CHAPTER 13

Compensation Agreements

Firms often struggle with identifying appropriate compensation packages for their employees. One important ingredient in designing such a package is stock ownership. By providing employees with the shares of the firm, or claims on the shares of the firm, management aligns the interests of employees with those of owners (i.e., the shareholders). Two common contracts used in this context are an *employee stock option* (ESO) and an *employee stock purchase plan* (ESPP). Like a warrant, an ESO is a call option contract issued by the firm. Typically, ESOs are at-the-money at the time of issuance (i.e., the exercise price is set equal to the stock price) and have terms to expiration of ten years. Over the first few (usually three) years, the options cannot be exercised. This is called the *vesting period*. If the employee leaves the firm during the vesting period, the options are forfeit. After the vesting date, the options can be exercised at any time but are *nontransferable*. Because they are nontransferable, the only way for the employee to capitalize on its value is to exercise the option. An ESPP allows the employee to buy the company's stock at a discount, usually 15%, within a certain period of time, typically six months. Some the ESPP includes a lookback provision that allows its holder to apply the discount to either the end-of-period or the beginning-of-period stock price, whichever is less.

The purpose of this chapter is to describe how to value some of the different types of stock compensation contracts that exist in practice. The first section shows that standard ESOs can be valued as call options, even though they have longer terms to expiration, greater dividend uncertainty, and vesting considerations. The second section addresses the valuation effects of the vesting period. As it turns out, unless the employee leaves the firm before the vesting period is up, being precluded from exercising the option has little effect on ESO value. The third section examines the consequences of the practice of using a constant dividend yield model to value ESOs. The resulting valuation errors may be quite large. The fourth section focuses on ESO valuation in circumstances where employees are known to exercise their options early, even though it is suboptimal to do so. We then turn to valuing two important, albeit more specialized, employee stock options—ESOs with indexed exercise prices in the fifth section and ESOs with reload features in the sixth. The last section of the chapter focuses on the valuation of ESPPs.

STANDARD EMPLOYEE STOCK OPTIONS

Like warrants and convertible bonds, the exercise of employee stock options (ESOs) dilutes the value of existing shares. For most employee stock option plans, however, the existing shareholder base is so large that the dilution factor and the effect on ESO valuation are small. Consequently, the BSM call option valuation equations/ methods that we applied to exchanged-traded stock options in Chapter 11 can also be applied here. All that is needed are estimates of the interest rate, the expected dividend stream, and the expected volatility rate.

Given the long-term nature of ESOs, we need to be especially careful in estimating the parameters that go into determining option value. Small changes in the parameter values can produce large changes in value. Probably the best estimate for the risk-free interest rate is the continuously compounded yield on a U.S. Treasury strip bond with the same maturity date as the ESO. Recall from Chapter 2 that this rate is computed using the transformation,

$$r = \frac{\ln(100/B)}{T}$$

where B is the price of the strip bond as a percentage of par, and T is the term to maturity of the bond expressed in years. The expected dividend stream for the underlying stock should account for the discrete nature of quarterly cash dividend payments. In Chapter 11, we discussed dividend payment practices by U.S. firms. In that discussion, we noted that firms tend to (1) pay the same cash dividend each quarter throughout the year; (2) pay the quarterly dividends at the same times each year; and (3) increase the annual total cash dividends at a constant rate through time. Consequently, projecting the amount and timing of quarterly cash dividends over the ESO's life is not as difficult as it might seem at first blush.[1] Finally, to estimate the volatility parameter, historical return data should be used.[2] It is probably a good idea to use weekly returns rather than daily returns to mitigate the effects of measurement errors in prices.[3] The length of the return history needs to be at least as long as the ESO's life to so that we can be comfortable that we are seeing the firm's share price across of the range of business cycles that it might face over such a long period. Where the firm is newly-listed and does not have a long price history, all available price data should be used. Then, it would then be wise to compare the estimate against the historical volatility estimates of the stocks in the firm's peer group.

[1] Naturally, more thought must be given to situations in which the firm does not currently pay dividends but may do so during the life of the option. One potentially useful source of information is the dividend yield levels of firms in a comparable peer group.

[2] The best estimate of volatility is the implied volatility from exchange-traded options on the firm's stock. But, this is true only if the exchange-traded options have the same time to expiration as the ESOs. In general, this is not the case—exchange-traded stock options have much shorter times to expiration than ESOs. Consequently, implied volatility from short-term options is a very noisy predictor of expected long-term volatility.

[3] Recall that the effects of measurement errors were discussed at the end of Chapter 7.

In valuing employee stock options, the effects of discrete cash dividend payments need to be recognized explicitly. Using a constant dividend yield assumption can produce significant errors, as we will show later in the chapter. Discrete dividends have a distinctly different effect on early exercise than continuous dividends. Recall that, in Chapter 6, it was shown that an American-style call will not optimally be exercised just prior to a dividend payment if the amount of the dividend is less than the present value of the interest that can be earned implicitly on the exercise price by holding the option,[4] that is,

$$D_i < X(1 - e^{-r(t_{i+1} - t_i)}) \tag{13.1}$$

A quick check of the projected cash dividends will tell us if early exercise is likely or not. Suppose that we are valuing a 10-year, at-the-money stock option. The stock price is 100, and the stock's volatility rate is 36%. The firm's next quarterly dividend is to be paid in 20 days and will be 1.00 per share. The same dividend is expected to be paid for the next three quarters. The quarterly dividends (the same each quarter) are expected to grow by 5% annually. The quarterly dividends in the second year, therefore, are projected to be 1.0513 (= $1.00e^{0.05}$). In the final year, the quarterly dividends are 1.568. Now, compare these dividend amounts to the present value of the interest income that will be earned implicitly by deferring exercise. The amount for all quarters except for the last is

$$PVInt_i = X(1 - e^{-r(t_{i+1} - t_i)}) = 100(1 - e^{-0.07(91/365)}) = 1.730$$

as is shown in Table 13.1. Comparing each quarterly dividend with this amount shows that early exercise will not be optimal until the very last quarter of the option's life, if at all. In the last quarter, the present value of the interest income is

$$PVInt_{40} = 100(1 - e^{-0.07(81/365)}) = 1.541$$

which means that early exercise just prior to the last dividend payment *may* be optimal. The maximum amount of the early exercise premium, however, is small. The maximum gain from exercising just prior to the last dividend is 1.568 – 1.541 = 0.027. The maximum gain is the present value of this amount or 0.014, as indicated in the table. Indeed, the actual early exercise premium is trivial. The OPTVAL Function Library contains binomial and trinomial routines for valuing European- and American-style options on dividend-paying stocks. The syntax of the function call for the binomial method is

OV_STOCK_OPTION_VALUE_BIN(*s, x, t, r, v, n, cp, ae, mthd, dvd, tdvd*)

where *s* is the current stock price, *x* is the exercise price, *t* is the time to expiration, *r* is the risk-free interest rate, *v* is the stock's volatility rate, *cp* is a (c)all/(p)ut indicator, *ae* is an (A)merican/(E)uropean-style option indicator, *mthd* is

[4] This was condition (6.16) in Chapter 6.

TABLE 13.1 Evaluating the optimality of early exercise for a long-term employee stock option.

Stock	
Price (S)	100
Current dividend (D_1)	1.0000
Dividend growth (g)	5.00%
No. of days to 1st divd.	20
Volatility rate (σ)	36.00%
Market	
Interest rate (r)	7.00%
Option parameters	
Exercise price (X)	100
Years to expiration (T)	10.00
Discrete dividend assumption:	
PVD	35.571
S-PVD	64.429
Call option value	
European-style, analytical (c)	32.529
European-style, binomial (c)	32.523
American-style, binomial (C)	32.523
Final dividend	1.568
Days remaining	81
Present value of interest	1.541
Maximum gain from exercise	0.027
Maximum exercise premium	0.014
Continuous dividend yield assumption:	
Implied dividend yield	4.396%
Call option value (yield)	
European-style, analytical (c)	32.529
European-style, binomial (c)	32.522
American-style, binomial (C)	37.271
Difference	4.748
Percent error	14.60%
No. of time steps	1,000
Method	2

Quarterly Dividends					
i	Days	t_i	D_i	$PVInt_i$	$PV(D_i)$
1	20	0.0548	1.0000	1.730	0.9962
2	111	0.3041	1.0000	1.730	0.9789
3	202	0.5534	1.0000	1.730	0.9620
4	293	0.8027	1.0000	1.730	0.9454
5	384	1.0521	1.0513	1.730	0.9766
6	475	1.3014	1.0513	1.730	0.9597
7	566	1.5507	1.0513	1.730	0.9431
8	657	1.8000	1.0513	1.730	0.9268
9	748	2.0493	1.1052	1.730	0.9575
10	839	2.2986	1.1052	1.730	0.9409
11	930	2.5479	1.1052	1.730	0.9246
12	1,021	2.7973	1.1052	1.730	0.9086
13	1,112	3.0466	1.1618	1.730	0.9387
14	1,203	3.2959	1.1618	1.730	0.9225
15	1,294	3.5452	1.1618	1.730	0.9065
16	1,385	3.7945	1.1618	1.730	0.8908
17	1,476	4.0438	1.2214	1.730	0.9203
18	1,567	4.2932	1.2214	1.730	0.9044
19	1,658	4.5425	1.2214	1.730	0.8887
20	1,749	4.7918	1.2214	1.730	0.8733
21	1,840	5.0411	1.2840	1.730	0.9022
22	1,931	5.2904	1.2840	1.730	0.8866
23	2,022	5.5397	1.2840	1.730	0.8713
24	2,113	5.7890	1.2840	1.730	0.8562
25	2,204	6.0384	1.3499	1.730	0.8845
26	2,295	6.2877	1.3499	1.730	0.8692
27	2,386	6.5370	1.3499	1.730	0.8542
28	2,477	6.7863	1.3499	1.730	0.8394
29	2,568	7.0356	1.4191	1.730	0.8672
30	2,659	7.2849	1.4191	1.730	0.8522
31	2,750	7.5342	1.4191	1.730	0.8374
32	2,841	7.7836	1.4191	1.730	0.8230
33	2,932	8.0329	1.4918	1.730	0.8502
34	3,023	8.2822	1.4918	1.730	0.8355
35	3,114	8.5315	1.4918	1.730	0.8210
36	3,205	8.7808	1.4918	1.730	0.8068
37	3,296	9.0301	1.5683	1.730	0.8335
38	3,387	9.2795	1.5683	1.730	0.8191
39	3,478	9.5288	1.5683	1.730	0.8049
40	3,569	9.7781	1.5683	1.541	0.7910

the choice of binomial coefficients (2 is JR coefficients),[5] *dvd* is a cash dividend vector, and *tdvd* is a vector containing the time to the dividend payments. Using this function, the values of European-style and American-style calls are both 32.523. In other words, the early exercise premium is less than 1/10 of one penny. It is also worth noting that the values obtained using the binomial model are slightly less than that obtained using the BSM European-style option valuation formula. This amount, less than a penny, is approximation error.

One last caveat—by most accounts, the assumed dividend payment stream is extremely generous. The implicit dividend yield exceeds 4% annually. This is high by U.S. standards. With smaller dividends, the probability of early exercise becomes even smaller. What this means is that little is lost by valuing the American-style employee stock option using the European-style valuation equation. Simply subtract the present value of the promised dividends over the option's life from the current stock price and apply the BSM formula. The formula value, 32.529, is also reported in Table 13.1.

VESTING PERIOD

Most employee stock options have a vesting period just after they are issued during which time option exercise is prohibited. Typically, the vesting period runs three years. While, intuitively, one might think that prohibiting the exercise of the ESO during the vesting period reduces the ESO value, it does not. In the ESO valuation problem shown in Table 13.1 and discussed above, we demonstrated that early exercise is seldom optimal. Indeed, using the binomial method, the value of the American-style call was identically equal to the value of the European-style call, so the early exercise premium was, for all practical purposes, valueless. Thus, for the valuation problem at hand, prohibiting exercise during the vesting period, or at any time during the ESO's life for that matter, has no economic value holding other factors constant. If early exercise may be optimal, the effects of vesting can be handled within the binomial framework by not checking the early exercise boundaries during the vesting period.

EARLY EXERCISE

Based on the discussion in the preceding sections, using a cash-dividend-adjusted version of the BSM call option valuation formula seems entirely appropriate for valuing ESOs. Some argue that this practice overstates the true value of the ESO because its assumes that the option holder will never exercise early since it is not optimal to do so. As a practical matter, this is not the case. If an employee voluntarily or involuntarily leaves the firm in the postvesting period, he must exercise the ESO since it is not transferable. This makes the BSM value using the stated expiration of the ESO too high. A quick-and-dirty fix to this

[5] Chapter 9 contains a description of three sets of coefficients that may be used in the binomial method.

problem is to replace the stated time for expiration with an expected time to expiration based on the historical employment records of the firm.[6]

Perhaps, more important, however, is that, for many ESO holders, the value of the options represents a significant portion of their wealth. Exercising early offers the employee the opportunity to cash-in and diversify a relatively undiversified portfolio. On this matter, there is empirical evidence to suggest that employees tend to exercise ESOs when the stock price reaches certain multiples of the option's exercise price. As it turns out, this type of behavior can be accommodated easily with lattice-based valuation procedures like the binomial method. We simply impose a barrier on the stock price lattice, and, where the stock price at a particular node in the lattice exceeds the barrier, we replace the option value at that node with the option's exercise proceeds.

ILLUSTRATION 13.1 Value ESO with maximum stock price.

Compute the value of a 10-year, at-the-money ESO where the holder plans to exercise when the stock price is twice as high as the option's exercise price. Assume that the underlying stock has a price of 50, a volatility rate of 40%, and pays no dividends. The risk-free rate of interest is 7%. Use the binomial method with the CRR coefficients and two time steps.

The first step in the binomial method is to compute the stock price lattice. To do so, we compute the up-step and down-step coefficients, u and d, for the CRR method outlined in Chapter 9. Using the problem parameters, the numerical values are

$$u = e^{0.40\sqrt{5}} = 2.4459 \quad \text{and} \quad d = \frac{1}{u} = \frac{1}{2.4459} = 0.4088$$

Starting at time 0 with a stock price of 50, the stock price lattice becomes

Stock Price Lattice			
Time	0	1	2
			299.130
		122.297	
	50.000		50.000
		20.442	
			8.358

The second step in the binomial method is to value the option at expiration at each stock price node. For a call, the value is the maximum of 0 and the difference between the stock price and the exercise price, $\max(0, S_{i,j} - X)$. The bold values at time 2 in the table below are the terminal option values.

[6] This adjustment is inexact because the BSM option value is a nonlinear function to time to expiration.

Option/Stock Price Lattice for European-Style Option

Time	0	1	2
			249.130
			299.130
		82.872	
		122.297	
	27.567		**0.000**
	50.000		50.000
		0.000	
		20.442	
			0.000
			8.358

The third step is to value the option at earlier nodes by taking the present value of the expected future value at each node. To compute the expected value, it is necessary to know the probabilities of an up-step and a down-step. The probability of an up-step is given by (9.8) in Chapter 9. The numerical value for the problem at hand is

$$p = \frac{1}{2} + \frac{1}{2}\left(\frac{0.07 - 0.5(0.40^2)}{0.40}\right)\sqrt{5} = 0.4720$$

The complementary probability of a down-step is 0.5280. The present value of the expected future value at the top stock price node at time 1 is therefore

$$e^{-0.07(5)}[0.4720(249.13) + 0.5280(0)] = 82.872$$

Similar computations can be performed to fill in the remaining option value nodes. Using two time steps, the value of a European-style call option is 27.567.

To incorporate the effects of the option holder's desire to exercise the option should the stock price exceed the exercise price by a factor of two, you need to impose boundary restrictions. Each time the stock price exceeds 100 in the lattice, you need to replace the computed option value with the option's immediate exercise proceeds. If you impose this constraint on the option values in the lattice, you obtain the following lattice:

Option/Stock Price Lattice for ESO with Early Exercise Constraint

Time	0	1	2
			50.000
			299.130
		50.000	
		122.297	
	16.632		**0.000**
	50.000		50.000
		0.000	
		20.442	
			0.000
			8.358

In other words, if we impose the constraint that the ESO will be exercised if the stock price exceeds 100, the ESO value drops from 27.567 to 16.632. The cost of exercising early can be quite significant. At 1,000 time steps, the value of the ESO with no early exercise constraint is 32.464, while the ESO value with the early exercise constraint is 22.375. Thus the value of transferability is 10.089.

CONSTANT DIVIDEND YIELD MODELS

Employee stock options are often valued under the assumption that the common stock pays a constant dividend yield over the life of the option. The only reason for doing this is convenience. Estimating the amount and timing of quarterly dividend payments is cumbersome. Assuming a single dividend yield for the underlying stock is easy. Unfortunately, although valuing options under a constant dividend yield assumption is easy, it is also prone to make serious errors.

Under the constant dividend yield assumption, the firm pays out dividends as a constant, continuous proportion of stock price. This means that dividends are paid continuously (not quarterly). It also means that, if the stock price goes up, dividend income goes up, and, if the stock price goes down, dividend income goes down. Clearly these attributes are inconsistent with actual dividend payment behavior. But more seriously, when this assumption is used for ESO valuation, the results can be misleading.

To illustrate, use the example summarized in Table 13.1. To apply the constant dividend yield model, we are first faced with the problem of estimating the constant dividend yield. How should this be done? The answer is not simple. The problem is that with the constant dividend yield model you do not know the dollar amount of dividends earned. While the dividend rate is constant, the dividend payments are random. Since we know the value of the European-style call under the assumption of discrete dividends, 32.529, we can set it equal to the formula for European-style call under a continuous dividend yield assumption and solve for dividend yield. Using the parameters from the above example,

$$32.5286 = 100e^{-i(10)}N(d_1) - 100e^{-0.07(10)}N(d_2)$$

where

$$d_1 = \frac{\ln(100e^{(0.07-i)10}/100) + 0.5(0.36^2)10}{0.36\sqrt{10}} \quad \text{and} \quad d_2 = d_1 - 0.36\sqrt{10}$$

and the implied dividend yield is 4.396% annually.

Now recall that the objective is to value the ESO and the ESO is American-style. If we use the binomial method for valuing the call under the assumption of a 4.396% dividend yield and the use of 1,000 time steps, the ESO value is 37.271—nearly 15% greater than the value you obtained by addressing the problem more realistically. The constant dividend yield approach says that the value of the early exercise premium in this ESO is about 4.742, when it is, in fact worthless. Using a constant dividend yield assumption in valuing American-style options on discrete dividend-paying stocks is a practice that should be avoided.

ESOs WITH INDEXED EXERCISE PRICES

Some employee options have exercise prices that vary with an index of stock prices of firms within the same industry. This type of contract makes a good deal of sense from the firm's perspective. With standard employee stock options, option values increase as the market rises even if the firm is not doing as well as its competitors. With an indexed exercise price, employees benefit based on stock price performance. If the stock price rises relative to the index, option value increases, independent of whether the market rises or falls.

Valuing options with an indexed exercise price can be handled using standard techniques, as we saw in Chapter 8. An exchange option is like a standard option except in place of paying (receiving) the exercise price in cash at expiration, we pay (receive) a second risky asset. For an indexed ESO, the second asset is the index. The value of the ESO with an indexed exercise price is

$$c = S_1^x N(d_1) - S_2^x N(d_2) \tag{13.2}$$

where

$$d_1 = \frac{\ln(S_1^x / S_2^x) + 0.5\sigma^2\sqrt{T}}{\sigma\sqrt{T}} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{T}$$

In the valuation formula (13.7), S_1^x is the current share price net of the present value of dividends paid during the option's life and S_2^x is the index level net of dividends.[7] The volatility rate, σ, is defined as

$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho_{12}\sigma_1\sigma_2}$$

where σ_1 and σ_2 are the return volatilities of the stock and the index, respectively, and ρ_{12} is the correlation between the return of the stock and the return of the index.

ILLUSTRATION 13.2 Value indexed employee stock option.

(1) Suppose that a firm decides to award employee stock options based on performance. More specifically, assume, that instead of awarding standard 10-year, at-the-money stock options, they award 10-year, at-the-money indexed stock options, where the index is created as a value-weighted average of the firm's competitors' stock prices standardized to the firm's current stock price. The firm's current stock price is 50 per share, it pays no divi-

[7] Another way to think of S_1^x and S_2^x is as the prices of prepaid forward contracts on assets 1 and 2.

dends, and its volatility rate is 40%. The index's current value is 50, it pays no dividends, and its volatility rate is 25%. Assume also that the correlation between the firm's returns and the index returns is 0.75. Compute the values of standard employee stock options and indexed stock options assuming both are European-style. Assume the interest rate is 7%.

The value of the standard employee stock option can be computed using the BSM model.

OV_OPTION_VALUE(50, 50, 10, .07, 0.00, 0.40, "c", "e") = 32.476

The value of the indexed employee stock option is

OV_NS_EXCHANGE_OPTION(50, 50, 10, 0.00, 0.00, 0.40, 0.25, 0.75) = 16.485

(2) Suppose that at the end of the first year, the market has fallen—the firm's share price is now at 45, and the index level is at 35. Compute the rate of return on the standard employee stock options vis-à-vis the indexed options. Assume that all other problem information remains the same.

The new option values are as follows:

OV_OPTION_VALUE(45, 50, 9, .07, .00, .40, "c", "e") = 26.665

and

OV_NS_EXCHANGE_OPTION(45, 35, 9, .00, .00, .40, .25, .75) = 18.115

The decline in stock price caused the standard ESO to drop in value by 17.89%. The indexed ESO, on the other hand, increased in value by 9.89%. Even though stock prices fell, the firm did well relative to the index, and the indexed options rewarded the employees accordingly.

(3) Alternatively, suppose that at the end of the first year, the market has risen—the firm's share price is now at 55, and the index level is at 60. Compute the rate of return on the standard employee stock options vis-à-vis the indexed options. Assume that all other problem information remains the same.

The new option values are as follows:

OV_OPTION_VALUE(55, 60, 9, 0.07, 0.00, 0.40, "c", "e") = 35.356

and

OV_NS_EXCHANGE_OPTION(55, 60, 9, 0.00, 0.00, 0.40, 0.25, 0.75) = 15.637

The rise in stock price caused the standard ESO to rise in value by 8.87%. The indexed ESO, on the other hand, fell by 5.14%. Again, the indexed option provided an appropriate reward. Even though the stock price rose, the firm did relatively less well than the index, and the indexed options rewarded the employees accordingly.

ESOs WITH RELOAD FEATURES

Reload options are like standard employee stock options, except that the holder has the right to exercise the option periodically, locking in the exercise proceeds from the original option issue and receiving new at-the-money stock options in their place. More specifically, upon the exercise of a reload option, the holder receives (1) cash proceeds equal to the difference between the stock price and the exercise price, $S - X$, for each original option owned, plus (2) X/S new at-the-money options with the same expiration date as the original options.[8] This reload feature adds significant value to a standard ESO.

To value a reload option, a binomial lattice framework can be used. To simplify the problem, assume the stock pays no dividends and that the option holder has a single opportunity to "reload" his option. With no dividends, the value of a standard employee stock option (with no reload feature) can be computed using the BSM formula from Table 13.3. To value the ESO with a reload feature, compute the stock price lattice for the binomial method in the usual fashion. With the computed stock price lattice in hand, start at the end of the option's life and work backward to the present. At the end of the option's life, the option value at each stock price node is the maximum of 0 and the exercise proceeds. With the expiration values computed, the procedure steps back one time increment, Δt, to time $n - 1$ and values the option at each node by taking the present value of the expected future value. Then, for each node, in place of checking for early exercise, we check whether any of the computed option values are less than the reload proceeds, $S_{i,j} - X + (X/S_{i,j})c_{i,j}$, where $S_{i,j}$ is the stock price at node j and time i, $c_{i,j}$ is the value of a European-style call, and $X/S_{i,j}$ is the new number of calls, as per the contract design. If so, replace the computed option value with the reload proceeds. The procedure is repeatedly recursively until time 0.

ILLUSTRATION 13.3 Value employee stock option with reload feature.

Compute the value of a ten-year, at-the-money employee stock option with a single opportunity to reload. Assume that the underlying stock has a price of 50, a volatility rate of 40%, and pays no dividends. The risk-free rate of interest is 7%. Use two time steps and the CRR method.

The first step in the binomial method is the same as in Illustration 11.1. Starting at time 0 with a stock price of 50, the stock price lattice becomes

Stock Price Lattice Underlying Reload Option

Time	0	1	2
			299.130
		122.297	
	50.000		50.000
		20.442	
			8.358

[8] Presumably, the reason that fewer options (i.e., X/S) are awarded at the time of reload is that the option has "cashed-in" a value of $S - X$ for each option. If the option had no time remaining to expiration, the option holder would receive $S - X$ for each underlying share and the X/S new, at-the-money options would be valueless. This would leave the shareholders of the firm indifferent about the reload feature. But, if the options have any time remaining to expiration, the option holder receives a windfall gain (and the shareholders a windfall loss) equal to the time value (i.e., the option value less its intrinsic value) of the newly issued options. The optimal reload exercise behavior for the option holder is therefore to exercise at every available opportunity should the option be in the money. Indeed, it may be beneficial to exercise when the option is slightly out of the money, particularly, if the time to expiration is long.

The second step in the binomial method is to value the option at expiration at each stock price node. For a call, the value is the maximum of 0 and the difference between the stock price and the exercise price, $\max(0, S_{i,j} - X)$. The bold values at time 2 in the following table are the terminal option values:

Option/Stock Price Lattice with No Reload

Time	0	1	2
			249.130
			299.130
		82.872	
		122.297	
	27.567		**0.000**
	50.000		50.000
		0.000	
		20.442	
			0.000
			8.358

The third step is to value the option at earlier nodes by taking the present value of the expected future value at each node. Using two time steps, the value of a European-style call option with no reload feature is 27.567.

To incorporate the reload feature, you need to check for the possibility of reloading at each node. Again, illustrating by focusing on the top node at time 1, you know that the value of the option in the absence of reloading is 82.872. If the option is reloaded, however, the holder receives cash proceeds equal to the difference between the stock price and the exercise price, $S_{i,j} - X$ plus $X/S_{i,j}$ new, at-the-money call options with no remaining opportunity to reload. The numerical value is therefore

$$122.297 - 50 + (50/122.297)56.562 = 95.422$$

where 56.562 is the value of an at-the-money European-style call with an exercise price of 122.297 and a five-year time to expiration. Similar checks are performed for the remaining nodes. The value of the ESO with the reload feature is 31.742, compared with the value of the ESO with no reload feature, 27.567. The opportunity to reload can have significant value:

Option/Stock Price Lattice with Reload

Time	0	1	2
			249.130
			299.130
		95.422	
		122.297	
	31.742		**0.000**
	50.000		50.000
		0.000	
		20.442	
			0.000
			8.358

From Chapter 9, you know that the binomial method is not particularly accurate with only two time steps. To increase the precision, you need to increase the number of time steps and revalue the ESO. The table below reports the results where the number of time steps is 200. Even with 200 time steps, however, the binomial method remains imprecise. With no reloads, the ESO value can be computed using the BSM formula. Its value is 32.476. The binomial method with 200 time steps produces a value of 32.417—about a six-cent error. Second, the value of the reload feature is 2.265 or about 6.5% of the ESO value. For the problem at hand, the reload feature is quite valuable:

Valuation of Employee Stock Options with Reload Feature

Stock		**Binomial parameters**	
Price (*S*)	50	No. of time steps	200
Volatility rate	40.00%	Method	1
Option		**Option valuation**	
Exercise price (*X*)	50	Analytical with no reload	32.476
Years to expitation (*T*)	10	Binomial with no reload	32.417
		Binomial with one reload	34.682
Market		Reload value	2.265
Interest rate (*r*)	7.00%		

EMPLOYEE STOCK PURCHASE PLANS

A typical employee stock purchase plan (ESPP) allows its holder to buy the company's stock at a discount within a certain period of time. The discount is usually 15%, and the investment period is typically six months. Many ESPPs also have an embedded lookback option that allows the holder to apply the discount to either the end-of-period stock price or the beginning-of-period price, whichever is less. To see how to value an ESPP, consider its value upon expiration. Assume that k is the discount, expressed as a proportion of the stock price (e.g., $k = 15\%$) and that the investment period ends at time T. The terminal value of the ESPP may be expressed as

$$ESPP_T = \begin{cases} \tilde{S}_T - (1-k)S & \text{if } S_T > S \\ \tilde{S}_T - (1-k)\tilde{S}_T & \text{if } S_T \leq S \end{cases} \tag{13.3}$$

If the end-of-period stock price S_T exceeds the beginning-of-period price S, the employee will choose to buy the shares at $(1 - k)$ times the beginning-of-period price, and, if the end-of-period stock price S_T is less than the beginning-of-period price S, the employee will choose to buy the shares at $(1 - k)$ times the end-of-period price.

With the payoff contingencies in hand, we will value the ESPP using *valuation-by-replication*. Recall that valuation-by-replication involves finding a portfolio of securities whose values we know has payoff contingencies identical to those of the instrument we wish to value. In the absence of costless arbitrage opportunities, the value of the instrument must equal the value of the portfolio. Consider a portfolio in which we (a) buy the stock, (b) borrow $(1 - k)Se^{-rT}$, and (c) buy $(1 - k)$ put options with an exercise price of S and a time until expiration of T. The terminal value of this portfolio is

$$\text{Portfolio}_T = \begin{cases} \tilde{S}_T - (1-k)S + 0 & \text{if } S_T > S \\ \tilde{S}_T - (1-k)S + (1-k)(S - \tilde{S}_T) & \text{if } S_T \leq S \end{cases} \tag{13.4}$$

With a little simplification, it becomes obvious that (13.4) is the same as (13.3). The value of the ESPP, therefore, must equal the sum of the values of the securities in the portfolio, that is,

$$\begin{aligned} ESPP &= S - (1-k)Se^{-rT} + (1-k)[Se^{-rT}N(-d_2) - SN(-d_1)] \\ &= S - (1-k)Se^{-rT} + (1-k)Se^{-rT}N(-d_2) - (1-k)SN(-d_1) \\ &= S - (1-k)S[1 - N(d_1)] - (1-k)Se^{-rT}N(d_2) \\ &= kS + (1-k)[SN(d_1) - Se^{-rT}N(d_2)] \end{aligned} \tag{13.5}$$

where

$$d_1 = \frac{\ln(Se^{rT}/S) + 0.5\sigma^2\sqrt{T}}{\sigma\sqrt{T}} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{T}$$

Interestingly enough, the reworking of equation (13.5) has produced another possible replicating portfolio. The value of an ESPP equals the value of a portfolio that consists of k shares of stock and $(1 - k)$ at-the-money call options.[9] To verify this conclusion, write the terminal value contingencies for this replicating portfolio:

$$\text{Portfolio}_T = \begin{cases} k\tilde{S}_T - (1-k)(\tilde{S}_T - S) & \text{if } S_T > S \\ k\tilde{S}_T + 0 & \text{if } S_T \leq S \end{cases} \tag{13.6}$$

A little algebra shows (13.6) is the same as (13.4). The intuition is that the *ESPP* provides its holder with an award of k percent of the stock price at the end of

[9] To verify this assertion, write the payoff contingencies of this two-security portfolio.

the investment period plus a "kicker" equal to $(1 - k)$ times the difference between the beginning- and ending-of-period stock prices if $S_T > S$ (i.e., the *ESPP* holder buys at the lower of S and S_T).

ILLUSTRATION 13.4 Value ESPP.

Suppose your employer provides gives you an ESPP that allows you to buy 10,000 shares of the firm's stock at a 15% discount at today's price or at the market price in six months. The current stock price is 50, the stock's volatility rate is 40%, and the risk-free interest rate is 5%. What is the value of the ESPP?

To determine the value of the ESPP, you can simply apply (13.3). Substituting the problem parameters, you get

$$ESPP = 0.15(50) + (1 - 0.15)[50N(d_1) - 50e^{-0.05(0.5)}N(d_2)] = 12.764$$

where

$$d_1 = \frac{\ln(50e^{0.05(0.5)}/50) + 0.5(0.40^2)0.5}{0.40\sqrt{0.5}} \quad \text{and} \quad d_2 = d_1 - 0.40\sqrt{0.5}$$

This value can also be computed using the OPTVAL Library function,

OV_STOCK_OPTION_ESPP(*s, k, t, r, v*)

where s is the stock price, k is the discount rate, t is the length of the investment period, r is the risk-free rate of interest, and v is the volatility rate.

SUMMARY

Designing appropriate employee compensation schemes is no easy task. One important ingredient in the mixture, however, is tying compensation to stock price performance. Employee stock options (ESOs) and employee stock purchase plans (EESPs) are such devices. This chapter examines the valuation of ESOs and ESPPs. The effects of vesting, early exercise, and discrete and continuous cash dividends are considered. In addition, two important new types of ESOs—ESOs with indexed exercise prices and ESOs with reload features—are valued.

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