CHAPTER 14

Stock Index Products: Futures and Options

Arguably the most exciting financial innovation of the 1980s was the development of stock index derivative contracts. Although derivatives on the Dow were contemplated by the Chicago Board of Trade (CBT) as early as the late 1960s, it was not until the early 1980s that index derivatives began trading. The Kansas City Board of Trade (KCBT) was the first by introducing the Value Line index futures in February 1982, and the Chicago Mercantile Exchange (CME) followed two months later with the S&P 500 index futures. On the options side, the CME launched trading in S&P 500 index futures options in January 1983, and the Chicago Board Options Exchange (CBOE) in S&P 100 index options in March 1983. Within a few years, stock index products began to appear on other major exchanges worldwide. The Sydney Futures Exchange (SFE) introduced the All Ordinaries index futures (options) in February 1983 (June 1985), the London International Financial Futures Exchange (LIFFE) the FT-SE 100 index futures (options) in May 1984 (October 1992), and the Hong Kong Futures Exchange (HKFE) introduced Hang Seng index futures (options) in May 1986 (March 1993). In spite of their relatively short history, the contracts have been a phenomonal success. Billions of dollars in equities change hands every day through index derivatives trading in nearly 30 different countries.

This chapter and the next focus on stock index derivatives product markets and portfolio return/risk management strategies. In this chapter, the primary focus is exchange-traded derivatives. We begin by describing the U.S. markets for stock index futures and options as well as providing the specifications of some popular index contracts. The second section focuses on the construction of stock indexes. In most cases, the underlying index is a market value-weighted combination of stocks, with the notable exception being the price-weighted Dow Jones Industrial Average (DJIA). The third section summarizes the no-arbitrage price relations and valuation principles for index derivatives. For the most part, these are the same as those of individual stocks since stock indexes are nothing more than portfolios of stocks. Occasionally, however, traders choose to model the dividend income on the index portfolio as a continuous rate rather than discrete flows. For completeness, we provide the no-arbitrage prices relations and valuation principles for derivatives written on an index with a continuous dividend yield rate. The fourth section contains two important return/risk management strategies using index derivatives. First, stock index futures are used to tailor the expected return-risk characteristics of a stock portfolio for purposes of market timing and asset allocation. Second, protected equity notes are analyzed. A protected equity note is an investment that allows individuals to protect the principal value of their investment, while, at the same time, share in the upside of a market index. Although these products are traded primarily in the OTC market, they can be created synthetically using risk-free bonds and exchange-traded index call options. Chapter 15 follows with descriptions of some advanced strategies/products including passive and dynamic portfolio insurance, buy-write ETFs, and market volatility derivatives.

MARKETS

Like stock derivatives, stock index derivatives trade both on exchanges and in the OTC market. Index futures and options have traded on exchanges for more than two decades. In addition, exchange-traded funds (ETFs) have recently attracted significant trading volume. ETFs are an effective, albeit indirect, means of trading stock portfolios. Each ETF is a basket of securities but trades like a single security. Forwards, options, and a wide variety of structured products are offered in the OTC market. The purpose of this section is to provide a broad overview of stock index products. We begin first with a brief history of the evolution of index products.

Evolution of Index Products

The idea of trading stock index derivatives contracts was contemplated as early as 1968. At the time, grain surpluses had driven grain prices down to governmental support levels. Without price volatility, trading activity in the futures market was substantially reduced. Rather than wait for the situation to recover, members of the CBT began to explore the possibility of creating futures contracts on assets other than physical commodities—assets with less cyclical price behavior. Their original notion was a cash-settled futures contract on the DJIA. Fears of running afoul with the SEC and the Illinois State gambling laws, however, caused the CBT to abandon the idea in favor of creating a market for stock options.[1] Subsequently, the idea of trading a stock index futures contract lay dormant for more than a decade. It was not until February 1982 (14 years later) that the first futures contract on a stock index was launched—the Kansas City Board of Trade's ill-fated Value Line Composite Index futures contract.

A brief digression on the "first-mover" advantage is probably worthwhile, as it pertains to the failure of the Value Line futures contract. As a rule of thumb, history has shown that the first exchange to launch futures trading in a new asset category captures the lion's share of trading volume, holding other factors constant. Similar products introduced later by other exchanges have difficulty gather-

[1] For a historical recount of the events surrounding the CBT's decision, see Falloon (1998).

ing market share because the primary means of exchanges competing is the size of market makers' bid/ask spread. In a competitive environment such as an exchange's futures pit, bid/ask spreads vary inversely with trading volume. The higher the volume, the lower the market maker's fixed cost per trade. Since the first-mover initially monopolizes trading activity, any new product offered by a competing exchange must either provide lower trading costs (i.e., lower bid/ask spreads) or change the contract specifications in such a way that attracts new market participants. In the case of the Value Index futures contract, the KCBT was the first-mover. The product was aimed at the institutional need to hedge stock market risk. The index underlying the futures contract could have been the level of any broad-based, well-diversified stock portfolio. From the discussion of the CAPM in Chapter 3, we know that well-diversified stock portfolios have returns that are highly correlated with one another. All futures contracts written on well-diversified portfolios will therefore be very close substitutes. But therein lies the reason for the failure of the Value Line futures contract market. While the Value Line index had in excess of 1,500 stocks and, by all accounts, should have been well-diversified, its construction was atypical in that it did not represent the value of a stock portfolio. Instead of taking a value-weighted arithmetic sum of the constituent stock prices (like the value of any well-diversified portfolio), the Value Line index was calculated by taking an equal-weighted geometric product of stock prices.[2] This has two unfortunate consequences. First, it means that the Value Line index returns will not be highly correlated with the returns of a well-diversified stock portfolio. Second, it means that it is impossible for the Value Line futures price and the Value Line index level to be linked by arbitrage because the index, itself, cannot be traded.[3] Without strong correlation between the returns of the futures and the underlying index and strong correlation between the returns of the index and well-diversified portfolios, the Value Line futures was an ineffective means of hedging stock market risk[4] and contract volume waned. Eventually, the index was redesigned as a value-weighted arithmetic sum, but, unfortunately it was too late. The first-mover advantage had been relinquished to the CME.

The first viable futures contract on a broad-based (arithmetic) index portfolio was the S&P 500 futures, launched by Chicago Mercantile Exchange in April 1982. Consistent with the first-mover principle, it remains by far the most active index futures contract today. Table 14.1 summarizes the trading volume of the eight most active index futures contracts on U.S. exchanges during the calendar year 2003. While a total of 35 index futures traded across U.S. exchanges, the eight most active accounted for 92.43% of total contract volume.[5] Measuring

[2] More specifically, the Value Index was computed by taking the product of the price of all of the stocks in the index and adjusting by a divisor.

[3] This is true only for the Value Line index. In general, the movements in a stock index can be replicated by trading the basket of underlying stocks. Because the Value Line index is generally weighted, replication is not possible.

[4] For an early evaluation of the properties of the Value Line Index futures, see Modest and Sundaresan (1983).

[5] Contract volumes for the calendar year 2003 were drawn from *Futures Industry Association Monthly Report* (December 2003). The total trading volume across all stock index futures listed on U.S. exchanges during 2003 was 296,694,711 contracts.

TABLE 14.1 Market value of the eight most active index futures contracts traded in the United States during the calendar year 2003.

Contract	Exchange	Underlying Index	Contract Mulitplier	Closing Level 12/31/2003	Contract Volume in 2003	Dollar Contract Volume in 2003
E-Mini S&P 500	CME	S&P 500	50	1111.92	161,176,639	8,960,776,421,844
S&P 500	CME	S&P 500	250	1111.92	20,175,462	5,608,374,926,760
E-Mini NASDAQ 100	CME	NASDAQ 100	20	1467.92	67,888,938	1,993,110,597,379
NASDAQ 100 Index	CME	NASDAQ 100	100	1467.92	4,421,221	648,999,873,032
Mini ($5) Dow Jones Industrial Index	CBT	DJIA	5	10453.9	10,859,690	567,630,566,455
Dow Jones Industrial Index	CBT	DJIA	10	10453.9	4,416,302	461,675,794,778
E-Mini Russell 2000	CME	Russell 2000	100	556.91	3,878,935	216,021,769,085
E-Mini S&P Midap 400 Index	CME	S&P Midcap 400	100	576.01	1,417,513	81,650,166,313
Total					274,234,700	18,538,240,115,646

Source: Data compiled from *Futures Industry Association Monthly Report* (December 2003) and *Datastream*.

the importance of trading activity in terms of numbers of contracts can be misleading, however, since different contracts have different contract multipliers and index levels. To accurately measure the economic significance of the trading volume for each contract, dollar contract volume (i.e., number of contracts traded times the contract multiplier times the index level) was computed and is reported in the last column of Table 14.1. The CME's S&P 500 contracts accounted for about 79% of the dollar contract volume in 2003.

To further elaborate on the evolution of exchange-traded stock index derivatives and the first-mover advantage, consider Table 14.2, which reports the correlation coefficients computed from the daily returns of the major stock market indexes in the U.S. during the calendar years 2002 and 2003. By way of history, the New York Futures Exchange (NYSE) was created in 1982 by the New York Stock Exchange (NYSE) for the exclusive purpose of trading futures contracts on the NYSE Composite index. In May 1982, a month after the launch of S&P 500 futures trading, the NYSE Composite index futures began trading. Other than having a different well-diversified portfolio serving as the underlying index, the contract specifications (e.g., contract size and expiration cycle) were very much like those of the S&P 500 index futures. Table 14.2 shows that the correlation between the returns of the S&P 500 and NYSE Composite indexes is 0.979. With virtually perfect correlation between the returns of the two indexes, futures contracts written on these indexes are nearly perfect substitutes.[6] It should not be surprising, therefore, to find that, while the NYSE Composite futures continues to trade, its contract volume is less than one percent of the S&P 500 contract volume. In a similar vein, the correlation between the S&P 500 and the S&P 100 is 0.994, again indicating that having futures contracts written on both indexes is redundant. Anecdotally, the CME launched S&P 100 futures contracts in the mid-1980s. The contract failed to attract significant trading volume and was delisted shortly thereafter.

The Chicago Board of Trade was late to step into the stock index futures market competition. The CBT's plans were to create index futures on the Dow Jones Industrial Average. At the same time, the American Exchange (AMEX) was planning to launch trading in index option contracts on the Dow. Unfortu-

TABLE 14.2 Correlation between daily index returns of major U.S. stock market indexes using data from the calendar years 2002 and 2003.

Index	DJIA	S&P500	NYSE	S&P 100	S&P 400	NASD 100
S&P 500	0.980					
NYSE	0.966	0.979				
S&P 100	0.980	0.994	0.969			
S&P 400	0.892	0.925	0.925	0.897		
NASD 100	0.857	0.905	0.853	0.894	0.883	
Russell 2000	0.809	0.854	0.850	0.825	0.946	0.861

[6] This presumes that there is the same amount of basis risk between each futures and its underlying index.

nately, Dow Jones refused to allow either exchange to proceed. In a retaliatory move, the AMEX created the Major Market Index (MMI)—an index designed to look like the DJIA. It was a price-weighted index (like the DJIA) and consisted of 20 "blue chip" stocks, 15 of which happened to be in the DJIA at the time. The CBT licensed the rights for trading futures and futures option contracts on the MMI from AMEX and began trading MMI futures in July 1984. The contract floundered and was later delisted.

In a reversal of its longstanding policy not to allow derivatives traded on its indexes, Dow Jones began to consider proposals to license its DJIA to serve as the index underlying index derivatives contracts in 1997. The CBT and the CME competed for the right to trade futures and futures option contracts, and the AMEX and the CBOE competed for the right to trade options. Dow Jones awarded the license for futures and futures option contracts to the CBT and the option contracts to the CBOE. On October 6, 1997, Dow options began trading on the CBOE and Dow futures and futures options on the CBT. The success of the Dow derivatives contracts, however, has been modest. In part, this is attributable to the high degree of correlation between the DJIA and the S&P 500 index. Table 14.2 shows that the correlation between the two indexes is 0.980. To maximize the probability of success, however, the CBT, in discussions with Dow Jones, attempted to differentiate the Dow contract from other index futures by making it considerably smaller. In this way, they aspired to attract retail (i.e., small investor) rather institutional (i.e., large investor) business. Unfortunately, the CME was quick to respond to this initiative. When Dow Jones awarded the license for DJIA futures contracts to the CBT, the CME immediately countered by creating a miniaturized version of its successful S&P 500 futures. The "E-mini S&P 500" contract is 1/5 the size of its big brother (and about half the size of the Dow contract) and began trading on September 9, 1997, about a month before the CBT was able to unveil the Dow futures. Judging by the contract volume figures reported in Table 14.1, there was pent up demand for a smaller index futures contract. Indeed, the E-mini S&P 500 futures contract now has greater dollar volume than its big brother. The E-mini S&P 500 futures also appears to have had a first-mover advantage in the sense that its dollar contract volume in 2003 was nearly ten times higher than the two futures contracts list on the DJIA.

Table 14.1 also shows that the NASDAQ 100 futures contracts have been quite successful, with trading volume in excess of 1.6 billion contracts in 2003. One reason for their success may be given in Table 14.2. The correlation between the returns of the S&P 500 index and the NASDAQ 100 index is only 0.905. Thus the S&P 500 index and NASDAQ 100 index are not perfect substitutes. Another reason may be that the NASDAQ contracts are about half the size of their S&P 500 counterparts and may be attracting more retail business. These cannot be the only reasons, however, since the Russell 2000 index has even lower correlation with the S&P 500 index, and the E-mini Russell 2000 futures is a very small contract. The remaining reason is that index arbitrage is more easily and cheaply executed using an index portfolio of 100 highly liquid stocks than with an index portfolio with 2,000 stock with varying degress of liquidity. The greater the arbitrage activity between the futures and its underlying index, the higher the correlation between the return of the futures and its underlying index and the more effective the futures is as a return/risk management tool.

The trading volumes of index options and index futures options traded on U.S. exchanges during the calendar year 2003 are reported in Table 14.3. Under the stock index options panel, the contract volume is greatest for exchange-traded funds. But this is aggregate trading volume across a number of different ETF option classes. By far the most active index option class is the S&P 500 index options traded on the CBOE. Excluding options on ETFs, S&P 500 options account for 47.6% of all index option trading. The second most active contract is the S&P 100 index options.

TABLE 14.3 Contract volume for stock index option and stock index futures option contracts in the United States during the calendar year 2003.

Contract	Exchange	Contract Volume	Percent of Total	Percent of Total (excl.)
Stock index options:				
Exchange Traded Funds	CBOE	41,146,233	34.8%	
S&P 500 Index Options (SPX)	CBOE	36,754,720	31.1%	47.6%
S&P 100 Index Options (OEX)	CBOE	14,343,992	12.1%	18.6%
Dow Jones Industrial Index (DJX)	CBOE	10,193,708	8.6%	13.2%
NASDAQ 100 Mini (MNX)	CBOE	4,034,201	3.4%	5.2%
Mini NASDAQ Non-Financial 100 Index (MNX)	AMEX	2,436,756	2.1%	3.2%
S&P 100 European Exercise (XEO)	CBOE	1,933,355	1.6%	2.5%
NASDAQ 100 (NDXCBO)	CBOE	1,622,687	1.4%	2.1%
Gold/Silver Index (XA U)	PHLX	1,130,430	1.0%	1.5%
Oil Service Sector (OSX)	PHLX	1,006,718	0.9%	1.3%
Other		3,713,816	3.1%	4.8%
TOTAL		118,316,616		
TOTAL (excluding ETFs)		77,170,383		
Stock index futures options:				
S&P 500 Index	CME	4,986,456	90.1%	
Dow Jones Industrial Index	CBT	263,629	4.8%	
E-Mini S&P 500	CME	112,864	2.0%	
Russell 1000	NYBOT	61,264	1.1%	
NASDAQ 100 Index	CME	50,439	0.9%	
NYSE Composite Index	NYBOT	25,320	0.5%	
Revised NYSE Composite	NYBOT	18,912	0.3%	
Other		16,062	0.3%	
TOTAL		5,534,946		

Source: Data compiled from *Futures Industry Association Monthly Report* (December 2003).

Interestingly, the first options on a stock index were the CME's S&P 500 futures options, which began trading in January 1983. Compared with other futures options activity, the S&P 500 contracts account for 90.1% of total index futures option trading volume in 2003. Compared with the S&P 500 index options traded on the CBOE, however, their volume is about 12.1%. Adjusting for contract size, the relative trading volume is 30.3%. One reason for the dominance of index options over index futures options is that many institutional investors can trade in securities markets but not futures markets. Another is that longer-term contracts are available in the index option market.

Security and futures exchanges tend to develop reputations as leaders in particular styles of contracts based on their relative trading volumes. Table 14.4 summarizes trading volume by U.S. exchange for the calendar year 2003. In terms of the reputation, the CME is the market leader in the stock index futures and the stock index futures options markets in the U.S. Table 14.5 shows that they account for 94.6% of stock index futures trading and 93.3% of stock index futures options trading in the U.S. The CBOE is the leader in index options trading, with 93.7% of the total index option contracts traded.

TABLE 14.4 Number of contracts listed and contract volume for stock index products traded on U.S. exchanges during the calendar year 2003.

Exchange	Symbol	No. of Contracts	Contract Volume	Percent of Total
Stock index futures:				
Chicago Mercantile Exchange	CME	22	280,649,663	94.6%
Chicago Board of Trade	CBT	3	15,319,313	5.2%
New York Board of Trade	NYBOT	7	720,147	0.2%
OneChicago	ONE	1	3,197	0.0%
Kansas City Board of Trade	KCBT	1	2,391	0.0%
TOTAL		34	296,694,711	
Stock index options:				
Chicago Board Options Exchange	CBOE	15	110,822,092	93.7%
American Exchange	AMEX	25	4,272,740	3.6%
Philadelphia Exchange	PHLX	16	3,221,784	2.7%
TOTAL		56	118,316,616	
Stock index futures options:				
Chicago Mercantile Exchange	CME	6	5,163,151	93.3%
Chicago Board of Trade	CBT	1	263,629	4.8%
New York Board of Trade	NYBOT	6	108,166	2.0%
TOTAL		13	5,534,946	

Source: Data compiled from *Futures Industry Association Monthly Report* (December 2003).

TABLE 14.5 Selected terms of S&P 500 index futures contract.

Exchange	Chicago Mercantile Exchange (CME)
Contract unit	$250 times S&P 500 index
Tick size	0.10
Tick value	$25
Contract months	Nearest eight months in the March quarterly expiration cycle (i.e., Mar./Jun./Sep./Dec.).
Trading hours	FLOOR trading: 8:30 AM to 3:15 PM CST. All contract months are traded. GLOBEX trading: 3:30 PM to 8:15 AM (the following morning) Monday through Thursday, and 5:30 PM to 8:15 AM (the following morning) Sundays and holidays. Shutdown period from 4:30 PM to 5:00 PM nightly. Only nearby contract month is traded.
Expiration day	Third Friday of the contract month.
Last day of trading	Business day immediately preceding the day of the determination of the final settlement price.
Final settlement price	Cash-settled at a special quotation of the index based on the opening prices of the index stocks on the expiration day.

Stock Index Futures

Index futures are standardized contracts, with a number of conventions regarding denomination, expiration, and method of settlement. Table 14.1, for example, shows the contract multiplier of the eight most active index futures traded in the U.S. The contract multiplier for the S&P 500 contract is 250 times the index futures price. A futures price of 1,110 implies that trading a single contract is like trading $275,000 in the S&P 500 index portfolio. The 250-multiplier has not been in effect for too long. From inception on April 21, 1982 through October 31, 1997, the multiplier was 500. The redenomination of the contract was an attempt by the CME to make the contract more accessible for investors.[7]

The contract specifications of the S&P 500 futures are presented in Table 14.5. As noted earlier, the contract multiplier is $250. Since the tick size is 0.10 index points, the minimum price movement in the contract is $25 (i.e., 0.10 × $250). The S&P 500 futures contract is on the March quarterly expiration cycle, which means that March, June, September, and December contracts are available. On any given date, eight contract months are listed. Hence, as of April 2004, June 2004 through March 2006 contract months are available. The last trading day of the S&P 500 futures contract is the third Thursday of the contract month. Cash settlement of the contract takes place at a special settlement quotation based on opening prices of the index stocks on Friday.

The S&P 500 futures trades on the floor of the exchange during regular trading hours as well electronically during the rest of the day. The floor trading

[7] For an analysis of the effect of the CME's re-denomination of the S&P 500 futures contract, see Bollen, Smith, and Whaley (2003).

hours of the S&P 500 futures are from 8:30 AM to 3:15 PM Central Standard Time (CST) Monday through Friday. All eight contract months are traded. Note that the hours of trading for the index futures are usually chosen to coincide with trading in the stock market, with the possibility of a short window before or after the stock market is opened or closed. For the S&P 500 futures, regular trading extends fifteen minutes beyond the close of the market, that is, trading on the NYSE is from 9:30 AM to 4:00 PM Eastern Standard Time (EST). Outside the floor trading hours, the nearby S&P 500 futures contract trades electronically on GLOBEX.[8] The electronic trading hours are from 3:30 PM (CST) until 8:15 AM (CST) the following morning Monday through Thursday, and 5:00 PM (CST) until 8:15 AM (CST) the following morning on Sundays and holidays.

The contract specifications of the E-mini S&P 500 futures are virtually identical to those of the S&P 500 futures. The only notable exception is that the contract multiplier is 50 instead of 250. The E-mini S&P 500 futures trades electronically virtually twenty-four hours a day—from 3:30 PM to 3:15 PM CST (on the following day) on Monday through Thursday and from 5:30 PM to 5:15 PM CST (on the following day) on Sunday and holidays Only the two nearby contract months are traded.

Stock Index Options

Stock index options are written on both stock index futures and the stock index. There are subtle differences in the contract designs, as discussed below.

Index Futures Options The first stock index futures option contracts to trade in the U.S. were the Chicago Mercantile Exchange's S&P 500 and the New York Futures Exchange's NYSE Composite futures option contracts. They began trading on January 28, 1983. Trading in index futures options is less active than index futures. Indeed, for the U.S. index futures contracts listed in Table 14.3, the only futures option contract to have trading volume greater than 500,000 contracts during 2003 was the S&P 500 futures option.

Table 14.6 contains the product specifications of the S&P 500 futures option contract. Each futures option is written on a single S&P 500 futures contract. Tick size, tick value, and trading hours conventions are the same as those of the underlying futures. S&P 500 futures options, like all futures options traded in the United States, are American-style. In the event of early exercise, the underlying futures contract is delivered. Exercising a long call position, for example, means that a long position in the underlying futures is delivered. A seller of the call, selected randomly from the outstanding short positions, would receive the offsetting short futures position. Both futures positions are marked-to-market at the exercise price of the call at the end of day.

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[8] GLOBEX is an electronic trading system developed by the CME (and Reuters) and began live trading June 25, 1992. On the first day of operation, 2,063 futures and futures option contracts on Deutsche marks and Japanese yen were traded. Today nearly all CME products are traded on GLOBEX, and trading activity averages more than one million contracts per day—a staggering 44% of total CME volume!

TABLE 14.6 Selected terms of S&P 500 index futures option contract.

Exchange	Chicago Mercantile Exchange (CME)
Contract unit	One S&P 500 futures contract
Tick size	0.10
Tick value	$25
Contract months	Four expirations on the quarterly cycle Mar./Jun./Sep./Dec. Also, two nearby contract months such that a total of six contract months are listed.
Trading hours	FLOOR trading: 8:30 AM to 3:15 PM CST. All contract months are traded. GLOBEX trading: 3:30 PM to 8:15 AM (the following morning) Monday through Thursday, and 5:30 PM to 8:15 AM (the following morning) Sundays and holidays. Shutdown period from 4:30 PM to 5:00 PM nightly. Only nearby contract month is traded.
Exercise style	American
Expiration day	Third Friday of the contract month.
Last day of trading	The same date and time as the underlying futures contract for the quarterly cycle and on the third Friday of the contract month for the other months.
Final settlement price	Quarterly expirations are cash-settled at a special quotation of the index based on the opening prices of the index stocks on the expiration day. Nonquarterly expirations call for the delivery of the underlying futures contract.

To illustrate the mechanics of exercising an American-style futures option, suppose you hold the June 2004 call with an exercise price 1050 listed in Table 14.7. On the morning of April 14, 2004, you decide to exercise the option. To do so, you must call your broker and tell him that you want to exercise your call. At the end of the day, what would appear in your futures account would include a long position in the June 2004 futures plus mark-to-market cash proceeds in the amount of $19,925 (i.e., the futures settlement price, 1,129.70, less the exercise price, 1050, times the contract denomination, $250).

Note that in the above illustration the exercise proceeds equal the difference between the settlement price and the exercise price at the end of the day even though you tendered exercise early in the day. Locking in the exercise proceeds earlier in the day is also possible. Suppose, for example, that the price of the June 2004 futures was at 1150 in the morning of April 14. To lock in the exercise proceeds at that futures price level, you call your broker and instruct him to (1) exercise the call and (2) sell the futures. Assuming the futures order is executed at 1150, your end-of-day settlement would include a mark-to-market gain of 1150 – 1129.70 times $250 on your short futures position, and a mark-to-market gain of 1129.70 – 1050 times $250 on the long futures position obtained when exercising the call. The total gain is $25,000, and the futures position is closed.

Table 14.6 describes both quarterly and nonquarterly expiration cycles. The quarterly expiration cycle patterns the futures—March, June, September, and December. The S&P 500 futures options with these contract months are written

TABLE 14.7 Settlement prices of selected S&P 500 index futures options drawn from www.cme.com on April 14, 2004. Settlement price of June 2004 S&P 500 futures was 1129.70.

Exercise Price	Call Options			Put Options		
	Apr/04	May/04	Jun/04	Apr/04	May/04	Jun/04
1000			134.20		1.70	4.90
1025	104.70	107.10	111.20	0.05	2.60	6.80
1050	79.80	83.80	89.10	0.10	4.20	9.60
1075	54.90	61.40	68.00	0.20	6.80	13.50
1100	30.20	41.00	48.90	0.55	11.40	19.30
1125	8.10	23.70	32.40	3.40	19.00	27.70
1150	0.30	11.10	19.20	20.60	31.40	39.40
1175	0.05	4.00	10.10	45.30	49.20	55.30
1200		1.00	4.60	70.30	71.20	74.70
1225		0.30	1.90	95.30		97.00
1250		0.15	0.80	120.30		120.80
1275		0.05	0.30			
1300			0.20			170.30

on the corresponding futures. Table 14.7, however, shows April and May contract months, where no April and May S&P 500 futures are traded. These are called serial months and are written on the June 2004 futures. Upon exercising a serial option, the option holder receives a position in the nearby futures contract, in this case the June 2004 futures, and is marked-to-market at the exercise price. If serial options are carried to their expiration on the third Friday of the contract month, they are automatically exercised if in the money. On the other hand, S&P 500 futures options expiring on the quarterly cycle are cash-settled at expiration—the June 2004 futures option expires at the same instant as the June 2000 futures.

Index Options The first stock index option contract to trade in the United States was the Chicago Board Options Exchange's S&P 100 index option.[9] They began trading on March 11, 1983. The CBOE launched trading in S&P 500 index options on July 1, 1983. Since the early 1980s, options on a number of narrowly-based industry indexes have also been introduced. Few have managed to generate significant trading volume. Nonetheless, options on more than fifty different stock indexes trade in the U.S. alone.

[9] Although the S&P 100 index is less well-known than the S&P 500 index, S&P 100 options had the greatest trading volume until only recently. By way of history, when the CBOE was initially considering introducing an index option contract in the early 1980's, it decided upon a value-weighted index of the one hundred largest stocks for which CBOE listed stock options. Originally, the index was called the "CBOE 100." Later, the CBOE reached an agreement for Standard & Poors' to track the portfolio composition, at which time, the index was renamed the S&P 100.

Table 14.3 summarized the trading volume of index options in the calendar year 2003. Most of their volume is concentrated in the broad-based indexes. The S&P 500 contract, for example, has 47.6% of the total non-ETF index option volume, and the S&P 100 contract has 18.6%. The only narrow-based index option to have significant volume is the Dow Jones index options, with 13.2% of total volume. To some degree, this is surprising considering that the market was launched on October 6, 1997—less than eight years ago. Part of this phenomenon, however, may be attributable to the fact that the Dow options have a smaller contract denomination. Finally, NASDAQ 100 index options account for about 7.3% of total trading volume.

All active index option contracts, except those on the S&P 100, are European-style. The S&P 100 index options are American-style. If an S&P 100 index option buyer exercises early, he or she receives the difference between the closing index level on that day and the exercise price of the option. The offsetting option seller, who is obliged to make the cash payment to the buyer, is randomly chosen from all of the open short positions in that option.

Table 14.8 contains selected terms of the S&P 500 option contract. In many ways, the terms of stock index options parallel those of stock options. The contract unit is $100 times the index level, mimicking the fact the stock options are written on 100 shares of stock. The tick size convention is also consistent with stock options. Option premiums at $3 and below have a minimum tick size of $.05 while options with premiums above $3 have a minimum tick size of $.10. The available contract months include the three near-term months followed by three additional months from the quarterly expiration cycle.[10] Leaps extending out three years are also offered. Also like stock options, the contracts expire on the Saturday after the third Friday of the contract month. Unlike stock options, however, stock index options are cash settlement rather than delivery contracts.

TABLE 14.8 Selected terms of S&P 100 index option contract.

Exchange	Chicago Board Options Exchange (CBOE)
Ticker symbol	SPX
Contract unit	$100 times the S&P 500 index
Tick size	0.05 point up to $3 premiums; 0.10 point over $3
Tick value	$5; $10
Contract months	Three near-term months followed by three additional months from the March quarterly cycle.
Trading hours	8:30 to 3:15 PM CST
Exercise style	American
Expiration day	Saturday following the third Friday of the contract month
Last day of trading	Business day (usually a Thursday) preceding the day on which the final settlement price is computed.
Final settlement price	Cash-settled at a special quotation of the index based on the opening prices of the index stocks on the expiration day.

[10] On October 28, 2005, the CBOE launched trading in one-week options on the S&P 500. The so-called "weeklys" are listed each Friday, and expire the following Friday.

Like the S&P 500 index futures, the S&P 500 index option is settled on the expiration day at a special morning settlement quotation based on opening prices of the index stocks on Friday.[11]

Exchange Traded Funds[12]

An *exchange traded fund* (ETF) is a hybrid security that behaves like an index portfolio but trades like a stock. To understand the popularity of ETFs, a brief review of the history of fund indexing is useful. The origin of fund indexing rests in the Sharpe (1964)/Lintner (1965) capital asset pricing model (CAPM), The CAPM says that investors should hold well-diversified portfolios that consist of all risky securities in the marketplace, with the proportion of wealth invested in each security equal to that security's market value relative to the total market value of all risky securities. Active portfolio management is unnecessary. Cash dividends are simply reinvested in the proportions dictated by the current market value weights. Other than that, investors "buy-and-hold."

Out of what seemed an esoteric theory in the early 1960s grew the practice of fund indexing. Early on, the most widely known, market value weighted index in the United States was the S&P 500.[13] Consequently, index funds began pegging their holdings to the S&P 500 portfolio, and the practice was born. The growth in the S&P 500 funds has been incredible. Perhaps the most well-known S&P 500 fund is the *Vanguard Index Trust—500 Portfolio*. The net asset value of the *500 Portfolio* was $14 million in 1976. At the end of December 2003, the amount was $64,368 million—an increase of nearly 460,000%! But this is only a single fund pegged to the S&P 500 portfolio. The total wealth invested in the S&P 500 index portfolio must account for Vanguard's other S&P 500 funds, other publicly traded S&P 500 funds managed by other investment companies, and privately held funds pegged to the S&P 500. In its 2003 *U.S. Indexed Assets Survey*, Standard & Poor's reported that assets tied to the S&P 500 exceeded the $1 trillion market, nearly 10% of the total market capitalization of the index.

Aside from the built-in diversification, index funds offer significant cost savings. Because the portfolio is passive, excessive trading costs (e.g., brokerage commissions and bid/ask spreads) associated with frequent turnover in actively managed portfolios are avoided. In addition, management fees are small. Since the portfolio composition is dictated by some third party (e.g., Standard and Poor's), index fund management *per se* is only a matter of taking new cash inflows (e.g., cash dividends) and allocating them across the index's constituent stocks. The only real disadvantage of traditional index funds is that they cannot be bought and sold on a real-time basis. Purchases and sales of the index fund occur only at

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[11] For the quarterly expiration cycle, the settlement quotation is the same for S&P 500 futures, S&P 500 futures options, and S&P 500 index options. The financial press sometimes refers to this as a "triple-witching" hour.

[12] For a lucid review of all aspects of exchange traded funds, see Gastineau (2002). The materials in this section are drawn from the source, as well as information and data from the American Stock Exchange.

[13] The earliest public advocate of fund indexing was John Bogle of the Vanguard Group. For his reflections of fund indexing and the mutual fund industry, see Bogle (1994).

end-of-day closing values. In addition, many important trading strategies require a short position in the market. Short selling index funds is not possible.

The basic idea underlying an ETF is trading an entire portfolio as if it were a stock. The first foray into this arena was in 1989 when the Philadelphia Stock Exchange (PHLX) and the American Stock Exchange (AMEX) launched the Index Participation Shares (IPS). While IPS on a number of indexes were available, the IPS on the S&P 500 were by far the most popular. The market showed significant promise, however, not without controversy. The Chicago Mercantile Exchange (CME) and the Commodity Futures and Trading Commission (CFTC) filed a lawsuit charging that IPS were futures contracts and must be traded on a futures exchange, not a securities exchange.[14] IPS were cleared by the Options Clearing Corporation and fell under the regulatory jurisdiction of the Securities and Exchange Commission (SEC). Unfortunately, from the securities exchanges' perspective, the IPS were like futures contracts in the sense that there was a zero net supply. For every long, there was a short, and vice versa. A federal court in Chicago ruled that the IPS were illegal futures contracts, and PHLX and AMEX were required to close down IPS trading.

The next significant event in the history of ETFs in the United States was AMEX's launch of Standard and Poor's Depository Receipts (SPDRs) on January 29, 1993. These receipts represent an interest in the S&P 500 index stocks held by a unit investment trust, and trade like shares of a common stock. They can be bought on margin, and can be sold short, even on a downtick. The key features that earmark SPDRs as a security rather than a futures are (1) they are both created from the securities of an underlying portfolio; and (2) they can be redeemed into the securities of an underlying portfolio during any trading day.[15] Because of the substitutability of SPDRs with S&P 500 index stocks, price discrepancies will be few.[16] Otherwise, arbitrageurs will quickly move in to profit.[17] ETF holders are eligible to receive their pro rata share of dividends, if any, accumulated on the stocks held in the portfolio.

Figure 14.1 shows the annual trading volume of the AMEX SPDRs since inception. While the initial pace of trading was modest, with less than an average of 100 million shares traded annually in the period 1993 through 1995, volume has grown to over 10.3 billion shares in 2003—a phenomenal success by most standards. The wealth invested in SPDRs now exceeds all S&P 500 index funds other than the Vanguard Group's *Index Trust—500 Portfolio*. The dollar trading volume of SPDRs still lags behind S&P 500 futures. The dollar value of

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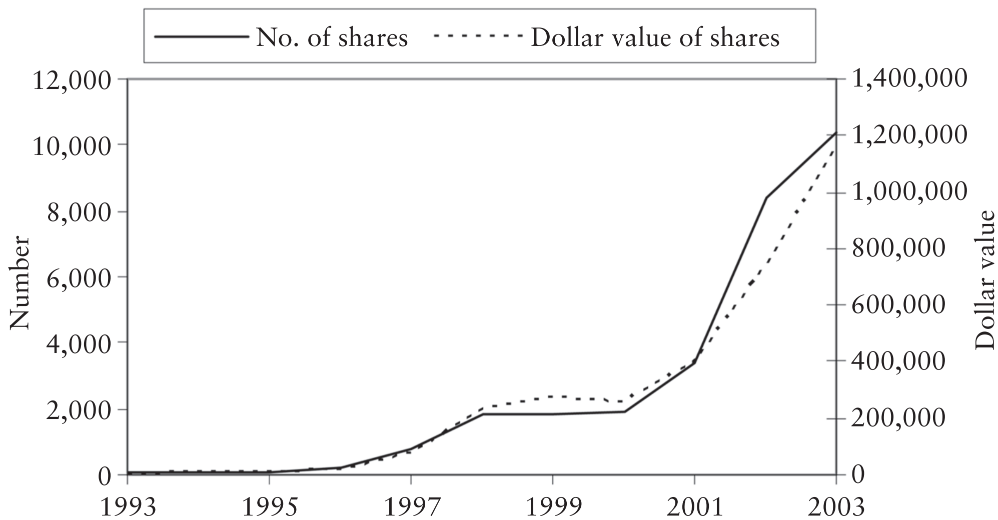
[14] Recall the discussion of competing regulatory authorities in Chapter 1.

[15] The AMEX was not the first securities exchange to adopt a unit trust style of ETF. Toronto Stock Exchange Index Participations or TIPS had introduced such an ETF in Canada a number of years earlier.

[16] AMEX has a webpage (that can be accessed from www.amex.com) for calculating summary statistics of the size of the premium/discount over the recent past. As of April 19, 2004, the mean (standard deviation) of the premium of the bid/ask midpoint over the net asset value of the fund over all trading days during the most recent 12 months was –0.01 (0.04). The minimum value observed over the period was –0.27, and the maximum was 0.14.

[17] ETF creations and redemptions are restricted to large transactions, typically in multiples of 50,000 shares but ranging from 25,000 to 600,000 shares, usually transacted by large investors and institutions.

FIGURE 14.1 Number of shares traded and dollar value of shares traded by year for AMEX SPDRs during the period January 1993 through December 2003.



the SPDRs traded in 2003 was $1.15 trillion, compared to $14.57 trillion for the S&P 500 futures (about 7.91%). But SPDRs are aimed at a retail customer market. S&P 500 futures are largely used by institutional customers.

SPDRs are not the only successful ETF. Table 14.9 shows that the daily dollar volume of the most ETFs traded on the AMEX as of the close on April 13, 2004. Although the trading volume of the SPDRs was nearly double that of the NASDAQ 100 QQQQs, the so-called "quadruple Qs" volume was respectable at $3.66 billion. Indeed, the dollar value of shares outstanding in the form of unit trusts is greater for the QQQs than the SPDRs. The table also shows that the AMEX now has 122 ETFs on a variety of indexes including broad-based stock portfolios, stock industry sectors, international stock portfolio, and bond indexes. The DIAMONDS are shares of the Dow Jones Industrial Average (DJIA). The iShares are a family of ETFs based created by Barclay Global Investors on a variety of different indexes.

COMPOSITION OF STOCK INDEXES

Before discussing no-arbitrage price relations and valuation equations for stock index derivatives, it is important to have a clear understanding of index construction and its implications for modeling dividend income. Generally speaking, stock indexes underlying derivative contracts are either (1) value-weighted or (2) price-weighted. With a value-weighted index, each stock is weighted by its market capitalization, while, with a price-weighted index, each stock is weighted by its price. In this section, the details of index construction are provided. Value-weighted indexes are discussed first, followed by price-weighted indexes. The section also examines the daily cash dividend payments of the S&P 500 and Dow Jones Industrial Average indexes in order to ascertain the most

TABLE 14.9 Summary of trading activity on April 13, 2004 for all AMEX ETFs with shares outstanding in excess of 1 billion dollars.

			In Dollars		In Shares	
Name	**Ticker Symbol**	**Last Trade**	**Daily Volume**	**Shares Outstanding**	**Daily Volume**	**Shares Outstanding**
NASDAQ-100 Index Tracking Stock	QQQQ	36.63	99,984,200	729,800,000	3,662,421,246	26,732,574,000
Vanguard Total Stock Market VIPERs	VTI	109.51	365,800	222,506,000	40,058,758	24,366,632,060
SPDRS	SPY	113.21	55,791,200	168,873,000	6,316,121,752	19,118,112,330
MidCap SPDRS	MDY	110.07	1,935,200	72,431,000	213,007,464	7,972,480,170
DIAMONDS	DIA	103.84	6,793,000	61,008,000	705,385,120	6,335,070,720
iShares MSCI-EAFE	EFA	142.45	324,300	35,800,000	46,196,535	5,099,710,000
iShares Russell 2000	IWM	116.64	8,225,000	38,800,000	959,364,000	4,525,632,000
iShares S&P 500	IVV	113.34	217,900	36,600,000	24,696,786	4,148,244,000
iShares Russell 1000 Value	IWD	59.41	181,900	44,400,000	10,806,679	2,637,804,000
iShares GS $ InvesTop Corp Bond Fn	LQD	110.31	91,200	22,200,000	10,060,272	2,448,882,000
iShares MSCI-Japan	EWJ	10.94	10,457,700	208,200,000	114,407,238	2,277,708,000
iShares Russell 1000	IWB	60.69	72,300	30,900,000	4,387,887	1,875,321,000
iShares S&P MidCap 400	IJH	120.48	113,800	14,150,000	13,710,624	1,704,792,000
iShares Lehman 1-3 Year Treasury B	SHY	82.31	181,500	18,700,000	14,939,265	1,539,197,000
iShares Russell 2000 Growth	IWO	62.72	1,825,600	24,450,000	114,501,632	1,533,504,000
iShares Russell 1000 Growth	IWF	47.58	191,900	31,750,000	9,130,602	1,510,665,000
iShares MSCI-Mexico	EWW	20.74	257,300	67,292,000	5,336,402	1,395,636,080
iShares Russell 3000	IWV	64.27	75,700	21,300,000	4,865,239	1,368,951,000
iShares S&P SmallCap 600	IJR	141.69	352,000	9,150,000	49,874,880	1,296,463,500
Select Sector SPDR-Technology	XLK	20.54	501,400	58,201,000	10,298,756	1,195,448,540
iShares Russell 2000 Value	IWN	168.10	385,900	5,950,000	64,869,790	1,000,195,000
Others			523,500	5,600,000	997,893,020	19,032,796,250
TOTAL			7,316,500	66,608,000	13,392,333,947	139,115,818,650
No. of funds		122				

Source: Data compiled from www.amex.com.

realistic way to model the dividend income of the index portfolio (i.e., as a discrete flow or as a continuous rate).

Value-Weighted Indexes[18]

The "value" of the common stocks in a value-weighted index refers to the total market capitalization of the firm's outstanding shares, that is, the number of shares outstanding, $n_{i,t}$, times the current price per share, $p_{i,t}$. The total market value of the index at time t is therefore

$$\text{Total market value of index}_t = \sum_{i=1}^{N} n_{i,t} p_{i,t} \tag{14.1}$$

where N is the number of stocks in the index. This market value is then scaled by a divisor so that the index in period t is

$$S_t = \frac{\sum_{i=1}^{N} n_{i,t} p_{i,t}}{\text{Divisor}_t} \tag{14.2}$$

The divisor represents what the stocks currently in the index would have been worth in the base period. In the base period the divisor is the market value of the stocks in the index,

$$\text{Divisor}_t = \sum_{i=1}^{N} n_{i,0} p_{i,0} \tag{14.3}$$

Note that stock splits and stock dividends have no effect on the index level because the increase in shares outstanding is proportionately offset by a reduction in share price.

Over time, the numerator of (14.2) changes because stocks enter or leave the index or because certain corporate actions such as restructurings, spinoffs, share issuance or repurchase affect the market value of a stock and hence the value of the index. Because such changes do not reflect market movements, an adjustment to the divisor is made on the day that a change occurs.[19] The new divisor on day t is just the old divisor on day t adjusted by the ratio of the market value of the new index composition on day t divided by the market value of the old index composition on day t,

[18] A value-weighted index is an inherently better measure of market performance. When the market is in equilibrium, the supply of stocks equals demand. The contribution of stock i to the performance of the market, therefore, equals the performance of stock i times the market value of all of i's shares outstanding as a proportion of the total market value of all stocks.

$$\text{New divisor}_t = \frac{\text{Market value of new index}_t}{\text{Market value of old index}_t} \times \text{Old divisor}_t \tag{14.4}$$

The best known value-weighted index in the United States is the S&P 500. The S&P 500 consists of 500 common stocks, 423 of which traded on the NYSE as of March 1, 2004 and 77 of which traded NASDAQ. The index was designed by Standard & Poors' to contain stocks from a broad variety of industry groupings. The market value for the base period of the S&P 500 is the average market values of the component stocks during the years 1941 through 1943. At that time, the index was set equal to 10. The S&P 500 index level at the close of trading on March 1, 2004 was 1,155.96. This is based on a total S&P 500 index market capitalization of $10,715,550,195,285 and a divisor of $9,269,805,842.

Value-weighted indexes can be heavily swayed by only a few stocks. Table 14.10 contains a list of the largest 50 stocks in the S&P 500 index as of the close of trading on March 1, 2004. Note that the largest 50 stocks account for nearly 52% of the total market capitalization of the index. The largest 10 stocks account for over 23%. To see the effect that a single large stock may have, consider the shares of General Electric, which accounted for 3.07% of the index on March 1, 2004. Using the information in Table 14.10 together with the above information about the total market capitalization and the divisor of the index, it is possible to show that a $1 move in the share price of General Electric will move the S&P 500 index by 1.08 points.

It is also worth noting that the stock indexes underlying derivatives contracts traded in non-U.S. countries are, in general, value-weighted indexes. These include Germany's DAX-30, France's CAC-40, the U.K.'s FT-SE 100, Australia's All Ordinaries Share Price index, Hong Kong's Hang Seng index, and Canada's TSE-35.

Price-Weighted Indexes

A *price-weighted index* is like a value-weighted index, except that the number of shares outstanding does not play a role. The price-weighted index is computed as

$$S_t = \frac{\sum_{i=1}^{N} p_{i,t}}{\text{Divisor}_t} \tag{14.5}$$

[19] An interesting phenomenon in its own right is the behavior of the price of the stock when it is added to or deleted from the S&P 500 index. Because about 10% of the market capitalization of the S&P 500 index portfolio is held as passive index mutual funds (e.g., The Vanguard Group's *500 Portfolio*) or exchange-traded funds (e.g., AMEX's SPDRs), an addition to (deletion from) implies that 10% of a stock's outstanding shares must be purchased (sold) on the day of the change. Such order imbalances generate abnormal price movements in the stock market. The early evidence indicated abnormal returns on order of two percent, See, for example, Harris and Gurel (1986) and Shliefer (1986). Because of substantial growth in indexing to the S&P 500 in recent years, the effect has become much larger. See, for example, Beneish and Whaley (1997, 2002).

TABLE 14.10 Fifty highest market value stocks in the S&P 500 index portfolio as of the close of trading on March 1, 2004.

Ticker	Company	Closing Price	No. of Shares	Market Value	Relative Weight	Cumulative Weight
GE	General Electric	32.79	10,041	329,240	3.07%	3.07%
MSFT	Microsoft Corp.	v26.70	10,812	288,693	2.69%	5.77%
PFE	Pfizer, Inc.	36.90	7,632	281,603	2.63%	8.39%
XOM	Exxon Mobil Corp.	42.52	6,610	281,049	2.62%	11.02%
WMT	Wal-Mart Stores	60.45	4,328	261,614	2.44%	13.46%
C	Citigroup Inc.	50.46	5,159	260,307	2.43%	15.89%
INTC	Intel Corp.	29.72	6,532	194,131	1.81%	17.70%
AIG	American Int'l. Group	74.09	2,608	193,223	1.80%	19.50%
IBM	International Bus. Machines	97.04	1,720	166,950	1.56%	21.06%
CSCO	Cisco Systems	23.54	6,903	162,505	1.52%	22.58%
JNJ	Johnson & Johnson	53.76	2,968	159,567	1.49%	24.07%
PG	Procter & Gamble	103.86	1,297	134,678	1.26%	25.32%
BAC	Bank of America Corp.	82.13	1,486	122,029	1.14%	26.46%
KO	Coca Cola Co.	49.62	2,452	121,644	1.14%	27.60%
MO	Altria Group, Inc.	58.18	2,031	118,174	1.10%	28.70%
MRK	Merck & Co.	48.45	2,225	107,801	1.01%	29.71%
VZ	Verizon Communications	38.70	2,762	106,872	1.00%	30.70%
WFC	Wells Fargo	57.63	1,692	97,512	0.91%	31.61%
CVX	ChevronTexaco Corp.	90.27	1,069	96,495	0.90%	32.51%
PEP	PepsiCo Inc.	52.16	1,717	89,546	0.84%	33.35%
DELL	Dell Inc.	33.52	2,560	85,825	0.80%	34.15%
JPM	J.P. Morgan Chase & Co.	41.53	2,040	84,732	0.79%	34.94%
HD	Home Depot	36.84	2,275	83,819	0.78%	35.72%
AMGN	Amgen	64.19	1,290	82,802	0.77%	36.50%
LLY	Lilly (Eli) & Co.	73.44	1,123	82,466	0.77%	37.27%
SBC	SBC Communications Inc.	24.25	3,311	80,284	0.75%	38.02%
UPS	United Parcel Service	69.85	1,124	78,528	0.73%	38.75%
TWX	Time Warner Inc.	17.23	4,522	77,910	0.73%	39.48%
FNM	Fannie Mae	77.25	972	75,061	0.70%	40.18%
HPQ	Hewlett-Packard	23.00	3,049	70,130	0.65%	40.83%
AXP	American Express	53.65	1,286	69,009	0.64%	41.47%
ORCL	Oracle Corp.	13.07	5,227	68,312	0.64%	42.11%
CMCSA	Comcast Corp.	30.33	2,251	68,271	0.64%	42.75%
ABT	Abbott Labs	43.40	1,563	67,850	0.63%	43.38%
VIA.B	Viacom Inc.	38.74	1,749	67,766	0.63%	44.01%
MWD	Morgan Stanley	60.75	1,083	65,795	0.61%	44.63%
WB	Wachovia Corp. (New)	48.66	1,324	64,414	0.60%	45.23%
MMM	3M Company	78.78	785	61,833	0.58%	45.81%
ONE	Bank One Corp.	54.44	1,118	60,860	0.57%	46.37%
MER	Merrill Lynch	62.29	945	58,883	0.55%	46.92%
TYC	Tyco International	29.11	1,999	58,194	0.54%	47.47%
MDT	Medtronic Inc.	47.25	1,212	57,270	0.53%	48.00%
USB	U.S. Bancorp	28.71	1,929	55,374	0.52%	48.52%
DIS	Walt Disney Co.	26.87	2,045	54,959	0.51%	49.03%
BMY	Bristol-Myers Squibb	28.24	1,939	54,765	0.51%	49.54%
TXN	Texas Instruments	31.02	1,731	53,684	0.50%	50.04%
WYE	Wyeth	39.64	1,332	52,793	0.49%	50.54%
BLS	BellSouth	27.68	1,848	51,156	0.48%	51.01%
GS	Goldman Sachs Group	107.35	473	50,827	0.47%	51.49%
QCOM	QUALCOMM Inc.	62.75	800	50,204	0.47%	51.96%

In a price-weighted index, the divisor in the base period equals the sum of the prices of the stocks in the base period, that is,

$$\text{Divisor}_0 = \sum_{i=1}^{N} p_{i,0} \tag{14.6}$$

Like a value-weighted index, the divisor of a price-weighted index is adjusted to reflect changes in composition, stock splits, stock dividends and spin-offs so that the index level remains unchanged. Unlike the value-weighted index, however, the divisor of the price-weighted index is not adjusted for new stock issues or share repurchases.

The best known price-weighted is the Dow Jones Industrial Average or DJIA or, simply, the Dow, in honor of Charles H. Dow who unveiled this average of industrial stock prices on May 26, 1896. The mechanics of the index had to be simple since, at the time, the index had to be computed by hand. The index was therefore a simple average of the prices of the constituent stocks. At inception, the Dow had only 12 stocks and a level of 40.94. In 1916, the number of stocks was increased to 20, and, in 1928, to 30, where it remains today. It was in 1928 that the Dow initiated the use of a divisor to handle changes in composition and corporate actions including stock splits, stock dividends, restructurings and spin-offs. Its level as of March 1, 2004 was 10,678.14. Of the 30 stocks in the Dow, 28 trade on the NYSE and 2 trade on NASDAQ.

The composition of the DJIA on March 1, 2004 is given in Table 14.11. Only one of the original 12 Dow stocks remains, General Electric. Note the implied weights of the stocks in the Dow. Proctor & Gamble has the most weight, followed by IBM. The last three columns of Table 14.11 construct the weights for each Dow stock if the index was value-weighted. Proctor & Gamble constitutes only 4.13% of the value-weighted Dow, considerably less than its 7.20% price-weight. General Electric's value-weighted contribution, on the other hand, is 10.11%, considerably more than its price-weighted contribution of 2.27%. Overall, the correlation between the price-weights and value-weights is only 0.41. The market value of the Dow is $3.258 trillion, about 30% of the market value of the S&P 500.

To illustrate the computation of the DJIA, consider the values reported at the bottom on Table 14.11. The sum of the prices of the Dow stocks on March 1, 2004 was $1,441.58. On the same day, the divisor was 0.13500289. The closing level of the Dow on March 1, 2004 was therefore 10,678.14.

Discrete or Continuous Dividend Income?

The decision about whether to model the dividend income of an index portfolio as discrete cash payments or as a continuous dividend yield must be based on an analysis of the actual dividend payments. Here, such an analysis is conducted for the S&P 500 and DJIA indexes, as they have two of the most active derivative contract markets.

TABLE 14.11 Thirty Dow Jones Industrial Average stocks as of the close of trading on March 1, 2004.

Ticker	Company	Closing Price	Price Weight	No. of Shares	Market Value	Value Weight
PG	Procter & Gamble Co.	103.86	7.20%	1,297	134,678	4.13%
IBM	International Business Machines Corp.	97.04	6.73%	1,720	166,950	5.12%
UTX	United Technologies Corp.	92.20	6.40%	515	47,517	1.46%
MMM	3M Co.	78.78	5.46%	785	61,833	1.90%
CAT	Caterpillar Inc.	76.84	5.33%	347	26,682	0.82%
WMT	Wal-Mart Stores Inc.	60.45	4.19%	4,328	261,614	8.03%
MO	Altria Group Inc.	58.18	4.04%	2,031	118,174	3.63%
JNJ	Johnson & Johnson	53.76	3.73%	2,968	159,567	4.90%
AXP	American Express Co.	53.65	3.72%	1,286	69,009	2.12%
C	Citigroup Inc.	50.46	3.50%	5,159	260,307	7.99%
KO	Coca-Cola Co.	49.62	3.44%	2,452	121,644	3.73%
GM	General Motors Corp.	48.65	3.37%	561	27,281	0.84%
MRK	Merck & Co. Inc.	48.45	3.36%	2,225	107,801	3.31%
DD	E.I. DuPont de Nemours & Co.	45.63	3.17%	997	45,483	1.40%
IP	International Paper Co.	44.50	3.09%	480	21,381	0.66%
BA	Boeing Co.	43.77	3.04%	841	36,821	1.13%
XOM	Exxon Mobil Corp.	42.52	2.95%	6,610	281,049	8.63%
JPM	J.P. Morgan Chase & Co.	41.53	2.88%	2,040	84,732	2.60%
AA	Alcoa Inc.	38.40	2.66%	865	33,230	1.02%
HD	Home Depot Inc.	36.84	2.56%	2,275	83,819	2.57%
HON	Honeywell International Inc.	35.31	2.45%	862	30,439	0.93%
GE	General Electric Co.	32.79	2.27%	10,041	329,240	10.11%
INTC	Intel Corp.	29.72	2.06%	6,532	194,131	5.96%
EK	Eastman Kodak Co.	29.03	2.01%	287	8,319	0.26%
MCD	McDonald's Corp.	28.41	1.97%	1,269	36,057	1.11%
DIS	Walt Disney Co.	26.87	1.86%	2,045	54,959	1.69%
MSFT	Microsoft Corp.	26.70	1.85%	10,812	288,693	8.86%
SBC	SBC Communications Inc.	24.25	1.68%	3,311	80,284	2.46%
HPQ	Hewlett-Packard Co.	23.00	1.60%	3,049	70,130	2.15%
T	AT&T Corp.	20.37	1.41%	790	16,090	0.49%
	Sum across closing prices on 3/1/4	1,441.58	100.00%		3,257,913	100.00%
	Divisor on 3/1/4	0.13500289				
	Closing DJIA 3/1/4	10,678.14				

In Chapter 8, cash dividends of individual U.S. stocks were shown to be paid on a quarterly cycle. Since a stock index is nothing more than a portfolio of stocks, the dividend income of a value-weighted index is simply the sum of the value-weighted dividends of the index stocks, that is,

$$d_{VW,t} = \frac{\sum_{i=1}^{N} n_{i,t} d_{i,t}}{\text{Divisor}_t} \tag{14.7}$$

and the dividend income of a price-weighted index is the sum of the equal-weighted dividends of the index stocks, that is,

$$d_{PW,t} = \frac{\sum_{i=1}^{N} d_{i,t}}{\text{Divisor}_t} \tag{14.8}$$

If the quarterly dividend payment cycles of the stocks comprising the index are randomly distributed throughout the year and if the number of stocks in the index is large, the dividend stream of the index will be reasonably smooth, and modeling the index dividends as a continuous yield would be appropriate. On the other hand, if the cash dividend payment cycles tend to cluster at different times during the year or if the number of stocks is small, modeling the index dividends as discrete cash payments is better.

S&P 500 Dividends Figure 14.2 shows the average daily cash dividends of the S&P 500 index by calendar month during the period 1989 through 2003. Note the prominence of the cash dividends in the Feb./May/Aug./Nov. cycle. The amount of the dividends paid during these four months is nearly as much as the other eight months combined. This seasonal pattern induces considerable variation in the index's dividend yield rate. Based on Figure 14.2, the dividend yield rate in the month of January is about half that of February. Depending on which calendar months the life of the derivatives contract spans, the dividend yield rate on the S&P 500 index will vary. This is not very comforting if you want to apply the constant dividend yield valuation framework.

FIGURE 14.2 Average daily cash dividends of the S&P 500 index by month of year during the period January 1989 through December 2003.

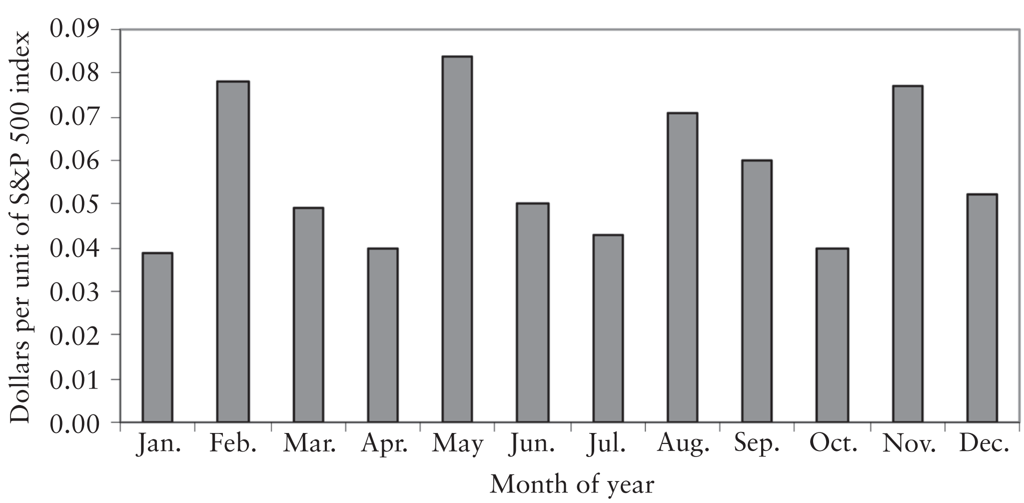
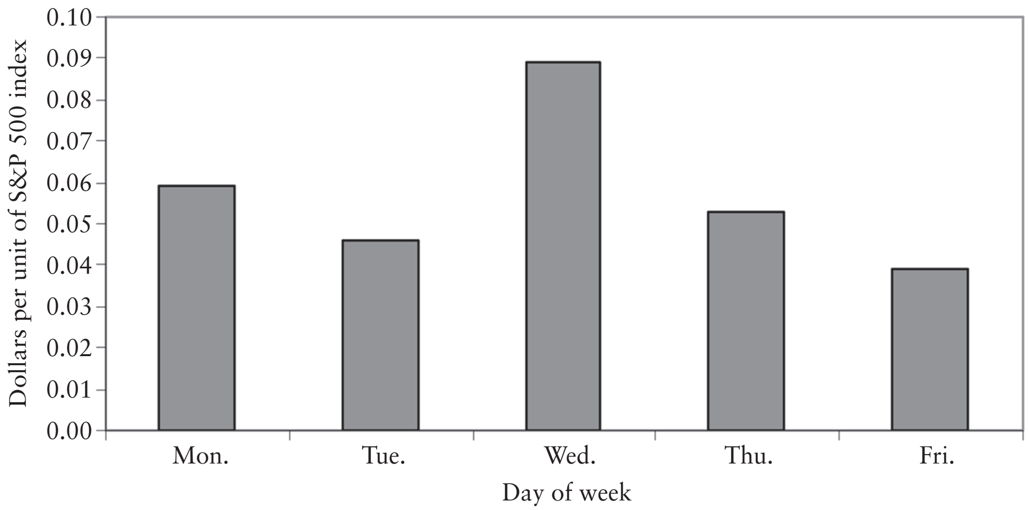


FIGURE 14.3 Average daily cash dividends of the S&P 500 index by day of week during the period January 1989 through December 2003.



The quarterly payment cycle is not the only pattern that appears in the S&P 500 index cash dividends. Figure 14.3 shows the average daily cash dividends of the S&P 500 index by day of week. More dividends are paid on Mondays and Wednesdays than other days of the week. For short-term derivative contracts, this variation in dividend payments can have a pronounced effect on valuation. Put differently, the dividend yield rate of the index will vary during the derivative contract's life. Taken together, the evidence showing monthly and daily variation in cash dividends supports the application of the discrete flow cost of carry framework.

One final note regarding the S&P 500 cash dividends is warranted. Over the period 1989 through 2003, the total cash dividends paid on the S&P 500 index has grown by about 58%, as is shown in Figure 14.4. The growth in the index itself, however, has been about 285%. Consequently, the annual dividend yield rate has fallen dramatically over the period, from 3.89% annually in 1989 to only 1.96% in 2003.[20] The smaller are the dividends relative to the index level, the less important modeling dividends accurately becomes. Hence, using the constant dividend yield model to value S&P 500 derivatives is not completely without merit.

DJIA Dividends Figure 14.5 shows the average daily cash dividends of the DJIA by calendar month during the period 1963 through 2003. For the DJIA, the cash dividends in the Feb./May/Aug./Nov. cycle are even more pronounced than they were for the S&P 500. The amount of the dividends paid during these four months easily exceeds the other eight months combined. Figure 14.6 shows the average daily cash dividends of the DJIA by day of week. Considerably more

[20] The reported dividend yield of the S&P 500 is a continuous rate computed as $\ln[(S + DVDS)/S]$, where $DVDS$ is the sum of the cash dividends paid during the year and S is the closing level of the index on the last day of trading of the previous year.

FIGURE 14.4 Total cash dividends and dividend yield of the S&P 500 index by year during the period January 1989 through December 2003.

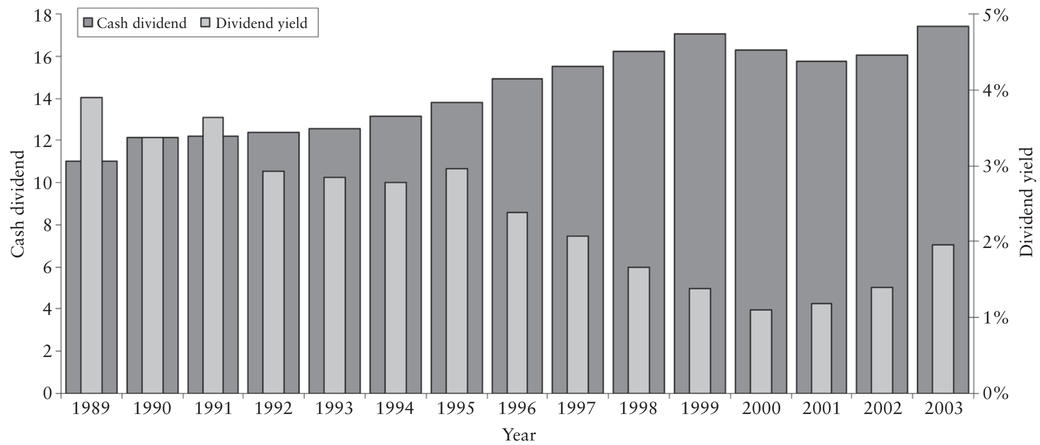
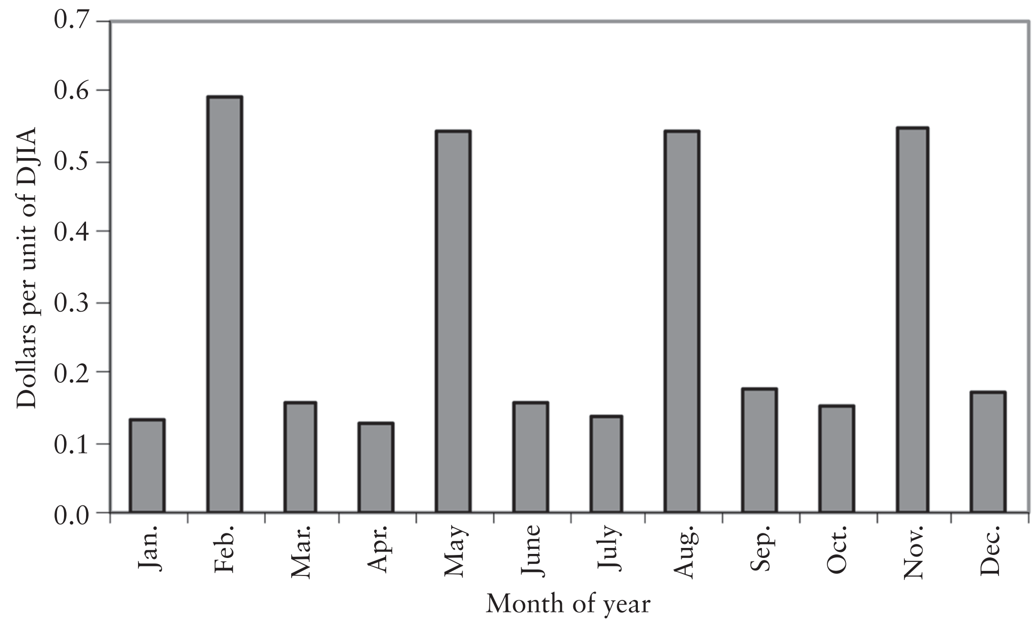
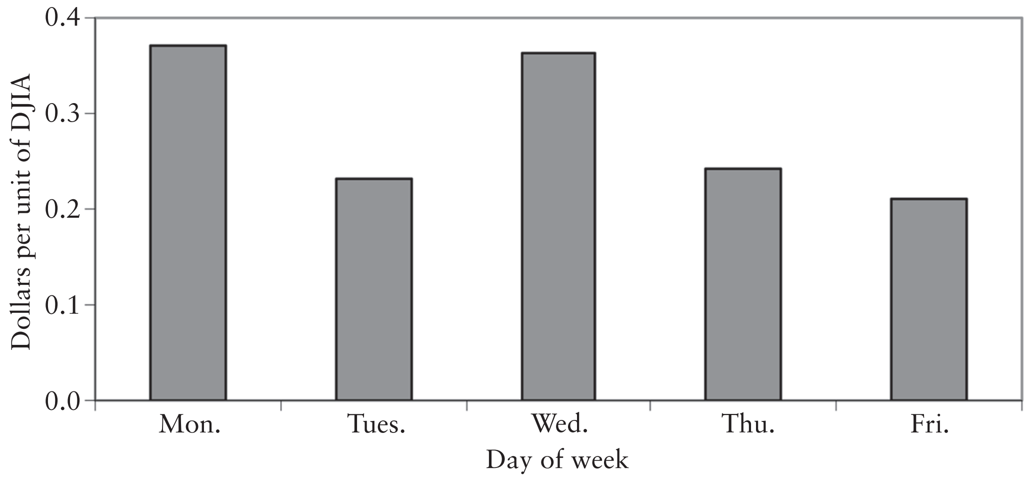


FIGURE 14.5 Average daily cash dividends of the DJIA index by month of year during the period January 1963 through December 2002.



dividends are paid on Mondays than other days of the week. For the DJIA, it is clearly inappropriate to apply a valuation framework that assumes the dividend yield rate is constant through time. The evidence clearly supports the application of the discrete flow valuation results.

FIGURE 14.6 Average daily cash dividends of the DJIA index by day of week during the period January 1963 through December 2002.



NO-ARBITRAGE RELATIONS AND VALUATION

Under the assumption of discrete dividend payments on the underlying index, the no-arbitrage price relations and valuation methods for stock index derivatives are summarized in Table 14.12. For completeness, the valuation results for an index with a continuous dividend yield rate are provided in Table 14.13. Below, the focus is primarily on the former.

Stock Index Futures

Index Arbitrage The cost of carry relation between the stock index futures price and the level of the underlying index under the assumption of known discrete dividends in future value form is

$$F = Se^{rT} - FVD \tag{14.9}$$

and, expressed as a prepaid forward contract, is

$$Fe^{-rT} = S - PVD \tag{14.10}$$

where

$$FVD = \sum_{i=1}^{n} D_i e^{r(T-t_i)}$$

is the future value of the cash dividends paid during the futures life and

TABLE 14.12 Summary of no-arbitrage price relations and valuation equations/ methods for derivatives on stock indexes with discrete dividends.

No-Arbitrage Price Relations		
Forward/Futures	$f = F = Se^{rT} - FVD$ or $fe^{-rT} = Fe^{-rT} = S - PVD$	
	where $FVD = \sum_{i=1}^{n} D_i e^{r(T-t_i)}$ and $PVD = e^{-rT} FVD = \sum_{i=1}^{n} D_i e^{-rt_i}$	
European-Style:	**Options**	**Futures Options**
Lower bound for call	$c \geq \max(0, S - PVD - Xe^{-rT})$	$c \geq \max[0, e^{-rT}(F - X)]$
Lower bound for put	$p \geq \max(0, Xe^{-rT} - S + PVD)$	$p \geq \max[0, e^{-rT}(X - F)]$
Put-call parity	$c - p = S - PVD - Xe^{-rT}$	$c - p = e^{-rT}(F - X)$
American-Style:		
Lower bound for call	$C \geq \max(0, S - PVD - Xe^{-rT}, S - X)$	$C \geq \max(0, F - X)$
Lower bound for put	$P \geq \max(0, Xe^{-rT} - S + PVD, X - S)$	$P \geq \max(0, X - F)]$
Put-call parity	$S - PVD - X \leq C - P \leq S - Xe^{-rT}$	$Fe^{-rT} - X \leq C - P \leq F - Xe^{-rT}$
Valuation Equations/Methods		
European-Style:	**Options**	**Futures Options**
Call value	$c = S^x N(d_1) - Xe^{-rT} N(d_2)$	$c = e^{-rT}[FN(d_1) - XN(d_2)]$
Put value	$p = Xe^{-rT} N(-d_2) - S^x N(-d_1)$	$p = e^{-rT}[XN(-d_2) - FN(-d_2)]$
	where $S^x = S - PVD$,	where
	$d_1 = \frac{\ln(S^x / Xe^{-rT}) + 0.5\sigma^2 T}{\sigma\sqrt{T}}$ and $d_2 = d_1 - \sigma\sqrt{T}$	$d_1 = \frac{\ln(F/X) + 0.5\sigma^2 T}{\sigma\sqrt{T}}$ and $d_2 = d_1 - \sigma\sqrt{T}$
American-Style:		
Call and put values	Numerical valuation: binomial method and trinomial method.	Numerical valuation: quadratic approximation, binomial method, and trinomial method.

TABLE 14.13 Summary of arbitrage price relations and valuation equations for derivatives on stock indexes with continuous dividend yields.

No-Arbitrage Price Relations		
Forward/Futures	Future value: $f = F = Se^{(r-\delta)T}$ Present value: $fe^{-rT} = Fe^{-rT} = Se^{-dT}$	
European-Style:	**Options**	**Futures Options**
Lower bound for call	$c \geq \max(0, Se^{-\delta T} - Xe^{-rT})$	$c \geq \max[0, e^{-rT}(F - X)]$
Lower bound for put	$p \geq \max(0, Xe^{-rT} - Se^{-\delta T})$	$p \geq \max[0, e^{-rT}(X - F)]$
Put-call parity	$c - p = Se^{-\delta T} - Xe^{-rT}$	$c - p = e^{-rT}(F - X)$
American-Style:		
Lower bound for call	$C \geq \max(0, Se^{-\delta T} - Xe^{-rT}, S - X)$	$C \geq \max(0, F - X)$
Lower bound for put	$P \geq \max(0, Xe^{-rT} - Se^{-\delta T}, X - S)$	$P \geq \max(0, X - F)]$
Put-call parity	$Se^{-\delta T} - X \leq C - P \leq S - Xe^{-rT}$	$Fe^{-rT} - X \leq C - P \leq F - Xe^{-rT}$
Valuation Equations/Methods		
European-Style:	**Options**	**Futures Options**
Call value	$c = Se^{-\delta T}N(d_1) - Xe^{-rT}N(d_2)$	$c = e^{-rT}[FN(d_1) - XN(d_2)]$
Put value	$p = Xe^{-rT}N(-d_2) - Se^{-\delta T}N(-d_1)$	$p = e^{-rT}[XN(-d_2) - FN(-d_2)]$
	where $S^x = S - PVD$,	where
	$d_1 = \dfrac{\ln(Se^{-\delta T}/Xe^{-rT}) + 0.5\sigma^2 T}{\sigma\sqrt{T}}$ and $d_2 = d_1 - \sigma\sqrt{T}$	$d_1 = \dfrac{\ln(F/X) + 0.5\sigma^2 T}{\sigma\sqrt{T}}$ and $d_2 = d_1 - \sigma\sqrt{T}$
American-Style:		
Call and put values	Numerical valuation: quadratic approximation, binomial method, and trinomial method.	Numerical valuation: quadratic approximation, binomial method, and trinomial method.

$$PVD = FVDe^{-rT} = \sum_{i=1}^{n} D_i e^{-rt_i}$$

is the present value of the cash dividends. The relation arises from the absence of costless arbitrage opportunities in the marketplace. The intuition for this relation is that we have two ways to have a stock portfolio on hand at time T at a price we know today. The first, represented by the left-hand side of (14.8), is to buy a futures contract with maturity T. At time T, we pay F and receive the stock portfolio. The second, represented by the right-hand side of (14.8), is to borrow at a rate r to buy the stock portfolio today at S, and then carry it until T has elapsed. At time T, we must repay our borrowings plus interest, Se^{rT}, which is partially offset by the accumulated cash dividends (plus accrued interest) received while holding the stock portfolio, FVD. Since the two alternatives are perfect substitutes, the two sides of (14.8) must be equal. The second formulation (14.9) is the same as (14.8), except that it is expressed in present value terms.

Fair Value The term *fair value* is often used in conjunction with stock index arbitrage. Unfortunately, it is not always used in a consistent manner, and this often leads to confusion. To some, the definition of fair value is the theoretical futures price given the current index level, the cash dividends promised during the futures' life, and the risk-free rate of interest. In the interest of clarity, we will call this definition the *fair value of the futures*. By virtue of the cost of carry relation (14.8), we know that

$$\text{Fair value of futures} = Se^{rT} - FVD \tag{14.11}$$

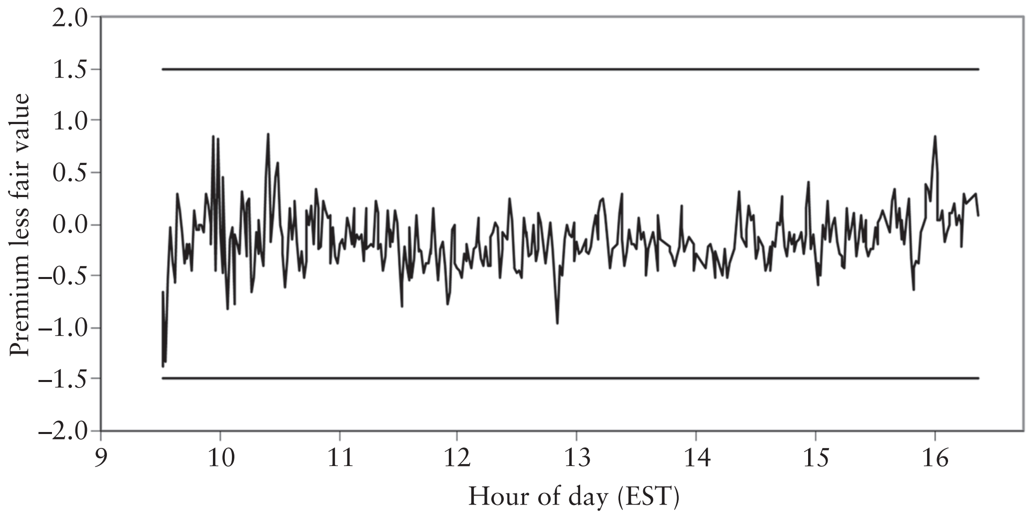
To others, fair value is the theoretical futures price less the current index level. We will call this definition the *fair value of the basis*. Subtracting the current index level from (14.10), we get

$$\text{Fair value of basis} = S(e^{rT} - 1) - FVD \tag{14.12}$$

It is important to recognize that fair values are theoretical values and may not correspond to *actual* prices reported in the marketplace. The *premium* (or *spread*) refers to the difference between the current prices of the futures and the index assuming both markets are open. Thus, if the premium is above the fair value of the basis, index arbitrageurs will sell futures and buy the underlying stocks, driving the price of the futures down and the prices of stocks up. The arbitrage will continue until where the premium equals fair value. On the other hand, if the premium is below fair value, index arbitrageurs will buy the futures and sell the underlying stocks, driving the price of the futures up and the prices of stocks down.

Figure 14.7 shows the difference between the premium and the fair value (i.e., the *basis mispricing*) for the S&P 500 index and index futures on a minute-to-minute basis throughout the trading day on August 29, 2003. The vertical axis is in index points. The solid horizontal lines at 1.5 and –1.5 represent trading cost

FIGURE 14.7 Intraday basis mispricing of the September 2003 S&P 500 futures on August 29, 2003.



bands. Index arbitrageurs look for differences between the premium and fair value of at least 1.5 index points in order to cover the trading costs of index arbitrage. Note that the premium seldom violated the trading costs bands during the day. This should not be surprising in the sense that a number of index arbitrageurs monitor the basis mispricing continuously throughout the day and trade quickly each time there is a profitable arbitrage opportunity appears. The large deviations that appear at the beginning of the day are illusory. The S&P 500 index is based on last trade prices, and, when the index begins getting computed each morning (9:30 AM EST), it is based largely on the closing prices of the previous trading day. A few minutes after the open, when all stocks have traded (and incorporated overnight news), the premium is back near fair value.

Program Trading Stock index arbitrage is unlike typical basis arbitrage in the sense that buying and selling the underlying asset means buying and selling a precisely weighted *portfolio* of common stocks. Engaging in index arbitrage with the S&P 500 index, for example, requires a mechanism for buying or selling quickly and simultaneously all 500 stocks in the S&P 500 index portfolio. Since the simultaneous purchase or sale of the stocks in a precisely weighted and timely fashion is cumbersome, computers and computer programs are usually used to place transaction orders as well as to assist in the execution of those orders. For this reason, trading of portfolios of stocks is called *program trading*.

Pre-Open Stock Market Predictions Financial news programs such as Squawk Box on CNBC use the fair value of the basis to generate predictions regarding the level at which the stock will open (relative to the previous day's close) based on the fair value of the basis. The arithmetic is simple. They report two numbers. The first is the change in the futures price expected due to the fact that the stock market and the futures market close at different times during the day (4:00 versus 4:15 PM EST). This gets reported with the caption "Fair value." For the sake of illustration, suppose that what appears on the screen is "Fair value ▲5.00." The second is the change in the futures price from its previous day's close at 4:15 PM EST. The index futures contract trades electronically virtually 24 hours a day. Suppose that as of 8:30 AM EST (one hour before the stock market opens), the futures price change is reported as "Futures ▲2.00." Based of these two values reported on the screen, the commentator might say, "Even with the futures up 2, they are well below fair value and are a negative for the opening. We need to get to plus 5 in order to be at fair value." The implicit arithmetic is simply the difference between the second number and the first, that is, 2.00 – 5.00 = –3.00. The stock market is expected to open 3 points lower.

The mechanics of their computation is this. First, they compute the fair value of the basis using (14.9b), where T is the time to expiration (expressed in years) of the futures contract and the index level is the previous day's close, $S_{4:00\ \text{PM}}$.[21] Assume, for the sake of argument, fair value is 6.00. Next they compute the difference between the closing futures price and the closing index level. When both markets are open, we would expect the difference between the two prices to hover around fair value due the presence of index arbitrageurs in the marketplace (e.g., see Figure 14.7). The stock market closes at 4:00 PM EST, however, and the index futures market closes at 4:15 PM EST. The futures price may move up or down during this 15-minute interval of time as new market information arrives, and the closing premium may be quite different from fair value. Suppose, for the sake of argument, the futures price closes at a 1.00 premium to the closing index while fair value is 6.00. What would appear on the television screen is "Fair value ▲5.00," which means, based on the closing index level and the computed fair value, the futures price should be five points higher. If the early morning futures price is only 2.00 points higher than its close ("Futures ▲2.00"), the stock market is expected to open 3.00 points lower than the previous day's close. If the futures price is 9.00 points higher ("Futures ▲9.00"), the stock market is expected to open 4.00 points higher, and, if the futures price is 7.00 points lower ("Futures ▼7.00"), the stock market is expected to open 12.00 points lower.

Special Settlement Quotation

S&P 500 index products use a special cash settlement quotation based on the opening prices of index stocks. As this number plays a key role in the profitability and risk of stock index arbitrage strategies, some discussion of the settlement procedure is warranted.

Probably the first question that arises in considering the settlement of the index derivatives contracts is why use the opening price rather than the closing price? After all, stock option contracts had expired at the close since they began trading a decade before index products were introduced.[22] The answer to the

[21] By this definition the fair value of the basis will be constant throughout the trading day.
[22] There is the subtle, but important, distinction that stock options settle through the delivery of the underlying stock, however.

question is that exchange-traded index products *were*, indeed, cash settled at the close when they were first introduced in the early 1980s. After about four years of closing settlements, the financial press and other market commentators uncovered the fact that stock indexes moved "abnormally" during the last hour of trading on the quarterly expiration of index futures, index futures options, and index options (the notorious "triple-witching hour"). Regulators quickly jumped into the fray, charging that index derivatives had become a destabilizing influence on the stock market and should be banned.

The key to understanding the controversy lies in the mechanics of index arbitrage and the cash settlement of index derivatives. Consider stock index futures arbitrage, for example. During the life of the futures contract, arbitrageurs tend to build up large positions in index futures and the stocks of the underlying index portfolio. If the premium tended to be above the fair value on average during the futures contract's life, arbitrageurs are likely be short index futures and long the index portfolio's stocks going into contract expiration, and vice versa. (Keep in mind that all index arbitrageurs see the same set of signals during the futures contract's life and are therefore likely to have similar positions.) Now consider the actions of the arbitrageurs at contract expiration. Because the index futures is cash-settled at the closing index level, arbitrageurs must unwind their stock portfolios at the same prices that go into the closing index level computation. To accomplish this, they place market-on-close orders.[23] If arbitrageurs are long stocks, they place market-on-close orders to sell and the excess selling pressure causes stock prices to fall at the close. If arbitrageurs are short stocks, the excess buying pressure causes stock prices to rise. Since the net positions of index arbitrageurs on the expiration day are not known, the direction of the price movement is not predictable. It is these uncertain price movements on the quarterly expiration cycle that are at the center of the triple-witching hour controversy.

Claims that index derivatives contract expirations destabilize the stock market have been refuted, however.[24] Stoll and Whaley (1987) examined movements in the S&P 500 index on the ten quarterly expirations (September 1983 through December 1985) and found they are roughly the same size as one would expect given trading costs in the marketplace. Indeed most of the observed index movement in the last hour of trading is not a real movement in stock prices. Consider the nature of the reported index level at any point in time during the day. The S&P 500 index, for example, is based on the last trade prices for each of the stocks within the index portfolio. The last trade price of a stock, in turn, may be at the bid or at the ask, depending on whether the trader sold or bought. Assuming the last trades of the 500 stocks in the index are approximately evenly balanced between buys and sells, the reported index level at any point in time can be thought of as being at a midpoint between the bid and ask

[23] A market-on-close (MOC) order is an order that is executed at the closing price of the day. Under current NYSE rules, a MOC order is assured of execution at the closing price if it is entered by a certain time during the trading day. There is no such mechanism to provide such assurance for NASDAQ trades, however.

[24] In this discussion, we focus primarily on U.S. stock markets, however, expiration-day effects have been analyzed empirically for Japan (see Karolyi (1996)), Australia (see Stoll and Whaley (1997)), and Hong Kong (see Bollen and Whaley (1999)).

levels of its constituent stocks. Now consider what happens when index arbitrageurs unwind their positions at the close on expiration day. Assuming they are long (short) stocks, they place MOC order to sell (buy). With all 500 stocks traded at bid (ask) prices, the reported index level moves, not because of selling pressure moving prices but only because the reported index level at this one instant in time is based entirely on bid (ask) prices.

Even though the evidence indicated that the price movements during the triple-witching hour were not abnormal, the Chicago Mercantile Exchange (CME) adopted a suggestion by the Securities and Exchange Commission (SEC) to move its S&P 500 futures and futures options expiration from the close of the trading day to the open beginning with the June 1987 contract expiration. The rationale was that at the open the NYSE specialists have the opportunity to disseminate information about large order imbalances to off-floor market participants, thereby minimizing price impact. The Chicago Board Options Exchange (CBOE) continued to settle its index options at the close. Put differently, as of June 1987, the triple-witching hour at the close of trading on the quarterly contract expiration cycle ceased to exist, and was replaced with a double-witching hour at the open and a single witching hour at the close.

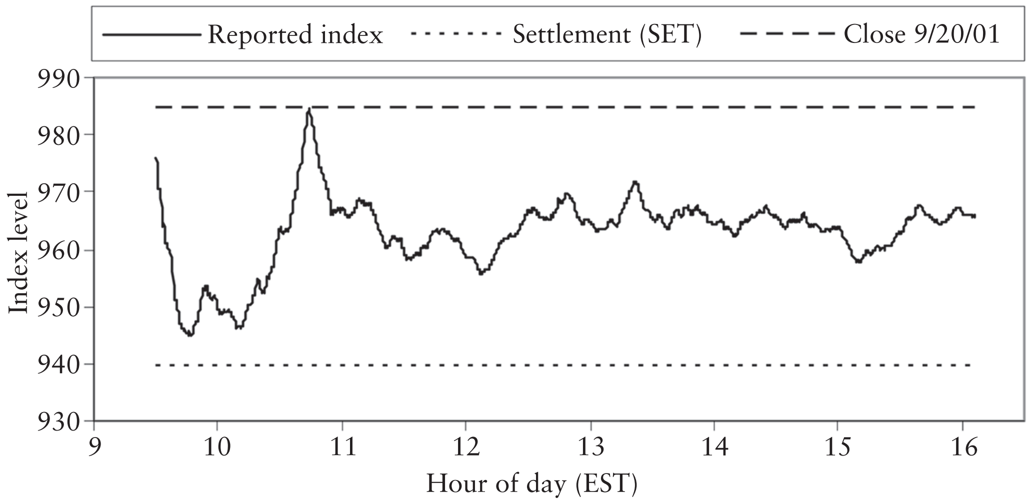
Stoll and Whaley (1991) assessed the effect of the change in procedure and found that the absolute size of the price movement at the open was slightly smaller than it was under the previous regime. One possible explanation for this result is that expiration-day trading was split between the open and the close. Another is that the change in settlement procedure accomplished its goal of reducing price impact. Over time, a consensus seemed to develop that the opening settlement worked best, and the CBOE adopted the practice for its S&P 500 index options. Now all contracts are settled at the special opening quotation, and the triple-witching hour has reemerged, albeit at the open of the trading day.

Index arbitrageurs do not care whether they have close or open settlement. As long as they can liquidate their stocks at the same prices that are used to compute the settlement index level of the futures, they can exit their arbitrage positions risklessly. But the special opening settlement quotation has introduced an interesting anomaly. Since the settlement quotation is computed on the basis of the opening trade prices and opening trades occur at different times in the morning, the settlement quotation may be quite different from *any* reported index level during the trading day. Figure 14.8 shows the reported levels of the S&P 500 index on September 21, 2001—the expiration day of the September 2001 S&P 500 futures, futures options, and index options. The special settlement quotation for the S&P 500 contracts, based on the opening trade prices of the index stocks, was 939.57. During the trading day, however, the reported S&P 500 index level, based on last trade prices, never fell below 944.75, 5.18 points higher than the settlement quotation. Imagine the confusion of someone holding a September 2001 call with an exercise price of 940!

Stock Index Options

The valuation equations/methods for index options are also provided in Tables 14.11 and 14.12. Most index options traded in the United States are European-

FIGURE 14.8 Reported intraday S&P 500 index levels on September 21, 2001 in relation to previous day's close and the special opening quotation for the S&P 500 index (SET).



style. The most notable exception is the S&P 100 index options, which are American-style. All of the index futures options traded in the U.S. are American-style.

ILLUSTRATION 14.1 Compute implied volatilities from at-the-money S&P 500 index option prices.

Assume that the S&P 500 index level is 1,100, the three-month S&P 500 futures price is 1,103, and the three-month S&P 500 at-the-money call and puts options have prices of 48.50 and 45.60, respectively. The risk-free rate of interest is 2.5%. Compute the market's perception of expected stock market volatility over the next three months.

The valuation formulas for European-style call and put options are given in Table 14.11. Since the present value of the cash dividends promised during the options' lives is not provided, you must find a way to deduce the amount. Since you are given the futures price, you can use the cost of carry relation (14.8a) to compute the future value of the dividends paid during the options' lives as

$$FVD = Se^{rT} - F = 1{,}100e^{0.025(3/12)} - 1{,}103 = 3.897$$

Based on the value of FVD, $PVD = 3.897e^{0.025(3/12)} = 3.872$.

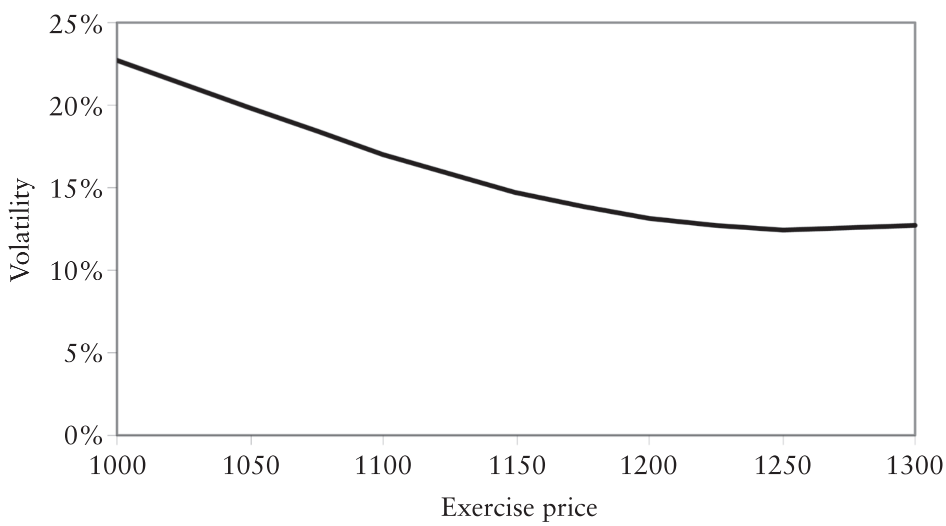
With the present value of dividends computed, you can compute the index level net of the present value of the dividends paid during the options' life, that is, $S^x = 1{,}100 - 3.872 = 1{,}096.128$. Now you have all the information you need to solve for the implied standard deviations of the call and the put. Using the OV_OPTION_ISD function from the OPTVAL Library, the implied volatility of the call is 21.53% and the implied volatility of the put is 21.57%. Given put-call parity, the implied volatility figures should be exactly equal to one another. Their values are slightly different because reported prices are discrete (option prices above $3 are reported in dimes). To mitigate part of this error, you may want to average the call and put implied volatilities to arrive at your estimate of expected future volatility, 21.55%.

ILLUSTRATION 14.2 Plot relation between implied volatility and exercise price.

Under the BSM assumptions, the prices of all options written on the same underlying asset or futures should have the same implied volatility. In practice, however, they are not. Based on the settlement prices of the S&P 500 futures options reported in Table 14.7, compute the implied volatilities for all June 2004 put options and plot them as a function of exercise price. The closing Jun/04 S&P 500 futures prices was 1129.70, and the risk-free rate of interest was 0.8879%. The options have 65 days remaining to expiration.

Futures options traded in the U.S. are American-style. To compute the implied volatilities for each option series, we use the OPTVAL function OV_FOPTION_ISD. Setting the option style argument in the function to "A" (for American-style) means that option valuation occurs using the quadratic approximation. The implied volatilities of the Jun/04 put option series are as shown:

Exercise Price	Jun/04 Put Prices	Implied Volatility
1000	4.90	22.71%
1025	6.80	21.24%
1050	9.60	19.85%
1075	13.50	18.39%
1100	19.30	17.06%
1125	27.70	15.83%
1150	39.40	14.67%
1175	55.30	13.85%
1200	74.70	13.12%
1225	97.00	12.72%
1250	120.80	12.38%
1300	170.30	12.70%



Implied Volatility Function (IVF) The relation between implied volatility and exercise price for index options shown in the above illustration is popularly referred to as the implied volatility "smile" or "sneer." Where this relation should be a horizontal line under the assumptions of the BSM model, implied volatility of S&P 500 futures options declines monotonically as exercise price rises. Most attempts to explain the shape of the IVF focus on relaxing the BSM assumption of constant volatility by allowing the local volatility rate of underlying security returns to evolve either deterministically or stochastically through time. Emanuel and MacBeth (1982) examine the power of the deterministic Cox and Ross (1976) constant elasticity of variance (CEV) model to explain the cross-sectional distribution of stock option prices. With its additional degree of freedom, the CEV model (necessarily) fits the observed structure of option prices better than the BSM constant volatility model. Out of sample, however, Emanuel and MacBeth conclude that the CEV model does no better than the BSM model. Similarly, the implied binomial tree framework of Dupire (1994), Derman and Kani (1994), and Rubinstein (1994) offers a deterministic local volatility structure so flexible that, in sample, it can describe the cross-section of options prices exactly at any point in time. Empirical tests by Dumas, Fleming, and Whaley (1998), however, show that a model based on a simple deterministic volatility structure has parameters that are highly unstable through time. Taken together, this evidence suggests that deterministic volatility models cannot explain the time-series variation in option prices or, equivalently, in the shape of the IVF.

Option valuation models based on stochastic volatility assumptions also have the potential to explain the shape of the IVF. In particular, a stochastic volatility model can generate the observed downward sloping IVF if innovations to volatility are negatively correlated with underlying asset returns. A negative relation between volatility and returns has been documented empirically by Black (1976) for individual stocks and Nelson (1991) for the index. Chernov et al. (2003) study a two-factor stochastic volatility model and find that it achieves a good fit to daily Dow Jones Industrial Average returns. Studies by Jorion (1989) and Anderson, Benzoni, and Lund (2002) report that randomly arriving jumps in security price in addition to stochastic volatility are required to capture the time-series dynamics of index returns.

Recent examinations of the performance of stochastic volatility option valuation models indicate that, at best, they can provide only a partial explanation of the shape of the index IVF. Bakshi, Cao, and Chen (1997), for example, advocate the use of a stochastic volatility model with jumps for valuing S&P 500 index options. While their model appears to perform better than the BSM formula, some of the implied parameter estimates, including the volatility of volatility coefficient, differ significantly from the ones estimated directly from returns. Similarly, Bates (2000) examines the ability of a stochastic volatility model, with and without jumps, to generate the negative skewness consistent with a steep IVF. He finds that the inclusion of a jump process can improve the model's ability to generate IVFs consistent with market prices, but in order to do so parameters must be set to unreasonable values. Along a similar line, Jackwerth attempts to recover risk aversion functions from S&P 500 index option prices and concludes that they are "irreconcilable with a representative investor" (2000, p. 450).

Another avenue of investigation that seems to lead to a better understanding of the IVF is the study of option market participants' supply and demand for different option series in different option markets. One way to think of the IVF is as a series of market clearing option prices quoted in terms of BSM implied volatilities. Under dynamic replication, the supply curve for each option series is a horizontal line. No matter how large the demand for buying a particular option, its price and implied volatility are unaffected. In reality, however, there are limits to arbitrage. Shleifer and Vishny (1997) describe how the ability of professional arbitrageurs to exploit mis-priced securities is limited by the responsiveness of investors to intermediate losses. Liu and Longstaff (2000) show that it is often optimal for a risk-averse investor to take a smaller position in a profitable arbitrage than his margin constraints allow, since intermediate mark-to-market losses may force liquidation of his position prior to convergence. In the same way, a market maker will not stand ready to sell an unlimited number of contracts in a particular option series. As his position grows large and imbalanced, his hedging costs and/or volatility risk exposure also increase, and he is forced to charge a higher price. With an upward sloping supply curve, differently shaped IVFs in different markets can be expected. The result of these limits to arbitrage is that market prices can diverge from model values, and that the divergence can persist. In effect, the no-arbitrage band within which prices can fluctuate can be quite wide, allowing price to be affected by supply and demand considerations.

Interacting with the market maker's willingness to supply options is investor demand. The level of implied volatility will be higher or lower depending upon whether net public demand for a particular option series is to buy or to sell. In the S&P 500 index option market, for example, it is well known that institutional investors buy index puts as portfolio insurance. Unfortunately, there are no natural counterparties to these trades, and market makers must step in to absorb the imbalance. With an upward sloping supply curve, implied volatility will exceed actual return volatility, with the difference being the market maker's compensation for hedging costs and/or exposure to volatility risk.[25] If institutional demand tends to be focused in a particular option series, such as out-of-the-money puts, the IVF will be downward sloping.

Bollen and Whaley (2004) investigate the role of supply and demand in the options market by exploring the possibility that market makers set option prices with a model not radically different from BSM and that the shape of the IVF is attributable to the buying pressure of specific option series and a limited ability of arbitrageurs to bring prices back into alignment. In particular, they document that daily changes in the implied volatility of an option series are significantly related to net buying pressure and that the changes are transitory, as market makers are gradually able to rebalance their portfolios. Buying pressure on index put options appears to drive the permanently downward sloping shape of the S&P 500 index option IVF, consistent with hedgers seeking portfolio insurance. In contrast, buying pressure on call options appears to drive the shape of stock option IVFs. A simulated trading strategy that sells options, and then delta-hedges the positions using the underlying security, generates significant paper profits for the index but

[25] In contrast, the ability to dynamically replicate option positions in the idealized (frictionless) BSM world ensures that the market maker earns the risk-free rate of return.

not for individual stocks. For index options, they find that profits are highest for the category of options that contain the OTM puts, which corresponds to the institutional demand for portfolio insurance. While the prices of these options are considerably higher than is suggested by the BSM formula and the actual level of volatility in the marketplace, they do not represent profitable arbitrage opportunities for the market maker once the costs of hedging volatility risk are considered.

RISK MANAGEMENT LESSONS: BARINGS BANK PLC

The collapse of Barings Bank PLC in 1995 has been described as one of the worst "derivatives disasters" in history.[26] Disaster to be sure. Unsanctioned index futures and options trading bankrupted Britain's oldest, most venerable, bank—a bank that had financed both the Louisiana Purchase in 1803 and the Napoleonic Wars. But, is it fair to characterize Barings Bank as a derivatives disaster? Not really. The exchange-traded futures and options contracts/markets behaved exactly as they should. The main problem was that senior management of Barings Bank allowed a single trader, Nick Leeson, to place huge, unauthorized bets on the direction of the Japanese stock market over a period of more than two years. By the time that Barings' senior management came to grips with the illicit trading activity, the bank had lost $1.2 billion and was essentially worthless.

The key player in the Barings Bank controversy was Nicholas William Leeson, a man of humble beginnings. He was born in Watford, Hertfordshire in England, the son of a self-employed plasterer. Upon completing high school, Leeson opted for finance career and took a job as a bank clerk at Coutts & Co. In June 1987, about two years later, he moved on to Morgan Stanley as a futures and options settlement clerk. In June 1989, he joined the settlements department of Barings Securities at an annual salary of £12,000.

Leeson's big break came in 1990 when he was assigned to the back-office operations[27] of the Indonesian branch of Barings Securities to sort out a large number of unreconciled stock trades that had stacked up in the bank's error account (the infamous "88888 account"). The bank's use of an error account was not uncommon. By isolating trade discrepancies, a bank can proceed with its remaining back office activities in an unencumbered and timely fashion. What was uncommon, however, is that the trade discrepancies were not reconciled and closed out within a day. In spite of its own internal guidelines, Barings allowed them to accumulate through time.

Over a period of many months, Leeson managed to clean up the back-office problems in Jakarta. Indeed, he was so successful in executing his duties that, in

[26] The story of Nick Leeson and the demise of Barings Bank has been reported in a number of venues including the HBO movie, *Rogue Trader*. Two particularly insightful recounts are Rawnsley (1995) and Marthinsen (2005, Ch. 7). Brown and Steenbeek (2001) analyze Leeson's trading strategy. Many of the details provided in this section are drawn from these sources.

[27] The back office handles the administrative functions of the bank such as trade confirmation, settlement, regulatory compliance, reconciling, and clearing. The front office, on the other hand, handles brokerage business (i.e., direct interface with customers and the execution of their orders) and proprietary trading (i.e., trading for the bank's own accounts).

April 1992, he was promoted and assigned to Barings Futures (Singapore) (hereafter BFS), a new indirect subsidiary of Barings Securities Limited, to set up accounting and settlement functions. Only July 1, 1992, BFS started trading on the SIMEX, with Leeson in charge of operations, including *both* the trading (front office) and the accounting and settlement (back office) activities. Apparently, the bank believed that it was unnecessary to separate the front-office and back-office operations because Leeson's trading was merely executing orders on behalf of clients. In the ensuing months, the brokerage business waned as Japanese clients began set up their own trading operations. To compensate for the loss in line of business, Barings turned to proprietary trading. By early 1993, Leeson was actively involved in stock index arbitrage—not between the futures and the underlying basket of stocks but rather between the Nikkei 225 futures contracts traded simultaneously on the Singapore International Monetary Exchange (SIMEX) and the Osaka Stock Exchange (OSE). Since the contract specifications are virtually identical, arbitraging between the two markets means profiting from (minor) contract price discrepancies between the two markets, selling the more expensive and buying cheaper. Being long and short the same contract, this trading activity is virtually riskless.[28] Curiously, Leeson quickly began reporting extraordinary profits. So large were the reported profits during 1993, BFS accounted for 20% of Barings' worldwide profits. His bonus for the year was £130,000.

The extraordinary profits reported by Leeson should have set off alarms. The strategy is relatively mindless and can be executed mechanically using a PC, real-time pricing information, and electronic links to the trading floors. Since the barriers to entry for engaging in this strategy are small, competition would quickly drive the revenue from this strategy down to a level at which it equals marginal costs of trading. Common sense dictates that reported abnormal profits from this strategy should have been a "red herring."

In truth, Leeson had been placing directional bets all along. The bets were relatively modest at the outset. Some paid off, others did not. As it turns out, Leeson had created the ability to hide losses early. On July 2, 1992, just a day after BFS commenced trading on SIMEX, Leeson gave specific instructions to change the back-office software to exclude the 88888 account from all market activity reports. Its only use was to be for determining futures/options margins. By reporting winning trades to management and hiding losing trades in the 88888 account, Leeson was able to convince the bank's management that he was a brilliant trader. His credibility became beyond reproach. Senior management (from his direct supervisor through the board of directors) turned a "blind eye" to virtually all of his activities. During the first half of 1994, Leeson's reported profits accounted for about 50% of Barings' worldwide profit. All the while, Leeson was *doubling*[29] his bets trying to recover the mounting losses. The charade continued through 1994. At yearend, his reported profits were 500% greater than expected, and his bonus for the year was £450,000. Hidden in the background, however, were accumulated losses of $835 million.

[28] The only risk is "legging into" the transactions. You must buy and sell the contracts in different markets at exactly the same time.

[29] *Doubling* refers to the gambling strategy of doubling the bet each time there is a loss. See Brown and Steenbeek (2001).

Leeson's fundamental bet was that the Japanese stock market would rise. From mid-1994 through the end of the year, the Nikkei 225 was on a steady path downward, as shown in Figure 14.9. The futures contracts were, of course, marked-to-market each day, requiring that variation margin be paid. Leeson requested funding from Barings Securities London and, to his surprise, they wired the money. They apparently beleived his stories that the transfers were needed mostly to meet the needs of BFS customers who operated in different time zones and had difficulty in clearing checks in time and that large margin calls were to be expected in his index arbitrage trading activity. Neither story was, in fact, true. The funding from Barings was not adequate to cover margin calls. Consequently, Leeson decided to write Nikkei 225 option straddles to generate additional cash.[30] At the same time, he also began to record fictitious trades and falsified internal transfer records to lower the size of margin calls by lowering his exposures.

In the first two weeks of January 1995, as the market declined, Leeson began to bet more and more heavily that the Nikkei 225 would not fall below a level of 19,000. Unfortunately, the Kobe earthquake hit on January 17, 1995, disrupting markets throughout Japan. The market fell through the 19,000 level on Friday, January 20. In an attempt to bid up the stock market to restore the profitability of his short straddles and long futures positions, Leeson bought more and more index futures. But, to no avail. Over the weekend, the Nikkei 225 dropped by more than 1,000 points, substantially worsening Leeson's plight. Although he had managed to recover his losses since the earthquake by January 30, he bought even more futures because of his belief that the market would recover further. In the early days of February, the market, again, turned against him. The stress became too much. On Thursday, February 23, Leeson attended work as usual,

FIGURE 14.9 Nikkei 225 during the Barings Bank scandal.

joined colleagues for drinks at a local bar after the market close, went home,

[30] From Chapter 10, we know that straddles generate cash and are profitable as long as the market does not move dramatically in one direction or the other.

packed his bags, and flew to Borneo for a vacation with his wife. At the time of his departure, his open long index futures position alone was 61,039 contracts.

To illustrate the scope of the risk of the position that Leeson had amassed as of the close of trading on February 23, we can compute the 5% value-at-risk (VAR) and conditional value-at-risk (CVAR) measures over one day.[31] The historical volatility rate of the Nikkei 255 over the most recent 30 days leading up to and including February 23 was 23.3% on an annualized basis. At an assumed index futures price of 17,830.02 and an assumed futures price appreciation rate of 0%, the critical level below which the index level may be within one day at 5% probability is

OV_OPTION_ASSET_PROB_INV(17830.02,1/365,0,0.233,"b",0.05) = 17,474.61

The futures contract denomination is JPY 2,500, and the size of the SIMEX position was 61,039 contracts. Hence the 5% VAR over one day was

$$(17{,}830.02 - 17{,}474.61) \times \text{JPY } 2{,}500 \times 61{,}039 = \text{JPY } 54.2 \text{ billion}$$

or USD 560.9 million.[32] In other words, standing at the close on February 23, 1995, there was a 5% chance that the open index futures position could lose about USD 561 million by the close the next day. The 5% tail VAR or CVAR was USD 700.7 million. That is, conditional upon a loss in the 5% tail, the expected loss is about USD 701 million.

In retrospect, Leeson was a rogue trader whose massive, unauthorized, speculation in the futures and options went unmonitored by his employer, Britain's oldest and most venerable bank. Could the situation have been avoided? Absolutely! Like in most derivatives fiascos, the main culprits were:

1. *Rogue trader.* Leeson became addicted to his own fame. In order to protect his reputation as a brilliant trader and keep reporting extraordinary profits for the bank (and earning extraordinary bonuses for himself), he accelerated his trading activity. Since the bank had placed him in a position in which he was responsible for trading and compliance, alarms did not go off early in 1993 when they should have. If the trading had stopped at that time, losses would have been miniscule by comparison.
2. *Lack of understanding by senior management.* The fact that the index arbitrage activity was producing extraordinary profits in early 1993 should have alarmed Barings' senior management, from Leeson's direct supervisor through the board of directors. Apparently, they did not have a basic understanding of the law of one price—two perfect substitutes must have the same price.[33] Simultaneously buying and selling the same futures contract on different exchanges is a virtually riskless activity. At best, the strategy should produce only small returns.

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[31] Recall the these measures were developed in Chapter 7 under the assumption that the underlying security has a lognormal price distribution. See Illustrations 7.5 and 7.7.

[32] The exchange rate at the time was JPY 96.7/USD.

[33] Recall that this assumption was introduced in Chapter 2 and is the foundational assumption of derivatives valuation and risk management.

3. *Lack of meaningful supervision by senior management.* Judging by their actions, Barings' senior management did not supervise Leeson's trading activities in any meaningful way. Indeed, because they had continued to allow Leeson to be responsible for proprietary trading and compliance, they were "allowing the fox to guard the henhouse." Effective risk management demands a clear separation of these two activities for obvious reasons.

From Borneo, Leeson then traveled to Frankfurt, where he had hoped to find safe haven. He was apprehended and extradited back to Singapore, where he pleaded guilty to fraud and spent three and a half years of a six and a half year sentence in a Singapore jail. Upon completing his sentence, he returned to England.

RETURN/RISK MANAGEMENT STRATEGIES

Exchange-traded stock index derivatives can be used in a variety of important trading strategies including market timing, asset allocation, and protected equity notes. The purpose of this section is to elaborate on each of these strategies, showing precisely what trades need to be executed.

Alter Market Risk of a Stock Portfolio Using Index Futures

The key to effective market timing and asset allocation is the ability to modify the expected return/risk characteristics of your portfolio quickly and efficiently. Stock index futures are ideally suited for this purpose. To understand exactly how to use them, we need to recall some of the principles from earlier chapters. First, in Chapter 5, we demonstrated that a stock portfolio's beta can be used to determine the optimal number of index futures to sell in order to minimize portfolio risk. The optimal hedge was

$$n_F = -\beta_P\left(\frac{P}{S}\right) \tag{14.13}$$

where β_P and P are the beta and the market value of the stock portfolio, and S is the market value of an index unit (i.e., the index level times the denomination of the futures). Second, in Chapter 4, we learned that the net cost of carry relation implies the return of the futures equals the return on the underlying index portfolio less the risk-free rate of interest, that is,

$$R_F = R_S - r \tag{14.14}$$

Third, by virtue of the CAPM, we know that the expected return of a risky asset (e.g., your portfolio P) equals the risk-free rate of return plus a market risk premium equal to the product of the difference between the expected rate of return on the market and the risk-free rate and the asset's beta, that is,

$$E_P = r + (E_S - r)\beta_P \tag{14.15}$$

With these tools in hand, we can now address the market timing/asset allocation problem.

First use (14.13) and (14.14) to verify that the optimal hedge ratio (14.12) is correct. You want to set the hedge so that the expected return on your hedge portfolio equals the risk-free rate of return, that is,

$$E_P + n_F E_F = r \tag{14.16}$$

But from (14.14) and (14.13) you know that $E_P = r + (E_S - r)\beta_P = r + E_F\beta_P$, so the left hand side of expected return of the hedge portfolio (14.15) becomes

$$r + \beta_P E_F + n_F E_F = r \tag{14.17}$$

We immediately see that the optimal number of futures is $n_F = -\beta_P$, which we then scale by the ratio of the market value of our portfolio relative to the market value of a unit of the index portfolio underlying the futures to arrive at (14.12). Thus we have verified a result from an earlier chapter using a different, albeit equivalent, approach.

Next, rather than set the expected return on your hedged portfolio equal to the risk-free rate as we did in (14.15), we set it equal to the expected return on a portfolio with our desired risk level, β^*, that is,

$$r + (E_S - r)\beta_P + n_F^* E_F = r + (E_S - r)\beta^* \tag{14.18}$$

Substituting (14.13) and simplifying, we find that the number of futures contracts to buy or sell in order to adjust our market risk exposure to β^* is $n_F^* = \beta^* - \beta_P$. Adjusting for the difference in size of our portfolio relative to an index portfolio unit, the general result is

$$n_F^* = (\beta^* - \beta_P)\left(\frac{P}{S}\right) \tag{14.19}$$

If our desired risk level is 0, we get the minimum variance hedge (14.12). If we want to increase our risk exposure (i.e., $\beta^* > \beta_P$), we buy futures rather than sell.

ILLUSTRATION 14.3 Alter risk of stock portfolio.

Suppose you manage a \$30 million stock portfolio with a beta of 1.50. The current level of the S&P 500 is 1,200, and the three-month S&P 500 futures price is 1,218.14. The risk-free interest rate is 6%. Find the appropriate number of the index futures to buy to bring the portfolio's risk exposure up to a level of $\beta^* = 2.50$.

To find the number of index futures to buy today, substitute the problem information into (14.18), that is,

$$n_F = (2.50 - 1.50)\left(\frac{30{,}000{,}000}{1{,}200(250)}\right) = 100$$

Creating Protected Equity Notes

A *protected equity note* (PEN) is a discount bond-like contract structured to provide a guaranteed rate of return on the principal invested plus a fraction of the any upside relative price appreciation (or total return) on an underlying equity security such as a stock index. PENs were introduced by the over-the-counter market in the late 1980s, and are known by a variety of other names including *principal-protected notes*, *capital-guarantee notes, safe-return certificates, equity-linked notes,* and *index-linked bonds*.

To value a PEN, we, again, apply the valuation by replication. We begin by describing the notation. Let V be the principal amount of the PEN, g be its guaranteed investment return, k be the proportion of price appreciation earned if the market rises (i.e., the "participation rate"), and T be the time remaining to expiration. Let the underlying index have a current price of S, a dividend yield rate of δ, and a volatility rate of σ. For the sake of convenience, we initially assume $S = V$.[34] The risk-free interest rate is denoted r.

Under the assumed notation, the terminal value of the PEN may be expressed as

$$PEN_T = \begin{cases} Ve^{gT} + k(\tilde{S}_T - Ve^{gT}) & \text{if } S_T > Ve^{gT} \\ Ve^{gT} & \text{if } S_T \le Ve^{gT} \end{cases} \tag{14.20}$$

As (14.34) shows, the protected equity note guarantees a minimum terminal value of Ve^{gT}. The only way to guarantee this minimum future value is to include risk-free bonds in the replicating portfolio. To provide a floor level of Ve^{gT} at time T, we need to buy $Ve^{-(r-g)T}$ in risk-free bonds. Next, in the event the equity index appreciates more than rate g over the life of the PEN, the protected equity note also pays k percent of any excess appreciation, $S_T - Ve^{gT}$. Obviously, this is nothing more than a European-style call option with an exercise price of Ve^{gT} and a time to expiration of T. Using the BSM model, the value of the call is

$$c = Se^{-\delta T}N(d_1) - Ve^{(g-r)T}N(d_2) \tag{14.21}$$

where

$$d_1 = \frac{\ln(Se^{-\delta T}/Ve^{(g-r)T}) + 0.5\sigma^2 T}{\sigma\sqrt{T}} \tag{14.21a}$$

and

$$d_2 = d_1 - \sigma\sqrt{T} \tag{14.21b}$$

[34] We relax this assumption in the illustrations that follow.

Since the replicating portfolio has terminal value contingencies exactly equal to those of the PEN, the value of the PEN must be

$$PEN = Ve^{-(r-g)T} + kc \tag{14.22}$$

In the event that the PEN is linked to the total return of the index rather than its price appreciation, we set the dividend yield rate equal to 0 in (14.20) even though the index, itself, may pay dividends. Naturally, such call will be more expensive and, hence, we can expect to receive a lower participation rate on the PEN, other factors being held constant.

ILLUSTRATION 14.4 Value protected equity note.

Suppose your bank offers protected equity notes to its customers. Under the terms of the agreement, you invest \$100,000 for one year. At the end of the year, you receive a guaranteed return of 2% and 30% of any price appreciation of the S&P 500 index over and above the guaranteed return. Value the PEN assuming that the S&P has a current level of 1,250, a dividend yield rate of 1.5%, and a volatility rate of 16%. Assume the risk-free rate of interest is 6%.

First, compute the guaranteed floor value of the investment at the end of one year, that is,

$$Ve^{gT} = 100{,}000e^{0.02(1)} = 102{,}020.13$$

The present value of the risk-free bonds necessary to provide a guaranteed return of 2% (i.e., the first term on the right hand-side of (14.36)) is

$$Ve^{-(r-g)T} = e^{-0.06(1)}(102{,}020.13) = 96{,}078.94$$

Second, recognize that the principal amount of the PEN and the index are at different levels—100,000 versus 1,250. This problem is overcome simply by scaling the current level of the index by a factor of 80. Now, compute the value of the call option embedded in the agreement. Using the BSM formula, the call value is

$$c = 100{,}000e^{-0.015(1)}N(d_1) - 100{,}000e^{(0.02-0.06)1}N(d_2) = 7{,}495.32$$

where

$$d_1 = \frac{\ln(100{,}000e^{-0.015(1)}/102{,}020.13e^{-0.06(1)}) + 0.16^2(1)}{0.16\sqrt{1}} = 0.2363$$

$$d_2 = d_1 - 0.16\sqrt{1} = 0.0763,$$
$$N(d_1) = 0.5934, \text{ and } N(d_2) = 0.5304$$

Finally, compute the value of the PEN as the sum of the value of the risk-free bonds and k times the value of the call option, that is,

$$PEN = 96{,}078.94 + 0.30 \times 7{,}495.32 = 98{,}327.54$$

Had the PEN be written on the total return of the index, the call option value is 8,405.68 and the value of the PEN is 98,600.65.

The computations can be verified using the OPTVAL Function Library. The function (and its syntax) that values a protected equity note is

OV_NS_PROTECTED_EQUITY_NOTE(*princ*, *g*, *k*, *t*, *r*, *i*, *v*, *rp$*)

where *princ* is the amount of principal of the PEN, *g* is the minimum guaranteed rate of return, *k* is the participation rate, *t* is the time to expiration, *r* is the risk-free interest rate, *i* is income rate of the underlying index, *v* is the index's volatility rate, and *rp$* is a (r)eturn/(p)rice indicator. If the PEN provides a share of the total return on the index, *rp$* = "r", and, the PEN provides a share of the price appreciation of the index, *rp$* = "p". An example of the function call is provided here:

B18 fx =OV_NS_PROTECTED_EQUITY_NOTE(B3,B4,B7,B5,B15,B11,B12,B6)

	A	B	C	D	E	F
1	**PROTECTED EQUITY NOTE ANALYSIS**					
2	***Equity note description***					
3	Principal (*V*)	100,000				
4	Minimum growth rate (*g*)	2.00%				
5	Years to expiration (*T*)	1.00				
6	(R)eturn/(P)rice appreciation	P				
7	Rate of participation	30.00%				
8						
9	***Underlying index/stock:***					
10	Level (*S*)	1250.00				
11	Dividend yield (*d*)	1.50%				
12	Volatility (*σ*)	16.00%				
13						
14	***Market parameters***					
15	Interest rate (*r*)	6.00%				
16						
17	***Protected equity note***					
18	Value	98,327.54				

Financial institutions that offer products such as PENs demand a fee for the contract that they are structuring for you. To deduce the size of the fee, you simply compare the principal of the note, *V*, with its economic value as determined by (14.36). In order to do so, you will have to estimate the dividend yield rate and volatility rate of the index and identify the risk-free interest rate on a discount bond of comparable. But these are tasks about which we are familiar. It is also important to note that the reason is that these products are popular is that many individuals are unfamiliar with index option markets and do not understand that they can form a portfolio with exactly the same payoff contingencies on their own.

SUMMARY

This chapter discusses exchange-traded stock index products. The first section is devoted to describing stock index derivatives markets worldwide. Contract specifications of selected index products are provided. The framework for valuing index derivatives depends critically on how the cash dividends of the index portfolio are paid through time. In the second section, the dividend payment patterns of the S&P 500 and DJIA stocks indexes are presented. The patterns indicate that accurate modeling of stock index derivatives requires using a discrete cash dividend framework. The third section provides the valuation principles under the discrete dividend (as well as the continuous dividend yield rate) frameworks.

The fourth section discusses two important index derivatives trading strategies. The first is tailoring the expected return/risk characteristics of a stock portfolio using index futures. Such adjustments can be made using the stock portfolio's current beta in relation to the desired level of risk exposure. Where the desired exposure is 0, the optimal number of futures to sell matches that of the minimum variance hedge developed in Chapter 2. Increasing the risk exposure of the portfolio in response to a prediction of a bull market (i.e., a market timing strategy) or to a desire to have more of the portfolio wealth invested in stocks (i.e., an asset allocation strategy) means buying rather than selling index futures contracts. The second return/risk management strategy discussion focuses on a structured product called a protected equity note in which the buyer is provided a guaranteed minimum rate of return on investment plus a share of the return (or price appreciation) in an index portfolio. Valuation by replication is used to demonstrate that this instrument is nothing more than risk-free bonds plus an index call option.

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