: Close of trading on Thursday of contract month that is not also termination for quarterly and serial European-style options. |
| Final settlement | Physical delivery of the underlying futures. |

TABLE 16.5 Selected terms of euro option contract.

| | |
|---|--|
| Exchange | Philadelphia Stock Exchange |
| Contract unit | 62,500 euro |
| Tick size | \$0.0001 per euro |
| Tick value | \$6.25 per contract |
| Trading hours | 2:30 AM to 2:30 PM (EST) Monday through Friday |
| Exercise style | American- and European-styles available. |
| Contract months | Four months in March quarterly cycle (Mar., Jun., Sep., Dec.) plus two-near months. |
| Last day of trading/ contract expiry | Friday before third Wednesday of expiring month provided it is a business day (otherwise day immediately prior). |
| Final settlement | Physical delivery of euro currency on day after expiry. |

options for its FX options as well as several expiration months. On any given date, four contracts on the March quarterly expiration cycle (March, June, September, December) and two near-months are available for trade. The options expire on the Friday preceding the third Wednesday of the contract month. All of the PHLX's FX options require the delivery of the underlying currency.

Interestingly, the most active FX options traded on the PHLX are its *customized currency options*. These options allow users to set most of the terms of the option contract including exercise price, expiration date (up to two years), and premium quotation as either units of the currency or percent of underlying value. The contract denominations are preset according to the underlying currency. For more information, go the PHLX's website at www.phlx.com.

VALUATION

The values of currency derivatives are best modeled under the continuous net carry cost assumption. The net cost of carry rate of a foreign currency is the difference between the domestic and foreign interest rates, that is, $b = r_d - r_f$. Substituting this definition into the valuation results of Chapters 4 through 9, we get the FX valuation principles summarized in Table 16.6. This section focuses on developing intuition for these results.

Forwards/Futures

To value foreign currency forwards and futures, we use the net cost of carry relation,

$$F = S e^{(r_d - r_f)T} \quad (16.1)$$

Unless otherwise stated, we assume that F and S are the forward and spot prices of the currency in USD per unit of foreign currency for ease of exposition. Thus r_d is the domestic (U.S.) risk-free interest rate and r_f is the risk-free rate of interest in the foreign market. The net cost of carry relation (16.1) arises from the absence of costless arbitrage opportunities in the marketplace. The intuition underlying the relation is that we have two ways to have one unit of the foreign currency on hand at time T at a price we know today. The first is represented on the left-hand side of the net carry relation (16.1), that is, we can buy a forward contract with maturity T today, and pay F at time T . The second is represented by the right-hand side of (16.1), that is, we can borrow domestically at a rate r_d to buy one unit of the foreign currency today at a cost of S , and then invest the currency at the prevailing foreign interest rate r_f until time, T . Under this second arrangement, the terminal cost is

$$S \times e^{r_d T} \times e^{-r_f T} = S e^{(r_d - r_f)T}$$

Since the two alternatives are perfect substitutes, the two sides of (16.1) must be equal.

ILLUSTRATION 16.1 Compute implied risk-free rate in Canada given spot rate, forward rate, and U.S. risk-free rate.

Compute the implied six-month risk-free rate in Canada given a spot exchange rate of USD 0.85510/CAD, and a six-month forward rate of USD 0.85948/CAD. (Note that these are the mid-rates implied by the bid/ask quotes from Table 16.1.) The six-month LIBOR rate is 4.90%.

First, you need to compute the continuously-compounded domestic risk-free rate of interest. The LIBOR rate is a nominal rate over a 180-day period. To transform it to a continuous six-month rate on an annualized basis, we solve

$$e^{r_d(0.5)} = 1 + 0.0490 \left(\frac{180}{360} \right)$$

TABLE 16.6 Summary of no-arbitrage price relations and valuation equations for options on foreign currencies.

| No-Arbitrage Price Relations | | |
|------------------------------|--|--|
| Forward/Futures | $f = F = Se^{(r_d - r_f)T}$ | |
| European-Style: | Futures Options | |
| Lower bound for call | $c \geq \max(0, Se^{-r_f T} - Xe^{-r_d T})$ | $c \geq \max[0, e^{-r_d T} (F - X)]$ |
| Lower bound for put | $p \geq \max(0, Xe^{-r_d T} - Se^{-r_f T})$ | $p \geq \max[0, e^{-r_d T} (X - F)]$ |
| Put-call parity | $c - p = Se^{-r_f T} - Xe^{-r_d T}$ | $c - p = e^{-r_d T} (F - X)$ |
| American-Style: | | |
| Lower bound for call | $c \geq \max(0, Se^{-r_f T} - Xe^{-r_d T}, S - X)$ | $c \geq \max(0, F - X)$ |
| Lower bound for put | $p \geq \max(0, Xe^{-r_d T} - Se^{-r_f T}, X - S)$ | $p \geq \max(0, X - F)$ |
| Put-call parity | $Se^{-r_f T} - X \leq C - P \leq S - Xe^{-r_d T}$ | $Fe^{-r_d T} - X \leq C - P \leq F - Xe^{-r_d T}$ |
| Valuation Equations/Methods | Options | Futures Options |
| European-Style: | Options | Futures Options |
| Call value | $c = Se^{-r_f T} N(d_1) - Xe^{-r_d T} N(d_2)$ | $c = e^{-r_d T} [FN(d_1) - XN(d_2)]$ |
| Put value | $p = Xe^{-r_d T} N(-d_2) - Se^{-r_f T} N(-d_1)$ | $p = e^{-r_d T} [XN(-d_2) - FN(-d_1)]$ |
| | where $d_1 = \frac{\ln(Se^{-r_f T} / Xe^{-r_d T}) + 0.5\sigma^2 T}{\sigma\sqrt{T}}$ and $d_2 = d_1 - \sigma\sqrt{T}$ | where $d_1 = \frac{\ln(F/X) + 0.5\sigma^2 T}{\sigma\sqrt{T}}$ and $d_2 = d_1 - \sigma\sqrt{T}$ |
| American-Style: | Numerical valuation: quadratic approximation, and binomial and trinomial methods. | Numerical valuation: quadratic approximation, and binomial and trinomial methods. |

to find that r_d is 4.908%. Next, substitute the problem information into the net cost of carry relation, that is,

$$0.85948 = 0.85510e^{(0.04908 - r_f)0.5}$$

which can be rearranged to yield

$$r_f = 0.04908 - 2 \ln\left(\frac{0.85948}{0.85510}\right) = 3.888\%$$

The six-month risk-free rate of interest in Canada is about 102 basis points lower than in the United States.

This computation can be verified using the forward pricing functions contained in the OPTVAL Function Library. To compute the implied income rate in the continuous version of the net cost of carry relation, use the function,

$$\text{OV_FORWARD_II}(s, f, r, t)$$

where s is the spot price, f is the forward price, r is the risk-free (domestic) interest rate, and t is time to expiration of the forward contract.

B11 ▼ fx =OV_FORWARD_II(\$B\$2,\$B\$3,\$B\$9,\$B\$4)

| | A | B | C | D |
|----|---|---------------|---------|---|
| 1 | Currency | USD/CAD | CAD/USD | |
| 2 | Spot exchange rate | 0.85510 | 1.16945 | |
| 3 | Forward exchange rate | 0.85948 | 1.16350 | |
| 4 | Time to forward expiration | 0.5000 | | |
| 5 | | | | |
| 6 | LIBOR rate | | | |
| 7 | No. of days to maturity | 180 | | |
| 8 | Quoted LIBOR rate | 4.900% | | |
| 9 | Continuous compounded rate | 4.908% | | |
| 10 | | | | |
| 11 | Implied risk-free rate in Canada | 3.888% | | |

Interest Rate Parity

In international finance literature, the relation (16.1) is sometimes called *interest rate parity* (IRP). The intuition underlying the IRP relation is developed as follows. Consider an investor who has one USD to invest. If the money is invested domestically at the risk-free rate, the value at time T is $e^{r_d T}$. On the other hand, the dollar can be used to buy a foreign currency, and then that currency can be invested at the foreign risk-free rate. At the same time, we can write a contract to convert the proceeds of the foreign investment back into dollars at maturity, using the FX forward market. At time T , the dollar cash proceeds of this hedged foreign investment are

$$\frac{1}{S} \times e^{r_f T} \times F$$

Since both investments provide a risk-free return in the domestic currency, their terminal values must be equal. Setting $e^{r_d T}$ equal to $1/S \times e^{r_f T} \times F$ and rearranging provides the net cost of carry relation (16.1).

Sometimes interest rate parity is expressed in relative terms, that is,

$$\frac{F - S}{S} = e^{(r_d - r_f)T} - 1 \quad (16.2)$$

The left-hand side of (16.2) goes by a variety of names including the *forward premium* or *swap rate*. The term *swap rate* comes from the fact that investors frequently buy a foreign currency and agree to swap it back for dollars at some future date. The swap rate specifies the percentage gain or loss on such a transaction. The right-hand side is the interest differential between the two countries.

Cross Rates and Triangular Arbitrage

The *cross-rate relation* is an arbitrage relation that involves three currencies. Suppose we buy (1) Canadian dollars using U.S. dollars, (2) euros using the Canadian dollars, and then (3) U.S. dollars using euros. In the absence of trading costs and costless arbitrage opportunities, we must be back exactly where we started, that is,

$$\left(\frac{\text{USD}}{\text{CAD}}\right)\left(\frac{\text{CAD}}{\text{EUR}}\right)\left(\frac{\text{EUR}}{\text{USD}}\right) = 1$$

Another way of thinking about it is the U.S. dollar cost of euros should be the same if we (1) used U.S. dollars to purchase Canadian dollars and then used the Canadian dollars to buy euros or (2) used U.S. dollars to buy euros directly, that is,

$$\left(\frac{\text{USD}}{\text{EUR}}\right) = \left(\frac{\text{USD}}{\text{CAD}}\right)\left(\frac{\text{CAD}}{\text{EUR}}\right)$$

To illustrate that the market is well aware of this cross-rate relation, consider the cross-rate relations reported in Table 16.7. The USD/EUR rate is reported as 1.2010. The USD/CAD rate is 0.85510, and the CAD/EUR rate is 1.4045. The product of 0.85510 and 1.4045 is 1.2010, exactly as expected. If the two methods gave different answers an opportunity for *triangular arbitrage* would exist. In the absence of triangular arbitrage opportunities, the following relation must hold for all triplets of currencies:

$$S_{i,j} = S_{i,k} S_{k,j} \quad (16.3)$$

where $S_{i,j}$ is the number of units of the i -th currency required to purchase one unit of the j -th currency.

TABLE 16.7 Key currency cross rates on Monday, March 27, 2006.

| | USD | JPY | EUR | CAD | CHF | GBP |
|-----|---------|-----------|---------|---------|---------|--------|
| GBP | 0.57220 | 0.0049040 | 0.68721 | 0.48929 | 0.43692 | |
| CHF | 1.3096 | 0.0112 | 1.5728 | 1.1198 | | 2.2887 |
| CAD | 1.1695 | 0.010023 | 1.4045 | | 0.89298 | 2.0438 |
| EUR | 0.83264 | 0.0071361 | | 0.71199 | 0.63580 | 1.4552 |
| JPY | 116.68 | | 140.13 | 99.773 | 89.096 | 203.92 |
| USD | | 0.0085704 | 1.2010 | 0.85510 | 0.76359 | 1.7477 |

ILLUSTRATION 16.2 Compute profit from triangular arbitrage opportunity.

Suppose you observe the following exchange rates:

USD/EUR 1.3500

CAD/EUR 1.6200

USD/CAD 0.8200

Is a costless arbitrage profit possible?

To examine whether a costless arbitrage opportunity exists, take any two rates, multiply them appropriately, and see if the product equals the other rate. Given the way the problem information is presented, it is easiest to check whether

$$\left(\frac{\text{USD}}{\text{EUR}}\right) = \left(\frac{\text{USD}}{\text{CAD}}\right)\left(\frac{\text{CAD}}{\text{EUR}}\right)$$

Substituting the problem information, you find that

$$1.3500 > (0.82)(1.62) = 1.3284$$

hence, a costless arbitrage profit is possible. The trades that you need to place are as follows:

- (1) Buy Canadian dollars with U.S. dollars at a rate of 0.82.
- (2) Buy euros with the Canadian dollars from part (a) at a rate of 1.62.
- (3) Sell euros from part (b) for U.S. dollars at a rate of 1.3500.

Assuming a trade size of USD 100,000, your risk-free profit is computed as follows:

- (1) pay USD 100,000 for CDN 121,951.22
- (2) pay CDN 121,951.22 for EUR 75,278.53
- (3) deliver EUR 75,278.53 for USD 101,626.02.

The risk-free profit is USD 1,626.02 per USD 100,000 of arbitrage activity.

RISK MANAGEMENT LESSON: AWA LTD.

Triangular arbitrage was nominally at the heart of the first modern-day derivatives “scandal.” Amalgamated Wireless Australasia Ltd. (now AWA Ltd.) manufactured, imported and exported electronic and electrical products. In order to hedge its foreign currency risk exposure from contracts it had in place for the

goods it imported, AWA decided in 1985 to buy currency contracts against actual or anticipated import requirements. At the time, however, the market between foreign currencies and Australian dollars was thin. Only the AD/USD forward market was liquid. To circumvent this problem, the firm decided to execute the hedge in two legs. In the first leg, AWA would go long in the foreign currency (e.g., Japanese yen) and short U.S. dollars, and, in the second, they would go long U.S. dollars and short Australian dollars. By virtue of the absence of triangular arbitrage opportunities, the risk management strategy was perfectly sensible.

So, how did AWA go about losing nearly AD 49.8 million? The answer is with the help of Andy Koval, a commerce graduate from the University of New South Wales. In 1984, Koval was hired as a trainee management accountant on AD 14,000 a year to help set up a money market operation for AWA. At the time, AWA's foreign currency (FX) operation was modest and, for the fiscal year ending June 30, 1985, they reported a pretax gain of only AD 282,000. During 1985, Andy was put in charge of FX operations. Things changed quickly. For the fiscal year ending June 30, 1986, the pretax FX gains rose to AD 7.5 million. Indeed, so successful was the program that, by September 1986, the firm had made its FX operation a profit center of the firm. The next year started off the same way. For the six-month period ending December 31, 1986, the FX operation posted a pretax profit of AD 10 million.

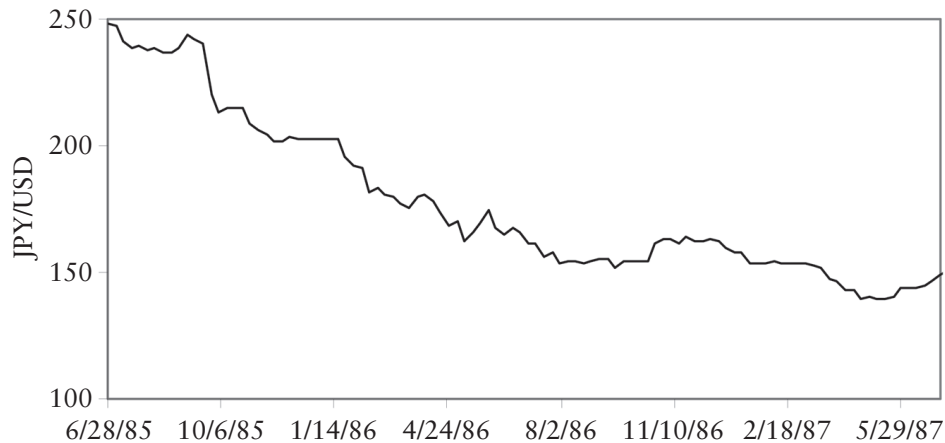
The amazing success of AWA's hedging program was well publicized. On March 10, 1987, the *Sydney Morning Herald* talked about "unprecedented returns from the foreign exchange operations" for AWA.² In the article, Mr. John Hooke, Chairman of AWA is reported as saying that "the forex profit had risen as the company had begun trading in currency futures, which it initially had taken out to hedge itself against movements in the Australian dollar against the yen." Two days later, the same newspaper featured an article titled "Andy Koval, AWA'S Forex Whiz-kid."³ Among other things, the article says that the "unassuming Andy appears to have discovered trading techniques the rest of the Forex market is clamouring for." Andy, himself is quoted as saying, "Our success is to remain covered and make money out of a trend, rather than punting. There is none of this cowboy stuff." The article goes on to say, "Asked whether he feels it is somewhat unusual for one so young to be responsible for such a huge chunk of one of Australia's larger, albeit somewhat sleepy, public companies, Andy Koval says: 'Yeah, sometimes. It's a bit funny with the board of directors. My parents think it's pretty amazing, too.'"

The public comments made by Hooke and Koval seemed to indicate that AWA was, in fact, hedging. Hooke's comment suggests that AWA had taken the hedges since it imported most of its components from Japan, while Koval clearly indicates that AWA's FX exposure "remains covered." But if hedges are in place, how can the firm be earning such extraordinary profits if they are hedging? As it turns out, they were not. The house of cards came tumbling down only a few months later when the Australian Stock Exchange discovered irregularities in AWA's reporting and determined that AWA had sustained substantial losses. After many months of investigation, a truer picture of Koval's actual trading activity emerged.

² *Sydney Morning Herald*, 10 March 1987, p. 21.

³ *Sydney Morning Herald*, 12 March 1987, p. 23.

1. *Koval was not hedging.* Apparently, Koval had a strong directional view that the USD would fall, so he entered only the first leg of the hedge—he bought Japanese yen and sold U.S. dollars. Unfortunately, he was wrong in his view, very wrong. As the figure below shows, the JPY depreciated steady in value relative to the USD during the period Koval managed FX operations, and thereby had to have incurred significant speculative losses.



2. *Koval disregarded trading limits and took positions well beyond hedging needs.* Even if Koval had executed both legs of the hedge (i.e., buying JPY with USD, and buying USD with AD), there were other telltale signs that he was speculating. At the end of December 1986, for example, AWA's hedge requirement for Japanese yen over the next year was estimated to be about JPY 10 billion. At the time, however, AWA records show that actual open positions exceeded JPY 75 billion.
3. *Koval entered the second leg of the hedge in reverse.* Another telltale sign that Koval was speculating came on the occasions in which he appeared to execute the second leg of the hedge. Recall that buying JPY with AD can be accomplished by buying JPY with USD, and then buying USD with AD. What appeared in AWA's books, however, were trades in which Koval was buying AD with USD, exactly the reverse of what should be done. In other words, Koval was betting not only that the USD was going to tank relative to the yen but also that the USD was going to tank relative to the AD. In both cases, he was wrong.

Could the situation have been avoided? Absolutely! While the hedge strategy was entirely appropriate for a company needing to manage its foreign currency risk exposure on the costs of imports whose prices were fixed, no one monitored whether the strategy was being implemented properly. The primary factors driving the extraordinary losses were:

1. *Lack of meaningful supervision:* Koval was permitted by AWA management to operate without effective control and supervision.
2. *Absence of a proper system of books and records and other internal controls.* Koval had generally disclosed only the contracts showing a profit. Loss-making contracts were disclosed either by rolling them over at historical rates or by

paying the losses out of what were claimed to be unauthorized borrowings by Koval from a number of different banks, the existence of these loans were also concealed from AWA.

3. *Managers and boards of directors did not understand risk management strategy.* The hedge strategy developed by AWA was perfectly sensible in light of the liquidity of available hedge instruments. But given the simplicity of this hedge strategy (i.e., buying forwards/futures to hedge the currency risk exposure of input costs), extraordinary profits should have been the first sign that something was amiss. Did they choose to turn a blind eye to the matter of how the money was being earned? After all, the FX profits were huge in a company that was not otherwise performing well. Or was the board incapable of understanding the nature of the FX hedge operations? After all, it was the board that approved a budget which treated FX trading as a major profit center.
4. *Hubris.* The quotes in the *Sydney Morning Herald* in March 1987 support the view that both Hooke and Koval were almost giddy in their optimism about future FX trading profits. When asked about the amazing feat of earning nearly AD 20 million in pretax profits over 18 months, Koval stated that it was “No fluke.”⁴ He went on to say “I thought about going out on my own, but am fairly happy here at the moment.” In the same article, Hooke is reported as saying that FX profits in the first half of 1987 “will be in line” with the AD 10 million earned in the last half of 1986.”

In the end, AWA lost slightly less than AD 50 million. As is typical when such events occur, litigation ensued. What makes this story different is that the Australian courts held AWA’s auditors liable for 80% of the damages because “they were negligent in their duties in failing to draw the attention of AWA’s board and senior management to the absence of sufficient internal controls.” They also held that “AWA was liable for contributory negligence and Hooke liable to contribute to the damages.

Swaps

No-arbitrage price relations for swaps were also developed in Chapter 4. A currency swap contract is an agreement to exchange a set of future cash flows with no cash flow occurring at inception. A plain-vanilla currency swap is usually regarded to be an exchange of a fixed payment for a floating payment, where the floating payment is tied to an exchange rate. The key information needed to value a swap contract is the *forward curve* for the underlying currency and the *zero-coupon yield curve* for domestic risk-free bonds. All rates on both curves are assumed to be tied to the prices of tradable securities.

To make currency swap valuation as clear as possible, assume that we import goods from the United Kingdom in a uniform manner throughout the year and sell them in the United States. At the beginning of each year, we negotiate a fixed price for each unit we import during the year, and the price is quoted in British pounds. The goods we import, however, are sold in U.S. dollars. If the British pound appre-

⁴ *Sydney Morning Herald*, 12 March 1987.

ciates in value relative to the dollar during the year and we cannot pass on the cost increase to our U.S. customers, our profit margin will fall. To manage this risk exposure, we may want to buy British pounds in the forward market. One alternative is to buy a *strip* of forward (or futures) contracts, one corresponding to each desired delivery date. Unfortunately, while the cost of the monthly delivery will be locked-in, it will be different each month, except in the special case in which the forward curve happens to be a horizontal line. If our customers' demands are uniform throughout the year, this means that our profit margin will vary from month to month, albeit in a deterministic way. A second alternative is to buy a swap contract that provides for a fixed delivery amount each month at a single fixed price for all deliveries. In the absence of costless arbitrage opportunities, the present value of the deliveries using the forward curve must be the same as the present value of the deliveries using the fixed price of the swap contract, that is,

$$\sum_{i=1}^n f_i e^{-r_i T_i} = \sum_{i=1}^n \bar{f} e^{-r_i T_i} \quad (16.4)$$

where n is the number of delivery dates, f_i is the price of a forward contract with time to expiration T_i , r_i is the risk-free rate of interest corresponding to time to expiration T_i ,⁵ and \bar{f} is the fixed price in the swap agreement.⁶ In an instance where the right-hand side of (16.4) is greater (less) than the left-hand side, an arbitrageur would buy (sell) the swap and sell (buy) the strip of forward contracts, pocketing the difference. Because such free money opportunities do not exist, (16.4) must hold as an equality.

Equation (16.4) can be rearranged to isolate the fixed price of the swap agreement, that is,

$$\bar{f} = \frac{\sum_{i=1}^n f_i e^{-r_i T_i}}{\sum_{i=1}^n e^{-r_i T_i}} = \sum_{i=1}^n f_i \left(\frac{e^{-r_i T_i}}{\sum_{i=1}^n e^{-r_i T_i}} \right) \quad (16.5)$$

Expressed in this fashion, it becomes obvious that the fixed price of a swap is a weighted average of forward prices, one corresponding to each delivery date.

ILLUSTRATION 16.3 Compute fixed exchange rate of swap based on forward exchange rate curve.

Suppose that you own a chain of Irish pubs in Boston. Your customers' favorite brew is, of course, Guinness Irish Stout. Your supplier is a distributor in the United Kingdom, and you have negotiated a fixed price of £50 per keg, delivered in Boston, for all deliveries during the next year. Based upon consumption over the past few years, you anticipate

⁵ Note that we are allowing for the fact that the risk-free rate may be term-specific.

⁶ The delivery quantity is irrelevant since it is the same on both sides of the equation. That is, equation (4.10) assumes that one unit is delivered on each delivery date.

that the average consumption rate will be 10,000 kegs per month. Your customers are accustomed to paying USD 5.00 per pint. If the British pound appreciates relative to the USD, you will not be able to pass on the price increase to your customers. They will simply switch over to a less expensive domestic brand whose margins are much lower. Consequently, you are considering different hedging alternatives. Currently, the forward curve for the USD/GBP exchange rate is

$$f_i = 1.92 - 0.035\ln(1 + T_i)$$

and the zero-coupon yield curve for risk-free U.S. bonds is

$$r_i = 0.03 + 0.01\ln(1 + T_i)$$

Determine the fixed forward exchange rate on a 12-month currency swap with uniform monthly deliveries.

Based on the forward curve and the yield curve, you can compute prepaid forward prices for each of the 12 delivery dates. You then sum the prepaid forward prices, and divide by the sum of the discount factors to determine the fixed price. The intermediate computations are as follows.

| Time to Payment | USD/GBP Forward Rate | Risk-Free Rate | Discount Factor | Prepaid Forward Price |
|---------------------|----------------------|----------------|-----------------|-----------------------|
| 0.0833 | 1.9171 | 3.08% | 0.9974 | 1.9122 |
| 0.1667 | 1.9142 | 3.15% | 0.9948 | 1.9041 |
| 0.2500 | 1.9113 | 3.22% | 0.9920 | 1.8959 |
| 0.3333 | 1.9083 | 3.29% | 0.9891 | 1.8875 |
| 0.4167 | 1.9054 | 3.35% | 0.9861 | 1.8790 |
| 0.5000 | 1.9025 | 3.41% | 0.9831 | 1.8704 |
| 0.5833 | 1.8996 | 3.46% | 0.9800 | 1.8616 |
| 0.6667 | 1.8967 | 3.51% | 0.9769 | 1.8528 |
| 0.7500 | 1.8938 | 3.56% | 0.9737 | 1.8439 |
| 0.8333 | 1.8908 | 3.61% | 0.9704 | 1.8349 |
| 0.9167 | 1.8879 | 3.65% | 0.9671 | 1.8258 |
| 1.0000 | 1.8850 | 3.69% | 0.9637 | 1.8167 |
| | | | 11.7743 | 22.3847 |
| Fixed price of swap | | | | 1.9012 |

Based on the forward curve, the fixed rate on the swap should be

$$\bar{f} = \frac{22.3847}{11.7743} = 1.9012 \text{ USD per GBP}$$

The OPTVAL Function Library contains a function that values a currency swap with uniform quantities each period. The function is

$$\text{OV_SWAP_CURRENCY}(t, f, r, vr)$$

where t is a vector containing the times to each delivery date, f is a vector of forward/futures prices corresponding to each date, r is a vector of zero-coupon risk-free rates corresponding to each delivery date, and vr is an indicator variable instructing the function

to compute (1) the sum of the present values of the prepaid forward contracts (v or V), (2) the sum of the discount factors (d or D), or (3) the break-even fixed price of the swap based on the forward curve (r or R). For the illustration at hand,

| E22 | | fx =OV_SWAP_CURRENCY(\$A\$4:\$A\$15,\$B\$4:\$B\$15,\$C\$4:\$C\$15,\$D\$22) | | | | | | |
|-----|-----------------------------------|--|-----------|----------|---------|---|---|---|
| | A | B | C | D | E | F | G | H |
| 1 | | USD/GBP | | | Prepaid | | | |
| 2 | Time to | forward | Risk-free | Discount | forward | | | |
| 3 | payment | rate | rate | factor | price | | | |
| 4 | 0.0833 | 1.9171 | 3.08% | 0.9974 | 1.9122 | | | |
| 5 | 0.1667 | 1.9142 | 3.15% | 0.9948 | 1.9041 | | | |
| 6 | 0.2500 | 1.9113 | 3.22% | 0.9920 | 1.8959 | | | |
| 7 | 0.3333 | 1.9083 | 3.29% | 0.9891 | 1.8875 | | | |
| 8 | 0.4167 | 1.9054 | 3.35% | 0.9861 | 1.8790 | | | |
| 9 | 0.5000 | 1.9025 | 3.41% | 0.9831 | 1.8704 | | | |
| 10 | 0.5833 | 1.8996 | 3.46% | 0.9800 | 1.8616 | | | |
| 11 | 0.6667 | 1.8967 | 3.51% | 0.9769 | 1.8528 | | | |
| 12 | 0.7500 | 1.8938 | 3.56% | 0.9737 | 1.8439 | | | |
| 13 | 0.8333 | 1.8908 | 3.61% | 0.9704 | 1.8349 | | | |
| 14 | 0.9167 | 1.8879 | 3.65% | 0.9671 | 1.8258 | | | |
| 15 | 1.0000 | 1.8850 | 3.69% | 0.9637 | 1.8167 | | | |
| 16 | | | | 11.7743 | 22.3847 | | | |
| 17 | | | | | | | | |
| 18 | Fixed price of swap | | | | 1.9012 | | | |
| 19 | | | | | | | | |
| 20 | Present value of prepaid forwards | | | V | 22.3847 | | | |
| 21 | Present value of discount factors | | | D | 11.7743 | | | |
| 22 | Break-even fixed price on swap | | | R | 1.9012 | | | |

The swap valuation framework provided above makes the assumption that the number of units of currency needed each period is the same throughout the life of the swap. There are many instances in which this is not the case, however. Suppose we let quantity, Q_i , vary from period to period. To determine a single fixed exchange rate for all periods, we again equate the present value of the deliveries using the forward curve to the present value of the deliveries using the fixed price of the swap contract, that is,

$$\sum_{i=1}^n Q_i f_i e^{-r_i T_i} = \sum_{i=1}^n Q_i \bar{f} e^{-r_i T_i} \tag{16.6}$$

Equation (16.6) can be rearranged to isolate the fixed price of the swap agreement, that is,

$$\bar{f} = \frac{\sum_{i=1}^n Q_i f_i e^{-r_i T_i}}{\sum_{i=1}^n Q_i e^{-r_i T_i}} = \sum_{i=1}^n Q_i f_i \left(\frac{e^{-r_i T_i}}{\sum_{i=1}^n e^{-r_i T_i}} \right) \tag{16.7}$$

Expressed in this fashion, it becomes obvious that the fixed price of a swap is based on a weighted average of forward payments, one corresponding to each delivery date. Note that the forward payment explicitly accounts for the delivery amount.

ILLUSTRATION 16.4 Compute fixed exchange rate of swap with time-varying quantities.

Based upon your computations in 16.3, you feel prepared to negotiate with an OTC derivatives dealer with respect to the fixed rate on a currency swap. Based on your computations, you believe the fair fixed rate is 1.9012 USD/GBP. Nonetheless, you are perfectly prepared to pay as much as 1.9020 for the convenience of having a single-hedge contract (rather than a portfolio of contracts). The dealer earns this fee. Considering you need to buy GBP 6 million over the next year, the total fee is on order \$5,000 (i.e., \$0.0008 times GBP 6 million).

You describe your commitment to buy 120,000 kegs of Guinness over the next year at £50 a keg and your need to hedge the currency exposure on a monthly basis. The dealer quotes you a fixed rate of 1.9052 USD/BP for GBP 6 million that you need over the next year. He says he is giving the swap to you at cost because he is Irish, visits your pubs frequently, and is impressed by your altruistic spirit in trying to keep the price of a pint of Guinness stable. You tell him that you were not born yesterday and that he is demanding five times the fee that you were prepared to pay. He denies your allegations (at least the second one), and shows you his computations.

Upon looking at his work, you see that in the third month of the year, the quantity of Guinness delivered is 54,000 kegs, but is only 6,000 in the remaining months of the year. You ask why, and he says it should be obvious. Total demand is 120,000 kegs per year. Everyone knows that as a result of St. Patrick's Day's celebrations, consumption of Guinness is nine times higher in March than any other month of the year. You realize, of course, that he is absolutely right. Accounting for the quantity delivered each month, what is the fair fixed rate on the currency swap?

To answer this question, you must weight the discount factors and prepaid forward prices by the monthly quantities. The computations are shown below. Based on the forward curve, the fair fixed rate is, indeed, 1.9052 USD/GBP. Note that the OPTVAL Function library has a quantity weighted swap valuation routine. The vector of monthly quantities in column G are used as an input to the function.

| E22 fx =OV_SWAP_CURRENCY_QUANTITY(\$A\$4:\$A\$15,\$B\$4:\$B\$15,\$C\$4:\$C\$15,\$G\$4:\$G\$15,D22) | | | | | | | | | | |
|---|-----------------------------------|---------|-----------|----------|------------|---------|-----------|-----------|------------|---|
| | A | B | C | D | E | F | G | H | I | J |
| 1 | | USD/GBP | | | Prepaid | Monthly | Monthly | Weighted | Weighted | |
| 2 | Time to | forward | Risk-free | Discount | forward | demand | demand | discount | prepaid | |
| 3 | payment | rate | rate | factor | price | (kegs) | (pounds) | factor | forward | |
| 4 | 0.0833 | 1.9171 | 3.08% | 0.9974 | 1.9122 | 6,000 | 300,000 | 299,231 | 573,651 | |
| 5 | 0.1667 | 1.9142 | 3.15% | 0.9948 | 1.9041 | 6,000 | 300,000 | 298,427 | 571,239 | |
| 6 | 0.2500 | 1.9113 | 3.22% | 0.9920 | 1.8959 | 54,000 | 2,700,000 | 2,678,331 | 5,118,961 | |
| 7 | 0.3333 | 1.9083 | 3.29% | 0.9891 | 1.8875 | 6,000 | 300,000 | 296,730 | 566,260 | |
| 8 | 0.4167 | 1.9054 | 3.35% | 0.9861 | 1.8790 | 6,000 | 300,000 | 295,844 | 563,705 | |
| 9 | 0.5000 | 1.9025 | 3.41% | 0.9831 | 1.8704 | 6,000 | 300,000 | 294,935 | 561,114 | |
| 10 | 0.5833 | 1.8996 | 3.46% | 0.9800 | 1.8616 | 6,000 | 300,000 | 294,006 | 558,490 | |
| 11 | 0.6667 | 1.8967 | 3.51% | 0.9769 | 1.8528 | 6,000 | 300,000 | 293,060 | 555,837 | |
| 12 | 0.7500 | 1.8938 | 3.56% | 0.9737 | 1.8439 | 6,000 | 300,000 | 292,097 | 553,158 | |
| 13 | 0.8333 | 1.8908 | 3.61% | 0.9704 | 1.8349 | 6,000 | 300,000 | 291,119 | 550,457 | |
| 14 | 0.9167 | 1.8879 | 3.65% | 0.9671 | 1.8258 | 6,000 | 300,000 | 290,127 | 547,736 | |
| 15 | 1.0000 | 1.8850 | 3.69% | 0.9637 | 1.8167 | 6,000 | 300,000 | 289,123 | 544,996 | |
| 16 | | | | 11.7743 | 22.3847 | 120,000 | 6,000,000 | 5,913,030 | 11,265,604 | |
| 17 | | | | | | | | | | |
| 18 | Fixed price of swap | | | | | | | | 1.9052 | |
| 19 | | | | | | | | | | |
| 20 | Present value of prepaid forwards | | | V | 11,265,604 | | | | | |
| 21 | Present value of discount factors | | | D | 5,913,030 | | | | | |
| 22 | Break-even fixed price on swap | | | R | 1.9052 | | | | | |

Purchasing Power Parity

Another important currency-related arbitrage relation ties together the prices of a particular commodity in two different countries. *Purchasing power parity*

(PPP) says that the prices of a commodity in two different countries must be the same after adjustment for the exchange rate, that is,

$$\text{Price}_{i,d} = S_{d,f} \text{Price}_{i,f} \quad (16.8)$$

where $\text{Price}_{i,d}$ and $\text{Price}_{i,f}$ are the domestic and foreign prices of commodity i , and $S_{d,f}$ is the spot exchange rate expressed as number of units of the domestic currency per unit of the foreign currency. The intuition underlying this relation is straightforward. In perfect markets, if the USD price of a Sony television in the United States (i.e., $\text{Price}_{i,USD}$) is more than the USD price in Japan (i.e., $USD/JPY \times \text{Price}_{i,JPY}$), arbitragers will buy televisions in Japan, and import and sell them in the United States, earning a costless arbitrage profit.

Naturally, the PPP relation is not expected to hold nearly as tightly as it does for interest rate parity or, for that matter, parity in the prices of any financial asset in two countries. The reason is that executing the arbitrage with physical assets may be cumbersome and costly. Transportation costs can be prohibitive. Shipping bulking goods such as televisions, for example, is expensive. In addition, governments may restrict trade to certain countries or impose import duties. Moreover, services such as labor are simply not traded as assets. The hourly rate of a car mechanic can only be “traded” by moving the mechanic from one country to another. Thus, restrictions on international migration may prevent arbitrage of services.

Nonetheless, PPP can provide important guidance in designing appropriate risk management strategies. Consider, for example, a U.S. firm that sells widgets in Ireland. If the U.S. firm fears that the euro will fall relative to the U.S. dollar and competes, in Ireland, with an Irish firm that also produces widgets, using forward contracts to hedge exchange rate risk will ensure that the U.S. firm can remain competitive. On the other hand, if widgets are unavailable elsewhere in Ireland, the U.S. firm may have the ability to simply increase price in accordance with the movement in the spot exchange rate to eliminate exchange rate risk exposure without using derivatives. In both cases, the PPP relation provides guidance.

Options

The arbitrage relations and valuation equations/methods for FX options are also summarized in Table 16.6. Before applying some of the valuation results, it is worth noting that there are a certain complementary relations that exist among FX options. Consider, for example, an individual who holds a call option to buy 60,000 euros at USD 1.250/EUR. If the spot exchange rate is USD 1.500/EUR at the option’s expiration, the call option holder earns USD 15,000 (i.e., EUR 60,000 times USD 0.25/EUR). At the same time, consider an individual in a EU member state who holds a put option to sell 75,000 (i.e., USD 1.250/EUR times EUR 60,000) USD at EUR 0.800/USD. If the spot exchange rate is EUR 0.6667/USD at expiration, the put option holder also earns EUR 10,000 or USD 15,000 (i.e., USD 75,000 times EUR 0.1333/USD times USD 1.500/EUR). Thus, a call option to buy currency A with currency B is nothing more than a put option to sell currency B for currency A.

ILLUSTRATION 16.5 Compute values of European-style USD/GBP and GBP/USD options on spot currency and on futures.

Suppose the USD/GBP exchange rate is 1.4912, the six-month USD/GBP futures price is 1.4968, the volatility rate of USD/GBP exchange rate is 10%, and the six-month U.S. risk-free rate of interest is 5.178%. Now do the following:

- (1) Compute the value of a European-style call to buy British pounds using U.S. dollars, assuming the option's time to expiration is six months, the option's exercise price is USD 1.40/GBP, and the denomination of the option contract is GBP 1,000,000.
- (2) Compute the value of a European-style put to sell U.S. dollars for British pounds, assuming the option's time to expiration is six months, the option's exercise price is GBP 0.7143/USD, and the denomination of the option contract is USD 1,400,000 (i.e., GBP 1,000,000 times USD 1.40/GBP since the contract sizes should be the same).
- (3) Compute the value of the European-style options in parts (1) and (2) assuming they are written on the futures price rather than the spot exchange rate.

Part (1): From Table 16.6, the valuation equation for a European-style call on a currency is

$$c = Se^{-r_f T} N(d_1) - Xe^{-r_d T} N(d_2)$$

where

$$d_1 = \frac{\ln(Se^{-r_f T}/Xe^{-r_d T}) + 0.5\sigma^2 T}{\sigma\sqrt{T}} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{T}$$

The problem information, however, does not include the foreign risk-free rate of interest. Fortunately, however, you know that six-month USD/GBP futures price is 1.4968. In the absence of costless arbitrage opportunities, the net cost of carry relation,

$$F = Se^{(r_d - r_f)T}$$

holds at all points in time. Substituting the problem information into the cost of carry relation and rearranging, you find that the risk-free interest rate in Britain is 4.428%, that is,

$$r_f = 0.05178 - \frac{\ln(1.4968/1.4912)}{0.5} = 4.428\%$$

Alternatively, you could have computed the integral limit directly as

$$d_1 = \frac{\ln(F/X) + 0.5\sigma^2 T}{\sigma\sqrt{T}}$$

Now that you have the risk-free rate on interest in Britain, you can compute the value of the call using the formula. Substituting into the formula, you get

$$c = 1.4912e^{-0.04428(0.5)} N(d_1) - 1.4000e^{-0.05178(0.5)} N(d_2)$$

where

$$d_1 = \frac{\ln(1.4912e^{-0.04428(0.5)}/1.4000e^{-0.05178(0.5)}) + 0.5(0.10^2)0.5}{0.10\sqrt{0.5}} = 0.9809$$

and

$$d_2 = 0.9809 - 0.10\sqrt{0.5} = 0.9101$$

The probability values are

$$N(d_1) = N(0.9809) = 0.8367 \quad \text{and} \quad N(d_2) = N(0.9101) = 0.8186$$

Thus, the value of the European-style call is

$$c = 1.4912e^{-0.04428(0.5)}(0.8367) - 1.4000e^{-0.05178(0.5)}(0.8186) = 0.10353 \text{ USD/GBP}$$

The contract denomination is GBP 1,000,000, so the value of the overall contract is USD 103,531, as summarized in the table below. The value of the corresponding European-style put is also provided, that is, USD 9,205.

The option value can be verified using the function OV_OPTION_VALUE from the OPTVAL Function library, that is,

$$\text{OV_OPTION_VALUE}(1.4912, 1.4000, 0.5, 0.05178, 0.04428, 0.10, \text{"c"}, \text{"e"}) = 0.10353$$

Note that, since you know the forward price, you could have also valued this European-style call as a forward option, that is,

$$\text{OV_FOPTION_VALUE}(1.4968, 1.4000, 0.5, 0.05178, 0.10, \text{"c"}, \text{"e"}) = 0.10353$$

Part (2): The valuation equation of a European-style put is given in Table 16.6. Note the domestic currency is now British pounds and the foreign currency is U.S. dollars. The value of a European-style put option to sell U.S. dollars for British pounds is

$$p = 0.7143e^{-0.04288(0.5)}N(-d_2) - 0.6706e^{-0.05178(0.5)}N(-d_1)$$

where

$$d_1 = \frac{\ln(0.6706e^{-0.05178(0.5)}/0.7143e^{-0.04428(0.5)}) + 0.5(0.10^2)0.5}{0.10\sqrt{0.5}} = -0.9101$$

and

$$d_2 = -0.9101 - 0.10\sqrt{0.5} = -0.9809$$

The probabilities are

$$N(-d_1) = N(0.9101) = 0.8186 \quad \text{and} \quad N(-d_2) = N(0.9811) = 0.8367$$

Thus, the value of the European-style put is

$$p = 0.7143e^{-0.04428(0.5)}(0.8367) - 0.6706e^{-0.05178(0.5)}(0.8186) = 0.04959 \text{ GBP/USD}$$

The OV_OPTION_VALUE function can be used to verify this result, that is,

$$\text{OV_OPTION_VALUE}(0.6706, 0.7143, 0.5, 0.04428, 0.05178, 0.10, \text{"p"}, \text{"e"}) = 0.04959$$

The contract denomination is USD 1,400,000, so the value of the overall contract is GBP 69,429, as shown in the table below. Converting this value to USD using the current exchange rate, the USD value of this European-style put to sell British pounds for U.S.

dollars is USD 103,530 (i.e., USD 1.4912/GBP times GBP 69,429), exactly the same value as the European-style call to buy British pounds using U.S. dollars. In the interest of completeness, the value of the corresponding European-style call is USD 9,205, exactly the same value as the European-style put to buy British pounds for U.S. dollars.

Part (3): The values may again be computed using the formulas in Table 16.6. You should be able to reproduce the following values:

| European-Style Futures Option Values | | |
|--------------------------------------|---------|---------|
| | U.S. | Britain |
| Spot price | 1.4912 | 0.6706 |
| Futures prices | 1.4968 | 0.6681 |
| Volatility rate | 10.00% | 10.00% |
| Exercise price | 1.4000 | 0.7143 |
| Time to expiration | 0.5000 | 0.5000 |
| Interest rate | 5.178% | 4.428% |
| Value of call | 0.10353 | 0.00441 |
| Value of put | 0.00921 | 0.04959 |

These values are exactly the same as when the options were written directly on the spot currency, as we proved in Chapter 6.

The table below summarizes the results of this illustration.

| | USD/GBP | BP/USD |
|----------------------|---------|--------|
| Spot rate | 1.4912 | 0.6706 |
| 6-month forward rate | 1.4968 | 0.6681 |
| Time to expiration | 0.5000 | |

| | USD | GBP |
|-----------------|--------|--------|
| Interest rates | 5.178% | 4.428% |
| Exercise price | 1.4000 | 0.7143 |
| Volatility rate | 10.00% | |

| Buy USD Option to Buy/Sell GBP | | | Buy GBP Option to Buy/Sell USD | | |
|--------------------------------|-----------|-----------|--------------------------------|-----------|-----------|
| | Call | Put | | Call | Put |
| Option value (USD/GBP) | 0.103531 | 0.009205 | Option value (GBP/USD) | 0.004409 | 0.049592 |
| Quantity (BP) | 1,000,000 | 1,000,000 | Quantity (USD) | 1,400,000 | 1,400,000 |
| Total value (USD) | 103,531 | 9,205 | Total value (GBP) | 6,173 | 69,428 |
| | | | Total value (USD) | 9,205 | 103,531 |

| Buy USD Futures Option to Buy/Sell GBP | | | Buy GBP Futures Option to Buy/Sell USD | | |
|--|-----------|-----------|--|-----------|-----------|
| | Call | Put | | Call | Put |
| Option value (USD/GBP) | 0.103531 | 0.009205 | Option value (GBP/USD) | 0.004409 | 0.049592 |
| Quantity (GBP) | 1,000,000 | 1,000,000 | Quantity (USD) | 1,400,000 | 1,400,000 |
| Total value (USD) | 103,531 | 9,205 | Total value (BP) | 6,173 | 69,428 |
| | | | Total value (USD) | 9,205 | 103,531 |

ILLUSTRATION 16.6 Compute values and early exercise premiums of American-style USD/GBP options on spot and forward exchange rates.

Using the USD/GBP options and their parameters from Illustration 16.5, compute the values and early exercise premiums of the corresponding American-style options, and explain why there are differences between the values of the currency options and the futures options. Use the quadratic approximation method to handle the computations.

The values of American-style FX options can be computed using the quadratic approximation method by calling the function `OV_OPTION_VALUE` from the `OPTVAL` function library.⁷ The value of an American-style call option to buy British pounds using U.S. dollars is, for example,

$$\text{OV_OPTION_VALUE}(1.4912, 1.4000, 0.5, 0.05178, 0.04428, 0.10, \text{"c"}, \text{"a"}) = 0.10364$$

Applying the function for the remaining American-style FX options, we get:

| | European-Style Value | American-Style Value | Early Exercise Premium |
|-----------------------------------|----------------------|----------------------|------------------------|
| USD/BP options on currency | | | |
| Call | 0.10353 | 0.10364 | 0.00011 |
| Put | 0.00921 | 0.00937 | 0.00017 |
| USD/BP options on futures | | | |
| Call | 0.10353 | 0.10453 | 0.000995 |
| Put | 0.00921 | 0.00927 | 0.000068 |
| Difference | | | |
| Call | 0.00000 | -0.00089 | -0.00089 |
| Put | 0.00000 | 0.00010 | 0.00010 |

In reviewing these figures, we see that the values of the American-style options exceed the values of the corresponding European-style options for both the options on the currency and the options on the futures. This is expected, since the American-style options have the same terms as the European-style option but provide the additional benefit of early exercise. Interestingly, the American-style call on the currency has a lower value than the American-style option on the futures. The reason for this is that the futures price, USD 1.4968/GBP, is above the spot currency rate, USD 1.4912/GBP. If the futures price is higher than the currency rate, then exercising the call on the futures early will provide greater proceeds than exercising the call on the spot currency early. The opposite is true for the put.

RISK MANAGEMENT

Currency derivatives provide an effective means of managing different types of currency risk exposures. In the previous section, we reviewed the valuation of forward, futures, and options contracts as they apply to currencies. In this section, we

⁷ A detailed example of all the computations embedded in the quadratic approximation method was provided in Chapter 6.

illustrate how these valuation/risk measurement tools can be used to manage currency risk exposures. In the first illustration, we show how to use a currency swap to redenominate the currency of a bond issue and potentially generate interest savings. We also show how to compare its cost effectiveness with buying a strip of forward contracts. The second illustration focuses on managing the risk of a large foreign currency transaction that is known to occur in the future. We compare the expected return/risk attributes of hedging using forward, options, and money market instruments. We also consider the effects when the transaction, itself, is uncertain. The third illustration considers the case where there are multiple transactions to be hedged. Here we consider not only currency swaps but also a nonstandard product—an average rate option. The fourth illustration focuses on the risk management of balance sheet risk, that is, the uncertainty of having certain assets and liabilities on the balance sheet being denominated in foreign currencies.

Using Currency Swaps to Obtain Foreign Financing

In the first chapter of the book, we described a *plain-vanilla interest rate swap* as being a convenient means of “swapping” out of fixed rate debt into floating rate debt and vice versa. A *currency swap* is also a convenient means of restructuring debt—in this case swapping out of debt (interest payments and repayment of principal) denominated in one currency into debt denominated in another. Such swaps may be useful, for example, to a multinational firm that finds it comparatively less expensive to borrow domestically even though its financing need is in a foreign country.

To illustrate, suppose Canuck Brewing Inc., a small microbrewery in Canada, is looking to expand internationally by setting up breweries in other countries. Market research in different regions of the United States indicates that Canuck’s products will be most popular in the Southeast region of the United States. Canuck therefore decides to build a new brewery in North Carolina and requires USD 5 million to acquire the land.

Canuck is currently evaluating different financing proposals. All else being equal, a USD-denominated loan would be best since the interest payments will be made from U.S. sales. In this way, Canuck avoids currency risk on the interest payments. The problem is that Canuck is not well known in the U.S. The lowest available coupon interest rate that it can obtain on a three-year, fixed rate USD 5 million bond is 7.5%. In Canada, where Canuck’s credit is first rate and its products are well known, it can issue three-year, fixed rate bonds at 6.0%. Given the current exchange rate of CAD 1.40/USD, the par value of the Canadian bonds will be CAD 7.0 million.

USD Bonds The two alternative bond issues have different currency exposures. The first is denominated in U.S. dollars and therefore has no currency risk. The semiannual cash flows are contained in the following table. The continuously compounded implied yield to maturity of the loan is 7.36%.⁸

⁸The implied yield to maturity is the continuously compounded discount rate that equates the present value of the promised bond payments to the par value of the bond.

| U.S. Bonds | | Year | | | | | | | |
|-----------------|------------|------------------|----------|----------|----------|----------|----------|----------|------------|
| Par value (USD) | 5,000,000 | | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 |
| Coupon rate | 7.50% | Cash flows (USD) | -187,500 | -187,500 | -187,500 | -187,500 | -187,500 | -187,500 | -5,187,500 |
| PV (cash flows) | -5,000,000 | PV (cash flows) | -180,723 | -174,191 | -167,895 | -161,826 | -155,977 | | -4,159,388 |
| Implied yield | 7.36% | | | | | | | | |

CAD Bonds Plus Currency Swap The second alternative is denominated in Canadian dollars. Consequently, Canuck faces currency risk when its U.S. dollar sales are used to cover the Canadian dollar interest payments and principal repayment. To undo the currency risk exposure, Canuck considers entering a currency swap. After some negotiation with an OTC swap dealer in Canada, Canuck finds that it can enter into a fixed-for-fixed currency swap in which it will receive interest at a rate of 6% on a CAD 7 million par amount and will pay interest at a rate of 7.25% on USD 5 million par. Payments will be made semiannually. Thus, if Canuck issues the Canadian bonds and enters the currency swap, it will have locked in a U.S. dollar denominated loan at a coupon interest rate of 7.25%, 25 basis points lower than it would have had it issued the U.S. bonds directly. The following table shows the combined cash flows of the Canadian dollar bond and the currency swap. Note that, unlike a plain-vanilla interest rate swap, the currency swap *requires* an exchange of principal. The continuously compounded implied yield to maturity of this alternative is 7.12%.

| Canadian Bonds | | Year | | | | | | | |
|-----------------------|------------|------------------|----------|----------|----------|----------|----------|----------|------------|
| Par value (CAD) | 7,000,000 | | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 |
| Coupon rate | 6.00% | Cash flows (CAD) | -210,000 | -210,000 | -210,000 | -210,000 | -210,000 | -210,000 | -7,210,000 |
| Swap Agreement | | | | | | | | | |
| Receive leg (CAD) | | | | | | | | | |
| Par value (CAD) | 7,000,000 | Cash flows (CAD) | 210,000 | 210,000 | 210,000 | 210,000 | 210,000 | 210,000 | 7,210,000 |
| Coupon rate | 6.00% | | | | | | | | |
| Pay Leg (US) | | | | | | | | | |
| Par value (USD) | 5,000,000 | Cash flows (US) | -181,250 | -181,250 | -181,250 | -181,250 | -181,250 | -181,250 | -5,181,250 |
| Coupon rate | 7.25% | | | | | | | | |
| Net Payments (USD) | | Cash flows (US) | -181,250 | -181,250 | -181,250 | -181,250 | -181,250 | -181,250 | -5,181,250 |
| PV (cash flows) | -5,000,000 | PV (cash flows) | -174,910 | -168,791 | -162,886 | -157,188 | -151,689 | | -4,184,536 |
| Implied yield | 7.12% | | | | | | | | |

By issuing Canadian bonds and engaging in a currency swap, Canuck has managed to reduce the effective cost of financing from 7.36% to 7.12%. How does this saving arise? One possibility is that the terms of the swap were favorable to Canuck. To examine this explanation, we need to value the swap at inception. For simplicity, assume the risk-free term structure of interest rates is

flat in both Canada and the United States, and the rates are 4.25% and 5.00%, respectively. Using these risk-free interest rates to discount the flows of each leg of the swap, we get the following:

| Risk-Free Rates | | Year | | | | | | |
|-----------------------|------------------|-----------------------|----------|----------|----------|----------|----------|------------|
| | | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 |
| CAD rate | 4.25% | | | | | | | |
| USD rate | 5.00% | | | | | | | |
| Swap Agreement | | | | | | | | |
| Receive leg (CAD) | Cash flows (CAD) | 210,000 | 210,000 | 210,000 | 210,000 | 210,000 | 210,000 | 7,210,000 |
| PV (receive) | 7,332,512 | PV (cash flows) | 205,585 | 201,262 | 197,030 | 192,888 | 188,832 | 6,346,916 |
| Pay leg (US) | Cash flows (USD) | -181,250 | -181,250 | -181,250 | -181,250 | -181,250 | -181,250 | -5,181,250 |
| PV (pay) | -5,300,836 | PV (cash flows) (USD) | -176,775 | -172,410 | -168,154 | -164,002 | -159,953 | -4,459,543 |
| PV (pay) (CAD) | -7,421,171 | | | | | | | |
| Swap value | -88,659 | | | | | | | |

The value of the currency swap at origination from Canuck's perspective is

$$V = B_d - SB_f$$

where B_d is the present value of what Canuck receives (in Canadian dollars), B_f is the present value of what Canuck pays in U.S. dollars, and S is the CAD/USD exchange rate. Using the numbers in the table above, the present value of the receive leg is CAD 7,332,512, and the present value of the pay leg is USD 5,300,836 or CAD 7,421,171. The value of the swap is therefore -CAD 88,659. The figure represents a trading cost implicitly paid by Canuck to the swap dealer. Clearly, the terms of the swap are not driving the cost savings.

CAD Bonds Plus Forward Strip Is there an alternative to the currency swap that Canuck can consider to avoid the swap dealer's margin? The answer is yes. We know all swaps can be decomposed into portfolios of forwards and/or options. The current exchange rate is CAD 1.40/USD, and the Canadian and U.S. risk-free interest rates are 4.25% and 5.00%, respectively. By interest rate parity, the six-month forward exchange rate (i.e., CAD/USD) must be $1.4000e^{(0.0425-0.0500)0.05} = 1.3948$, the one-year forward rate $1.4000e^{(0.0425-0.0500)0.05} = 1.3895$, and so on. These CAD/USD forward rates can be inverted to get USD/CAD forward rates of 0.7143, 0.7170, and so on. Canuck can buy a strip of USD/CAD forwards, each with a contracted amount and time to delivery corresponding to the Canadian dollar payments to bondholders. These trades would commit Canuck to a stream of USD payments beginning with USD 150,564 in six months, USD 151,129 in 1 year, and so on. In return, Canuck would receive Canadian dollar payments in the amounts it needs to service the Canadian bondholders, as is shown in the next table. The implied yield to maturity under this arrangement is only 6.66%.

| Exchange Rates | | | Year | | | | | | |
|--------------------|-----------|---------------------------|--------|----------|----------|----------|----------|----------|------------|
| | | | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 |
| Spot rate | 1.4000 | CAD/USD | 1.4000 | 1.3948 | 1.3895 | 1.3843 | 1.3792 | 1.3740 | 1.3689 |
| CAD risk-free rate | 4.25% | USD/CAD | 0.7143 | 0.7170 | 0.7197 | 0.7224 | 0.7251 | 0.7278 | 0.7305 |
| USD risk-free rate | 5.00% | | | | | | | | |
| Canadian bonds | | | | | | | | | |
| Par value (CAD) | 7,000,000 | Coupon payments (CAD) | | -210,000 | -210,000 | -210,000 | -210,000 | -210,000 | -7,210,000 |
| Interest rate | 6.00% | Received on forward (CAD) | | 210,000 | 210,000 | 210,000 | 210,000 | 210,000 | 7,210,000 |
| Par value (USD) | 5,000,000 | Paid of forward (USD) | | -150,564 | -151,129 | -151,697 | -152,267 | -152,839 | -5,267,188 |
| Implied yield | 6.66% | PV (cash flows) (USD) | | -145,631 | -141,389 | -137,271 | -133,273 | -129,391 | -4,313,044 |

Using FX Futures and Options to Manage Transaction Risk— Single Flow

Transaction risk refers to the currency risk of a particular future transaction denominated in a foreign currency. Suppose, for example, Jetmaker, Inc., a U.S. jet manufacturer, receives an order for its new X626 plane from Alps Air, Inc., a Swiss airline. Payment for the new X626 is specified in Swiss francs and is to be made when the plane is delivered in six months. Jetmaker is exposed to significant currency risk. The Swiss franc may depreciate relative to the U.S. dollar over the next six months (i.e., the value of a Swiss franc, USD/SF, may fall), driving the U.S. dollar proceeds from the Swiss franc payment downward. Consider the following short hedging strategies will allow Jetmaker to reduce its transaction risk exposure.

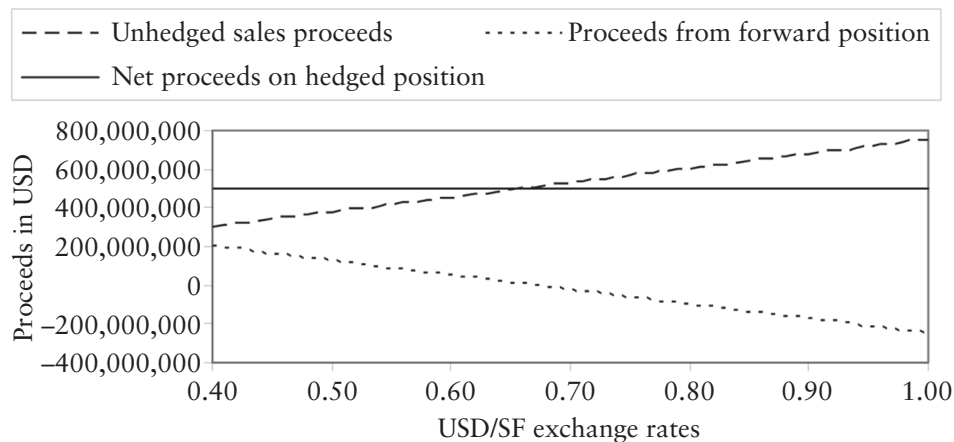
Short-Hedging Using a Forward Contract⁹ Suppose the cost of the X626 is SF 750 million. Assume also that the current exchange rate is USD 0.66/SF and that the six-month forward rate is USD 0.66667/SF. If Jetmaker chooses not to hedge, the USD value of the contract is subject to fluctuations in the exchange rate. If the Swiss franc depreciates relative to the U.S. dollar and the exchange rate is USD 0.60/SF in six months, Jetmaker receives USD 450 million (i.e., USD 0.60 times SF 750 million). On the other hand, if the Swiss franc appreciates to, say, USD 0.70/SF in six months, the firm receives USD 525 million. Jetmaker wants to eliminate this risk exposure.

To do so, Jetmaker can hedge by selling the SF 750 million exposure in the forward market. The current six-month forward rate is USD 0.66667. Selling the forward implies that Jetmaker will receive exactly USD 500 million in six months. If the exchange rate falls to USD 0.60/SF, for example, the net proceeds are USD 500 million—USD 450 million from the sale of the plane plus USD 50

⁹ Futures contracts could also be used, but the size and maturity of a futures contract may not match the hedging need exactly.

million (i.e., USD 0.06667/SF times SF 750 million) from the short forward position. If the exchange rate rises to USD 0.70/SF, the net proceeds remain at USD 500 million—USD 525 million from the sale of the plane and –USD 25 million (i.e., –USD 0.03333/SF times SF 750 million) from the short forward position.

The following figure summarizes the net proceeds in six months over a wider range of USD/SF exchange rates. As the Swiss franc appreciates (depreciates) relative to the U.S. dollar, Jetmaker's unhedged sales proceeds increase (decrease). The proceeds from the short forward position, however, fall (rise) by an equal amount. The combination of the unhedged and forward positions, therefore, creates a certain net proceeds of USD 500 million in six months.



Short-Hedging in the Money Market By the cost of carry relation (or, in this case, interest rate parity), an equivalent hedge can be executed in the money market. Assume the U.S. risk-free rate of interest is 5.25%. The current exchange rate is USD 0.66/SF and the six-month forward exchange rate is USD 0.66667/SF. By interest rate parity, therefore, the six-month risk-free rate in Switzerland must be 3.24%.

Under a *money market hedge*, Jetmaker borrows against the SF 750 million payment that it will receive in six months. The amount of the loan is SF 737,947,886 (i.e., $\text{SF } 750,000,000e^{-0.0324(0.5)}$). Jetmaker then converts the Swiss franc proceeds into U.S. dollars at the current exchange rate and gets USD 487,045,605 (i.e., USD 0.66/SF times SF 737,947,886). The U.S. dollars are then invested at the U.S. risk-free rate. In six months, Jetmaker uses the Alps Air payment of SF 750 million to retire its loan and enjoys a USD 500 million (i.e., $\text{USD } 487,045,768e^{0.0525(0.5)}$) balance in its U.S. account.

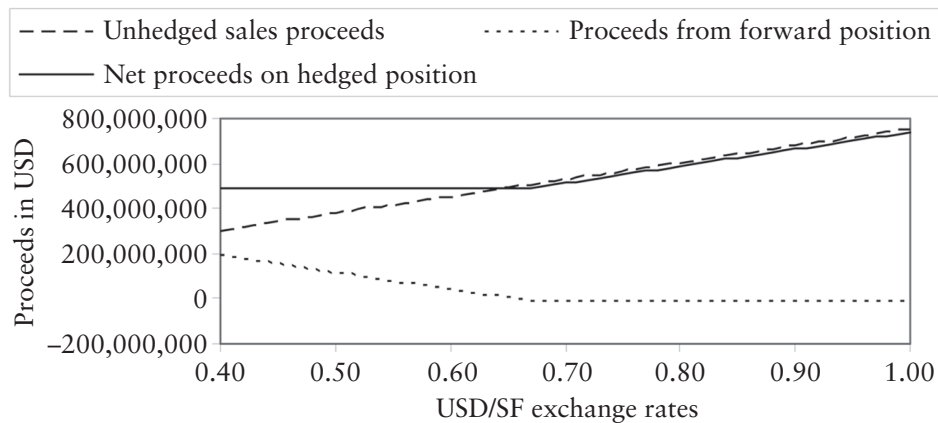
Short-Hedging Using an Option Yet another alternative is to hedge by buying a European-style USD/SF put option. The benefit of doing so is that, if the Swiss franc appreciates relative to the U.S. dollar (i.e., USD/SF rises), Jetmaker receives the gain. On the other hand, if the Swiss franc depreciates relative to the U.S. dollar (i.e., USD/SF falls), Jetmaker's reduced sales proceeds are offset by the exercise proceeds of the put. Nothing is free, however. Jetmaker must pay for the put.

To illustrate, suppose that a six-month put with an exercise price of USD 0.66667/SF and a denomination of SF 750 million costs USD 0.01465/SF or

USD 10,990,032 in total. The cost of the put carried forward six months is USD $10,990,032e^{0.0525(0.5)}$, which must be paid regardless of the movement in the exchange rate. If the spot exchange rate is USD 0.60/SF in six months, Jetmaker receives USD 450 million from Alps Air and USD 50 million on its put position, thereby netting USD 488,717,660. On the other hand, if the spot exchange rate is USD0.70/SF in six months, Jetmaker receives USD 525 million from the sale of the X626, lets the put expire out of the money, and thereby nets USD 513,717,660, as shown in this table:

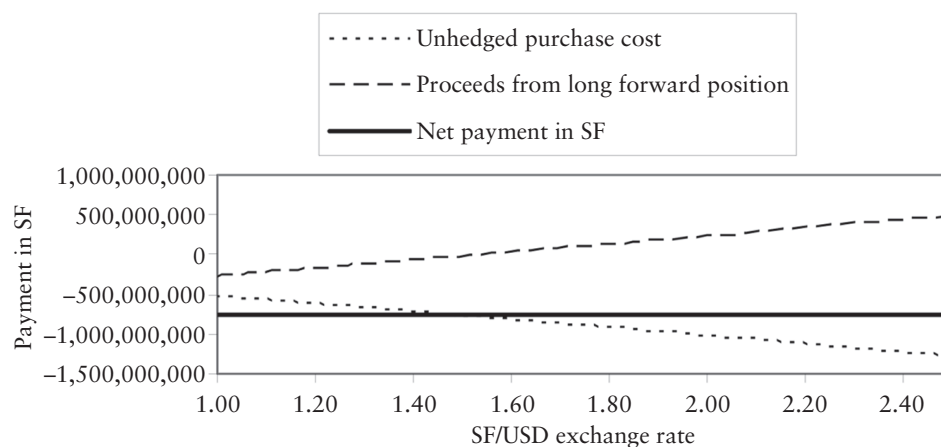
| | Spot Rate in 6 Months | |
|--------------------------------------|-----------------------|-------------|
| | 0.60000 | 0.70000 |
| Cost of put carried forward 6 months | -11,282,340 | -11,282,340 |
| Sales proceeds in USD | 450,000,000 | 525,000,000 |
| Proceeds from exercising put | 50,000,000 | 0 |
| Net proceeds | 488,717,660 | 513,717,660 |

The next figure summarizes the net proceeds from the put option hedge for a wider range of exchange rates. As the USD/SF exchange rate rises, the net proceeds from the sale of the jet increase. At the same time, the put option expires worthless allowing Jetmaker to enjoy the gain (net of the cost of the put). On the other hand, if the USD/SF rate falls, the unhedged proceeds of the X626 sale fall, however, they are exactly offset by the exercise proceeds from the put. The maximum loss is USD 11,282,340—the purchase price of the at-the-money put.



Long-Hedging Using a Forward Managing the transaction risk befalls Alps Air if the cost of the plane had been quoted in U.S. dollars. Suppose that all of the parameters in our illustration remain the same, except the cost of the X626 is USD 500 million payable in six months at the time of delivery. Alps Air faces the risk that the Swiss franc will depreciate relative to the U.S. dollar over the next six months, which means paying more Swiss francs to buy the plane. One long-hedging strategy is to buy a six-month forward contract on U.S. dollars. Since the current six-month forward rate to buy Swiss francs with U.S. dollars is USD 0.66667/SF, the current six-month forward rate to buy U.S. dollars with Swiss francs must be SF 1.50/USD. By

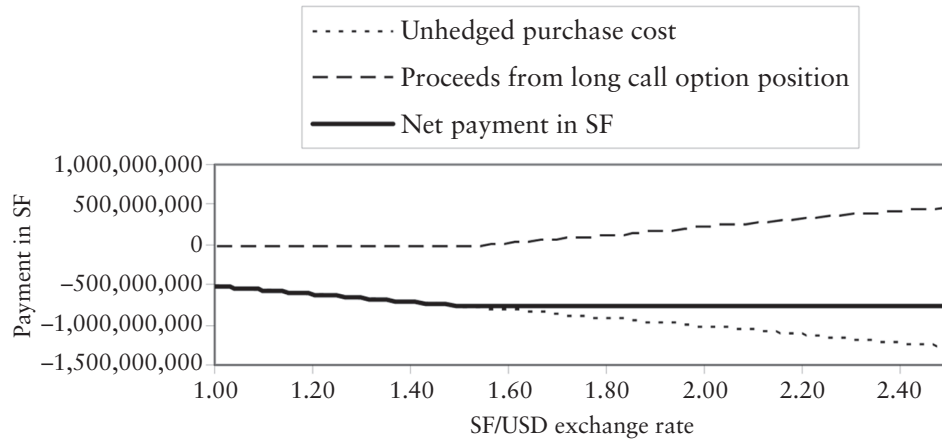
buying a forward contract at SF 1.50/USD, the cost of the plane is locked in at SF 750 million. If the Swiss franc depreciates relative to the U.S. dollar (i.e., USD/SF falls and SF/USD rises) and the spot exchange rate is, say, SF 1.66667/USD (i.e., USD0.60/SF) in six months, Alps Air pays SF 833,333,333 million to buy the plane but has earned SF 83,333,333 from its long forward position. Alps Air's net payment is, of course, SF 750 million. On the other hand, if the Swiss franc appreciates relative to the U.S. dollar (i.e., USD/SF rises and SF/USD falls) to, say, SF 1.42857/USD (i.e., USD 0.70/SF) in six months, the firm must pay SF 714,285,714 to buy the plane and SF 35,714,286 to cover its forward obligation. The net payment is again SF 750 million. Thus Alps Air can eliminate all of its transaction risk by buying forward, as is shown in the following figure. By interest rate parity, we know that risk elimination can also be accomplished using a money market hedge.



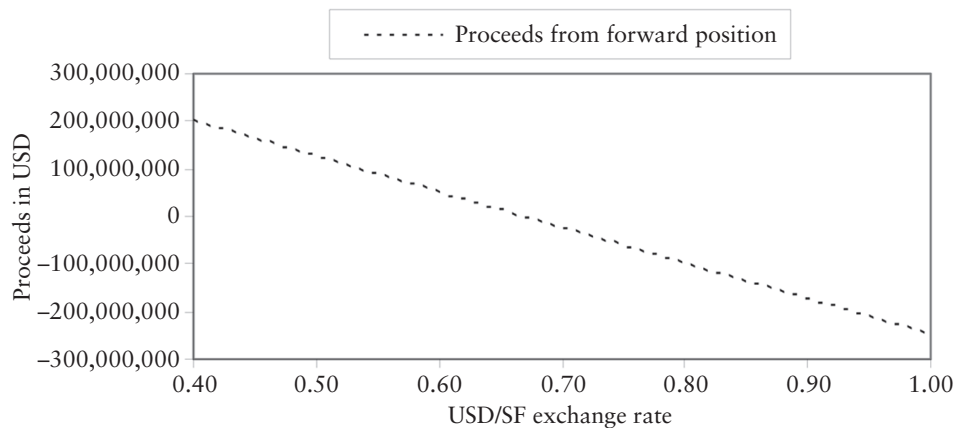
Long-Hedging Using an Option Alps Air can also long hedge by buying a European-style call option. The benefits are twofold. First, if the U.S. dollar appreciates relative to the Swiss franc (i.e., the SF/USD rate increases), Alps Air receives a subsidy in the form of the exercise proceeds on call. Second, if the U.S. dollar falls, the Swiss franc payment is reduced and the call expires out of the money. The cost of the hedge is, of course, that the firm must pay for the call.

To illustrate, suppose that a six-month call with an exercise price of SF 1.50/USD and a denomination of USD 500 million costs SF 0.03330/USD or SF 16,651,563 in total. Carrying this forward six months implies a terminal cost of the call of SF 16,923,510 (i.e., $SF 16,651,563e^{0.0324(0.5)}$), as is indicated in the first row of the table below. If the exchange rate falls to SF 1.42857, the net payment is SF 731,209,224—the purchase price of USD 500 million times SF 1.42857 or SF 714,285,714, plus the cost of the call carried forward, SF 16,923,510. The call expires worthless. On the other hand, if the exchange rate rises to, say, SF 1.66667/USD, the net payment is SF 766,923,510—the cost of the call carried forward, SF 16,923,510, plus purchase price of SF 833,333,333, less the exercise proceeds of the call, SF 83,333,333. Note that the exercise proceeds of the call always reduce the purchase price of the X626 to SF 750 million. The maximum that Alps Air will pay for the acquisition of the plane is SF 750 million plus the cost of the call carried forward, SF 16,923,510, as is shown in this next figure.

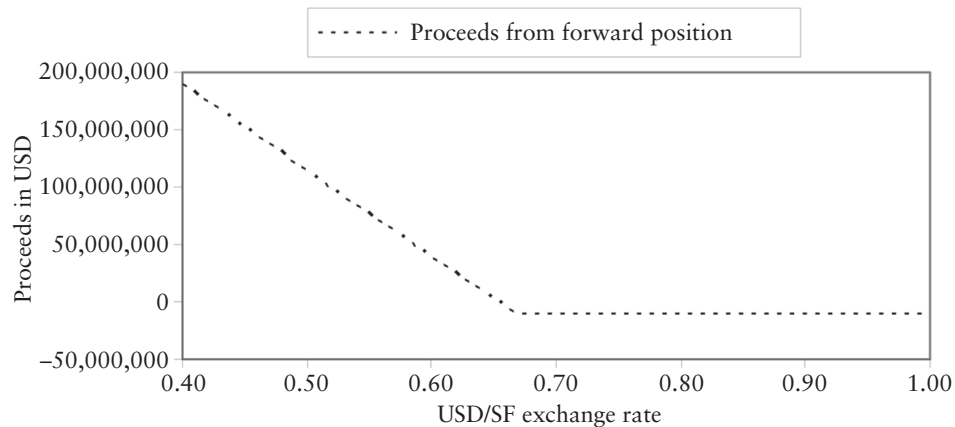
| | Spot Rate in 6 Months | |
|---------------------------------------|-----------------------|--------------|
| | 1.42857 | 1.66667 |
| Cost of call carried forward 6 months | -16,923,510 | -16,923,510 |
| Purchase price in SF | -714,285,714 | -833,333,333 |
| Exercise proceeds from call in SF | 0 | 83,333,333 |
| Net payment | -731,209,224 | -766,923,510 |



Hedging an Uncertain Transaction The hedging strategies discussed above presume that the purchase/sale of the plane will take place in six months. Hedging using a forward contract eliminates all transaction price risk, and hedging using an option contract, while costly, eliminates the downside transaction price risk and retains the upside transaction price risk. In many instances, however, the transaction may be uncertain. Suppose, for example, Alps Air has the right to cancel the agreement with Jetmaker at any time during the next six months. Jetmaker faces the risk that the Swiss franc will depreciate in value (i.e., USD/SF falls and SF/USD rises), but, if it sells forward to short hedge the foreign exchange risk, and the agreement is cancelled, it is left with an open currency forward position that may have to be liquidated at a loss as indicated in the following figure. Jetmaker’s losses are unlimited as the Swiss franc depreciates without limit.



Under put option short-hedging strategy, however, Jetmaker locks in its maximum exposure at the cost of the put option, USD 11,282,340. If the sale is consummated, the least that Jetmaker will receive is USD 488,717,660. If the Swiss franc appreciates in value (i.e., USD/SF rises and SF/USD falls), Jetmaker receives more. On the other hand, if the sales agreement is cancelled, Jetmaker's loss is limited to the cost of the put. In the event that the Swiss franc appreciates, Jetmaker may even gain, as shown in the figure that follows. In effect, Jetmaker is buying a put option to hedge the cancellation option they have given Alps Air. Jetmaker has implicitly given Alps Air a put option to sell SF 750 million in return for the X626. Jetmaster hedges that risk by buying a put.



Using FX Futures and Options to Manage Transaction Risk— Multiple Flows

Multinational firms often wish to create a package of hedges to manage the price risk of its inputs or outputs. Consider an Australian firm that produces goods to sell in Australia but imports its raw materials from Japan. Assuming that the Australian firm has negotiated the sales price of its production over the next 12 months, it may want to hedge the currency risk of its Japanese input costs. If its production input needs are predictable, fixed rate swaps and average rate option contracts may be added to the risk management arsenal.

ILLUSTRATION 16.7 Comparing option alternatives.

Suppose a U.S. firm wants to hedge its foreign input costs by buying insurance. More specifically, the firm has entered into a contract to buy 100,000 widgets per month over the next two years at EUR 0.50 per widget. The current exchange rate is USD 1.20/EUR, and its volatility rate is 10%. The risk-free rate of interest in the U.S. is 5%, and the rate in Europe is 6%. Compare the costs of (1) buying a portfolio of at-the-money call options expiring at the end of each month through the two-year period, (2) buying a single two-year at-the-money call option for the contract amount, and (3) buying an arithmetic average-rate at-the-money call option where the underlying exchange rate is an average of the month-end exchanges rates over the life of the contract.

The firm has entered an agreement to buy widgets at a fixed price of EUR 0.50 per widget and at the rate of 100,000 per month for 24 months. This means that the firm will make annuity payments of EUR 50,000 each month, or EUR 120,000 in total over the life of the agreement. To hedge against a depreciating USD, it is considering different call option alternatives.

The first alternative is to buy a strip of at-the-money call options, one expiring each month. In the event the exchange rate rises to, say, USD 1.30/EUR by the end of the first month, for example, the option will pay USD $1.30 - 1.20 = 0.10$. The net cost of buying euros at the end of the first month is therefore USD $1.30 - 0.10 = 1.20$ /EUR, or 50,000 times USD 1.20 or USD 60,000 in total. The value of the one-month call per euro is

$$\text{OV_OPTION_VALUE}(1.20, 1.20, 1/12, 0.05, 0.06, 0.10, \text{"C"}, \text{"E"}) = 0.05450$$

or USD 0.05450/EUR times 50,000 euros or USD 2,725 in total. If we repeat this computation 23 more times, once for each monthly cash flow, the total value of all option premiums is USD 199,853.

The second alternative is to buy a single 24-month at-the-money call option. The value of this option per euro is

$$\text{OV_OPTION_VALUE}(1.20, 1.20, 2, 0.05, 0.06, 0.10, \text{"C"}, \text{"E"}) = 0.2288$$

or USD 0.2288/EUR times 1,200,000 euros or USD 274,575 in total.

The final alternative is to buy an at-the-money average rate call option. The option's time to expiration is two years, and the final cash settlement price is the difference between the arithmetic average month-end exchange rate and the exercise price of the option, that is,

$$\max\left(\frac{1}{24} \sum_{t=1}^{24} S_t - 1.20\right)$$

where S_t is the USD/EUR exchange rate at the end of each month. To value an at-the-money, arithmetic average rate option, an approximation method is necessary. In Chapter 7, we showed how Monte Carlo simulation can be used to value so-called "Asian-style options," one of which is an option on an average-rate. The OPTVAL function

$$\text{OV_APPROX_ASIAN_OPT_MC}(s, x, t, r, i, v, n, ntrial, cp, sx, ag)$$

can be used. The first six parameters were defined earlier. The parameter n is the total number of observations used in computing the average. If the option's life is two years and monthly observations are used, n is set equal to 24.¹⁰ The parameter $ntrial$ is the number of simulation runs. The parameter cp is either "C" or "P," depending upon whether you are valuing a call or a put. The parameter sx is either "S" or "X," depending upon whether you are averaging the asset price to replace the asset price or the exercise price of the average rate option. Finally, the parameter ag is either "A" or "G," depending upon whether the average rate of the option is arithmetic or geometric. For the illustration at hand,

$$\begin{aligned} \text{OV_APPROX_ASIAN_OPT_MC}(1.20, 1.20, 2, 0.05, 0.06, 0.10, 24, 10000, \text{"C"}, \text{"S"}, \text{"A"}) \\ = 0.1370 \end{aligned}$$

This means that the cost of the average-rate option alternative is USD 0.1370/EUR, or 0.1370 times 1,200,000 or USD 164,459 in total.

¹⁰ If n is set equal to one, the value of an average rate option should equal the value of a European-style option. They will not be exactly the same, however, since the Monte Carlo simulation is an approximation method.

Using FX Futures and Options to Manage Balance Sheet Risk

In many cases, a firm faces currency risk that is not tied to a particular transaction but rather to a particular asset or liability on the firm's balance sheet. A firm exporting to Ireland, for example, is likely to have both significant accounts receivable and inventory denominated in euros. Indeed, a firm's balance sheet may have both assets and liabilities denominated in various foreign currencies.

Managing *balance sheet risk* depends on whether the foreign currency obligation is contractual or not. A *contractual* obligation denominated in a foreign currency is subject to exchange rate risk because the contract price is set and cannot be changed. If the value of the currency falls, the foreign currency price cannot be adjusted. On the other hand, a *noncontractual* business operation is less subject to exchange risk because changes in exchange rates may be partially offset by price changes in the foreign currency. A U.S. widget manufacturer with an Irish subsidiary, for example, may find it possible to offset declines in the value of the euro by increasing the price of widgets sold in Ireland.

The nature of the balance sheet hedge depends on the nature of the underlying asset/liability. In the normal course of operation, a U.S. firm operating an Irish subsidiary might find it necessary to have a large euro balance in the subsidiary's cash account. Worried about a possible decline in the euro, the parent can hedge the cash position by selling futures. If the USD/EUR exchange rate falls, the decline in the U.S. dollar value of the cash will be offset by the profit on the short futures position. This hedge is analogous to the transaction risk hedge in the sense that both are contractual. This balance sheet hedge will not be as effective, however. The mismatch in the terms to maturity of the cash balance (term to maturity = 0) and futures contracts (term to maturity > 0) means that slippage will be incurred due to basis risk.¹¹ In addition, unlike a fixed transaction price agreement, the cash balance changes day to day from normal business operations.

An example of a balance sheet hedge of a noncontractual asset is hedging finished goods inventory denominated in euros. This hedge may be more complicated because a decline in the USD/EUR exchange rate will reduce the U.S. dollar value of the inventory. This, however, might be offset by an increase in the price of the finished goods in Ireland. Indeed, under purchasing power parity, a full price adjustment is expected. But the subsidiary may not have the freedom to change prices in a dramatic way. To illustrate, suppose prices may be increased by only 50% of the amount they should increase under PPP. Since the ability to re-price provides a partial (50%) natural hedge of the finished goods inventory, only half the inventory balance needs to be hedged in the futures market.

SUMMARY

This chapter focuses on the management of currency risk using derivatives contracts. In the first section, currency derivative markets are discussed. The lion's

¹¹ Deciding the appropriate term to maturity of the futures contract is not straightforward. The shorter the term to maturity of the futures, the less the basis risk but the greater the trading costs associated with frequent rollovers in the futures position.

share of currency derivatives trading takes place in the OTC market, although the trading volumes on futures and options exchanges are respectable. In the second section, the principles of currency derivatives valuation and risk measurement are provided. No-arbitrage price relations and valuation equations/methods are provided for currency forwards, futures, options, and swaps. All of them are on the continuous net cost of carry results developed in Chapters 4 through 9. The continuous net cost of carry rate for currencies is the domestic risk-free interest rate less the foreign risk-free interest rate. The third section illustrates a number of important currency risk management strategies. Among them are using a currency swap or a strip of currency forwards to re-denominate fixed-rate debt in one currency into another, using forward/options to manage the price risks of single and multiple transactions, and using forward/options to manage balance sheet risk.

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