8 INTEREST RATE FUTURES CONTRACTS

Like stock index futures contracts, interest rate futures have been extremely successful financial futures contracts. Interest rate futures provide a means of trading future loan commitments and are important and useful risk management tools for fixed-income portfolio managers. This chapter begins with a description of interest rate futures markets and the specifications of the contracts traded in those markets. Section 2 analyzes the instruments underlying interest rate futures contracts. In section 3, a key measure of interest rate risk, duration, is explained. In section 4, the term structure of interest rates is examined. We focus on the effect of the term structure on the value of coupon and non-coupon bonds. Spot rates, forward rates, and yield to maturity are defined. Examples illustrate most of the important concepts. Sections 5 and 6 describe, in detail, the interest rate futures contracts corresponding to short-term and long-term interest rates. The most active short-term interest rate futures contracts in the U.S. are the Chicago Mercantile Exchange's Treasury bill and Eurodollar contracts, while the most active long-term interest rate futures contract is the Chicago Board of Trade's Treasury bond contract. Contracts in the intermediate-term range, the CBT's five-year and ten-year Treasury note futures, are also discussed. Section 7 describes the cost-of-carry equilibrium in the T-bond futures market. Section 8 contains three illustrations of the use of interest rate futures contracts, and section 9 contains a summary.

8.1 INTEREST RATE FUTURES MARKETS

Table 8.1 contains the contract specifications of the most active interest rate futures contracts traded in the U.S. Of the contracts listed in Table 8.1, the CME's T-bill

TABLE 8.1 Interest rate futures contracts specifications (most active contracts in U.S. markets).

Security (Exchange)	Trading Hours	Contract Months[a]	Units/ Minimum Price Fluctuation	Last Day of Trading	Deliverable Grade[b]
T-Bond (CBT)	8:00–2:00 (CST)	3,6,9,12	\$100,000/ 1/32 (\$31.25)	Seven business days prior to last business day of month	Nominal 8% coupon with 15 years to maturity or first call date
10-yr. T-Note (CBT)	8:00–2:00 (CST)	3,6,9,12	\$100,000/ 1/32 (\$31.25)	Seven business days prior to last business day of month	Nominal 8% coupon with 6.5 to 10 years to maturity
5-yr. T-Note (CBT)	8:00–2:00 (CST)	3,6,9,12	\$100,000/ 1/32 (\$31.25)	Seven business days prior to last business day of month	Any of the four most recently auctioned 5-year Treasury notes
91-day T-Bill (CME)	7:20–2:00 (CST)	3,6,9,12	\$1,000,000/ .01 (\$25.00)	Business day preceding issue date of new 91-day T-bill	Any of the three T-bills 91-days from maturity
Eurodollar Time Deposit (CME)	7:20–2:00 (CST)	3,6,9,12 plus current month	\$1,000,000/ .01 (\$25.00)	Second London business day before third Wednesday	Cash settled

a. The notation used in this column corresponds to the month of the calendar year (e.g., 1 is January, 2 is February, and so on).
b. All interest rate futures contracts other than the Eurodollar contract call for delivery of the underlying security.

futures contract is the oldest, with trading beginning in January 1976. The CBT had introduced a futures contract on GNMA pass-through certificates in the fall of 1975, but it was later delisted.

Unlike stock index futures contracts, most interest rate futures contracts call for the delivery of the underlying interest rate instrument. The CBT's T-bond futures contract, for example, requires the delivery of a nominal eight-percent coupon, \$100,000 face value, U.S. Treasury bond. Delivery may take place at any time during the delivery month, at the discretion of the short. The last day of trading of the futures contract is the eighth-to-last business day of the contract month.

Table 8.2 shows prices of these contracts as of the close of trading on November 13, 1991. The open interest figures in Table 8.2 show that the T-bill, Eurodollar, and T-bond futures contracts are the most active interest rate futures contracts pres-

TABLE 8.2 Interest rate futures contract prices.

INTEREST RATE INSTRUMENTS

FUTURES

TREASURY BONDS (CBT)—$100,000; pts. 32nds of 100%

	Open	High	Low	Settle	Chg	Yield Settle	Yield Chg	Open Interest
Dec	101-04	101-05	99-15	100-03	− 29	7.991	+ .091	280,722
Mr92	100-08	100-10	98-20	99-07	− 30	8.079	+ .095	31,420
June	99-12	99-12	97-30	98-10	− 30	8.173	+ .097	10,197
Sept	98-13	98-13	97-06	97-14	− 31	8.264	+ .101	2,717
Dec	97-19	97-19	96-10	96-20	− 31	8.350	+ .102	4,487
Mr93	96-00	96-05	95-28	95-28	− 32	8.430	+ .107	511

Est vol 370,000; vol Tues 245,191; op Int 330,091, +8,975.

TREASURY BONDS (MCE)—$50,000; pts. 32nds of 100%

	Open	High	Low	Settle	Chg	Yield Settle	Yield Chg	Open Interest
Dec	101-02	101-02	99-15	100-04	− 32	7.987	+ .100	13,542

Est vol 6,600; vol Tues 6,396; open Int 13,641, −132.

T-BONDS (LIFFE) U.S. $100,000; pts of 100%

	Open	High	Low	Settle	Chg			Open Interest
Dec	101-02	101-03	99-21	100-05	− 0-23	101-03	96-24	5,443

Est vol 2,273; vol Tues 4,086; open Int 5,480, +721.

GERMAN GOV'T. BOND (LIFFE)
250,000 marks; $ per mark (.01)

	Open	High	Low	Settle	Chg			Open Interest
Dec	86.23	86.25	86.02	86.19	+ .02	86.44	83.73	75,176
Mr92	n.a.	n.a.	n.a.	n.a.	n.a.	86.70	85.39	6,879

Est vol 47,392; vol Tues 51,218; open Int 82,055, −2,487.

TREASURY NOTES (CBT)—$100,000; pts. 32nds of 100%

	Open	High	Low	Settle	Chg	Yield Settle	Yield Chg	Open Interest
Dec	103-23	103-23	102-18	103-06	− 14	7.540	+ .061	86,289
Mr92	102-29	102-29	101-26	102-13	− 14	7.651	+ .062	12,194
June	....	....	....	101-18	− 13	7.772	+ .058	418

Est vol 30,000; vol Tues 25,215; open Int 98,902, +4,714.

5 YR TREAS NOTES (CBT)—$100,000; pts. 32nds of 100%

	Open	High	Low	Settle	Chg	Yield Settle	Yield Chg	Open Interest
Dec	04-275	104-28	104-12	04-215	− 5.5	6.880	+ .040	91,919
Mr92	04-015	04-015	103-19	03-275	− 6.0	7.071	+ .045	10,105

Est vol 19,429; vol Tues 16,648; open Int 102,024, +2,602.

2 YR TREAS NOTES (CBT)—$200,000, pts. 32nds of 100%

	Open	High	Low	Settle	Chg	Yield Settle	Yield Chg	Open Interest
Dec	103-26	103-26	103-17	03-255	− ¼	....	....	13,800
Mr92	103-11	103-11	03-057	03-105	− ½	....	....	3,553

Est vol 1,500; vol Tues 885; open Int 17,353, +153.

30-DAY INTEREST RATE (CBT)-$5 million; pts. of 100%

	Open	High	Low	Settle	Chg	Yield Settle	Yield Chg	Open Interest
Nov	95.14	95.14	95.13	95.14	− .01	4.86	+ .01	1,254
Dec	95.12	95.12	95.08	95.09	− .05	4.91	+ .05	1,232
Ja92	95.16	95.17	95.15	95.17	− .05	4.83	+ .05	1,100
Feb	95.23	95.27	95.23	95.26	− .04	4.74	+ .04	962
Mar	95.18	95.21	95.18	95.21	− .05	4.79	+ .05	570
Apr	95.20	95.20	95.20	95.20	− .05	4.80	+ .05	107
June	95.10	95.11	95.10	95.11	− .04	4.89	+ .04	189

Est vol 725; vol Tues 529; open Int 5,464, +185.

TREASURY BILLS (IMM)—$1 mil.; pts. of 100%

	Open	High	Low	Settle	Chg	Discount Settle	Discount Chg	Open Interest
Dec	95.38	95.38	95.31	95.34	− .03	4.66	+ .03	21,996
Mr92	95.52	95.52	95.42	95.50	− .02	4.50	+ .02	29,371
June	95.23	95.34	95.23	95.32	− .03	4.68	+ .03	3,892
Sept	95.09	95.09	95.05	95.08	− .02	4.92	+ .02	338
Dec	94.64	94.64	94.64	94.64	....	5.36	....	156

Est vol 6,328; vol Tues 5,830; open Int 55,771, +261.

LIBOR-1 MO. (IMM)—$3,000,000; points of 100%

	Open	High	Low	Settle	Chg	Discount Settle	Discount Chg	Open Interest
Nov	95.05	95.05	94.98	95.02	− .04	4.98	+ .04	6,950
Dec	94.60	94.60	94.50	94.55	− .10	5.45	+ .10	7,336
Ja92	95.11	95.11	95.02	95.08	− .05	4.92	+ .05	9,651
Feb	95.01	95.08	94.99	95.06	− .05	4.94	+ .05	2,515
Mar	94.96	95.01	94.94	95.00	− .05	5.00	+ .05	1,390
Apr	....	....	....	95.01	− .04	4.99	+ .04	163

Est vol 1,577; vol Tues 2,063; open Int 28,005, +300.

MUNI BOND INDEX (CBT)-$1,000; times Bond Buyer MBI

	Open	High	Low	Settle	Chg	High	Low	Open Interest
Dec	95-23	95-23	95-01	95-08	− 15	95-25	88-16	12,542
Mr92	95-04	95-04	94-07	94-13	− 19	95-04	88-00	827

Est vol 2,500; vol Tues 2,607; open Int 13,370, +850.
The Index: Close 95-09; Yield 6.82.

EURODOLLAR (IMM)—$1 million; pts of 100%

	Open	High	Low	Settle	Chg	Yield Settle	Yield Chg	Open Interest
Dec	94.85	94.86	94.75	94.80	− .07	5.20	+ .07	242,049
Mr92	94.99	94.99	94.84	94.94	− .05	5.06	+ .05	252,314
June	94.77	94.78	94.62	94.74	− .04	5.26	+ .04	145,943
Sept	94.48	94.50	94.34	94.46	− .03	5.54	+ .03	100,739
Dec	93.94	93.96	93.83	93.93	− .03	6.07	+ .03	71,656
Mr93	93.76	93.77	93.65	93.74	− .03	6.26	+ .03	55,423
June	93.46	93.46	93.36	93.43	− .02	6.57	+ .02	44,243
Sept	93.17	93.19	93.09	93.17	− .01	6.83	+ .01	31,658
Dec	92.79	92.82	92.72	92.82	+ .02	7.18	− .02	21,132
Mr94	92.76	92.81	92.71	92.80	+ .03	7.20	− .03	27,888
June	92.50	92.58	92.46	92.57	+ .05	7.43	− .05	17,956
Sept	92.28	92.37	92.23	92.35	+ .06	7.65	− .06	11,828
Dec	91.97	92.06	91.92	92.05	+ .07	7.95	− .07	9,816
Mr95	91.96	92.05	91.93	92.04	+ .07	7.96	− .07	7,178
June	91.84	91.93	91.82	91.92	+ .07	8.08	− .07	6,868
Sept	91.69	91.78	91.67	91.77	+ .07	8.23	− .07	6,419

Est vol 283,796; vol Tues 130,709; open Int 1,053,277, +4,629.

EURODOLLAR (LIFFE)—$1 million; pts of 100%

	Open	High	Low	Settle	Change	Lifetime High	Lifetime Low	Open Interest
Dec	94.86	94.87	94.75	94.83	− .02	94.94	90.58	17,546
Mr92	94.97	94.99	94.85	94.96	....	95.06	90.60	10,098
June	94.77	94.78	94.66	94.75	....	94.83	90.97	5,226
Sept	94.49	94.49	94.47	94.46	....	94.53	90.97	2,724
Dec	93.97	93.97	93.95	93.93	....	94.00	91.54	614
Mr93	93.80	93.80	93.78	93.74	+ .02	93.80	91.55	545
June		....	....	93.43	− .23	93.44	92.60	405
Sept		....	....	93.17	+ .08	93.09	92.82	137

Est vol 3,771; vol Tues 4,227; open Int 37,295, +327.

STERLING (LIFFE)—£500,000; pts of 100%

	Open	High	Low	Settle	Change	Lifetime High	Lifetime Low	Open Interest
Dec	89.79	89.82	89.78	89.81	+ .02	90.35	86.52	52,369
Mr92	90.25	90.29	90.24	90.28	+ .04	90.49	86.68	45,849
June	90.34	90.37	90.33	90.36	+ .03	90.46	87.45	34,626
Sept	90.31	90.33	90.29	90.32	+ .02	90.41	87.46	10,853
Dec	90.22	90.23	90.20	90.23	+ .02	90.32	87.55	6,664
Mr93	90.07	90.10	90.07	90.09	+ .02	90.16	87.50	4,548
June	89.97	89.97	89.97	89.98	+ .02	90.09	87.58	2,095
Sept	89.93	89.95	89.93	89.95	+ .02	90.08	88.20	1,746
Dec	89.88	89.90	89.88	89.92	+ .04	90.02	88.95	1,641

Est vol 20,529; vol 20,507; open Int 160,391, −1,238.

LONG GILT (LIFFE)—£50,000; 32nds of 100%

	Open	High	Low	Settle	Change	Lifetime High	Lifetime Low	Open Interest
Dec	95-17	95-23	95-09	95-17	+ 0-03	97-17	89-10	43,299
Mr92	95-24	95-24	95-18	94-22	+ 0-05	96-06	94-18	2,524

Est vol 24,368; vol Tues 23,953; open Int 45,823, +1,160.

OTHER INTEREST RATE FUTURES

Settlement prices of selected contracts. Volume and open interest of all contract months.

Mortgage-Backed (CBT)—$100,000, pts. & 64ths of 100%
Nov Cpn 8.5 102-04 −6; Est. vol. 0; Open Int. 90
5-Yr. Int. Rate Swap (CBT)—$25 per ½ b.p.; pts of 100%
Dec 92.770 −.010; Est. vol. 50; Open Int. 707
3-Yr. Int. Rate Swap (CBT)—$25 per ½ b.p.; pts of 100%
Dec 93.490 −.010; Est. vol. 0; Open Int. 456
Treas. Auction 5 Yr (FINEX)—$250,000, 100 minus yield
Dec 93.22 −4.0; Est. vol. 100; Open Int. 4

CBT—Chicago Board of Trade. FINEX—Financial Instrument Exchange, a division of the New York Cotton Exchange. IMM—International Monetary Market at Chicago Mercantile Exchange. LIFFE—London International Financial Futures Exchange. MCE—MidAmerica Commodity Exchange.

ently trading. These contracts are the focus of the dominant part of this chapter. The T-bill and Eurodollar futures contracts are on short-term debt instruments and are discussed first. The T-bond (and T-note) futures are written on long-term interest rates and are discussed second. Prior to beginning either discussion, however, it is useful to review the pricing mechanics of the fixed-income securities that underlie these futures contracts.

8.2 UNDERLYING BONDS

Two types of interest rate or fixed-income securities underlie interest rate futures contracts. One type is a *zero-coupon* or a *discount bond.* A discount bond provides no explicit interest payments. It is traded at prices below the face value or par value of the security. Security income results from price appreciation. The other type of fixed-income security is a *coupon-bearing bond.* Like a discount bond, a coupon-bearing bond pays the face or par value at the end of the bond's life. In addition, a coupon-bearing bond has prespecified coupon interest payments at regular intervals throughout the bond's life.

Zero-Coupon or Discount Bonds

The price of a discount bond, B_d, is computed by taking the present value of the promised payment of face value, F_n, at the end of the bond's life, n periods from now, that is,

$$B_d = \frac{F_n}{(1+y)^n}, \tag{8.1}$$

where y is the yield to maturity or rate of return on the bond.

U.S. Treasury bills are discount bonds. T-bills are short-term debt instruments issued by the U.S. Government. New 91-day and 182-day bills are issued every Thursday. New 364-day bills are issued every fourth Thursday. The minimum face value of a T-bill is $10,000.

T-bill quotes are unusual in that their prices are not reported directly. Table 8.3 contains market information about the T-bills that were active on November 13, 1991. Note that, in this table, bid and ask "discounts" appear. These are *bank discount price quotations.* The definition of a bank discount is

$$\text{Bank discount} = (360/n)(100 - B_d), \tag{8.2}$$

where n is the number of days to maturity of the bill, and B_d is the bill's price expressed as a percentage of par. (Traditionally, bankers have assumed a 360-day year.) To compute the price of the T-bill, simply rearrange equation (8.2) to isolate B_d, that is,

$$B_d = 100 - \text{Bank discount}(n/360). \tag{8.3}$$

TABLE 8.3 Treasury bond, note, and bill prices.

TREASURY BONDS, NOTES & BILLS

Wednesday, November 13, 1991

Representative Over-the-Counter quotations based on transactions of $1 million or more.

Treasury bond, note and bill quotes are as of mid-afternoon. Colons in bid-and-asked quotes represent 32nds; 101:01 means 101 1/32. Net changes in 32nds. n-Treasury note. Treasury bill quotes in hundredths, quoted on terms of a rate of discount. Days to maturity calculated from settlement date. All yields are to maturity and based on the asked quote. Latest 13-week and 26-week bills are boldfaced. For bonds callable prior to maturity, yields are computed to the earliest call date for issues quoted above par and to the maturity date for issues below par. *-When issued.

Source: Federal Reserve Bank of New York.

U.S. Treasury strips as of 3 p.m. Eastern time, also based on transactions of $1 million or more. Colons in bid-and-asked quotes represent 32nds; 101:01 means 101 1/32. Net changes in 32nds. Yields calculated on the bid quotation. ci-stripped coupon interest. bp-Treasury bond, stripped principal. np-Treasury note, stripped principal. For bonds callable prior to maturity, yields are computed to the earliest call date for issues quoted above par and to the maturity date for issues below par.

Source: Bear, Stearns & Co. via Street Software Technology Inc.

GOVT. BONDS & NOTES

Rate	Maturity Mo/Yr	Bid	Asked	Chg.	Ask Yld.
6½	Nov 91n	100:00	100:02	....	0.00
8½	Nov 91n	100:00	100:02	....	0.00
14¼	Nov 91n	100:01	100:03	....	0.00
7¾	Nov 91n	100:04	100:06	....	3.06
7⅝	Dec 91n	100:11	100:13	....	4.24
8¼	Dec 91n	100:14	100:16	....	4.10
11⅝	Jan 92n	101:03	101:05	– 1	4.43
8⅛	Jan 92n	100:22	100:24	– 1	4.40
6⅝	Feb 92n	100:13	100:15	....	4.65
9⅛	Feb 92n	101:01	101:03	....	4.59
14⅝	Feb 92n	102:13	102:15	– 1	4.48
8½	Feb 92n	101:01	101:03	....	4.61
7⅞	Mar 92n	101:03	101:05	– 1	4.69
8½	Mar 92n	101:11	101:13	....	4.63
11¾	Apr 92n	102:25	102:27	– 2	4.72
8⅞	Apr 92n	101:26	101:28	....	4.68
6⅝	May 92n	100:27	100:29	....	4.77
9	May 92n	102:00	102:02	– 1	4.78
13¾	May 92n	104:11	104:13	– 1	4.73
8½	May 92n	101:28	101:30	– 1	4.83
8¼	Jun 92n	102:01	102:03	– 1	4.82
8⅜	Jun 92n	102:03	102:05	....	4.84
10⅜	Jul 92n	103:16	103:18	....	4.88
8	Jul 92n	102:02	102:04	– 1	4.92
4¼	Aug 87-92	98:13	99:13	– 1	5.07
7¼	Aug 92	101:18	101:22	....	4.94
7⅞	Aug 92n	102:00	102:02	– 1	5.04
8¼	Aug 92n	102:09	102:11	– 1	5.03
8⅛	Aug 92n	102:11	102:13	– 1	4.99
8⅛	Sep 92n	102:18	102:20	– 1	5.02
8¾	Sep 92n	103:03	103:05	– 1	5.02
9¾	Oct 92n	104:04	104:06	– 1	5.02
7¾	Oct 92n	102:12	102:14	– 1	5.11
7¾	Nov 92n	102:15	102:17	– 1	5.12
8⅜	Nov 92n	103:02	103:04	– 1	5.13
10½	Nov 92n	105:04	105:06	– 1	5.11
7⅜	Nov 92n	102:06	102:08	– 1	5.13
7¼	Dec 92n	102:06	102:08	....	5.17
9⅛	Dec 92n	104:06	104:08	– 1	5.19
8¾	Jan 93n	103:28	103:30	– 2	5.23
7	Jan 93n	101:30	102:00	– 1	5.27
4	Feb 88-93	96:22	97:22	– 1	5.94
6¾	Feb 93	101:22	101:26	– 1	5.24
7⅞	Feb 93	103:00	103:04	– 1	5.26
8¼	Feb 93n	103:15	103:17	– 1	5.30
8⅜	Feb 93n	103:20	103:22	– 1	5.29
10⅞	Feb 93n	106:19	106:21	– 2	5.31
6¾	Feb 93n	101:22	101:24	– 2	5.33
7⅛	Mar 93n	102:08	102:10	– 2	5.36
9⅝	Mar 93n	105:17	105:19	– 1	5.35
7⅜	Apr 93n	102:20	102:22	– 1	5.38
7	Apr 93n	102:05	102:07	– 1	5.40
7⅝	May 93n	103:02	103:04	....	5.43
8⅝	May 93n	104:15	104:17	....	5.44
8½	Jul 97n	107:07	107:09	– 5	6.92
8⅝	Aug 97n	107:26	107:28	– 6	6.94
8¾	Oct 97n	108:16	108:18	– 4	6.96
8⅞	Nov 97n	109:04	109:06	– 7	6.97
7⅞	Jan 98n	104:06	104:08	– 7	7.01
8⅛	Feb 98n	105:13	105:15	– 6	7.03
7⅞	Apr 98n	104:06	104:08	– 5	7.04
7	May 93-98	100:16	100:24	....	6.47
9	May 98n	109:27	109:29	– 8	7.07
8¼	Jul 98n	105:30	106:00	– 7	7.10
9¼	Aug 98n	111:04	111:06	– 8	7.13
7⅛	Oct 98n	100:09	100:11	– 5	7.06
3½	Nov 98	97:24	98:24	+ 24	3.70
8⅞	Nov 98n	109:09	109:11	– 6	7.15
8⅞	Feb 99n	109:09	109:11	– 6	7.20
8½	May 94-99	105:00	105:08	– 4	6.20
9⅛	May 99n	110:23	110:25	– 7	7.24
8	Aug 99n	104:08	104:10	– 6	7.26
7⅞	Nov 99n	103:15	103:17	– 6	7.28
7⅞	Feb 95-00	102:15	102:19	– 1	6.97
8½	Feb 00n	107:01	107:03	– 9	7.34
8⅞	May 00n	109:06	109:08	– 12	7.39
8⅜	Aug 95-00	104:18	104:22	– 3	6.93
8¾	Aug 00n	108:12	108:14	– 13	7.42
8½	Nov 00n	106:27	106:29	– 13	7.43
7¾	Feb 01n	101:31	102:01	– 11	7.44
11¾	Feb 01	128:14	128:18	– 10	7.43
8	May 01n	103:22	103:24	– 10	7.44
13⅛	May 01	138:04	138:08	– 13	7.44
7⅞	Aug 01n	102:29	102:31	– 14	7.44
8	Aug 96-01	103:19	103:23	– 15	7.07
13⅜	Aug 01	140:16	140:20	– 8	7.44
7½	Nov 01n*	100:19	100:20	– 11	7.41
15¾	Nov 01	157:21	157:25	– 10	7.45
14¼	Feb 02	147:25	147:29	– 14	7.48
11⅝	Nov 02	130:10	130:14	– 13	7.51
10¾	Feb 03	123:31	124:03	– 12	7.54
10¾	May 03	124:06	124:10	– 12	7.55
11⅛	Aug 03	127:04	127:08	– 15	7.58
11⅞	Nov 03	133:07	133:11	– 23	7.59
12⅜	May 04	137:30	138:02	– 29	7.60
13¾	Aug 04	149:07	149:11	– 29	7.63
11⅝	Nov 04	132:02	132:06	– 23	7.67
8¼	May 00-05	104:10	104:14	– 8	7.53
12	May 05	135:23	135:27	– 26	7.69
10¾	Aug 05	125:12	125:16	– 19	7.71
9⅜	Feb 06	114:14	114:18	– 18	7.68
7⅝	Feb 02-07	99:20	99:24	– 16	7.65
7⅞	Nov 02-07	101:14	101:18	– 16	7.66
8⅜	Aug 03-08	104:28	105:00	– 12	7.72
8¾	Nov 03-08	107:19	107:23	– 19	7.75
9⅛	May 04-09	110:21	110:25	– 24	7.76
10⅜	Nov 04-09	120:28	121:00	– 24	7.78
11¾	Feb 05-10	132:10	132:14	– 25	7.78

Mat.	Type	Bid	Asked	Chg.	Bid Yld.
Nov 99	np	55:03	55:07	– 7	7.59
Feb 00	ci	53:28	54:00	– 8	7.64
Feb 00	np	53:22	53:27	– 9	7.68
May 00	ci	52:28	53:00	– 7	7.64
May 00	np	52:22	52:27	– 7	7.68
Aug 00	ci.	51:26	51:30	– 7	7.66
Aug 00	np	51:23	51:27	– 7	7.68
Nov 00	ci	50:24	50:28	– 8	7.68
Nov 00	np	50:24	50:28	– 7	7.68
Feb 01	ci	49:14	49:19	– 7	7.76
Feb 01	np	49:09	49:13	– 8	7.80
May 01	ci	48:14	48:18	– 8	7.78
May 01	np	48:08	48:12	– 9	7.82
Aug 01	ci	47:16	47:21	– 9	7.78
Aug 01	np	47:11	47:15	– 8	7.82
Nov 01	ci	46:20	46:24	– 7	7.78
Feb 02	ci	45:09	45:13	– 12	7.88
May 02	ci	44:11	44:15	– 11	7.90
Aug 02	ci	43:12	43:17	– 12	7.92
Nov 02	ci	42:15	42:19	– 11	7.94
Feb 03	ci	41:16	41:21	– 12	7.97
May 03	ci	40:21	40:26	– 12	7.98
Aug 03	ci	39:27	39:31	– 11	7.99
Nov 03	ci	39:00	39:05	– 12	8.00
Feb 04	ci	38:04	38:08	– 11	8.03
May 04	ci	37:09	37:13	– 12	8.05
Aug 04	ci	36:16	36:21	– 12	8.06
Nov 04	ci	35:24	35:28	– 12	8.07
Nov 04	bp	35:23	35:27	– 11	8.08
Feb 05	ci	34:31	35:03	– 11	8.09
May 05	ci	34:08	34:12	– 11	8.10
May 05	bp	34:09	34:13	– 11	8.09
Aug 05	ci	33:18	33:22	– 11	8.10
Aug 05	bp	33:19	33:24	– 12	8.09
Nov 05	ci	32:29	33:01	– 11	8.10
Feb 06	ci	32:05	32:10	– 12	8.12
Feb 06	bp	32:04	32:08	– 11	8.13
May 06	ci	31:17	31:21	– 11	8.12
Aug 06	ci	30:29	31:01	– 11	8.12
Nov 06	ci	30:10	30:14	– 11	8.12
Feb 07	ci	29:16	29:20	– 12	8.17
May 07	ci	28:29	29:01	– 12	8.17
Aug 07	ci	28:11	28:15	– 12	8.17
Nov 07	ci	27:25	27:29	– 12	8.17
Feb 08	ci	27:03	27:07	– 11	8.20
May 08	ci	26:15	26:19	– 11	8.22
Aug 08	ci	25:30	26:02	– 11	8.22
Nov 08	ci	25:14	25:18	– 10	8.22
Feb 09	ci	24:25	24:29	– 11	8.25
May 09	ci	24:10	24:13	– 10	8.25
Aug 09	ci	23:26	23:30	– 10	8.25
Nov 09	ci	23:11	23:15	– 10	8.25
Nov 09	bp	22:30	23:02	– 9	8.35
Feb 10	ci	22:28	23:00	– 10	8.25
May 10	ci	22:13	22:17	– 10	8.25
Aug 10	ci	21:30	22:01	– 10	8.26
Nov 10	ci	21:15	21:19	– 11	8.26
Feb 11	ci	21:02	21:05	– 10	8.26
May 11	ci	20:20	20:24	– 10	8.26
Aug 11	ci	20:07	20:11	– 10	8.26
Nov 11	ci	19:26	19:30	– 10	8.26
Feb 12	ci	19:12	19:16	– 10	8.27
May 12	ci	19:00	19:03	– 9	8.27
Aug 12	ci	18:20	18:23	– 9	8.27
Nov 12	ci	18:08	18:11	– 9	8.27
Feb 13	ci	17:28	17:31	– 9	8.27
May 13	ci	17:16	17:20	– 10	8.27
Aug 13	ci	17:05	17:09	– 9	8.27
Nov 13	ci	16:26	16:30	– 9	8.27
Feb 14	ci	16:15	16:19	– 9	8.27
May 14	ci	16:05	16:08	– 9	8.27
Aug 14	ci	15:26	15:30	– 9	8.27
Nov 14	ci	15:16	15:20	– 9	8.27
Feb 15	ci	15:06	15:10	– 9	8.27
Feb 15	bp	15:07	15:11	– 8	8.26
May 15	ci	14:29	15:00	– 8	8.27

TABLE 8.3 continued

10 1/8	May 93n	106:18	106:20	− 2	5.46
6 3/4	May 93n	101:26	101:28	− 1	5.47
7	Jun 93n	102:08	102:10	− 1	5.49
8 1/8	Jun 93n	103:31	104:01	− 1	5.50
7 1/4	Jul 93n	102:21	102:23	− 1	5.52
6 7/8	Jul 93n	102:02	102:04	− 1	5.56
7 1/2	Aug 88-93	100:20	100:24	+ 2	7.04
8	Aug 93n	103:30	104:00	....	5.57
8 5/8	Aug 93	104:30	105:02	....	5.55
8 3/4	Aug 93n	105:05	105:07	....	5.58
11 7/8	Aug 93n	110:11	110:13	....	5.56
6 3/8	Aug 93n	101:09	101:11	....	5.58
6 1/8	Sep 93n	100:27	100:29	....	5.61
8 1/4	Sep 93n	104:19	104:21	− 1	5.60
7 1/8	Oct 93n	102:21	102:23	− 1	5.61
6	Oct 93n	100:21	100:23	....	5.61
7 3/4	Nov 93n	103:27	103:29	− 1	5.66
8 5/8	Nov 93	105:15	105:19	+ 1	5.63
9	Nov 93n	106:05	106:07	− 1	5.67
11 3/4	Nov 93n	111:10	111:12	− 1	5.65
7 5/8	Dec 93n	103:24	103:26	− 1	5.70
7	Jan 94n	102:15	102:17	− 2	5.74
6 7/8	Feb 94n	102:09	102:11	....	5.75
8 7/8	Feb 94n	106:10	106:12	− 3	5.81
9	Feb 94	106:18	106:22	− 2	5.79
8 1/2	Mar 94n	105:24	105:26	− 2	5.84
7	Apr 94n	102:19	102:21	− 1	5.81
4 1/8	May 89-94	96:24	97:24	− 3	5.09
7	May 94n	102:17	102:19	− 2	5.87
9 1/2	May 94n	108:06	108:08	− 3	5.90
13 1/8	May 94n	116:17	116:19	− 3	5.89
8 1/2	Jun 94n	106:03	106:05	− 1	5.93
8	Jul 94n	104:30	105:00	− 2	5.95
6 7/8	Aug 94n	102:08	102:10	− 3	5.95
8 5/8	Aug 94n	106:16	106:18	− 2	6.00
8 3/4	Aug 94	106:25	106:29	− 2	5.99
12 5/8	Aug 94n	116:17	116:19	− 2	5.99
8 1/2	Sep 94n	106:12	106:14	− 1	6.03
9 1/2	Oct 94n	109:00	109:02	− 2	6.06
6	Nov 94n*	100:00	100:01	− 2	5.99
8 1/4	Nov 94n	105:27	105:29	− 2	6.07
10 1/8	Nov 94	110:28	111:00	− 3	6.06
11 5/8	Nov 94n	114:31	115:01	− 2	6.07
7 5/8	Dec 94n	104:10	104:12	− 2	6.07
8 5/8	Jan 95n	106:30	107:00	− 2	6.16
3	Feb 95	97:00	98:00	− 3	3.66
7 3/4	Feb 95n	104:17	104:19	− 3	6.17
10 1/2	Feb 95	112:10	112:14	− 3	6.22
11 1/4	Feb 95n	114:17	114:19	− 5	6.22
8 3/8	Apr 95n	106:08	106:10	− 4	6.29
8 1/2	May 95n	106:24	106:26	− 2	6.30
10 3/8	May 95	112:14	112:18	− 3	6.32
11 1/4	May 95n	115:03	115:05	− 4	6.35
12 5/8	May 95	119:14	119:18	− 2	6.31
8 7/8	Jul 95n	107:28	107:30	− 4	6.41
8 1/2	Aug 95n	106:24	106:26	− 4	6.43
10 1/2	Aug 95n	113:07	113:09	− 4	6.46
8 5/8	Oct 95n	107:08	107:10	− 4	6.48
8 1/2	Nov 95n	106:28	106:30	− 4	6.50
9 1/2	Nov 95n	110:10	110:12	− 6	6.51
11 1/2	Nov 95	117:12	117:16	− 4	6.47
9 1/4	Jan 96n	109:18	109:20	− 5	6.57
7 1/2	Jan 96n	103:09	103:11	− 5	6.58
7 7/8	Feb 96n	104:18	104:20	− 5	6.61
8 7/8	Feb 96n	108:07	108:09	− 6	6.61
7 1/2	Feb 96n	103:07	103:09	− 5	6.61
7 3/4	Mar 96n	104:03	104:05	− 5	6.64
9 3/8	Apr 96n	110:08	110:10	− 5	6.64
7 5/8	Apr 96n	103:22	103:24	− 5	6.64
7 3/8	May 96n	102:22	102:24	− 5	6.66
7 5/8	May 96n	103:21	103:23	− 5	6.66
7 7/8	Jun 96n	104:18	104:20	− 5	6.70
7 7/8	Jul 96n	104:19	104:21	− 6	6.70
7 7/8	Jul 96n	104:23	104:25	− 5	6.68
7 1/4	Aug 96n	102:06	102:08	− 5	6.69
7	Sep 96n	101:06	101:08	− 5	6.70
8	Oct 96n	105:06	105:08	− 5	6.73
6 7/8	Oct 96n	100:26	100:28	− 5	6.66
7 1/4	Nov 96n	102:04	102:06	− 5	6.73
8	Jan 97n	105:04	105:06	− 5	6.79
8 1/2	Apr 97n	107:07	107:09	− 5	6.87
8 1/2	May 97n	107:07	107:09	− 5	6.89
10	May 05-10	118:15	118:19	− 23	7.75
12 3/4	Nov 05-10	141:24	141:28	− 29	7.79
13 7/8	May 06-11	152:06	152:10	− 30	7.79
14	Nov 06-11	154:08	154:12	− 31	7.79
10 3/8	Nov 07-12	122:13	122:17	− 25	7.87
12	Aug 08-13	137:25	137:29	− 30	7.88
13 1/4	May 09-14	150:07	150:11	− 36	7.89
12 1/2	Aug 09-14	143:13	143:17	− 34	7.90
11 3/4	Nov 09-14	136:27	136:31	− 35	7.87
11 1/4	Feb 15	135:01	135:05	− 36	7.92
10 5/8	Aug 15	128:19	128:23	− 33	7.92
9 7/8	Nov 15	120:23	120:25	− 30	7.93
9 1/4	Feb 16	113:30	114:00	− 33	7.94
7 1/4	May 16	92:21	92:23	− 27	7.93
7 1/2	Nov 16	95:07	95:09	− 28	7.94
8 3/4	May 17	108:25	108:27	− 31	7.94
8 7/8	Aug 17	110:05	110:07	− 31	7.94
9 1/8	May 18	113:00	113:02	− 32	7.94
9	Nov 18	111:21	111:23	− 32	7.94
8 7/8	Feb 19	110:09	110:11	− 32	7.94
8 1/8	Aug 19	101:31	102:01	− 30	7.94
8 1/2	Feb 20	106:06	106:08	− 32	7.94
8 3/4	May 20	109:02	109:04	− 32	7.94
8 3/4	Aug 20	109:02	109:04	− 30	7.94
7 7/8	Feb 21	99:07	99:09	− 30	7.94
8 1/8	May 21	102:04	102:06	− 31	7.93
8 1/8	Aug 21	102:09	102:11	− 30	7.92
8	Nov 21*	101:12	101:13	− 28	7.88

U.S. TREASURY STRIPS

Mat.	Type	Bid	Asked	Chg.	Bid Yld.
Feb 92	ci	98:26	98:26	+ 1	4.87
May 92	ci	97:18	97:19	− 1	4.99
Aug 92	ci	96:11	96:11	− 1	5.04
Nov 92	ci	95:04	95:05	− 2	5.05
Feb 93	ci	93:19	93:20	− 1	5.38
May 93	ci	92:08	92:09	− 2	5.46
Aug 93	ci	90:26	90:27	− 3	5.59
Nov 93	ci	89:13	89:15	− 2	5.68
Feb 94	ci	87:24	87:26	− 1	5.89
May 94	ci	86:13	86:15	− 1	5.93
Aug 94	ci	84:30	85:00	− 3	6.03
Nov 94	ci	83:12	83:15	− 3	6.15
Nov 94	np	83:08	83:11	− 3	6.20
Feb 95	ci	81:18	81:20	− 4	6.37
Feb 95	np	81:24	81:26	− 4	6.30
May 95	ci	80:02	80:05	− 5	6.45
May 95	np	80:01	80:03	− 4	6.47
Aug 95	ci	78:20	78:22	− 4	6.52
Aug 95	np	78:14	78:17	− 5	6.58
Nov 95	ci	77:08	77:11	− 5	6.56
Nov 95	np	77:02	77:05	− 5	6.62
Feb 96	ci	75:14	75:17	− 5	6.74
Feb 96	np	75:09	75:12	− 5	6.79
May 96	ci	74:04	74:07	− 5	6.77
May 96	np	74:01	74:05	− 6	6.79
Aug 96	ci	72:19	72:22	− 5	6.86
Nov 96	ci	71:14	71:17	− 6	6.84
Nov 96	np	70:10	70:13	− 3	7.17
Feb 97	ci	69:20	69:23	− 4	7.02
May 97	ci	68:07	68:10	− 3	7.08
May 97	np	68:05	68:09	− 4	7.09
Aug 97	ci	66:27	66:31	− 4	7.13
Aug 97	np	66:26	66:29	− 3	7.14
Nov 97	ci	65:24	65:28	− 1	7.11
Nov 97	np	65:18	65:22	− 4	7.16
Feb 98	ci	63:31	64:02	− 8	7.28
Feb 98	np	63:23	63:27	− 4	7.34
May 98	ci	62:24	62:28	− 8	7.30
May 98	np	62:16	62:20	− 4	7.36
Aug 98	ci	61:15	61:19	− 8	7.34
Aug 98	np	61:06	61:10	− 4	7.41
Nov 98	ci	60:12	60:16	− 8	7.34
Nov 98	np	60:05	60:09	− 7	7.39
Feb 99	ci	58:19	58:23	− 7	7.51
Feb 99	np	58:14	58:18	− 8	7.55
May 99	ci	57:14	57:18	− 7	7.53
May 99	np	57:09	57:13	− 8	7.57
Aug 99	ci	56:07	56:11	− 7	7.57
Aug 99	np	56:04	56:08	− 7	7.59
Nov 99	ci	55:06	55:10	− 8	7.57
Aug 15	ci	14:19	14:22	− 9	8.27
Aug 15	bp	14:20	14:23	− 8	8.26
Nov 15	ci	14:10	14:13	− 8	8.27
Nov 15	bp	14:11	14:14	− 7	8.26
Feb 16	ci	14:00	14:04	− 9	8.27
Feb 16	bp	14:03	14:06	− 8	8.26
May 16	ci	13:23	13:27	− 9	8.27
May 16	bp	14:00	14:03	− 8	8.19
Aug 16	ci	13:15	13:18	− 8	8.27
Nov 16	ci	13:06	13:09	− 8	8.27
Nov 16	bp	13:12	13:15	− 6	8.21
Feb 17	ci	12:31	13:02	− 8	8.26
May 17	ci	12:22	12:25	− 8	8.26
May 17	bp	12:26	12:29	− 8	8.22
Aug 17	ci	12:14	12:17	− 8	8.26
Aug 17	bp	12:18	12:21	− 8	8.22
Nov 17	ci	12:05	12:08	− 8	8.27
Feb 18	ci	11:31	12:02	− 8	8.25
May 18	ci	11:24	11:26	− 7	8.25
May 18	bp	11:27	11:30	− 8	8.21
Aug 18	ci	11:16	11:19	− 8	8.25
Nov 18	ci	11:09	11:12	− 7	8.25
Nov 18	bp	11:12	11:15	− 9	8.21
Feb 19	ci	11:03	11:06	− 8	8.23
Feb 19	bp	11:08	11:11	− 8	8.18
May 19	ci	10:29	11:00	− 8	8.22
Aug 19	ci	10:22	10:25	− 8	8.22
Aug 19	bp	10:27	10:30	− 7	8.17
Nov 19	ci	10:16	10:19	− 8	8.21
Feb 20	ci	10:11	10:13	− 7	8.20
Feb 20	bp	10:14	10:17	− 7	8.16
May 20	ci	10:05	10:08	− 7	8.19
May 20	bp	10:08	10:10	− 7	8.16
Aug 20	ci	9:30	10:01	− 7	8.19
Aug 20	bp	10:02	10:05	− 9	8.15
Nov 20	ci	9:26	9:28	− 7	8.17
Feb 21	ci	9:20	9:22	− 7	8.17
Feb 21	bp	9:23	9:26	− 8	8.13
May 21	ci	9:16	9:19	− 7	8.14
May 21	bp	9:19	9:21	− 7	8.11
Aug 21	ci	9:19	9:21	− 7	8.04
Aug 21	bp	9:18	9:20	− 9	8.05
Nov 21	bp	9:19	9:22	− 6	7.97

TREASURY BILLS

Maturity	Days to Mat.	Bid	Asked	Chg.	Ask Yld.
Nov 21 '91	6	4.68	4.58	− 0.02	4.65
Nov 29 '91	14	4.54	4.44	− 0.01	4.52
Dec 05 '91	20	4.41	4.31	− 0.03	4.39
Dec 12 '91	27	4.36	4.26	− 0.08	4.34
Dec 19 '91	34	4.39	4.35	− 0.05	4.43
Dec 26 '91	41	4.40	4.36	− 0.04	4.45
Jan 02 '92	48	4.43	4.39	− 0.02	4.49
Jan 09 '92	55	4.50	4.46	− 0.02	4.57
Jan 16 '92	62	4.55	4.53	− 0.01	4.63
Jan 23 '92	69	4.57	4.55	− 0.01	4.67
Jan 30 '92	76	4.57	4.55	− 0.01	4.67
Feb 06 '92	83	4.62	4.60	....	4.73
Feb 13 '92	90	4.63	4.61	....	4.73
Feb 20 '92	97	4.62	4.60	....	4.74
Feb 27 '92	104	4.62	4.60	....	4.74
Mar 05 '92	111	4.64	4.62	....	4.76
Mar 12 '92	118	4.66	4.64	+ 0.01	4.79
Mar 19 '92	125	4.66	4.64	+ 0.01	4.79
Mar 26 '92	132	4.64	4.62	....	4.78
Apr 02 '92	139	4.66	4.64	....	4.80
Apr 09 '92	146	4.69	4.67	....	4.84
Apr 16 '92	153	4.69	4.67	+ 0.01	4.84
Apr 23 '92	160	4.72	4.70	....	4.88
Apr 30 '92	167	4.68	4.66	....	4.84
May 07 '92	174	4.72	4.70	+ 0.02	4.89
May 14 '92	181	4.70	4.68	− 0.01	4.87
Jun 04 '92	202	4.65	4.63	+ 0.02	4.82
Jul 02 '92	230	4.71	4.69	+ 0.01	4.89
Jul 30 '92	258	4.74	4.72	+ 0.03	4.93
Aug 27 '92	286	4.74	4.72	+ 0.02	4.94
Sep 24 '92	314	4.73	4.71	+ 0.04	4.94
Oct 22 '92	342	4.75	4.73	+ 0.03	4.98

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To illustrate the bank discount convention, consider the T-bill in Table 8.3 that matures on February 13, 1992. If this bill is purchased, it would be at the ask price,[1] as reflected by the ask discount of 4.61. The number of days to maturity of this bill is 92. Therefore, the ask price of the bill is

$$B_d = 100 - 4.61(92/360) = 98.822$$

percent of par. If the par value is \$10,000, the price of the T-bill is \$9,882.20.

To compare the rate of return on the T-bill with the rate of return on other instruments, the *effective annual rate of return* is often computed. If the price of a 92-day T-bill is 98.822, the effective annual rate of return compounded on a daily basis[2] is

$$r = (100/98.822)^{365/92} - 1 = 4.81\%.$$

Eurodollar certificates of deposit (CD's) are also discount bonds. Eurodollar deposits are U.S. dollar deposits in any foreign bank, although Eurodollar deposits are most typically thought of as being U.S. dollar deposits in London. The rate on these CD's is referred to as the *London Interbank Offer Rate* (LIBOR). The usual denomination of Eurodollar CD's is \$1,000,000. The usual maturities are in the three- to six-month range, however, maturities as long as five years are not uncommon. Simple interest on Eurodollar deposits is calculated for the actual number of days on a 360-day year basis and is paid at maturity on deposits with terms of less than one year. For longer-term deposits, interest is paid annually.[3]

Sample Eurodollar rates are contained in Table 8.4. The rates reported are averages of the rates quoted by five major banks in London. The three-month rate, for example, is reported to be 5⅛ percent. That implies that a \$1,000,000 Eurodollar deposit on November 13, 1991, will be worth \$1,000,000 + 1,000,000(.05125)(90/360), or \$1,012,812.50, on February 11, 1992.[4] The effective annual rate of return on this investment may be computed as follows:

$$r = (1,012,812.50/1,000,000)^{365/90} - 1 = 5.30\%.$$

Note that although this Eurodollar deposit has approximately the same maturity as the T-bill in the previous example, the effective annual rate of return is 49 basis

[1]The bid/ask spread is the cost of immediate trade execution. A market-maker stands ready to buy immediately at her bid price and sell immediately at her ask price. The bid/ask spread is her compensation for providing market liquidity.

[2]The effective annual return compounded continuously is $r = \ln(100/98.822)(365/92) = 4.70\%$.

[3]For more information on Eurodollar certificates of deposit, see Stigum (1990).

[4]The number of days from November 13, 1991, to February 11, 1992, is 90.

TABLE 8.4 Money market rates.

MONEY RATES

Wednesday, November 13, 1991

The key U.S. and foreign annual interest rates below are a guide to general levels but don't always represent actual transactions.

PRIME RATE: 7½%. The base rate on corporate loans at large U.S. money center commercial banks.

FEDERAL FUNDS: 5⅛% high, 2% low, 2% near closing bid, 2½% offered. Reserves traded among commercial banks for overnight use in amounts of $1 million or more. Source: Babcock Fulton Prebon (U.S.A.) Inc.

DISCOUNT RATE: 4.50%. The charge on loans to depository institutions by the Federal Reserve Banks.

CALL MONEY: 6¾% to 7%. The charge on loans to brokers on stock exchange collateral.

COMMERCIAL PAPER placed directly by General Electric Capital Corp.: 4.85% 15 to 36 days; 4.70% 37 to 59 days; 4.93% 60 to 89 days; 4.90% 90 to 149 days; 4.87% 150 to 179 days; 4.70% 180 to 189 days; 4.87% 190 to 270 days. Commercial Paper placed directly by General Motors Acceptance Corp.: 4.85% 30 to 44 days; 4.80% 45 to 59 days; 5% 60 to 270 days.

COMMERCIAL PAPER: High-grade unsecured notes sold through dealers by major corporations in multiples of $1,000: 5% 30 days; 5.10% 60 days; 5.02% 90 days.

CERTIFICATES OF DEPOSIT: 4.48% one month; 4.61% two months; 4.60% three months; 4.62% six months; 4.98% one year. Average of top rates paid by major New York banks on primary new issues of negotiable C.D.s, usually on amounts of $1 million and more. The minimum unit is $100,000. Typical rates in the secondary market: 4.90% one month; 5% three months; 5% six months.

BANKERS ACCEPTANCES: 4.80% 30 days; 5.03% 60 days; 4.91% 90 days; 4.87% 120 days; 4.85% 150 days; 4.81% 180 days. Negotiable, bank-backed business credit instruments typically financing an import order.

LONDON LATE EURODOLLARS: 5% - 4⅞% one month; 5¼% - 5⅛% two months; 5 3/16% - 5 1/16% three months; 5 3/16% - 5 1/16% four months; 5 3/16% - 5 1/16% five months; 5⅛% - 5% six months.

LONDON INTERBANK OFFERED RATES (LIBOR): 4 15/16% one month; 5⅛% three months; 5⅛% six months; 5¼% one year. The average of interbank offered rates for dollar deposits in the London market based on quotations at five major banks. Effective rate for contracts entered into two days from date appearing at top of this column.

FOREIGN PRIME RATES: Canada 8.50%; Germany 11.50%; Japan 7%; Switzerland 10%; Britain 10.50%. These rate indications aren't directly comparable; lending practices vary widely by location.

TREASURY BILLS: Results of the Tuesday, November 12, 1991, auction of short-term U.S. government bills, sold at a discount from face value in units of $10,000 to $1 million: 4.64% 13 weeks; 4.71% 26 weeks.

FEDERAL HOME LOAN MORTGAGE CORP. (Freddie Mac): Posted yields on 30-year mortgage commitments. Delivery within 30 days 8.54%, 60 days 8.60%, standard conventional fixed-rate mortgages; 6%, 2% rate capped one-year adjustable rate mortgages. Source: Telerate Systems Inc.

FEDERAL NATIONAL MORTGAGE ASSOCIATION (Fannie Mae): Posted yields on 30 year mortgage commitments for delivery within 30 days (priced at par). 8.47%, standard conventional fixed rate-mortgages; 6.10%, 6/2 rate capped one-year adjustable rate mortgages. Source: Telerate Systems Inc.

MERRILL LYNCH READY ASSETS TRUST: 4.90%. Annualized average rate of return after expenses for the past 30 days; not a forecast of future returns.

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points higher.[5] This reflects the higher default risk of these deposits vis-à-vis U.S. government securities.

T-bills and CD's are short-term discount bonds. Table 8.3 also contains prices of Treasury strips, which are long-term discount bonds. Strips get their name from the fact that, for each year, coupons of several government coupon bonds are "stripped" from the principal and combined to create a discount bond that makes only one payment. Table 8.3 provides the prices and yields to maturity for these bonds.

Coupon Bonds

The price of a coupon-bearing bond, B, is computed by summing the present values of the fixed periodic coupon payments, C_t, and the present value of the terminal face value, F_n, that is,

$$B = \sum_{t=1}^{n} \frac{C_t}{(1+y)^t} + \frac{F_n}{(1+y)^n}. \tag{8.4}$$

In general, the coupon payments are the same fixed amount each period, so the bond price may also be written as the present value of an annuity plus the present value of the final amount:

$$B = (C/y)[1 - (1+y)^{-n}] + F(1+y)^{-n}. \tag{8.5}$$

The Treasury bonds and notes in Table 8.3 are coupon-bearing bonds. Treasury notes are issued with maturities of two to ten years and Treasury bonds with maturities longer than ten years. The denominations of bonds and notes range from \$1,000 to \$1 million. Their prices are quoted as a percentage of par, so a reported price of 96:00 for a \$100,000 face-value bond is actually 96 percent of \$100,000 or \$96,000. In addition, the decimal part of a T-bond or T-note price is the number of 32nds, so a reported price of 94:8 is actually 94⁸⁄₃₂, or 94.25 in decimal form. Finally, all T-bonds and T-notes have semiannual coupon payments. The "9s of November 2018" in Table 8.3, for example, pay coupon interest at a rate of 4.5 percent of par in May and November of each year. The last coupon is paid with the repayment of the face value in November 2018.[6]

The fact that Treasury bonds pay semiannual coupons implies that the yield to maturity, y, in (8.4) and (8.5) is an effective rate over a six-month period. To annualize this rate, the *effective annual yield to maturity*, y_A, may be computed as

[5] A basis point is 1/100 of one percent.

[6] Fabozzi and Fabozzi (1989, pp. 83–84) provide a summary of the various types of bill, note, and bond issues of the U.S. government.

$$y_A = (1 + y)^2 - 1. \tag{8.6}$$

While (8.6) is technically correct, some people prefer to use the *bond equivalent yield,* which is simply 2 × *y*, to measure of the expected annual rate of return on the bond.[7]

One last convention about Treasury bonds and notes must be discussed. The bond price reported in the financial press (i.e., the *quoted bond price*) excludes the *accrued interest* of the current coupon period. The price that you would pay for the bond if you decided to buy it would be the quoted price plus the coupon interest that has accrued on a straight-line basis during the current coupon period. That is, accrued interest equals the proportion of the current coupon period that has elapsed times the amount of the coupon,

$$AI = C\left(\frac{\text{number of days since last coupon was paid}}{\text{total number of days in current coupon period}}\right). \tag{8.7}$$

To illustrate, consider the 9s of November 2018, whose price is reported in Table 8.3. The bond is reported to have a bid price of 111:21 and an ask price of 111:23. Treasury bonds pay coupon interest every six months and do so on the 15th of the month. For the 9s of November 2018, coupon interest of 4.5 percent of par is paid on May 15th and November 15th. On November 13, 1991, the number of days since the last coupon payment is 182. The total number of days from May 15, 1991, to November 15, 1991, is 184. The accrued interest on this bond as of November 13 is, therefore, 4.5(182/184) or 4.45. If this bond were bought at the ask price, we would pay the reported ask price of $111^{23}/_{32}$ plus accrued interest of 4.45, or a total amount 116.17.

To verify that 116.17 is actually the ask price of the 9s of November 2018, compute the present value of the promised coupons and face value as of November 13, 1991, using the reported bond equivalent yield of 7.94 percent. We cannot apply the bond valuation formulas (8.4) and (8.5) directly because those formulas assume that the next coupon payment is in exactly six months. For the 9s of November 2018, we are part way through the coupon period, so the bond valuation formula needs to be modified:

$$B = \frac{1}{(1+y)^p}\left[\sum_{t=0}^{n-1} \frac{C_t}{(1+y)^t} + \frac{F_n}{(1+y)^{n-1}}\right], \tag{8.8}$$

where p is the ratio of the number of days remaining in the current coupon period to the total number of days during the current coupon period. The term in brackets

[7] Yet others rely on *coupon yield,* which is simply the annual coupon amount divided by the bond price.

represents the present value, just prior to the next coupon, of all future payments to the bondholder. This amount is then discounted back p coupon periods to the current date. Substituting the example parameters,

$$B = \frac{1}{(1.0397)^{2/184}} \left[\sum_{t=0}^{54} \frac{4.50}{(1.0397)^t} + \frac{100}{(1.0397)^{54}} \right] = 116.17.$$

Subtracting the accrued interest yields the quoted ask price of the bond reported in Table 8.3.

Finally, some Treasury bonds do not have a fixed maturity date. The issues denoted by the hyphenated maturity date in Table 8.3 are *callable bonds,* which the Treasury has the right to "call," or redeem, at any time during a prespecified period in the future. The 11¾ of August 2009-14, for example, are callable bonds which may be redeemed during the period November 2009 through 2014. Callable bonds may be delivered on certain futures contracts, as long as they satisfy some minimum amount of time before the first call date.

8.3 INTEREST RATE RISK

Bondholders (lenders) face the risk that interest rates will rise, causing a decline in the value of their bonds. Bond issuers (borrowers) face the risk that interest rates will fall, causing an increase in the value of their debt obligation. Interest rate risk is a key concern of all financial institutions that operate in the debt markets. These include banks, pension funds, and insurance companies. In this section, we analyze the risk of the default-free fixed income securities (such as U.S. Treasury securities) that these institutions hold.[8]

Duration

One measure of a fixed income security's interest rate risk is its *duration.* Duration specifies the sensitivity of the bond price to movements in yield. A specific formula for computing duration may be obtained by taking the derivative of B with respect to y in equation (8.4). First, for the sake of mathematical convenience, rewrite (8.4) as

$$B = \sum_{t=1}^{n} C_t (1+y)^{-t}. \qquad \textbf{(8.9)}$$

[8]Default risk is also an important consideration in fixed-income security risk management. Recall that in Chapter 4, we showed how to hedge both the interest rate and default risk exposure of Mobil Oil bonds using T-bond and stock index futures contracts.

Under this specification, we let $C_t = C$ for $t = 1, \cdots, n - 1$ and $C_t = C + F_n$ for $t = n$. Now, take the derivative of B with respect to y,

$$\frac{dB}{dy} = -\sum_{t=1}^{n} tC_t(1+y)^{-t-1}. \qquad \textbf{(8.10)}$$

Multiply (8.10) by $(1 + y)$,

$$(1+y)\frac{dB}{dy} = -\sum_{t=1}^{n} tC_t(1+y)^{-t}. \qquad \textbf{(8.11)}$$

Finally, divide (8.11) by B,

$$\frac{dB/B}{dy/(1+y)} = -\sum_{t=1}^{n} t\left[\frac{C_t(1+y)^{-t}}{B}\right] = -\sum_{t=1}^{n} tw_t \equiv -D, \qquad \textbf{(8.12)}$$

where $w_t = C_t(1 + y)^{-t}/B$ and where duration is defined as

$$D \equiv \sum_{t=1}^{n} tw_t. \qquad \textbf{(8.13)}$$

Equations (8.12) and (8.13) tell us two important things about duration. First, as the left-hand side of (8.12) indicates, minus duration, $-D$, can be interpreted as the percentage change in the bond price, dB/B, induced by a change in the bond's yield to maturity, dy, scaled by $1/(1 + y)$. Second, duration is the *weighted average time to maturity* of a bond, where the weights, w_t, are the present values of the payments in each period.[9] For coupon bonds, duration is less than time to maturity because some of the bond payments—the coupons—are received in years prior to maturity of the bond. For non-coupon bonds, duration equals time to maturity.

The scale factor $1/(1 + y)$ on the left-hand side of (8.12) is cumbersome to account for. Most fixed income portfolio managers prefer to use a measure of dura-

[9]Note that by virtue of equation (8.9), the weights, w_t, sum to one, that is,

$$\sum_{t=1}^{n} w_t = \sum_{t=1}^{n} \frac{C_t(1+y)^{-t}}{B} = \frac{B}{B} = 1.$$

tion that is simply the percentage change in bond price, *dB/B*, induced by a change in yield, *dy*. To find this expression, divide (8.12) by $(1 + y)$,

$$\frac{dB/B}{dy} = -\sum_{t=1}^{n} tw_t/(1+y) = -D/(1+y) \equiv -D_m. \tag{8.14}$$

D_m is called *modified duration* and is more commonly used in interest risk management strategies. It is simply the duration, as defined in (8.13), divided by $(1 + y)$. We use modified duration in a hedging application later in this chapter. We now turn to a numerical example that uses the duration formulas (8.13) and (8.14).

EXAMPLE 8.1

What is the duration and the modified duration of an 8 percent, ten-year, \$1000 bond, assuming annual coupon payments and a required yield to maturity of 7 percent?

From equation (8.4), we know the bond price is

$$B = \sum_{t=1}^{10} \frac{80}{(1.07)^t} + \frac{1,000}{(1.07)^{10}} = \$1,070.24.$$

This present value consists of the present value of 10 payments. In tabular form,

t	C_t	$(1+y)^{-t}$	$C_t(1+y)^{-t}$	w_t	tw_t
1	80.00	0.9346	74.77	0.0699	0.0699
2	80.00	0.8734	69.88	0.0653	0.1306
3	80.00	0.8163	65.30	0.0610	0.1831
4	80.00	0.7629	61.03	0.0570	0.2281
5	80.00	0.7130	57.04	0.0533	0.2665
6	80.00	0.6663	53.31	0.0498	0.2989
7	80.00	0.6227	49.82	0.0466	0.3259
8	80.00	0.5820	46.56	0.0435	0.3480
9	80.00	0.5439	43.51	0.0407	0.3659
10	1080.00	0.5083	549.02	0.5130	5.1299
Total			1,070.24	1.0000	7.3466

The duration of the bond is 7.3466, and the modified duration is 7.3466/1.07 = 6.8660.

The modified duration figure computed in the example predicts that if interest rates increase by 100 basis points, the bond price will change by

$$dB/B = -D_m \times dy = -6.8660 \times .01 = -6.8660\%.$$

Conversely, a decrease in yield of 100 basis points implies a 6.8660% increase in bond price.

The formula for the duration of a bond shows that duration—price sensitivity—depends on the maturity of the bond, on the coupon level, and on the yield to maturity. First, the greater the maturity, the greater the duration, holding constant other characteristics of the bond. Second, the larger a bond's coupon, the smaller the duration. Coupon payments cause weight to be put on the early years in the duration formula. In the case of a zero-coupon bond, duration equals maturity. Third, duration decreases with increases in the yield to maturity. This is because an increase in the yield has a greater effect on the present value of a distant coupon than on the present value of a nearby coupon.

Convexity

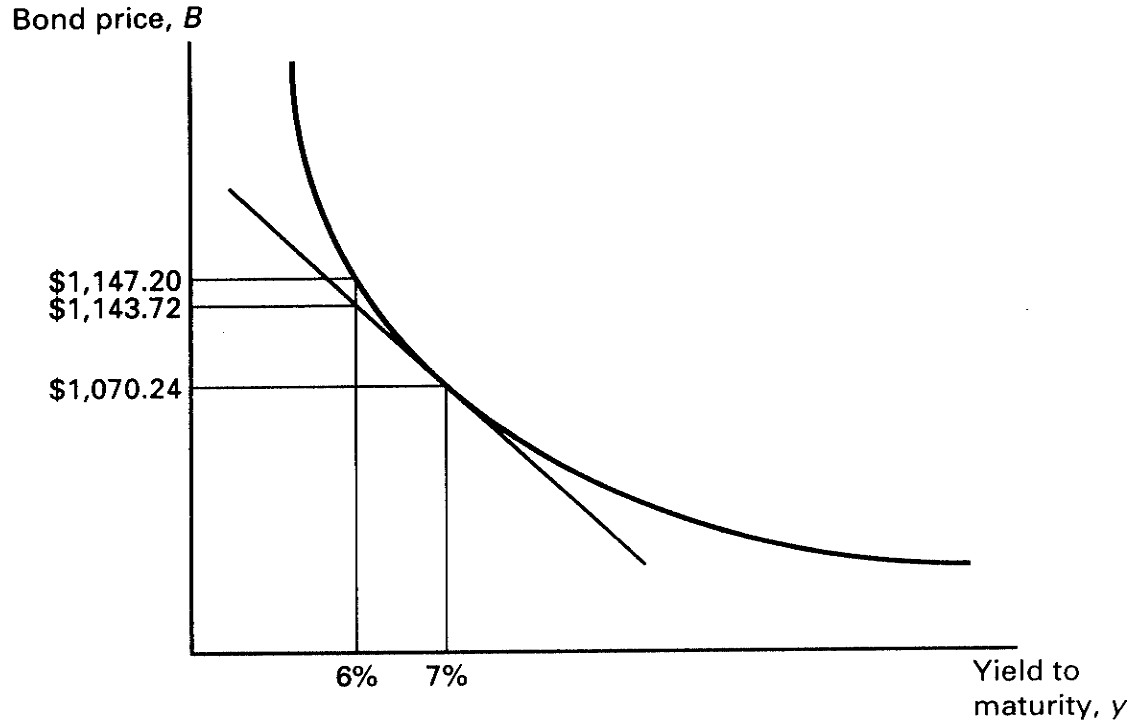
Modified duration is only an approximation of the percentage change in bond price for a given change in yield. In fact, it is accurate only for *infinitesimal* shifts in yield. To assess the degree of error that may be introduced, reconsider the results of Example 8.1. The level of modifed duration predicts that the bond price will increase to \$1,070.24(1.068660) = \$1,143.72 if the yield drops to 6 percent. By using the bond valuation formula (8.4), however, we know that the bond value is exactly \$1,147.20 at a yield of 6 percent. The difference between these prices is attributable to the fact that bond price is a nonlinear function of yield to maturity.

Figure 8.1 illustrates the approximation error in this example. At a yield of 7 percent, the bond's price is \$1,070.24. Modified duration depends on the derivative, *dB/dy*, evaluated at 7 percent. The derivative is depicted in the figure by the straight line tangent to the bond price curve at 7 percent. To estimate the change in bond price due to a 100 basis point drop in yield, we draw a vertical line from 6 percent on the horizontal axis to the straight line depicting the derivative, and then draw across horizontally to the vertical axis. The estimated value of the bond based on modified duration is \$1,143.72. If the vertical line is continued upward to the bond price curve, we find the exact value of the bond at a 6-percent yield, \$1,147.20. The pricing error, \$3.48, is attributable to the failure of modified duration to account for the convex nature of the bond pricing function.

We can achieve greater precision in measuring the bond's responsiveness to yield shifts by also accounting for the bond's convexity. To understand convexity, expand the bond price as represented by (8.9) into a Taylor series:

$$dB = \frac{dB}{dy}dy + \frac{1}{2}\frac{d^2B}{dy^2}(dy)^2 + \epsilon. \tag{8.15}$$

FIGURE 8.1 Bond Price as a Function of Yield



The error term, ϵ, recognizes the fact that we have used only the first two terms of the Taylor series expansion. Hence, our approximation for bond price changes will remain only approximate. Now, we drop the error term and divide both sides by B,

$$\frac{dB}{B} = \frac{dB/B}{dy}dy + \frac{1}{2}\frac{d^2B}{dy^2}\frac{1}{B}(dy)^2. \tag{8.16}$$

We now define *convexity* as

$$\text{Convexity} \equiv \frac{1}{2}\frac{d^2B}{dy^2}\frac{1}{B}, \tag{8.17}$$

and rewrite (8.16), the percentage change in bond price, using (8.14) and (8.17), as

$$\frac{dB}{B} = -D_m dy + \text{Convexity}(dy)^2. \tag{8.18}$$

The value of the second derivative of the bond price with respect to a change in yield is

$$\frac{d^2B}{dy^2} = \sum_{t=1}^{n} t(t+1)\left[\frac{C_t}{(1+y)^{t+2}}\right]. \qquad \textbf{(8.19)}$$

EXAMPLE 8.2

What is the convexity of an 8 percent, ten-year, $1000 bond, assuming annual coupon payments and a required yield to maturity of 7 percent?

From Example (8.1), we know the bond price is $1,070.24. The components of convexity (8.19) are

t	$t(t+1)$	$\frac{C_t}{(1+y)^{t+2}}$	$\frac{t(t+1)C_t}{(1+y)^{t+2}}$
1	2	65.3038	130.61
2	6	61.0316	366.19
3	12	57.0389	684.47
4	20	53.3074	1,066.15
5	30	49.8200	1,494.60
6	42	46.5607	1,955.55
7	56	43.5147	2,436.82
8	72	40.6679	2,928.09
9	90	38.0074	3,420.67
10	110	479.5329	52,748.62
Total			67,231.77

The value of d^2B/dy^2 for this bond is 67,231.77, so the convexity is ½ × 67,231.77 × 1/1,070.24 = 31.4098.

To complete our illustration, we now use convexity in conjunction with modified duration to arrive at a more accurate estimate of the percentage change in bond price attributable to a 100 basis point fall in yield.

$$\begin{aligned}\frac{dB}{B} &= -D_m dy + \text{Convexity}(dy)^2 \\ &= -6.8662 \times (-.01) + 31.4098 \times (-.01)^2 = -7.180\%.\end{aligned}$$

In other words, based upon modified duration and convexity, the bond price is expected to rise to $1,147.08 if yield falls by 100 basis points. Note that the error in prediction has been reduced from $3.48 to $0.12 by accounting for convexity.

All bonds with fixed payment schedules plot as convex curves in y, as shown in Figure 8.1, although different bonds have different degrees of convexity. Mortgage backed securities (MBS's), however, sometimes exhibit "negative convexity." That is, they plot as curves that are concave to the origin. This is because their payment schedules are uncertain. The underlying mortgages have the option to prepay before maturity, and they will choose to prepay just when it is most inconvenient for the holder of the MBS. When interest rates fall, mortgages are prepaid, so the duration and price sensitivity of the MBS are reduced. Consequently, while fixed-payment bonds increase in price, the price of MBS's increases little and may even decline. When interest rates rise, mortgage prepayments slow, resulting in duration and price sensitivity increases for the MBS. Consequently, the price decline resulting from an increase in interest rates is relatively greater for a MBS than for a fixed-payment bond.

8.4 TERM STRUCTURE OF INTEREST RATES

Up to this point in the chapter, we have assumed that all cash flows of a fixed-income security are discounted at a single rate—the yield to maturity. This assumption may be inappropriate since lenders may demand different rates on short-term loans than on long-term loans. The relation between the level of interest rates and the time to maturity of the loan is the focus of this section.

Yield Curve

The simplest way to consider the relation between expected bond return and time to maturity is to plot the yield to maturity of U.S. Treasury obligations versus the term to maturity (making sure that the bonds have the same default risk, no imbedded options such as callability, and no differential tax privileges). This relation is referred to as the *yield curve.* The yield curve is usually plotted using Treasury bonds and notes. The yield to maturity differs by maturity because expected future interest rates are different for different maturities and because risk premia differ by maturity.

The yield to maturity of a coupon-bearing bond can be a misleading reflection of its expected rate of return, however. To see this, consider the following situation. Suppose there exist two discount Treasury bonds, one with a one-year maturity and a price of 90.91, and the other with a two-year maturity and a price of 81.16. Assuming each of these bonds is redeemed at a par value of 100 at their respective maturities, the expected yield on the one-year bond is

$$100/90.91 - 1 = 10.00\%,$$

and the expected yield on the two-year bond is

$$(100/81.16)^{1/2} - 1 = 11.00\%.$$

Note that if the Treasury decided to issue other one- and two-year discount bonds, they must have the same prices as the existing issues, otherwise, costless arbitrage profits could be earned.

Now, suppose that the Treasury also has a two-year, 12-percent coupon bond. The 12-percent issue pays 12 at the end of one year, and 112 at the end of two years. The price of this coupon-bearing Treasury bond has to be

$$B = \frac{12}{1.10} + \frac{112}{(1.11)^2} = 101.81,$$

otherwise, costless arbitrage profits are possible.[10] But, if the two-year coupon-bearing bond has a price of 101.81, its yield to maturity is 10.94 percent, that is, the solution to

$$101.81 = \frac{12}{1+y} + \frac{112}{(1+y)^2}.$$

On the surface, it might appear that the two-year coupon bond expected yield to maturity of 10.94 percent conflicts with the two-year discount bond expected yield to maturity of 11 percent. However, arbitrage profits are not possible. The discrepancy between the rates arises because the two-year coupon bond has an "actual" term to maturity of less than two years. If you buy the coupon bond, you will receive 12 at the end of one year and 112 at the end of two years; so, in essence, you have a portfolio of two discount bonds. The first discount bond is worth

$$B_{d_1} = \frac{12}{1.10} = 10.91,$$

and the second discount bond is worth

$$B_{d_2} = \frac{112}{1.11^2} = 90.90.$$

The average term to maturity of the bond portfolio (or the coupon-bearing bond) is, therefore,

$$\text{Average term to maturity} = \left(\frac{10.91}{101.81}\right) 1 + \left(\frac{90.90}{101.81}\right) 2 = 1.893 \text{ years.}$$

[10]Note that buying .12 units of the one-year discount bond and 1.12 units of the two-year discount bond produces a cash flow stream exactly the same as the two-year coupon-bearing bond at a cost of .12(90.91) + 1.12(81.16) = 101.81.

The 10.94-percent yield is not an expected return on a bond with two years to maturity, but rather on a bond with a maturity of approximately 1.893 years.

Two principles underlie this discussion. First, to value coupon-bearing bonds precisely, all cash flows of the bond should not be discounted at the same rate. Instead, each cash flow should be discounted at the zero-coupon or discount bond rate, r_t, which coincides with the timing of the cash flow,

$$B = \sum_{t=1}^{n} \frac{C_t}{(1+r_t)^t} + \frac{F_n}{(1+r_n)^n}. \tag{8.20}$$

The rate r_t is called the *spot rate of interest* on a t-period loan. Second, to estimate the relation between expected bond returns and time to maturity, only zero-coupon or spot rates of interest should be used. The yields to maturity on coupon bonds should *not* be used because the term to maturity of the bond overstates its true "economic life." The economic life of a coupon bond is a weighted average life of its constituent discount bonds, that is,

$$\text{Average term to maturity} = \sum_{t=1}^{n} \left(\frac{\frac{C_t}{(1+r_t)^t}}{B} \right) t + \left(\frac{\frac{F_n}{(1+r_n)^n}}{B} \right) n. \tag{8.21}$$

Note that this weighted average term to maturity is the same as the bond's duration defined in (8.12).[11]

In practice, the relation between zero-coupon or spot rates of interest and time to maturity is referred to as the *zero-coupon yield curve*. In this section, we refer to the relation as the *term structure of interest rates*. The term structure can be read directly from the yields to maturity of Treasury strips. For example, Table 8.3 shows that the term structure was upward sloping on November 13, 1991. The one-year, two-year, five-year, and ten-year spot rates were 5.05, 5.68, 6.84, and 7.78, respectively.

The shape of the term structure of spot rates affects the relative values of bonds with different coupons. We have already illustrated this point when we showed that a two-year coupon bond would yield 10.94 percent to maturity while a two-year discount bond yielded 11 percent. In general, for bonds of the same maturity, when the term structure is upward sloping, the yield to maturity decreases as the coupon payment increases; when the term structure is downward sloping, the yield to maturity increases with a larger coupon. This result can be explained in two ways. First, a coupon bond has a duration that is less than its maturity. Its

[11] Duration and maturity are the same only for a discount bond because the coupon terms, C_t, in the summation of (8.21) are zero.

yield to maturity corresponds to its duration, not its maturity. Second, when the term structure is upward sloping, investors prefer coupon bonds because it is expected that the coupon payments can be invested at higher future rates. They, therefore, bid up prices of coupon bonds and lower their yield to maturity. Correspondingly, when the term structure is downward sloping, investors prefer low-coupon bonds because it is expected that the coupons can be invested only at lower future rates. Only when the term structure of interest rates is flat do all bonds have the same yield to maturity.

Spot Rates and Forward Rates

Spot rates of interest are rates observable today. The spot rate on a three-month, default-free security is the rate of return promised on a three-month T-bill. The notation that we use to describe the spot rate on a t-period bond is r_t. *Forward rates of interest* are interest rates on loans in the future and are implied from the current term structure of spot rates. The forward rate on a t-period loan in period n, denoted ${}_nf_t$, is computed using

$$(1 +_n f_t)^t = \frac{(1 + r_{n+t})^{n+t}}{(1 + r_n)^n}. \quad \textbf{(8.22)}$$

The forward rate on a one-year loan today, for example, is computed as

$$(1 +_0 f_1)^1 = (1 + r_1)^1,$$

and the forward rate on a one-year loan in three years is

$$(1 +_3 f_1)^1 = \frac{(1 + r_{3+1})^{3+1}}{(1 + r_3)^3}.$$

EXAMPLE 8.3

Suppose the one-year spot rate of interest is 10 percent and the two-year spot rate is 11 percent. What is the implied one-year forward rate in one year?

Substituting the parameters of the example into equation (8.22), we find that

$$1 +_1 f_1 = \frac{(1 + r_2)^2}{(1 + r_1)} = \frac{(1.11)^2}{(1.10)},$$

or

$${}_1f_1 = 12.01\%.$$

EXAMPLE 8.4

Using the values reported in Table 8.3, find the 91-day forward rate implied by selling the December 19, 1991 T-bill and buying March 19, 1992 T-bill. The effective annual rate of return of the December 19th T-bill is

$$r = \left[\frac{100}{100 - 4.39(36/360)}\right]^{365/36} - 1 = 4.562\%.$$

The effective annual rate of return of the March 19th T-bill is

$$r = \left[\frac{100}{100 - 4.64(127/360)}\right]^{365/127} - 1 = 4.858\%.$$

Given these two spot rates of interest, the implied forward rate of interest on a 91-day T-bill in 36 days is the solution to

$$(1 +_{36} f_{91})^{91/365} = \frac{(1.04858)^{127/365}}{(1.04562)^{36/365}}.$$

The implied forward rate of interest is 4.975 percent.

8.5 SHORT-TERM INTEREST RATE FUTURES CONTRACTS

The most actively traded short-term interest rate futures contracts are the Chicago Mercantile Exchange's T-bill and Eurodollar futures contracts. The *Treasury bill futures contract* requires the delivery of a $1,000,000 face value T-bill with 91-days to maturity. The contract expires the business day before the date on which the new 91-day T-bill is issued. The newly issued 91-day T-bill, the seasoned 182-day, and the seasoned 364-day T-bills are eligible for delivery on this futures contract.

The price of a T-bill futures contract is an index value based on the bank discount. The price reported for the December 1991 T-bill futures in Table 8.2, for example, is 95.34. This does not mean that the futures buyer will pay 95.34 percent of par when she buys the T-bill at the futures maturity. The index price, 95.34, implies that the bank discount on the bill is 100 − 95.34 = 4.66 on an annualized basis. Since the futures contract requires the delivery of a 91-day T-bill, the annualized discount is adjusted to a 91-day period using the banker's convention of a

360-day year. The 91-day discount is 4.66(91/360) = 1.178. Thus, if we bought the December 1991 T-bill futures contract on November 13, 1991, at the reported price of 95.34, we would be entering into a commitment to buy a 91-day, $1,000,000 T-bill on March 19, 1992, at a price of 98.822 percent of the face value of the T-bill or $988,220. The implied forward rate of interest on this T-bill is

$$\left(\frac{100}{98.822}\right)^{365/91} - 1 = 4.868\%.$$

It is interesting to note that the implied forward rate of interest from the futures contract, 4.868 percent, is less than the 4.975 percent rate implied by the T-bills in Example 8.4. Both rates apply to the 91-day period beginning on December 19, 1991. If one could borrow at 4.868 percent and lend at 4.975 percent, an arbitrage profit could be earned. In effect, this can be done by the following arbitrage transactions: (a) sell the December 19, 1991 T-bill for $9,956.10; (b) buy the March 19, 1992 T-bill for $9,836.31; and (c) sell the December 1991 T-bill futures contract at $9,882.20. Transactions (a) and (b) have the net effect of lending money over the period December 19, 1991 to March 19, 1992; transaction (c) commits the investor to borrowing money for the same period. The borrowing and lending, however, may never occur because the transactions can be closed out on December 19, 1991. On that date, the arbitrageur covers the short position in the maturing December 19 T-bill by paying $10,000, thereby incurring a cost of $43.90. The March 19 T-bill purchased on November 13 now has 91 days to maturity and can be delivered against the futures contract for $9,882.20, a net gain of $45.89. Finally, the net proceeds from the T-bill transactions on November 13, 1991, $119.79, have earned interest at, say, 5 percent and, after 36 days, are now worth $120.37. The arbitrage profit realized on December 19, therefore, totals

$$\$120.37 + \$45.89 - \$43.90 = \$122.36.$$

The *Eurodollar futures contract* is a commitment to transact a $1,000,000, three-month Eurodollar deposit. Delivery never takes place since a Eurodollar futures contract is cash settled. Cash settlement occurs on the second London business day before the third Wednesday of the contract month.

The settlement price of the Eurodollar futures contract on the expiration day is computed in such a way as to minimize the variation in the quoted Eurodollar deposit rates. During the last day of trading, a random sample of approximately twelve rates are taken from the twenty-plus approved banks in the London Eurodollar market. The rates are then ranked from highest to lowest, and the highest and lowest rates are discarded. The remaining ten rates are averaged, and the average rate is subtracted from 100 to determine the settlement price.

As noted in the previous paragraph, quoted Eurodollar futures prices are actually index values, that is, the value reported in the financial press is 100 less the Eurodollar interest rate. Thus, if we buy the March 1992 Eurodollar futures at the price reported in Table 8.2, 94.94, the implicit agreement that we are entering into is to buy a $1,000,000 three-month Eurodollar certificate of deposit on March 16, 1992 (the second London business day before the third Wednesday of the futures contract month), where the stated interest rate on the deposit is 100 − 94.94, or 5.06 percent. The effective three-month forward interest rate on such a deposit is, therefore,

$$r = \left[\frac{100 + 5.06(\frac{92}{360})}{100}\right]^{365/92} - 1 = 5.23\%.$$

Note that the three-month interval from March 16, 1992, through June 16, 1992, has 92 days.

8.6 LONG-TERM INTEREST RATE FUTURES CONTRACTS

The most active long-term interest rate futures contract is the T-bond contract on the Chicago Board of Trade.[12] The CBT's *Treasury bond futures contract* is a commitment to deliver a nominal 8 percent, $100,000 face-value U.S. Treasury bond with a least fifteen years to maturity or to first call date, whichever comes first. The seller of the futures contract has the option to deliver any of the eligible issues on any date during the delivery month. Whether an 8-percent coupon issue is available for delivery is unimportant since the futures contract allows for the delivery of any T-bond with a long enough maturity. To remove the effects of different bonds having different coupon rates, the CBT designed a system of conversion factors.

Conversion Factor and Invoice Price

To understand the CBT's system of conversion, recall first that the lower the coupon rate, the lower the bond's price, other factors being held constant [see equation (8.4)]. Since the seller of the T-bond futures contract can deliver *any* U.S. Treasury bond with at least fifteen years to maturity or to the first call date, a method of conversion is needed to offset the economic incentive to deliver the lowest coupon bond. The CBT's system adjusts the futures price, which is based on an 8-percent coupon, to a price that corresponds to the coupon of the issue being delivered. To illustrate the principle underlying the conversion, consider the price of an 8-percent, fifteen-year bond with annual coupons and a yield to maturity of 8 percent. Using equation (8.4), the bond price is

[12] For an interesting analysis of why this contract supplanted the earlier GNMA contract, see Johnston and McConnell (1989).

$$B = \sum_{t=1}^{15} \frac{8}{(1.08)^t} + \frac{100}{(1.08)^{15}} = 100.00.$$

Now, consider the price of a 12-percent, fifteen-year bond with fifteen years to maturity and the same 8-percent yield. The bond price is

$$B = \sum_{t=1}^{15} \frac{12}{(1.08)^t} + \frac{100}{(1.08)^{15}} = 134.24.$$

Note that the only difference between the two bonds is that the second bond has higher coupon payments. Owning the 12-percent coupon bond is like owning 1.3424 8-percent bonds. Since the futures price is based on an 8-percent coupon bond, the futures price is multiplied by a conversion factor of 1.3424 to compute the amount paid (delivery price) by the long to the short if the short delivers the 12-percent coupon issue.

The actual formula for computing the conversion factor is slightly more complex than what is demonstrated in the above example because coupons are paid on a semi-annual basis, and, in general, the next coupon payment is made in less than six months (i.e., we are part of the way through the current coupon period). The actual formula for the conversion factor, *CF*, is

$$CF = (1 + y/2)^{-X/6} \left(\frac{C}{2} + \left\{ \frac{C}{y} \left[1 - (1 + y/2)^{-2n}\right] + (1 + y/2)^{-2n} \right\} \right) - \frac{C}{2} \frac{(6 - X)}{6}, \quad \textbf{(8.23)}$$

where C is the annual coupon rate of the bond in decimal form, y equals 0.08, n is the number of whole years to first call, if the bond is callable, or the number of years to maturity, and X is the number of months that the maturity exceeds n, rounded down to the nearest quarter (e.g., $X = 0, 3, 6, 9$). Note that if $X = 0, 3, 6$, the formula (8.23) is used directly. If $X = 9$, set $2n = 2n + 1$, $X = 3$, and calculate as above.[13] Computer programs are available to perform the computation (8.23). The values in Table 8.5, for example, were generated using a program called OPTVAL. Alternatively, the CBT and others publish and distribute conversion factor tables.

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[13]Note that if $X = 0$ the formula (8.23) reduces to

$$CF = \left\{ \frac{C}{y} \left[1 - (1 + y/2)^{-2n}\right] + (1 + y/2)^{-2n} \right\}.$$

TABLE 8.5 Conversion factors for the U.S. Treasury Bonds eligible for delivery on the CBT's T-bond futures contract. These factors convert different coupon issues to yield 8 percent.

Years-Months	9%	$9\frac{1}{8}$%	$9\frac{1}{4}$%	$9\frac{3}{8}$%	$9\frac{1}{2}$%	$9\frac{5}{8}$%	$9\frac{3}{4}$%	$9\frac{7}{8}$%
25-0	1.1074	1.1208	1.1343	1.1477	1.1611	1.1745	1.1880	1.2014
25-3	1.1075	1.1210	1.1345	1.1479	1.1614	1.1749	1.1883	1.2018
25-6	1.1081	1.1216	1.1351	1.1486	1.1621	1.1756	1.1892	1.2027
25-9	1.1082	1.1217	1.1353	1.1488	1.1624	1.1759	1.1895	1.2030
26-0	1.1087	1.1223	1.1359	1.1495	1.1631	1.1767	1.1903	1.2039
26-3	1.1088	1.1225	1.1361	1.1497	1.1633	1.1770	1.1906	1.2042
26-6	1.1094	1.1230	1.1367	1.1504	1.1640	1.1777	1.1914	1.2051
26-9	1.1094	1.1232	1.1369	1.1506	1.1643	1.1780	1.1917	1.2054
27-0	1.1100	1.1237	1.1375	1.1512	1.1649	1.1787	1.1924	1.2062
27-3	1.1100	1.1238	1.1376	1.1514	1.1652	1.1789	1.1927	1.2065
27-6	1.1105	1.1244	1.1382	1.1520	1.1658	1.1796	1.1935	1.2073
27-9	1.1106	1.1245	1.1383	1.1522	1.1660	1.1799	1.1937	1.2076
28-0	1.1111	1.1250	1.1389	1.1528	1.1666	1.1805	1.1944	1.2083
28-3	1.1111	1.1251	1.1390	1.1529	1.1668	1.1807	1.1947	1.2086
28-6	1.1116	1.1256	1.1395	1.1535	1.1675	1.1814	1.1954	1.2093
28-9	1.1117	1.1257	1.1396	1.1536	1.1676	1.1816	1.1956	1.2096
29-0	1.1121	1.1262	1.1402	1.1542	1.1682	1.1822	1.1963	1.2103
29-3	1.1122	1.1262	1.1403	1.1543	1.1684	1.1824	1.1965	1.2105
29-6	1.1126	1.1267	1.1408	1.1549	1.1690	1.1830	1.1971	1.2112
29-9	1.1127	1.1268	1.1409	1.1550	1.1691	1.1832	1.1973	1.2114
30-0	1.1131	1.1273	1.1414	1.1555	1.1697	1.1838	1.1980	1.2121
30-3	1.1131	1.1273	1.1415	1.1556	1.1698	1.1840	1.1981	1.2123
30-6	1.1136	1.1278	1.1420	1.1562	1.1704	1.1846	1.1988	1.2130
30-9	1.1136	1.1278	1.1420	1.1562	1.1705	1.1847	1.1989	1.2131

To illustrate the use of the conversion factor system, suppose that we are considering delivery of the 9s of November 2018 on the March 1992 T-bond futures contract. This bond is eligible for delivery because on March 1, 1992, it has more than fifteen years to maturity. Specifically, on March 1, 1992, the 9s of November 2018 have 26.50 years to maturity (rounded down to the nearest quarter). Using Table 8.5, the conversion factor of this bond is 1.1094. In other words, in place of delivering the hypothetical 8-percent, fifteen-year bond on the March 1992 futures contract, we can deliver the 9s of November 2018, but the buyer is going to have to pay 1.1094 times the prevailing futures price.

On the delivery date, the seller of the T-bond futures delivers an eligible T-bond to the buyer of the T-bond futures contract. In return, the buyer must pay the *invoice price* to the bond seller. The amount of the invoice price will be the sum of the futures price times the conversion factor of the delivered bond and the accrued interest on the delivered bond. For example, suppose that on March 15, 1992, the March 1992 futures contract is priced at 96-18. Like the underlying bonds, the decimal part of the price is the number of 32nds, so the futures price is 96.5625. If we sell the futures and promptly deliver the 9s of November 2018 to the futures contract buyer, the invoice price paid by the buyer equals .965625 (the futures price in decimal form) times 100,000 (the denomination of the futures contract) times 1.1094 (the conversion factor of the 9s of November 2018 as of March 1, 1992), or $107,126.44, plus the accrued interest on the 9s of November 2018 as of March 15, 1992, $2,991.76 [i.e., .045(121/182)(100,000)]. The total invoice price is

$$\$107,126.44 + \$2,991.76 = \$110,118.20.$$

Cheapest to Deliver

In principal, the system of conversion factors is intended to make the short indifferent about which bond he delivers. This means that if we are at time T—the end of the futures contract life—the profits from selling the futures and buying and delivering bond i, computed as

$$\pi_{i,T} = F_T(CF_i) + AI_{i,T} - B_{i,T} - AI_{i,T} = F_T(CF_i) - B_{i,T},$$

where $F_T(CF_i) + AI_{i,T}$ is the invoice price received from delivering bond i and $B_{i,T} + AI_{i,T}$ is the price paid for the purchase of bond i, should equal zero.

In practice, however, one of the eligible delivery bonds is "cheapest to deliver" because the system of conversion factors is not exact. Each bond will have a different value of $\pi_{i,T}$. The bond with the highest $\pi_{i,T}$ is the cheapest to deliver. Its value of $\pi_{i,T}$, however, will be equal to zero. If it were positive, costless arbitrage profits could be earned by the short at the expense of the long. If it were negative, costless arbitrage profits could be earned by the long at the expense of the short. The computed profits for all other deliverable bonds will be negative. However, negative profits for these bonds do not imply arbitrage opportunities. To capture these "gains," the long would need to take delivery of a bond issue that is not the cheapest to deliver. Since it is not rational for the short to deliver any issue other than the cheapest to deliver, no arbitrage gains are possible.

A cheapest-to-deliver issue arises because the conversion factors are derived by discounting the cash flows of all bonds at 8 percent. Using 8 percent assumes that coupon payments can be reinvested at 8 percent. If the average future interest rate at which the coupons can be reinvested exceeds 8 percent, however, investors prefer high-coupon bonds over low-coupon bonds (when each bond is valued by discounting at 8 percent). The cheapest-to-deliver bond is, therefore, the low-coupon bond. If the average future interest rate at which coupons can be reinvested is less than 8 percent, investors prefer low-coupon bonds over high-coupon bonds, so the cheapest to deliver is the high-coupon bond. Only when the term structure of interest rates is flat at 8 percent, will all bonds be equally desirable for delivery. If the yield curve is above 8 percent, low-coupon bonds are the cheapest to deliver, and if the yield curve is below 8 percent, high-coupon bonds are the cheapest to deliver.

8.7 COST-OF-CARRY RELATION

Under the assumptions that the cheapest-to-deliver bond issue i is known and that it does not change through time, the cost-of-carry relation between the futures price and the cheapest-to-deliver bond may be written

$$F_0 = \frac{(B_{i,0} + AI_{i,0})e^{rT} - AI_{i,T} - \sum_{t=0}^{T} C_{i,t}e^{r(T-t)}}{CF_i}. \tag{8.24}$$

The left-hand side of the equation is the futures price at time 0. The maturity of the futures contract is T periods hence. The first term in the numerator of the right-hand side of the equation, $(B_{i,0} + AI_{i,0})e^{rT}$, is the time 0 cost of the bond taken forward to time T at the riskless rate of interest (i.e., the bond purchase is financed at the short-term riskless interest rate). The second and third terms in the numerator represent interest earned on the bond—accrued interest, $AI_{i,T}$, received when the bond is delivered against the futures contract and the future value of the coupons received (if any), $\sum_{t=0}^{T} C_{i,t}e^{r(T-t)}$. The conversion factor CF_i in the denominator "converts" bond i into the hypothetical 8-percent coupon issue upon which the futures contract is designed.

Prior to discussing the cost-of-carry relation in more detail, it is worth noting that the "short-term riskless rate of interest" used to finance the purchase of bonds required in the arbitrage transactions that drive (8.24) is usually the rate on a *repurchase agreement* or "repo." Repurchase agreements are collateralized loans. They involve a commitment to sell and then later to buy back a specific bond issue (presumably at the maturity of the futures contract).[14] The agreement specifies the date on which the bond will be repurchased,[15] as well as the interest rate that will be paid on the loan. The dollar interest paid on the loan is computed as

[14] Repurchase and reverse repurchase agreements are discussed at length in Stigum (1990).

[15] When the term of the loan is one day, it is called an *overnight repo*. Terms of greater than one day are *term repos*.

$$\text{Interest} = \text{Principal amount} \times \text{Repo rate} \times \frac{\text{Days repo is outstanding}}{360}.$$

Note that, under this arrangement, the lender has commited to consummating actions opposite the borrower, that is, the lender has entered into an agreement to buy and then later to sell the underlying bond. For this reason, the lender is said to have a *reverse repurchase agreement*, a "reverse repo" or, simply, a "reverse." Both the borrower and lender gain from these agreements. The borrower gets a lower rate than he might otherwise get at the bank, and the lender gets a higher rate than he might otherwise get on short-term, highly liquid investments.

Before maturity, as at maturity, the futures price is based on the price of the cheapest to deliver, and the cheapest to deliver is determined by finding the bond with the highest "cash and carry" portfolio profit,

$$\pi_{i,0} = F_0(CF_i) + AI_{i,T} + \sum_{t=0}^{T} C_{i,t}e^{r(T-t)} - (B_{i,0} + AI_{i,0})e^{rT}. \tag{8.25}$$

Again, the highest value of profit equals zero; otherwise arbitrage profits are possible. This identifies the cheapest-to-deliver issue. All other profits will be less than zero. In other words, the futures price will be less than in (8.24) for all bonds other than the cheapest to deliver.

Although (8.25) allows us to identify which bond is cheapest to deliver at time 0, there is no assurance that this bond will also be cheapest to deliver at time T. Since the identity of the cheapest to deliver at time T is uncertain, (8.24) does not hold as an equality, even for the bond that is currently cheapest to deliver. Indeed, the short has a valuable *quality option* that gives him the right to choose which bond to deliver at time T. Although the short may have entered a cash-and-carry position when bond i was cheapest to deliver, if, at maturity, bond j is cheapest, the short can profit by selling bond i, buying bond j, and then delivering bond j on his short futures commitment. Because this option to switch bonds is valuable, the investor doing cash-and-carry arbitrage is willing to sell futures at a price below the price specified by the cost of carry on the right-hand side of (8.24), that is,

$$F_0 < \frac{(B_{i,0} + AI_{i,0})e^{rT} - AI_{i,T} - \sum_{t=0}^{T} C_{i,t}e^{r(T-t)}}{CF_i}, \tag{8.26}$$

or, alternatively,

$$F_0 = \frac{(B_{i,0} + AI_{i,0})e^{rT} - AI_{i,T} - \sum_{t=0}^{T} C_{i,t}e^{r(T-t)}}{CF_i} - \text{Quality option}, \tag{8.27}$$

where bond i is the current cheapest to deliver. The value of the quality option embedded in the T-bond futures contract is estimated in Chapter 11.

The short futures also has a *timing option* that allows a choice about when during the contract month to deliver. The most valuable element in the timing option is called the *wildcard option.* In the delivery month, the futures price at which delivery is made is the settlement price established when the market closes at 2:00 PM. The short has until 8:00 PM to declare delivery. Obviously, if news arrives that justifies a decline in bond prices, the short will choose to make delivery at the already established settlement price.

8.8 HEDGING WITH INTEREST RATE FUTURES CONTRACTS

Short-Term, Long Hedge

Interest rate futures can be used to lock in forward interest rates. Suppose, for example, that on November 13, 1991, a company anticipates a cash inflow of $1,000,000 on March 16, 1992. The cash, when it is received, will be placed in a three-month certificate of deposit until summer when it will be used to partially finance a major capital expenditure that the firm plans. Suppose also that the company's financial analyst expects three-month CD rates to fall to a level of 4 percent by March, while the current implied three-month forward rate of the March 1992 Eurodollar futures, based on its reported price of 94.94, is 5.06 percent. What can the company do to lock in the higher rate of interest?

A simple solution to this problem is to buy the March 1992 Eurodollar futures contract at the reported price of 94.94. When the $1,000,000 is received on March 16, 1992, the price of the Eurodollar futures will be approximately the same as the spot Eurodollar rate since the futures is near expiration. Assume that our analyst is correct in her prediction, and the spot rate is 4 percent. When we sell the futures position, our profit is (9600 − 9494) basis points times $25, or $2,650. The Eurodollar deposit on March 16, 1992, is, therefore, $1,002,650, which, at a 4-percent rate implies a deposit balance of $1,012,787.91 on June 15, 1992. Thus, in spite of the fact that the nominal rate is lower, the earned interest income amounts to a nominal rate of 1.2788(360/91), or 5.06 percent, exactly the desired result. Note that it does not matter what the spot rate is on March 16, 1992—the 5.06-percent rate is locked in regardless.

Long-Term, Short Hedge

Earlier we developed the concept of modified duration to assess the interest rate risk of a bond. Recall that modified duration is an approximation for the percentage change in bond price with respect to a change in yield. From a fixed-income security portfolio risk management standpoint, it is useful to recognize that the duration of a portfolio of bonds or fixed-income securities is simply the market-value-weighted average of the durations of the constituent bonds. It is also useful to know that, in the absence of the options imbedded in the T-bond futures contract, the duration of the T-bond futures is approximately equal to the duration of the cheapest-to-deliver T-bond.

To develop a framework for using T-bond futures to manage the risk of a fixed-income security portfolio, define the following notation:

$D_P \equiv$ modified duration of fixed-income security portfolio P.
$D_F \equiv$ modified duration of T-bond futures contract.
$D^* \equiv$ modified desired duration exposure for fixed-income portfolio.
$P \equiv$ current market value of fixed-income portfolio.
$F \equiv$ current futures price.
$h \equiv$ optimal number of futures contracts to buy (sell).

Under this notation, the dollar change in the value of the unhedged fixed-income portfolio is $D_P P$ times the interest rate change. If we buy h futures contracts against this fixed-income investment, the dollar change in the overall portfolio is $(D_P P + hD_F F)$ times the interest rate change, which we equate to the dollar change in the hedged portfolio at the desired duration level, D^*P, that is,

$$D^*P = D_P P + hD_F F, \tag{8.28}$$

in order to determine the optimal hedge ratio. Rearranging to isolate h, we get

$$h = \frac{P(D^* - D_P)}{D_F F}. \tag{8.29}$$

EXAMPLE 8.5

Suppose that the cheapest-to-deliver bond (and hence the T-bond futures) has a duration of 12.50. Suppose also that the duration of the bond portfolio that we are managing is 10.00 and that the market value of the portfolio is \$50,000,000. If today's date is November 13, 1991, and we wish to hedge completely against movements in the level of long-term rates until the end of February 1992, how many March 1992 T-bond futures contracts should we sell?

Substituting the example values into equation (8.29), we get

$$h = \frac{50,000,000(0 - 10.00)}{12.50 \times .9921875 \times 100,000} = -403.15.$$

Asset Allocation

At its most basic level, portfolio management involves a decision concerning what types of assets should be purchased. For example, a fund manager might choose to invest 40 percent of the fund in stocks, 40 percent in bonds, and 20 percent in real estate. Deciding what proportion of fund wealth to place in each asset category is called the *asset allocation decision.*

Once the asset allocation decision is made and fund wealth is invested, dramatic changes to the allocation are usually avoided because the transaction costs of liquidating assets in one category and buying assets in another are excessive. Instead, fund managers use futures contracts to change the asset allocation indirectly.

To demonstrate, assume that a fund consists of S in stocks and B in bonds, for a total value of $V = S + B$. Now, suppose the fund manager wants to change the amount invested in long-term bonds from B to B^*. The bond portfolio has a modified duration of D_B. Rather than selling (buying) stocks to buy (sell) bonds, the portfolio manager can effectuate the change by buying (selling) T-bond futures contracts. She wants her bond portfolio to have income $D_B B^*$ if interest rates fall 1 percent. She plans on generating that amount with income from the current bond portfolio, $D_B B$, and income from a T-bond futures position, $hD_F F$, that is,

$$D_B B^* = D_B B + hD_F F. \tag{8.30}$$

Rearranging (8.30) to isolate h, we get

$$h = \frac{D_B(B^* - B)}{D_F F}. \tag{8.31}$$

Note that if the dollar investment in bonds is to be reduced (i.e., $B^* < B$), T-bonds futures contracts are sold, and, if the dollar investment in bonds is to be increased (i.e., $B^* > B$), T-bond futures contracts are purchased. Presumably, the reduction (increase) in bond investment is then transferred to stocks through buying (selling) stock index futures contracts.

EXAMPLE 8.6

A fund manager currently has $50,000,000 in a stock portfolio whose composition matches the S&P 500 and $50,000,000 in a bond portfolio whose modified duration is 12.00. Believing that stocks are going to do extraordinarily well over the next three months, the fund manager wants to take advantage of the impending stock market rise and to eliminate his interest rate risk exposure. Unfortunately, liquidating bonds and buying stocks is expensive, particularly if, at the end of the three months, the manager wants to return to his fifty-fifty portfolio mix. How can the fund manager use T-bond and S&P 500 futures to carry out his plans? Assume that the cheapest-to-deliver bond (and hence the T-bond futures) has a duration of 9.00 and that the price of a three-month T-bond futures contract is 96.00. Also, assume that the three-month S&P 500 futures is priced at 325.

First, with respect to eliminating the interest rate exposure, the number of T-bond futures to sell is 694.44:

$$h = \frac{12.00(0 - 50,000,000)}{9 \times .96 \times 100,000} = -694.44.$$

This action is tantamount to liquidating the bond investment. Second, to take a long position of $50,000,000 in stocks using the S&P 500 futures, the number of contracts to buy is

$$h = \frac{50,000,000}{325.00 \times 500} = 307.69.$$

8.9 SUMMARY

Following an introduction to the particular interest rate futures markets in the U.S., specific pricing and yield conventions governing the trading of fixed-income securities are discussed. The major focus of this chapter is interest rate risk management. To this end, the notions of modified duration and convexity are introduced and applied. Following that, the relation between short-term and long-term rates, that is, the term structure of spot interest rates, is presented. From the spot rates, forward rates of interest are derived, and the forward rates implied by cash and futures prices are compared. A detailed discussion of the specifics of T-bond futures contract delivery and pricing is provided. The chapter concludes with two interest rate risk management illustrations.