16 OTHER OPTION CONTRACTS

The last three chapters have focused on specific option contracts on stocks, stock indexes, and interest rate instruments. Where the valuation procedures of Chapters 10 and 11 did not directly apply to these specific option contracts, the procedures were modified. For example, we showed how the quadratic approximation could be used to price American-style index options and how the binomial method could be used to value options on assets with discrete cash flows during the option's life. In this chapter, we discuss options on currencies and on physical commodities. Since no new valuation procedures are needed for these contracts, sections 1 and 2 review only the nature of the exchange-traded options on currencies and physical commodities, and refer the reader to the appropriate valuation equations from the previous chapters. The third section discusses some exotic OTC options that are currently traded. Examples of these are options on options, options on the maximum and the minimum of two risky commodities, lookback options and barrier options. For these four types of options, we present valuation equations. Other types of exotic options include options on the average price of a commodity, deferred-start options, deferred-payment American options, and all-or-nothing options. For these instruments, we describe only the essence of the option contracts. The chapter concludes with a summary.

16.1 CURRENCY AND CURRENCY FUTURES OPTIONS

Option contracts on spot currencies have been actively traded on the Philadelphia Exchange since late 1982. Both European-style and American-style option contracts are traded, with the American-style contracts having the greatest trading volume and open interest. The most active contracts are for British pounds, German marks, Japanese yen, and Swiss francs, although options on other currencies also trade. Table 16.1 contains a listing of the currently active, exchange-traded currency option contracts. The spot currency must be delivered upon exercise of these options. The current spot prices are also reported in Table 16.1. Currency options expire on the Saturday before the third Wednesday of the contract month.

Valuation of Currency Options

The valuation of currency options is relatively straightforward. First, all of the arbitrage pricing principles developed in Chapter 10 apply. The cost-of-carry rate equals the difference between the domestic interest rate, r_d, and the foreign interest rate, r_f. Second, all of the valuation equations and approximations discussed in Chapter 11 and the trading strategies discussed in Chapter 12 also apply, assuming that the spot exchange rate has a lognormal price distribution at the option's expiration. For

TABLE 16.1 Foreign currency exchange rates and foreign currency options.

CURRENCY TRADING

EXCHANGE RATES

Wednesday, November 13, 1991

The New York foreign exchange selling rates below apply to trading among banks in amounts of $1 million and more, as quoted at 3 p.m. Eastern time by Bankers Trust Co.and other sources. Retail transactions provide fewer units of foreign currency per dollar.

	U.S. $ equiv.		Currency per U.S. $	
Country	Wed.	Tues.	Wed.	Tues.
Argentina (Austral) ...	.0001008	.0001008	9918.67	9918.67
Australia (Dollar)	.7860	.7870	1.2723	1.2706
Austria (Schilling)	.08681	.08681	11.52	11.52
Bahrain (Dinar)	2.6539	2.6539	.3768	.3768
Belgium (Franc)	.02966	.02966	33.72	33.72
Brazil (Cruzeiro)	.00144	.00146	694.71	685.60
Britain (Pound)	1.7730	1.7725	.5640	.5642
30-Day Forward	1.7648	1.7640	.5666	.5669
90-Day Forward	1.7504	1.7496	.5713	.5716
180-Day Forward	1.7299	1.7291	.5781	.5783
Canada (Dollar)	.8842	.8838	1.1310	1.1315
30-Day Forward	.8815	.8814	1.1344	1.1346
90-Day Forward	.8784	.8779	1.1384	1.1391
180-Day Forward	.8737	.8733	1.1445	1.1451
Chile (Peso)	.002844	.002780	351.56	359.65
China (Renminbi)	.185642	.185642	5.3867	5.3867
Colombia (Peso)	.001753	.001753	570.38	570.38
Denmark (Krone)	.1573	.1573	6.3570	6.3555
Ecuador (Sucre)				
Floating rate	.000966	.000966	1035.00	1035.00
Finland (Markka)	.24984	.24941	4.0025	4.0095
France (Franc)	.17881	.17879	5.5925	5.5930
30-Day Forward	.17813	.17808	5.6140	5.6156
90-Day Forward	.17690	.17685	5.6529	5.6545
180-Day Forward	.17510	.17504	5.7110	5.7130
Germany (Mark)	.6112	.6111	1.6362	1.6365
30-Day Forward	.6090	.6088	1.6421	1.6426
90-Day Forward	.6045	.6044	1.6543	1.6544
180-Day Forward	.5982	.5982	1.6717	1.6718
Greece (Drachma)	.005405	.005405	185.00	185.00
Hong Kong (Dollar) ...	.12884	.12884	7.7615	7.7615
India (Rupee)	.03880	.03880	25.77	25.77
Indonesia (Rupiah)	.0005056	.0005056	1978.00	1978.00
Ireland (Punt)	1.6330	1.6318	.6124	.6128
Israel (Shekel)	.4308	.4321	2.3215	2.3142
Italy (Lira)	.0008121	.0008117	1231.41	1232.01
Japan (Yen)	.007686	.007707	130.10	129.75
30-Day Forward	.007678	.007698	130.24	129.90
90-Day Forward	.007666	.007686	130.45	130.10
180-Day Forward	.007656	.007677	130.62	130.26
Jordan (Dinar)	1.4500	1.4500	.6897	.6897
Kuwait (Dinar)	3.4965	3.4965	.2860	.2860
Lebanon (Pound)	.001134	.001134	881.50	881.50
Malaysia (Ringgit)	.3650	.3647	2.7400	2.7420
Malta (Lira)	3.1250	3.1250	.3200	.3200
Mexico (Peso)				
Floating rate	.0003254	.0003254	3073.01	3073.01
Netherland (Guilder)	.5423	.5422	1.8440	1.8445
New Zealand (Dollar)	.5610	.5620	1.7825	1.7794
Norway (Krone)	.1558	.1558	6.4175	6.4185
Pakistan (Rupee)	.0405	.0405	24.72	24.72
Peru (New Sol)	1.0152	1.0051	.99	.99
Philippines (Peso)	.03839	.03839	26.05	26.05
Portugal (Escudo)	.007067	.007063	141.50	141.59
Saudi Arabia (Riyal) ..	.26663	.26663	3.7505	3.7505
Singapore (Dollar)	.5958	.5959	1.6785	1.6780
South Africa (Rand)				
Commercial rate	.3568	.3574	2.8023	2.7981
Financial rate	.3248	.3240	3.0790	3.0860
South Korea (Won)	.0013310	.0013310	751.30	751.30
Spain (Peseta)	.009723	.009699	102.85	103.10
Sweden (Krona)	.1673	.1672	5.9775	5.9815
Switzerland (Franc) ..	.6888	.6892	1.4517	1.4510
30-Day Forward	.6872	.6875	1.4552	1.4546
90-Day Forward	.6835	.6839	1.4631	1.4621
180-Day Forward	.6788	.6792	1.4732	1.4724
Taiwan (Dollar)	.038850	.037908	25.74	26.38
Thailand (Baht)	.03926	.03926	25.47	25.47
Turkey (Lira)	.0002044	.0002020	4892.01	4950.00
United Arab (Dirham)	.2723	.2723	3.6725	3.6725
Uruguay (New Peso)				
Financial	.000425	.000425	2352.94	2352.94
Venezuela (Bolivar)				
Floating rate	.01695	.01661	59.00	60.20
SDR	1.38023	1.38189	.72452	.72365
ECU	1.24952	1.25088	....	....

Special Drawing Rights (SDR) are based on exchange rates for the U.S., German, British, French and Japanese currencies. Source: International Monetary Fund.

European Currency Unit (ECU) is based on a basket of community currencies. Source: European Community Commission.

continued

TABLE 16.1

OPTIONS
PHILADELPHIA

Wednesday, November 13, 1991

Option & Underlying	Strike Price	Calls—Last Nov	Calls—Last Dec	Calls—Last Mar	Puts—Last Nov	Puts—Last Dec	Puts—Last Mar
50,000 Australian Dollars-cents per unit.							
ADollr.....	78	0.63	0.84	r	r	r	r
78.66	81	r	r	r	r	r	3.53
78.66	83	r	r	r	r	4.56	r
31,250 British Pounds-European Style.							
BPound..	150	r	r	23.80	r	r	r
177.12 ..	167½	r	r	r	r	r	2.20
177.12 ..	175	r	3.00	4.25	r	r	r
177.12 ..	177½	r	r	3.25	r	r	r
177.12 ..	187½	r	r	r	r	r	14.20
31,250 British Pounds-cents per unit.							
BPound..	167½	r	10.00	r	r	0.18	r
177.12 ..	172½	r	4.90	r	r	0.92	r
177.12 ..	175	2.20	2.95	r	0.27	1.70	r
177.12 ..	177½	0.63	2.10	r	1.05	r	r
177.12 ..	180	0.07	r	r	r	r	r
177.12 ..	182½	r	0.52	r	r	r	r
177.12 ..	190	s	r	r	s	13.77	r
50,000 Canadian Dollars-European Style.							
CDollar....	87½	r	r	r	r	0.30	r
88.23	88	r	r	r	0.08	r	r
88.23	88½	r	r	r	0.25	r	r
50,000 Canadian Dollars-cents per unit.							
CDollr.....	84½	r	r	r	r	r	0.12
88.23	85	r	r	r	r	r	0.17
88.23	86	r	r	r	r	r	0.41
88.23	87½	r	r	r	r	0.24	r
88.23	88	r	0.35	r	0.07	0.55	r
88.23	88½	r	0.20	r	0.62	r	r
88.23	89	r	0.13	0.34	0.70	r	r
88.23	89½	r	r	r	r	1.48	r
62,500 German Marks-cents per unit.							
DMark	56	r	r	r	r	0.02	r
61.03	57	r	4.05	r	r	r	0.51
61.03	58	r	r	r	r	0.10	0.81
61.03	58½	2.59	r	s	r	r	s
61.03	59	2.03	2.19	r	r	0.20	r
61.03	59½	r	r	s	r	0.29	s
61.03	60	r	r	r	0.04	0.44	r
61.03	60½	0.68	r	s	0.10	0.67	s
61.03	61	0.33	0.73	r	0.23	0.90	r
61.03	61½	0.15	0.53	s	r	r	s
61.03	62	0.06	0.46	r	r	r	r
61.03	62½	r	0.30	s	r	r	s
6,250,000 Japanese Yen-100ths of a cent per unit.							
JYen......	73	r	r	r	r	0.04	r
76.99	74	2.96	r	r	r	r	r
76.99	75	r	1.98	r	r	r	r
76.99	76	r	r	r	0.04	0.43	r
76.99	76½	r	r	s	0.13	0.62	s
76.99	77	0.22	r	r	0.34	0.86	r
76.99	77½	0.08	0.46	s	r	r	s
76.99	78	0.03	0.31	r	r	r	r
76.99	79	r	r	0.82	r	r	r
76.99	85	r	r	0.09	r	r	r
76.99	86	r	r	0.07	r	r	r
6,250,000 Japanese Yen-European Style.							
JYen......	74	r	r	r	r	r	0.63
76.99	76½	r	r	s	0.09	r	s
76.99	78	r	r	1.10	r	r	r
62,500 Swiss Francs-European Style.							
SFranc....	66	r	r	r	r	r	0.94
68.85	69	r	r	1.60	r	r	r
62,500 Swiss Francs-cents per unit.							
SFranc....	65	r	r	r	r	0.07	r
68.85	68	r	r	r	r	0.58	r
68.85	68½	r	r	s	0.17	0.79	s
68.85	69	0.24	r	r	0.25	1.08	r
68.85	78½	s	r	r	s	0.33	r

Total Call Vol 14,647 Call Open Int 476,994
Total Put Vol 17,077 Put Open Int 516,307

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European-style currency options, valuation equations (11.25) and (11.28) can be used, where $b = r_d - r_f$.[1]

For American-style currency options, the quadratic approximation from Chapter 13 is recommended. Again, the cost-of-carry rate is set equal to the difference between the domestic and foreign interest rates.

Option contracts on currency futures were developed in late 1984 and early 1985 by the Chicago Mercantile Exchange. The only such options available are American-style. Upon exercise, a long (call) or short (put) position in the futures is obtained. The expiration date of these options is the second Friday before the third Wednesday of the contract month. (The futures expires two business days before the third Wednesday.) Like the currency options, the most active currency futures options are on British pounds, German marks, Japanese yen, and Swiss francs. Table 16.2 contains current foreign currency futures and futures option contract prices.

The valuation of currency futures options is even more straightforward than the valuation of currency options. The cost-of-carry rate for any currency futures

[1] For an approach to foreign currency option valuation that permits interest rates to be stochastic, see Grabbe (1983).

TABLE 16.2 Foreign currency futures and futures options.

CURRENCY TRADING

FUTURES

JAPAN YEN (IMM)—12.5 million yen; $ per yen (.00)

	Open	High	Low	Settle	Change	Lifetime High	Lifetime Low	Open Interest
Dec	.7691	.7699	.7671	.7679	− .0012	.7770	.6997	69,869
Mr92	.7666	.7684	.7659	.7665	− .0011	.7737	.7000	3,572
June		....	....	.7659	− .0010	.7730	.7015	917
Sept		....	....	.7659	− .0010	.7710	.7265	599
Dec		....	....	.7662	− .0009	.7700	.7512	1,290

Est vol 19,740; vol Tues 19,486; open Int 76,247, +756.

DEUTSCHEMARK (IMM)—125,000 marks; $ per mark

	Open	High	Low	Settle	Change	Lifetime High	Lifetime Low	Open Interest
Dec	.6088	.6108	.6060	.6088	+ .0007	.6770	.5365	72,328
Mr92	.6012	.6045	.5998	.6024	+ .0007	.6065	.5353	6,380
June	.5965	.5970	.5960	.5963	+ .0007	.5985	.5322	715

Est vol 56,177; vol Tues 36,905; open Int 79,626, −1,188.

CANADIAN DOLLAR (IMM)—100,000 dlrs.; $ per Can $

	Open	High	Low	Settle	Change	Lifetime High	Lifetime Low	Open Interest
Dec	.8771	.8817	.8763	.8812	+ .0019	.8906	.8175	20,341
Mr92	.8720	.8769	.8713	.8767	+ .0020	.8857	.8253	4,840
June	.8675	.8725	.8675	.8725	+ .0018	.8820	.8330	734
Sept	.8630	.8685	.8630	.8685	+ .0016	.8774	.8348	105

Est vol 13,890; vol Tues 7,534; open Int 26,078, −782.

BRITISH POUND (IMM)—62,500 pds.; $ per pound

	Open	High	Low	Settle	Change	Lifetime High	Lifetime Low	Open Interest
Dec	1.7640	1.7696	1.7560	1.7650	+.0024	1.7900	1.5670	27,784
Mr92	1.7430	1.7490	1.7370	1.7436	+.0024	1.7570	1.5560	2,964

Est vol 13,723; vol Tues 7,681; open Int 30,780, −899.

SWISS FRANC (IMM)—125,000 francs; $ per franc

	Open	High	Low	Settle	Change	Lifetime High	Lifetime Low	Open Interest
Dec	.6877	.6905	.6849	.6869	+ .0002	.8090	.6235	29,074
Mr92	.6829	.6852	.6797	.6819	+ .0004	.6995	.6225	2,196
June	.6766	.6795	.6750	.6771	+ .0004	.6840	.6546	296

Est vol 23,401; vol Tues 16,538; open Int 31,566, −1,353.

AUSTRALIAN DOLLAR (IMM)—100,000 dlrs.; $ per A.$

	Open	High	Low	Settle	Change	Lifetime High	Lifetime Low	Open Interest
Dec	.7825	.7839	.7822	.7832	− .0006	.7960	.7380	1,204

Est vol 113; vol Tues 164; open Int 1,221, −246.

U.S. DOLLAR INDEX (FINEX)—500 times USDX

	Open	High	Low	Settle	Change	Lifetime High	Lifetime Low	Open Interest
Dec	89.04	89.40	88.78	88.99	− .11	98.96	88.47	5,019
Mr92	90.07	90.30	89.90	90.08	− .10	98.90	89.60	1,045

Est vol 1,896; vol Tues 2,675; open Int 6,090, −671.
The Index: High 88.86; Low 88.34; Close 88.54 −.08

OTHER FUTURES

Settlement prices of selected contracts. Volume and open interest of all contract months.

British Pound (MCE) 12,500 pounds; $ per pound
Dec 1.7650 +.0024; Est. vol. 120; Open Int. 422
Japanese Yen (MCE) 6.25 million yen; $ per yen (.00)
Dec .7679 −.0012; Est. vol. 240; Open Int. 353
Swiss Franc (MCE) 62,500 francs; $ per franc
Dec .6869 +.0002; Est. vol. 1,020; Open Int. 253
Deutschemark (MCE) 62,500 marks; $ per mark
Dec .6088 +.0007; Est. vol. 360; Open Int. 837
BP/DM Cross Rate (IMM) US $50,000 times BP/DM
Dec 2.8990 +.0005; Est. vol. 80; Open Int. 245
DM/JY Cross Rate (IMM) US $125,000 times DM/JY
Dec .7928 +.0022; Est. vol. 6; Open Int. 583

FINEX—Financial Instrument Exchange, a division of the New York Cotton Exchange. IMM—International Monetary Market at the Chicago Mercantile Exchange. MCE—MidAmerica Commodity Exchange.

FUTURES OPTIONS

JAPANESE YEN (IMM) 12,500,000 yen; cents per 100 yen

Strike	Calls—Settle			Puts—Settle		
Price	Dec-c	Jan-c	Mar-c	Dec-p	Jan-p	Mar-p
7550	1.50	1.68	....	0.21	0.54	....
7600	1.13	1.36	1.92	0.34	0.72	1.28
7650	0.81	1.09	....	0.52	0.94	....
7700	0.56	0.85	1.42	0.77	1.20	1.76
7750	0.39	0.66	....	....	....	....
7800	0.26	0.50	1.03	1.46	....	2.36

Est. vol. 9,179, Tues vol. 3,905 calls, 3,377 puts
Open Interest Tues 47,829 calls, 48,508 puts

DEUTSCHEMARK (IMM) 125,000 marks; cents per mark

Strike	Calls—Settle			Puts—Settle		
Price	Dec-c	Jan-c	Mar-c	Dec-p	Jan-p	Mar-p
6000	1.26	1.20	1.75	0.38	0.96	1.51
6050	0.94	0.97	....	0.56	1.22	....
6100	0.68	0.77	1.30	0.80	....	2.04
6150	0.48	0.60	....	1.10	....	....
6200	0.32	0.46	0.95	1.44	....	2.68
6250	0.21	....	....	1.83	....	....

Est. vol. 23,758, Tues vol. 8,623 calls, 10,510 puts
Open Interest Tues 79,104 calls, 103,796 puts

CANADIAN DOLLAR (IMM) 100,000 Can.$, cents per Can.$

Strike	Calls—Settle			Puts—Settle		
Price	Dec-c	Jan-c	Mar-c	Dec-p	Jan-p	Mar-p
8700	1.20	....	1.12	0.09	0.26	0.51
8750	0.78	0.60	0.84	0.16	0.46	0.70
8800	0.43	0.37	0.61	0.31	0.72	0.96
8850	0.21	0.21	0.43	0.59	1.06	1.26
8900	0.08	0.11	0.29	0.97	....	1.62
8950	0.02	....	0.18	....	....	....

Est. vol. 4,010, Tues vol. 913 calls, 1,284 puts
Open Interest Tues 13,838 calls, 17,860 puts

BRITISH POUND (IMM) 62,500 pounds; cents per pound

Strike	Calls—Settle			Puts—Settle		
Price	Dec-c	Jan-c	Mar-c	Dec-p	Jan-p	Mar-p
1725	4.64	....	5.54	0.66	....	3.68
1750	2.88	2.78	4.28	1.40	3.40	4.88
1775	1.60	1.80	3.24	2.60	4.92	6.30
1800	0.82	1.12	2.40	4.28	....	7.92
1825	0.36	0.68	1.74	6.34	....	....
1850	0.16	....	1.22	8.62	....	11.66

Est. vol. 1,706, Tues vol. 856 calls, 681 puts
Open Interest Tues 9,526 calls, 10,067 puts

SWISS FRANC (IMM) 125,000 francs; cents per franc

Strike	Calls—Settle			Puts—Settle		
Price	Dec-c	Jan-c	Mar-c	Dec-p	Jan-p	Mar-p
6750	1.55	....	....	0.35	0.91	....
6800	1.20	1.32	1.92	0.50	1.13	1.73
6850	0.91	1.08	....	0.72	1.39	....
6900	0.68	0.87	1.46	0.98	....	2.27
6950	0.49	0.70	....	1.28	....	....
7000	0.34	0.56	1.10	1.63	....	....

Est. vol. 1,746, Tues vol. 965 calls, 706 puts
Open Interest Tues 12,608 calls, 16,145 puts

U.S. DOLLAR INDEX (FINEX) 500 times Index

Strike	Calls—Settle			Puts—Settle		
Price	Dec-c	Jan-c	Feb-c	Dec-p	Jan-p	Feb-p
87	....	3.40	....	0.27	....	....
88	1.51	2.64	....	0.52	0.59	....
89	0.92	1.97	....	0.93	0.92	....
90	0.52	1.42	....	1.53	1.35	....
91	0.27	0.98	....	2.28	....	....
92	0.13	0.66	....	3.13	....	....

Est. vol. 570, Tues vol. 60 calls, 292 puts
Open Interest Tues 12,054 calls, 16,739 puts

OTHER FUTURES OPTIONS

Final or settlement prices of selected contracts. Volume and open interest are totals in all contract months.

Australian Dollar (IMM) $100,000; $ per $

Strike	Dec-c	Jan-c	Mar-c	Dec-p	Jan-p	Mar-p
7850	0.47	....	....	0.65	....	....

Est. vol. 3. Tues vol. 55. Op. Int. 1,668.

FINEX—Financial Instrument Exchange, a division of the New York Cotton Exchange. IMM-International Monetary Market at Chicago Mercantile Exchange. LIFFE-London International Financing Futures Exchange.

is zero, so we set $b = 0$ in all of the pricing relations of Chapters 10 and 11 and the trading strategies of Chapter 12.

Uses of Currency Options

Currency options are helpful in managing foreign exchange risk that arises in international trade or in the management of international investment portfolios. Currency options allow more flexible hedging of exchange risk than is possible with currency futures alone.

As an example, consider a U.S. importer of German machinery that costs DM500,000 and is to be delivered in March 1992. Payment in German marks is to be made upon delivery. At the March futures price of $0.6024 shown in Table 16.2, the dollar cost of the machinery will be $301,200. Of course, if the price of the German mark increases, the dollar cost of the machinery will increase. One way to hedge this exchange risk is to buy four futures contracts (each contract applies to DM125,000) or to enter into an appropriate forward contract with a bank. If the price of the D-mark does increase, the increased equivalent dollar cost of the machinery is offset by the profit on the futures position. On the other hand, if the price of the D-mark decreases, the lower dollar cost is offset by the loss on the futures position.

An alternative way to hedge against D-mark price increases is to buy March call options on the D-mark spot currency or on the D-mark futures. If the D-mark appreciates, the increase in the dollar cost of the machinery is offset by the profit on the call options. On the other hand, if the D-mark depreciates, the lower cost of the machinery is a pure gain. This type of hedge provides insurance against increases in the exchange rate without an offsetting penalty should the exchange rate drop. Naturally, the call premium reflects the value of this insurance. To reduce the premium cost, one might buy out-of-the-money D-mark futures options. For example, Table 16.2 indicates it would have been possible to buy call options on the D-mark futures with an exercise price of 0.61 at a cost of 1.30 cents per mark. Since each contract at the Chicago Mercantile Exchange is DM125,000, four contracts are necessary, and the total premium is $6,500. This option position would cap the total dollar cost at (0.61)(500,000) = $305,000, while retaining the possibility of gain if the D-mark should depreciate. Increases in the D-mark above 0.61 would be offset by profits on the futures option position.

Options can provide a useful hedge if there is uncertainty about the underlying import or export contract. For example, consider a U.S. company that bids a price of 350,000 pounds to install a computer system in Great Britain and suppose the British company has a month in which to accept or reject the bid. The U.S. company is concerned about a depreciation of the British pound, but if it sells futures to hedge the foreign exchange risk, and the bid is not accepted, the company is left with an open currency futures position that may have to be liquidated at a loss. An alternative hedge is for the U.S. company to buy put contracts on 350,000 British pounds. By purchasing puts, the company guarantees the price at which pounds can be sold if the bid is accepted. If the bid is rejected, the put option is not exercised and is sold. In effect, the U.S. company is using an option to hedge a contract that has an option feature. The U.S. company has given the British company the put option to sell 350,000 pounds to the U.S. company in return for the computer system. The U.S. company hedges that risk by buying a put.[2]

In addition to hedging import and export contracts, currency options are useful in international investment and portfolio management. Investment in a foreign country exposes a portfolio to exchange rate risk as well as the usual risk of capital losses. Currency options can be used to modify that risk. For example, an investor in Australian bonds could hedge principal and/or interest payments by purchasing puts on the Australian dollar. Over-the-counter options written by banks are frequently used to tailor such hedges to the needs of the investor, particularly when longer maturities are necessary and/or when a sequence of options is required (as when a stream of coupon payments is hedged). Some fixed-income securities are offered with imbedded currency options. For example, a bond might offer to pay interest and/or principal in either of two currencies at a fixed exchange rate, with the investor having the option to choose the currency. Complex or exotic options, which are discussed below, are often created to deal with currency risk. For example, a bond could offer to pay principal and interest in dollars or in two other currencies at fixed exchange rates established in the bond indenture. The holder of the bond thus has a dollar bond plus the option of choosing the most valuable of the three currencies in which payment may be received.

16.2 PHYSICAL COMMODITY FUTURES OPTIONS

Markets for option contracts on physical commodity futures became active in 1982 with the introduction of sugar futures options by the Coffee, Sugar, and Cocoa Exchange and of gold futures options by the Commodity Exchange. These option contracts are American-style and settle through delivery of a position in the underlying futures. The grain contracts trade predominantly on the Chicago Board of Trade; the livestock contracts on the Chicago Mercantile Exchange; oil and oil-related products on the New York Mercantile Exchange; and metals at the Commodity Exchange. Table 16.3 contains a listing of the currently active, exchange-traded commodity options. As with all futures option contracts, the valuation principles follow from Chapters 10 through 12 once the cost-of-carry rate is set to zero ($b = 0$).

16.3 EXOTIC OPTIONS

Exotic options are complex options that typically incorporate two or more option features. A compound option, for example, is considered an exotic option. It pro-

[2]The hedging uses of currency options in the kind of situation described here are also discussed in Giddy (1983) and in Feiger and Jacquillat (1979).

TABLE 16.3 Commodity futures options.

COMMODITY FUTURES OPTIONS

Wednesday, November 13, 1991.

—AGRICULTURAL—

CORN (CBT) 5,000 bu.; cents per bu.

Strike	Calls—Settle			Puts—Settle		
Price	Dec-c	Mar-c	May-c	Dec-p	Mar-p	May-p
230	16	25½	....	⅛	½	¾
240	6¼	17	24	⅝	2¼	2½
250	⅞	11¼	17⅛	5	5½	5
260	c4	7	12	14⅛	11⅝	9¾
270	c2	4⅛	8	24	18½	15¼
280	c1	2⅜	5¼	34	27	23

Est. vol. 8,000, Tues vol. 4,078 calls, 3,862 puts
Open interest Tues 112,688 calls, 73,185 puts

SOYBEANS (CBT) 5,000 bu.; cents per bu.

Strike	Calls—Settle			Puts—Settle		
Price	Jan-c	Mar-c	May-c	Jan-p	Mar-p	May-c
500	57½	65½	....	¼	1	2
525	33¼	43½	....	1½	3¾	5½
550	15⅞	26	35	8	11¾	13¼
575	6	15¼	23½	23½	25	25½
600	2⅜	9	16½	44½	43	41½
625	1	5¾	10¾	68¼	65	60½

Est. vol. 6,000, Tues vol. 4,563 calls, 1,671 puts
Open interest Tues 51,853 calls, 19,669 puts

SOYBEAN MEAL (CBT) 100 tons; $ per ton

Strike	Calls—Settle			Puts—Settle		
Price	Dec-c	Jan-c	Mar-c	Dec-p	Jan-p	Mar-c
170	10.90	9.30	....	.05	.50	2.35
175	5.90	5.55	6.35	.30	1.75	4.60
180	2.00	3.50	4.55	1.10	4.20	7.60
185	.40	1.70	3.25	4.40	7.75	11.20
190	c3	1.00	2.20	9.10	12.00	....
195	.05	.55	1.60	14.10	....	....

Est. vol. 300, Tues vol. 405 calls, 415 puts
Open interest Tues 6,814 calls, 6,286 puts

SOYBEAN OIL (CBT) 60,000 lbs.; cents per lb.

Strike	Calls—Settle			Puts—Settle		
Price	Dec-c	Jan-c	Mar-c	Dec-p	Jan-p	Mar-c
17	1.880	....	....	.005	....	....
18	.900	....	....	.020	.060	.220
19	.100	.400	....	.220	.470	.490
20	.010	.140	.450	1.130	1.200	1.180
21	.005	.050	.210	2.130	2.150	1.950
22	.005	.020	.130	6.130	3.100	2.850

Est. vol. 300, Tues vol. 147 calls, 58 puts
Open interest Tues 5,687 calls, 3,020 puts

WHEAT (CBT) 5,000 bu.; cents per bu.

Strike	Calls—Settle			Puts—Settle		
Price	Dec-c	Mar-c	May-c	Dec-p	Mar-p	May-p
330	22½	26¾	21½	⅛	4	13½
340	12⅝	19¾	16	½	7	18
350	4½	14¼	12	2	10¾	24
360	⅝	10	10	8¼	16¼	....
370	⅛	7	6½	17½	22¾	....
380	⅛	4¾	5¼	....	30½	....

Est. vol. 3,500, Tues vol. 2,186 calls, 2,861 puts
Open interest Tues 28,890 calls, 32,465 puts

WHEAT (KC) 5,000 bu.; cents per bu.

Strike	Calls—Settle			Puts—Settle		
Price	Dec-c	Mar-c	May-c	Dec-p	Mar-p	May-p
340	16	20½	16½	⅝	6	16¾
350	6	15	13½	1¼	10	....
360	1	11½	8¾	5¼	14¼	....
370	⅜	6½	6½	14¾	21½	....
380	....	4	5	24½	....	....
390	....	2½	3¼	....	....	....

Est. vol. 372, Tues vol. 168 calls, 1,030 puts
Open interest Tues 6,414 calls, 7,637 puts

COTTON (CTN) 50,000 lbs.; cents per lb.

Strike	Calls—Settle			Puts—Settle		
Price	Mar-c	May-c	Jl-c	Mar-p	May-p	Jly-p
57	....	....	....	1.25	1.50	....
58	2.90	....	....	1.60	1.90	....
59	....	3.25	....	2.02	2.25	....
60	1.90	2.75	....	2.60	2.75	....
61	1.50	2.30	....	3.20	3.30	3.50
62	1.15	1.95	2.55	3.85	3.85	....

Est. vol. 1,400; Tues vol. 1,065 calls; 534 puts
Open interest Tues; 17,101 calls; 13,990 puts

ORANGE JUICE (CTN) 15,000 lbs.; cents per lb.

Strike	Calls—Settle			Puts—Settle		
Price	Jan-c	Mar-c	My-c	Jan-p	Mar-p	May-p
165	13.25	18.05	....	1.60	7.50	11.10
170	9.65	16.40	....	3.00	9.75	13.25
175	6.65	13.85	....	....	12.20	15.50
180	4.35	11.70	....	....	....	....
185	1.95	9.65	....	....	....	....
190	1.55	....	....	....	....	....

Est. vol. 375; Tues vol. 98 calls; 116 puts
Open interest Tues; 4,002 calls; 5,065 puts

COFFEE (CSCE) 37,500 lbs.; cents per lb.

Strike	Calls—Settle			Puts—Settle		
Price	Mar-c	May-c	Jl-c	Mar-p	May-p	Jly-p
75	9.03	11.75	14.10	0.78	0.95	0.95
80	5.38	8.20	10.40	2.25	2.40	2.25
85	3.32	5.15	6.80	5.00	4.60	4.20
90	2.10	3.45	4.95	8.78	7.65	6.80
95	1.33	2.35	3.50	13.08	11.55	10.35
100	0.90	1.60	2.53	17.65	15.80	14.38

Est. vol. 745; Tues vol. 930 calls; 68 puts
Open interest Tues; 15,495 calls; puts

SUGAR—WORLD (CSCE) 112,000 lbs.; cents per lb.

Strike	Calls—Settle			Puts—Settle		
Price	Dec-c	Mar-c	My-c	Dec-p	Mar-p	May-p
7.50	0.83	0.98	....	0.06	0.21	....
8.00	0.42	0.68	0.75	0.16	0.40	0.51
8.50	0.19	0.45	....	0.43	0.67	....
9.00	0.07	0.30	0.41	0.79	1.00	1.11
9.50	0.03	0.20	....	1.26	1.43	....
10.00	0.02	0.12	0.20	1.75	1.85	1.94

Est. vol. 7,719; Tues vol. 1,431 calls; 498 puts
Open interest Tues; 73,733 calls; 31,856 puts

COCOA (CSCE) 10 metric tons; $ per ton

Strike	Calls—Settle			Puts—Settle		
Price	Mar-c	May-c	Jl-c	Mar-p	May-p	Jly-p
1100	174	214	257	10	14	23
1200	96	139	184	35	39	50
1300	50	84	121	90	84	91
1400	23	48	81	159	148	147
1500	12	29	54	248	229	220
1600	5	19	44	341	319	310

Est. vol. 408; Tues vol. 519 calls; 155 puts
Open interest Tues; 9,019 calls; 10,478 puts

—OIL—

CRUDE OIL (NYM) 1,000 bbls.; $ per bbl.

Strike	Calls—Settle			Puts—Settle		
Price	Jan-c	Feb-c	Mr-c	Jan-p	Feb-p	Mar-p
20	2.28	....	....	.06	.18	.31
21	1.38	1.33	1.25	.16	.37	.55
22	.63	.68	.69	.41	.72	.98
23	.23	.30	.35	1.01	1.33	1.62
24	.07	.13	.16	1.85	2.16	....
25	.03	.06	.07	....	....	....

Est. vol. 18,873; Tues vol. 10,016 calls; 23,569 puts
Open interest Tues; 104,742 calls; 130,839 puts

vides its holder with the right to buy another option. Options on the maximum of two (or more) risky commodities are also considered to be exotic options. With this option, the investor has the right to buy "the better of two commodities." Because exotic options are complex and are often tailored to the needs of the customer, they are available primarily in the OTC market.

TABLE 16.3 continued

HEATING OIL No.2 (NYM) 42,000 gal.; $ per gal.

Strike Price	Calls–Settle Jan-c	Feb-c	Mr-c	Puts–Settle Jan-p	Feb-p	Mar-p
62	.0478	.0454	.0301	.0030	.0116	.0255
64	.0323	.0329	.0215	.0075	.0190	.0369
66	.0198	.0219	.0160	.0150	.0280	.0514
68	.0115	.0160	.0115	.0267	.0421	....
70	.0070	.0110	.0085	.0422	.0571	.0839
72	.0042	.0077	.0065	.0594	....	....

Est. vol. 8,615; Tues vol. 5,373 calls; 4,349 puts
Open interest Tues; 57,423 calls; 19,720 puts

GASOLINE–Unleaded (NYM) 42,000 gal.; $ per gal.

Strike Price	Calls–Settle Jan-c	Feb-c	Mr-c	Puts–Settle Jan-p	Feb-p	Mar-p
58	.0344	.0386	....	.0030	.0060	.0065
60	.0194	.0246	.0366	.0080	.0120	.0120
62	.0095	.0145	.0246	.0181	.0219	.0200
64	.0045	.0080	.0160	.0331	.0354	.0314
66	.0025	.0043	.0100	.0511	.0517	.0454
68	.0015	.0022	.0062	....	....	....

Est. vol. 2,275; Tues vol. 4,287 calls; 3,129 puts
Open interest Tues; 26,183 calls; 12,417 puts

–LIVESTOCK–

CATTLE-FEEDER (CME) 44,000 lbs.; cents per lb.

Strike Price	Calls–Settle Nov-c	Jan-c	Mar-c	Puts–Settle Nov-p	Jan-p	Mar-p
80	4.00	3.70	2.92	0.00	0.70	1.60
82	2.00	2.20	1.87	0.02	1.20	2.45
84	0.30	1.17	0.85	0.30	2.17	3.52
86	0.02	0.52	0.40	2.02	3.50	5.07
88	0.00	0.17	0.25	4.00	5.17	....
90	0.00	0.05	0.10	6.00	7.00	....

Est. vol. 337, Tues vol. 92 calls, 187 puts
Open interest Tues 3,250 calls, 5,929 puts

CATTLE-LIVE (CME) 40,000 lbs.; cents per lb.

Strike Price	Calls–Settle Dec-c	Feb-c	Apr-c	Puts–Settle Dec-p	Feb-p	Apr-p
70	5.00	....	....	0.05	0.35	0.60
72	3.10	3.62	....	0.15	0.62	0.95
74	1.45	2.12	2.20	0.50	1.10	1.52
76	0.35	1.02	1.10	1.40	1.95	2.42
78	0.05	0.32	0.47	3.10	....	....
80	0.00	0.10	0.15	5.05	....	....

Est. vol. 2,527, Tues vol. 1,242 calls, 2,291 puts
Open interest Tues 17,961 calls, 31,004 puts

HOGS–LIVE (CME) 40,000 lbs.; cents per lb.

Strike Price	Calls–Settle Dec-c	Feb-c	Apr-c	Puts–Settle Dec-p	Feb-p	Apr-p
38	3.70	4.97	....	0.05	0.15	0.60
40	1.75	3.32	1.85	0.20	0.50	1.30
42	0.55	1.87	1.00	0.90	1.05	2.45
44	0.12	0.90	0.52	2.47	2.07	3.85
46	0.05	0.42	0.22	4.40	3.60	5.55
48	0.00	0.15	0.10	6.35	5.32	....

Est. vol. 252, Tues vol. 337 calls, 177 puts
Open interest Tues 5,053 calls, 2,941 puts

–METALS–

COPPER (CMX) 25,000 lbs.; cents per lb.

Strike Price	Calls–Last Mar-c	May-c	Jly-c	Puts–Last Mar-p	May-p	Jly-p
98	6.10	6.40	6.30	1.50	2.90	3.65
100	4.60	5.10	5.20	2.05	3.70	4.60
102	3.60	4.05	4.25	2.95	4.65	5.65
104	2.85	3.25	3.55	4.20	5.75	6.85
105	2.30	2.90	3.20	4.60	6.40	7.50
106	2.00	2.60	2.95	5.30	7.10	8.25

Est. vol. 260, Tues vol. 50 calls, 141 puts
Open interest Tues 2,616 calls, 2,041 puts

GOLD (CMX) 100 troy ounces; dollars per troy ounce

Strike Price	Calls–Last Jan-c	Feb-c	Apr-c	Puts–Last Jan-p	Feb-p	Apr-p
340	20.20	20.50	23.70	0.20	0.60	1.50
350	10.60	11.70	15.50	0.60	1.70	3.30
360	3.30	5.00	9.10	3.20	4.90	6.70
370	0.70	1.90	5.00	10.60	11.80	12.50
380	0.30	0.80	2.70	20.10	20.50	19.90
390	0.20	0.50	1.60	30.00	30.00	28.60

Est. vol. 4,000, Tues vol. 2,387 calls, 1,392 puts
Open interest Tues 49,979 calls, 18,110 puts

SILVER (CMX) 5,000 troy ounces; cents per troy ounce

Strike Price	Calls–Last Jan-c	Feb-c	Mar-c	Puts–Last Jan-p	Feb-p	Mar-p
350	60.7	61.0	61.7	0.1	0.4	1.0
375	36.2	37.0	39.1	0.5	1.8	3.4
400	14.4	17.5	21.0	3.7	7.0	10.3
425	3.5	7.0	21.0	3.7	7.0	10.3
450	1.1	3.0	4.8	40.3	42.0	44.1
475	0.7	1.50	2.7	64.8	65.0	67.0

Est. vol. 12,000, Tues vol. 2,739 calls, 603 puts
Open interest Tues 34,209 calls, 8,172 puts

OTHER FUTURES OPTIONS

Final or settlement prices of selected contracts. Volume and open interest are totals in all contract months.

Lumber (CME) 160,000 bd .ft., $ per 1,000 bd.ft.

Strike	Jan-c	Mar-c	May-c	Jan-p	Mar-p	May-p
210	6.70	....	....	5.20	....	....

Est. vol. 10. Tues vol. 0. Op. Int. 149.

Oats (CBT) 5,000 bu.; cents per bu.

Strike	Dec-c	Mar-c	May-c	Dec-p	Mar-p	May-p
130	½	....	....	1¾	3	....

Est. vol. 5. Tues vol. 2. Op. Int. 439.

Platinum (NYM) 50 troy oz.; $ per troy oz.

Strike	Jan-c	Feb-c	Mar-c	Jan-p	Feb-p	Mar-p
360	10.00	....	....	5.50	....	....

Est. vol. 76. Tues vol. 76. Op. Int. n.a..

Pork Bellies (CME) 40,000 lbs.; cents per lb.

Strike	Nov-c	Feb-c	Mar-c	Nov-p	Feb-p	Mar-p
40	....	2.60	3.05	0.75	2.92	3.42

Est. vol. 146. Tues vol. 494. Op. Int. 5,668.

Silver (CBT) 1,000 troy oz.; cents per troy oz.

Strike	Dec-c	Feb-c	Apr-c	Dec-p	Feb-p	Apr-p
400	4.0	....	23.0	1.5	....	....

Est. vol. 5. Tues vol. 3. Op. Int. 208.

Soybeans (MCE) 1,000 bu.; cents per bu.

Strike	Jan-c	Mar-c	May-c	Jan-p	Mar-p	May-p
550	15⅞	26	35	8	11¾	13¼

Est. vol. 150. Tues vol. 179. Op. Int. 3,694.

Wheat (MPLS) 5,000 bu.; cents per bu.

Strike	Dec-c	Mar-c	May-c	Dec-p	Mar-p	May-p
340	1½	12½	15	5	13	....

Est. vol. 5. Tues vol. 16. Op. Int. 712.

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In this section, we apply the lognormal price distribution mechanics used in Chapters 11 and 12 to price compound options (or, more commonly, "options on options"), options on the maximum or the minimum, lookback options, and barrier options. Illustrations are provided. Following these discussions, we describe some other types of exotic options that currently trade. Our list of exotic options is necessarily incomplete, since new option contracts are designed and traded almost every day. The descriptions included will give a flavor for the ingenuity of some current option contract designs.

Options on Options

Compound options or *options on options* fall in the category of exotic options. Call (put) options providing the right to buy (sell) call options (i.e., calls on calls or puts on calls) and call (put) options providing the right to buy (sell) put options (i.e., calls on puts or puts on puts) are the most common forms. To value these options on options,[3] we adopt the assumptions and notation used in Chapter 11. The critical assumptions are that the terminal commodity price distribution is lognormal and that the principles of risk-neutral valuation apply. The call and put options that we are valuing are assumed to be European-style with exercise prices, c_t^* and p_t^*, respectively, and with time to expiration t. The notation representing the right to buy a call at time t (i.e., a call on a call) is $c(c,t;c_t^*)$, the right to sell a call at time t (i.e., a put on a call) is $p(c,t;c_t^*)$, the right to buy a put at time t (i.e., a call on a put) is $c(p,t;p_t^*)$, and the right to sell a put at time t (i.e., a put on a put) is $p(p,t;p_t^*)$. The option received or delivered at expiration from the exercise of an option on an option has exercise price X and time to expiration T. The notation used to describe the underlying options is $c(S_t, T;X)$ and $p(S_t, T;X)$, respectively. Conditional upon knowing S_t, these European-style options can be valued using equations (11.25) and (11.28) from Chapter 11.

To demonstrate how to value a compound option, we use a call on a call. The first step in the risk-neutral valuation approach is to formulate the option's payoff contingencies. For the call on a call, the payoff contingencies at time t are

$$c_t = \begin{cases} c(S_t, T; X) - c_t^* & \text{if} \quad c_t > c_t^* \\ 0 & \text{if} \quad c_t \leq c_t^*. \end{cases} \tag{16.1}$$

That is, if the value of the call to be received at time t, $c(S_t, T;X)$, is greater than the exercise price, c_t^*, the call option holder will exercise his right to buy the call. If the value is less, he will let it expire worthless.

The second step involves restating the contingent payoffs in (16.1) in terms of the underlying commodity price at time t, S_t, in order to make the problem more

[3] The models presented in this section are based on the work of Geske (1979).

tractable mathematically. The commodity price above which the call option holder will choose to exercise his call at time t is given by

$$c(S_t^*, T; X) = c_t^*, \tag{16.2}$$

where $c(S_t, T; X)$ represents the European-style option valuation equation (11.25) evaluated at $S = S_t^*$. Note that the value of S_t^* may be solved iteratively in the same manner that we have computed critical commodity prices in earlier chapters.[4] With S_t^* known, the payoff contingencies expressed in (16.1) may be written as

$$c_t = \begin{cases} c(S_t, T; X) - c_t^* & \text{if} \quad S_t > S_t^* \\ 0 & \text{if} \quad S_t \leq S_t^*. \end{cases} \tag{16.3}$$

Call on Call. Under risk-neutral valuation, the value of a call on a call may be written as the present value of the expected terminal value of the option, where the discount rate is the riskless rate of interest, r:

$$c(c_t, t; c_t^*) = e^{-rt} E[c(S_t, T; X) - c_t^* | S_t > S_t^*] \text{Prob}(S_t > S_t^*). \tag{16.4}$$

Expressing $c(S_t, T; X)$ in terms of its terminal commodity price payoffs and isolating the cost of exercising the option at time t, equation (16.4) becomes

$$\begin{aligned} c(c_t, t; c_t^*) &= e^{-r(t+T)} E(S_T | S_T > X \text{ and } S_t > S_t^*) \text{Prob}(S_T > X \text{ and } S_t > S_t^*) \\ &\quad - e^{-r(t+T)} X \text{Prob}(S_T > X \text{ and } S_t > S_t^*) \\ &\quad - e^{-rt} c_t^* \text{Prob}(S_t > S_t^*). \end{aligned} \tag{16.5}$$

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[4] See, for example, the valuation of American-style call options on dividend-paying stocks in Chapter 13 or the valuation of American-style options using the quadratic approximation method in Chapter 14.

Under the assumption that future commodity prices are lognormally distributed, the *value of a European-style call on a call* is

$$\begin{aligned} c(c_t, t; c_t^*) &= Se^{(b-r)(t+T)} N_2(a_1, b_1; \sqrt{t/(t+T)}) \\ &\quad - Xe^{-r(t+T)} N_2(a_2, b_2; \sqrt{t/(t+T)}) \\ &\quad - e^{-rt} c_t^* N_1(b_2), \end{aligned} \tag{16.6}$$

where

$$a_1 = \frac{\ln(S/X) + (b + .5\sigma^2)(t+T)}{\sigma\sqrt{t+T}}, \quad a_2 = a_1 - \sigma\sqrt{t+T},$$

$$b_1 = \frac{\ln(S/S_t^*) + (b + .5\sigma^2)t}{\sigma\sqrt{t}}, \quad b_2 = b_1 - \sigma\sqrt{t},$$

and $N_1(\cdot)$ and $N_2(\cdot)$, are the cumulative univariate and bivariate unit normal density functions described in Chapters 11 and 13, respectively.

In equation (16.6), the term $N_2(a_1, b_1; \sqrt{t/(t + T)})$ is the delta value of the call option on a call option. It describes the call option price movement for a small change in the commodity price. Recall that in Chapter 12 we showed how delta values are used for hedging purposes. The term $N_1(b_2)$ is the probability that the commodity price will exceed the critical commodity price at time t. The term $N_2(a_2, b_2; \sqrt{t/(t + T)})$ is the probability that the commodity price will exceed S_t^* at time t and the exercise price X at time $t + T$.

Put on Call. The simplest way to derive the valuation equation for a put on a call is to deduce the valuation formula from known results. In Chapter 12, we showed that a long-call/short-commodity position is tantamount to a long-put position. Here, the underlying commodity position is a call option, so a long-call-on-a-call/short-call position should be tantamount to a put on a call. Since we have the valuation equation for a call on a call (16.6) and for a European-call (11.25), the *valuation equation for a put on a call* is

$$\begin{aligned} p(c_t, t; c_t^*) &= Se^{(b-r)(t+T)} N_2(a_1, b_1; \sqrt{t/(t+T)}) \\ &\quad - Xe^{-r(t+T)} N_2(a_2, b_2; \sqrt{t/(t+T)}) - e^{-rt} c_t^* N_1(b_2) \\ &\quad - Se^{(b-r)(t+T)} N_1(a_1) + Xe^{-r(t+T)} N_1(a_2) + e^{-rt} c_t^* \\ &= Xe^{-r(t+T)} N_2(a_2, -b_2; -\sqrt{t/(t+T)}) \\ &\quad - Se^{(b-r)(t+T)} N_2(a_1, -b_1; -\sqrt{t/(t+T)}) \\ &\quad + e^{-rt} c_t^* N_1(-b_2), \end{aligned} \tag{16.7}$$

where all notation is defined in (16.6).

Call on Put. The risk-neutral valuation framework shown above can also be applied to value a call on a put. The value of a *European-style call on a put* is

$$\begin{aligned} c(p_t, t; p_t^*) = & Xe^{-r(t+T)} N_2(-a_2, -b_2; \sqrt{t/(t+T)}) \\ & - Se^{(b-r)(t+T)} N_2(-a_1, -b_1; \sqrt{t/(t+T)}) \\ & - e^{-rt} p_t^* N_1(-b_2). \end{aligned} \tag{16.8}$$

The critical commodity price below which the call option holder will choose to exercise the call to buy the put at time t is determined by solving

$$p(S_t^*, T; X) = p_t^*. \tag{16.9}$$

$p(S_t, T; X)$ represents the European-style option valuation equation (11.28) evaluated at $S = S_t^*$. All other notation is as previously defined. The term $N_2(-a_1, -b_1; \sqrt{t/(t+T)})$ is the delta value of a call option on a put option delta value. The term $N_1(-b_2)$ is the probability that the commodity price will be below the critical commodity price at time t. The term $N_2(-a_2, -b_2; \sqrt{t/(t+T)})$ is the probability that the commodity price will be below S_t^* at time t and the exercise price X at time $t + T$.

Put on Put. A put on a put has the same payoff contingencies as a long-call on-a-put/short-put position. Using equations (11.28) and (16.8), it can be shown that the *value of a put on a put* is

$$\begin{aligned} p(p_t, t; p_t^*) = & Se^{(b-r)(t+T)} N_2(-a_1, b_1; -\sqrt{t/(t+T)}) \\ & - Xe^{-r(t+T)} N_2(-a_2, b_2; -\sqrt{t/(t+T)}) \\ & + e^{-rt} p_t^* N_1(b_2), \end{aligned} \tag{16.10}$$

where all notation is defined above.

EXAMPLE 16.1

Consider a call option that provides its holder with the right to buy a put option on the S&P 500 index portfolio. The put that would be delivered against the call if the call is exercised has an exercise price of $400 and a time to expiration of six months. The call has an exercise price of $10 and a time to expiration of three months. The S&P 500 index is currently at 390, pays dividends at a constant rate of 4 percent annually, and has a volatility rate of 28 percent. The riskless rate of interest is 7 percent.

The first step in valuating the compound option is to compute the critical commodity price below which the call will be exercised to take delivery of the put. This is done by solving

$$p(S_t^*, .5; 400) = 10.00.$$

The critical commodity price, S_t^*, is 497.814. The next step is to apply the valuation formula (16.8). Here, we get

$$\begin{aligned} c = & 400e^{-.07(.25+.5)}N_2(-a_2, -b_2; \sqrt{.25/.75}) \\ & - 390e^{.03(.25+.5)}N_2(-a_1, -b_1; \sqrt{.25/.75}) \\ & - 10e^{-.07(.25)}N_1(-b_2) = 27.722, \end{aligned}$$

where

$$a_1 = \frac{\ln(390/400) + [.03 + .5(.28)^2](.75)}{.28\sqrt{.75}} = .1096,$$

$$a_2 = .1096 - .28\sqrt{.75} = -.1329,$$

$$b_1 = \frac{\ln(390/497.814) + [.03 + .5(.28)^2](.25)}{.28\sqrt{.25}} = -1.620,$$

$$b_2 = -1.620 - .28\sqrt{.25} = -1.760.$$

The probability that the commodity price will be below the critical commodity price at time t, $N_1(-b_2)$, is .961. The probability that the commodity price will be below S_t^* at time t and below the exercise price, X, at time $t + T$, $N_2(a_2, b_2; \sqrt{t/(t + T)})$, is .453. The value of a call on a call with the same terms as the put is 27.012. (The critical index price is 342.424.)

Options on the Maximum and the Minimum

Options on the maximum and the minimum of two or more risky commodities are popular exotic options.[5] For example, someone may buy the right to buy the S&P 500 index or gold for $400, depending on which commodity is worth more at the option's expiration. As in the case of compound options, options on the maximum and the minimum can be valued straightforwardly, assuming that both commodity prices have lognormal price distributions at the option's expiration. Under the

[5]Other names for the option on the maximum are "the better of two assets" or "outperformance options." The models presented here are on the maximum or the minimum of two risky commodities and the valuation models are based on Stulz (1982). To generalize these models to three or more risky assets, see Johnson (1987).

risk-neutral valuation approach, the value of a call option on the maximum, for example, may be written as

$$\begin{aligned} c_{\max}(S_1, S_2; X) = \\ & e^{-rT} E(\tilde{S}_{1,T} | S_{1,T} > X \text{ and } S_{1,T} > S_{2,T}) \text{Prob}(S_{1,T} > X \text{ and } S_{1,T} > S_{2,T}) \\ & + e^{-rT} E(\tilde{S}_{2,T} | S_{2,T} > X \text{ and } S_{2,T} > S_{1,T}) \text{Prob}(S_{2,T} > X \text{ and } S_{2,T} > S_{1,T}) \\ & - X e^{-rT} \text{Prob}(S_{1,T} > X \text{ or } S_{2,T} > X). \end{aligned} \quad (16.11)$$

Under the assumption that future commodity prices are lognormally distributed, the *value of a European-style call on the maximum* is

$$\begin{aligned} c_{\max}(S_1, S_2; X) = \\ & S_1 e^{(b_1 - r)T} N_2(d_{11}, d_1'; \rho_1') + S_2 e^{(b_2 - r)T} N_2(d_{12}, d_2'; \rho_2') \\ & - X e^{-rT}[1 - N_2(-d_{21}, -d_{22}; \rho_{12})], \end{aligned} \quad (16.12)$$

where

$$d_{11} = \frac{\ln(S_1/X_1) + (b_1 + .5\sigma_1^2)T}{\sigma_1\sqrt{T}}, \quad d_{21} = d_{11} - \sigma_1\sqrt{T},$$

$$d_{12} = \frac{\ln(S_2/X_2) + (b_2 + .5\sigma_2^2)T}{\sigma_2\sqrt{T}}, \quad d_{22} = d_{12} - \sigma_2\sqrt{T},$$

$$d_1' = \frac{\ln(S_1/S_2) + (b_1 - b_2 + .5\sigma^2)T}{\sigma\sqrt{T}}, \quad d_2' = -(d_1' - \sigma\sqrt{T}),$$

$$\sigma^2 = \sigma_1^2 + \sigma_2^2 - 2\rho_{12}\sigma_1\sigma_2, \quad \rho_1' = \frac{\sigma_1 - \rho_{12}\sigma_2}{\sigma}, \quad \text{and} \quad \rho_2' = \frac{\sigma_2 - \rho_{12}\sigma_1}{\sigma}.$$

In equation (16.12), the term $1 - N_2(-d_{21}, -d_{22}; \rho_{12})$ is the probability that one of the two commodity prices will exceed the exercise price at time T or, alternatively, one minus the probability that neither commodity will have a price greater than the exercise price at the option's expiration.

EXAMPLE 16.2

Consider a call option that provides its holder the right to buy \$100,000 worth of the S&P 500 index portfolio at an exercise price of \$400 or \$100,000 worth of a particular T-bond at an exercise price of \$100, whichever is worth more at the end of three months. The S&P 500 index is currently priced at \$360, pays dividends at a rate of 4 percent annually, and has a return volatility of 28 percent. The T-bond is currently priced at \$98, pays a coupon yield of 10 percent, and has a return volatility of 15 percent. The correlation between the rates of return of the S&P 500 and the T-bond is .5. The riskless rate of interest is 7 percent.

Before applying the option on the maximum formula, it is important to recognize that there are two exercise prices in this problem: \$400 for the S&P index portfolio and \$100 for the T-bond. What this implies is that we can buy \$100,000/\$400 = 250 "units" of the index portfolio or \$100,000/\$100 = 1,000 T-bond "units" at the end of three months, depending on which is worth more. At this juncture, we must decide whether to work with the valuation equation (16.12) in units of the S&P 500 index portfolio, in which case we multiply the current T-bond price and its exercise price by 4, and then multiply the computed option price by 250, or to work with the valuation equation (16.12) in units of the T-bond, in which case we divide the current S&P 500 price and the option's S&P 500 exercise price by 4, and then multiply the computed option price by 1,000.[6] In this exercise, we choose to work in units of the S&P 500 index portfolio, so we adjust the T-bond prices: the current T-bond price is assumed to be 392, and the T-bond exercise price is 400.

With the units of the two underlying assets comparable, we now apply equation (16.12):

$$\begin{aligned} c_{\max} = {} & 360e^{-.04(.25)}N_2(d_{11}, d_1'; \rho_1') + 392e^{-.10(.25)}N_2(d_{12}, -d_2'; \rho_2') \\ & - 400e^{-.07(.25)}[1 - N_2(-d_{21}, -d_{22}; .5)] = 11.962, \end{aligned}$$

where

$$d_{11} = \frac{\ln(360/400) + [.07 - .04 + .5(.28)^2](.25)}{.28\sqrt{.25}} = -.6290,$$

$$d_{21} = -.6290 - .28\sqrt{.25} = -.7690,$$

$$d_{12} = \frac{\ln(392/400) + [.07 - .10 + .5(.15)^2](.25)}{.15\sqrt{.25}} = -.3319,$$

$$d_{22} = -.3319 - .15\sqrt{.25} = -.4069,$$

$$d_1' = \frac{\ln(360/392) + [.06 + .5\sigma^2](.25)}{\sigma\sqrt{.25}} = -.5175,$$

$$d_2' = -(-.5175 - \sigma\sqrt{.25}) = .6388,$$

$$\sigma = \sqrt{.28^2 + .15^2 - 2(.5)(.28)(.15)} = .2427,$$

$$\rho_1' = \frac{.28 - .5(.15)}{.2427} = .8447,$$

$$\rho_2' = \frac{.15 - .5(.28)}{.2427} = .0412.$$

[6]These types of adjustments can be made freely because the option price is linearly homogeneous in both the commodity price and the exercise price. See Merton (1973).

The computed option price is 11.962, which implies the value of the option contract is \$11.962 × 250, or \$2,990.50. The probability that either or both components of the option are in-the-money at expiration is 1 − $N_2(.7690,.4069;.5)$, or 42.72 percent.

Under the same assumptions, the *value of a European-style call on the minimum* is

$$c_{\min}(S_1, S_2; X) = S_1 e^{(b_1-r)T} N_2(d_{11}, -d_1'; -\rho_1') + S_2 e^{(b_2-r)T} N_2(d_{12}, -d_2'; -\rho_2') - X e^{-rT} N_2(d_{21}, d_{22}; \rho_{12}), \tag{16.13}$$

where all notation is as previously defined.

Lookback Options

Aside from compound options and options on the maximum and the minimum, many other exotic options trade in OTC markets. Some of the options are backward looking. A *lookback call option* provides its holder with settlement proceeds equal to the difference between the highest commodity price during the life of the option less the exercise price, and a *lookback put option* provides its holder with settlement proceeds equal to the difference between the exercise price and the lowest commodity price during the life of the option. It should come as no surprise, therefore, that these options are sometimes referred to as "no-regret options."

In a sense, lookback options are like American-style options because the option holder is guaranteed the most advantageous exercise price. Lookback call options can be valued analytically using the risk-neutral valuation mechanics.[7] The reason for this is that it never pays to exercise a lookback option prior to expiration. Independent of how low the commodity price has been thus far during the option's life, there is always some positive probability that it will fall further. For this reason, the option holder will always defer early exercise in the hope of recognizing higher exercise proceeds in the future.

Under the assumptions of risk-neutral valuation and lognormally distributed future commodity prices, the *value of a lookback call* may be written as

$$c_{LB} = S e^{(b-r)T} N_1(d_1) - X e^{-rT} N_1(d_2) + S e^{(b-r)T} \lambda \left[e^{-b[T + \frac{2\ln(S/X)}{\sigma^2}]} N_1(d_3) - N_1(-d_1) \right], \tag{16.14}$$

where X is the current minimum price of the commodity during the life of the option, $\lambda = .5\sigma^2/b$, $d_1 = \dfrac{\ln(S/X) + (b + .5\sigma^2)T}{\sigma\sqrt{T}}$, $d_2 = d_1 - \sigma\sqrt{T}$ and $d_3 = \dfrac{(b - .5\sigma^2)T}{\sigma\sqrt{T}}$. Note that the first two terms of the option are the value of a European-

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[7]The pricing equations provided here are based on the work of Goldman, Sosin, and Gatto (1979).

style call option whose exercise price is the current minimum value of the underlying commodity. This is the least the lookback call can be worth since the commodity price may fall below X, thereby driving the "exercise price" down further.

EXAMPLE 16.3

Consider a lookback call option that provides its holder with the right to buy the S&P 500 index at any time during the next three months. The S&P 500 index is currently at a level of 390, pays dividends at a constant rate of 4 percent annually, and has a volatility rate of 28 percent. The riskless rate of interest is 7 percent.

The cost-of-carry rate is .07 − .04 = .03. The value of the lookback call is, therefore,

$$\begin{aligned} c_{\text{LB}} = & 390e^{(.03-.07).25}N_1(d_1) - 390e^{-.07(.25)}N_1(d_2) \\ & + 390e^{(.03-.07).25}\lambda\left[e^{-.03[.25+\frac{2\ln(390/390)}{.28^2}]}N_1(d_3) - N_1(-d_1)\right] = 42.583, \end{aligned}$$

where

$$\lambda = .5(.28)^2/.03 = 1.3067,$$

$$d_1 = \frac{\ln(390/390) + [.03 + .5(.28)^2].25}{.28\sqrt{.25}} = .1236,$$

$$d_2 = .1236 - .28\sqrt{.25} = -.0164,$$

and

$$d_3 = \frac{[.03 - .5(.28)^2].25}{.28\sqrt{.25}} = -.0164.$$

Note that the price of the lookback call is considerably higher than an at-the-money index call option. The value of a European-style call (i.e., the sum of the first two terms in the valuation equation) is only 22.941.

The *value of a European-style lookback put option* is

$$\begin{aligned} p_{\text{LB}} = & Xe^{-rT}N_1(-d_2) - Se^{(b-r)T}N_1(-d_1) \\ & + Se^{(b-r)T}\lambda\left[N_1(d_1) - e^{-b[T+\frac{2\ln(S/X)}{\sigma^2}}N_1(-d_3)]\right], \end{aligned} \quad \textbf{(16.15)}$$

where all notation is as defined for the lookback call. Note that a standard European-style put option is the lower bound for the price of the lookback put option. The third term is necessarily positive. Using the same parameters as in Example

16.3, the value of a lookback put option is \$43.468, with the underlying ordinary European-style put being valued at \$20.056.

Other backward-looking options are also traded. For example, *average price* or *Asian options* are based on the average (either arithmetic or geometric) commodity price during the option's life. The average commodity price may be used as the exercise price of the option, in which case the settlement value of the call will be the terminal commodity price less the average price, or it may be used as the terminal commodity price, in which case the settlement value will be the average price less the exercise price. Unfortunately, most Asian options do not have closed-form valuation equations. Accurate pricing involves the use of numerical methods.[8]

Barrier Options

Barrier options are options whose existence depends on the underlying commodity price. A *down-and-out call,* for example, is a call that expires if the commodity price falls below a prespecified "out" barrier, *H*.[9] At that time, the option buyer may receive a cash rebate, *R*. A *down-and-in call* is a call that comes into existence if the commodity price falls below the "in" barrier at any time during the option's life. Note that if we buy a down-and-out call and a down-and-in call with the same barrier price, *H*, exercise price, *X*, and time to expiration, *T*, the portfolio has the same payoff contingencies as a standard call option. For this reason, we automatically know how to value a down-and-in call if we can value a down-and-out call.

Under the assumptions of risk-neutral valuation and lognormally distributed commodity prices, the valuation equation for a down-and-out call option is

$$\begin{aligned} c_{\text{DO}} =& Se^{(b-r)T}N_1(a_1) - Xe^{-rT}N_1(b_2) \\ &- Se^{(b-r)T}(H/S)^{2(\eta+1)}N_1(b_1) + Xe^{-rT}(H/S)^{2\eta}N_1(b_2) \\ &+ R(H/S)^{\eta+\gamma}N_1(c_1) + R(H/S)^{\eta-\gamma}N_1(c_2), \end{aligned} \qquad \textbf{(16.16)}$$

H is the barrier commodity price below which the call option life ends; *R* is the rebate, if any, received by the option buyer should the option terminate,

$$\eta = \frac{b}{\sigma^2} - 1/2, \quad \gamma = \sqrt{\eta^2 + \frac{2r}{\sigma^2}},$$

$$a_1 = \frac{\ln(S/X)}{\sigma\sqrt{T}} + (1+\eta)\sigma\sqrt{T}, \quad a_2 = a_1 - \sigma\sqrt{T},$$

$$b_1 = \frac{\ln(H^2/SX)}{\sigma\sqrt{T}} + (1+\eta)\sigma\sqrt{T}, \quad b_2 = b_1 - \sigma\sqrt{T},$$

$$c_1 = \frac{\ln(H/S)}{\sigma\sqrt{T}} + \gamma\sigma\sqrt{T}, \quad \text{and} \quad c_2 = c_1 - 2\gamma\sigma\sqrt{T}.$$

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[8]There are a number of useful background readings for those interested in pricing Asian options. Among them are Boyle (1977) and Boyle and Emanuel (1985).

[9]The valuation equation for the down-and-out call option was first provided in Cox and Rubinstein (1985, Ch. 7). The valuation equation presented here is a modified version of the formula presented in Rubinstein (1990).

The valuation equation for a down-and-in call is simply equation (11.25) less (16.16).

EXAMPLE 16.4

Consider a down-and-in call option that provides its holder with the right to buy the S&P 500 index at 380 any time during the next three months, should the index level fall below 375. The S&P 500 index is currently at a level of 390, pays dividends at a constant rate of 4 percent annually, and has a volatility rate of 28 percent. The riskless rate of interest is 7 percent.

The cost-of-carry rate is .07 − .04 = .03. The value of the down-and-out call is

$$\begin{aligned} c_{DO} =& 390e^{(.03-.07).25}N_1(a_1) - 380e^{-.07(.25)}N_1(b_2) \\ & - 390e^{(.03-.07).25}(375/390)^{2(\eta+1)}N_1(b_1) \\ & + 380e^{-.07(.25)}(375/390)^{2\eta}N_1(b_2) = 14.817, \end{aligned}$$

where

$$\eta = \frac{.03}{.28^2} - 1/2 = -.1173,$$

$$\gamma = \sqrt{.1173^2 + \frac{2(.07)}{.28^2}} = 1.3414,$$

$$a_1 = \frac{\ln(390/380)}{.28\sqrt{.25}} + (1 - .1173).28\sqrt{.25} = .3091,$$

$$a_2 = a_1 - .28\sqrt{.25} = .1691,$$

$$b_1 = \frac{\ln[375^2/(390 \times 380)]}{.28\sqrt{.25}} + .28(1 - .1173)\sqrt{.25} = -.2512,$$

$$b_2 = b_1 - .28\sqrt{.25} = -.3912,$$

$$c_1 = \frac{\ln(375/390)}{.28\sqrt{.25}} + .28(1.3414)\sqrt{.25} = -.0924,$$

and

$$c_2 = c_1 - .56(1.3414)\sqrt{.25} = -.4680.$$

The value of a standard European-style call option is 28.151, using equation (11.25). The value of the down-and-in call is, therefore, 28.151 − 14.817 = 13.334.

An *up-and-out put* and an *up-and-in put* can be valued in a similar manner. An up-and-out put is a put that expires if the commodity price rises above the "out" barrier. Its valuation equation is

$$\begin{aligned}p_{UO} =& Xe^{-rT}N_1(-b_2) - Se^{(b-r)T}N_1(-a_1) \\ &+ Se^{(b-r)T}(H/S)^{2(\eta+1)}N_1(-b_1) - Xe^{-rT}(H/S)^{2\eta}N_1(-b_2) \\ &- R(H/S)^{\eta+\gamma}N_1(-c_1) - R(H/S)^{\eta-\gamma}N_1(-c_2). \qquad \textbf{(16.17)}\end{aligned}$$

An up-and-in put comes into existence when the commodity price rises above H. Its valuation equation is simply (11.28) less (16.17).

Other Exotic Options

Exotic options abound.[10] Among those not yet mentioned are those involving deferred features. A *deferred-start option*, for example, is an option which is purchased before its life actually begins. A *deferred payment American option* is like a standard American-style option except, if the option is exercised early, the option buyer does not receive the exercise proceeds until the end of the option's life. Yet others involve lump sum payoffs. An *all-or-nothing call (put) option*, for example, pays a predetermined amount (i.e., the "all") should the underlying commodity price be above (below) the exercise price at the option's expiration. A *one-touch all-or-nothing call (put)* pays a predetermined amount if the commodity price touches the exercise price at any time during the option's life.

16.4 SUMMARY

This chapter concludes the presentation of option valuation principles and applications. First, we discussed currency and currency futures options. Contract specifications were provided, and we noted that the valuation of these options is a straightforward application of the constant cost-of-carry framework developed in Chapters 10 and 11. The cost-of-carry rate for currency options is the domestic rate of interest less the foreign rate of interest, and the cost-of-carry rate for currency futures options is zero. We discussed, as well, the use of currency options in hedging the currency risk that arises in international trade or investment. Second, we discussed physical commodity futures options. In general, no options on physical commodities trade, only options on physical commodity futures. Hence, the valuation principles for these options also follow straightforwardly from the constant cost-of-carry framework of the earlier chapters. The cost-of-carry rate for physical commodity futures is zero.

[10]For a brief review of a range of exotic options, see Hudson (1991).

The remainder of the chapter focuses on exotic options. These are not exchange-traded options but are unusual options that trade in OTC markets. We show how options on options, options on the maximum and the minimum of two commodities, lookback options, and barrier options may be valued within a log-normal price distribution framework. But, these are only four of a myriad of option contract designs that exist in the OTC markets. We discuss others; however, the list is certainly incomplete given the pace with which these new contracts are introduced.