DER 04.3

Monte Carlo simulation

1

Monte Carlo simulation

- Context:
  - Monte Carlo simulation is single most powerful financial decision-making tool.
    - Involves repeated random sampling.

2

Monte Carlo simulation

- Purpose:
  - Describe and apply Monte Carlo simulation for financial decision-making.
    - Standard financial analysis uses single number (i.e., expected value) to represent uncertain outcome.
      - E.g., in corporate finance, NPV discounts expected net cash flows.
    - Results are static (or deterministic).
      - Ignore range of possible outcomes.

3

Ad hoc solutions

- Scenario analysis:
  - Compute three outcomes using input values:
    - Most-likely (e.g., mean)
    - Best-case
    - Worst-case
- What-if analysis:
  - Compute multiple outcomes by using even increments of inputs.
    - Provides range of outcomes but no associated probabilities.

4

Monte Carlo simulation

- Monte Carlo simulation captures range of outcomes and associated probabilities.
- Two popular Monte Carlo simulation Excel add-in packages.
  - @Risk – Most comprehensive. Most widely adopted in industry.
  - Crystal Ball – Slightly easier to use.

5

Capital budgeting decision

- Compute NPV of buying new equipment.
  - Equipment cost today is $500,000.
  - Annual net cash flows (NCFs)
    - Normally distributed (i.e., negative cash flows are possible).
    - Expected value of NCF is $140,000 per year for 5 years.
    - Standard deviation of NCF is $50,000.
  - Equipment sale in 5 years.
    - Lognormally distributed.
    - Expected sales price (SP) is $40,000.
    - Standard deviation of (SP) is $20,000.

6

Capital budgeting decision

- Compute NPV of buying new equipment.
  - Expected cost of capital is 12%.
    - Note: Marginal cost of capital is random variable.
  - Support file: MC NPV.xlsx

7

Capital budgeting decision

Evaluation of new equipment purchase

Net future cash flows	
Expected NCF	140,000
Standard deviation NCF	50,000
Number of years	5
Equipment cost today	-500,000
Expected sale price in 5 years	40,000
StDev of sale price	20,000
Firm	
Cost of capital	12.00%

Time line of NCF

		0	1	2	3	4	5
Net cash flow		-500,000	140,000	140,000	140,000	140,000	140,000
Equipment sale							40,000
Discounted cash flows (DCFs)		-500,000	125,000	111,607	99,649	88,973	102,137
NPV (Sum of DCFs)	27,366						

Show cash flows on timeline.

8

Capital budgeting decision

Evaluation of new equipment purchase

Net future cash flows	
Expected NCF	140,000
Standard deviation NCF	50,000
Number of years	5
Equipment cost today	-500,000
Expected sale price in 5 years	40,000
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Cost of capital	12.00%

Time line of NCF

		0	1	2	3	4	5
Net cash flow		-500,000	140,000	140,000	140,000	140,000	140,000
Equipment sale							40,000
Discounted cash flows (DCFs)		-500,000	125,000	111,607	99,649	88,973	102,137
NPV (Sum of DCFs)	27,366						

Discount cash flows to compute NPV.
Decision rule: Accept since NPV > 0.

9

Capital budgeting decision

- Problems with standard NPV approach.
  - Implicitly assumes net cash flows are deterministic.
    - Does not recognize net cash flows are uncertain.
  - To properly evaluate, must recognize risk.
    - Monte Carlo simulation is appropriate tool.

10

Capital budgeting decision

Evaluation of new equipment purchase	
Net future cash flows	
Expected NCF	140,000
Standard deviation NCF	50,000
Number of years	5
Equipment cost today	-500,000
Expected sale price in 5 years	40,000
StDev of sale price	20,000
Firm	
Cost of capital	12.00%

Assume NCFs are normally distributed with mean 140,000 and standard deviation 50,000.

Time line of NCF		0	1	2	3	4	5
Net cash flow		-500,000	140,000	140,000	140,000	140,000	140,000
Equipment sale							40,000
Discounted cash flows (DCFs)		-500,000	125,000	111,607	99,649	88,973	102,137
NPV (Sum of DCFs)	27,366						

11

Capital budgeting decision

Evaluation of new equipment purchase	
Net future cash flows	
Expected NCF	140,000
Standard deviation NCF	50,000
Number of years	5
Equipment cost today	-500,000
Expected sale price in 5 years	40,000
StDev of sale price	20,000
Firm	
Cost of capital	12.00%

Assume equipment will be sold in 5 years. Sale price is lognormally distributed with mean 40,000 and standard deviation 20,000.

Time line of NCF		0	1	2	3	4	5
Net cash flow		-500,000	140,000	140,000	140,000	140,000	140,000
Equipment sale							40,000
Discounted cash flows (DCFs)		-500,000	125,000	111,607	99,649	88,973	102,137
NPV (Sum of DCFs)	27,366						

12

Capital budgeting decision

- Use RiskNormal(Mean,StDev) function.

D16 `=@RiskNormal($B$3,$B$4)`

	A	B	C	D
1		Evaluation of new equipment pur		
2	*Net future cash flows*			
3	Expected NCF	140,000		
4	Standard deviation NCF	50,000		
5	Number of years	5		
6				
7	Equipment cost today	-500,000		
8	Expected sale price in 5 years	40,000		
9	StDev of sale price	20,000		
10				
11	*Firm*			
12	Cost of capital	12.00%		
13				
14				T
15			0	1
16	Net cash flow		-500,000	140,000
17	Equipment sale			

13

Capital budgeting decision

- Use RiskLognorm(Mean,StDev) function.

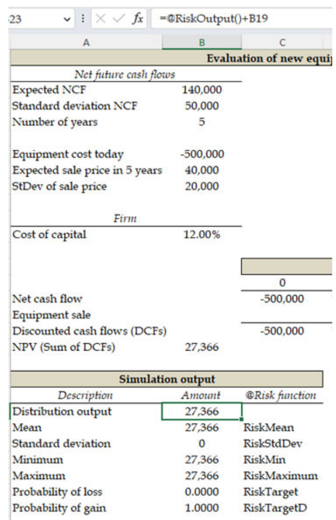
`=@RiskLognorm($B$8,$B$9)`

	B	C	D	E	F	G	H
	Evaluation of new equipment purchase						
flows							
	140,000						
	50,000						
	5						
	-500,000						
urs	40,000						
	20,000						
	12.00%						
		Time line of NCF					
		0	1	2	3	4	5
		-500,000	140,000	140,000	140,000	140,000	140,000
							40,000
CFs)		-500,000	125,000	111,607	99,649	88,973	102,137
	27,366						

14

Capital budgeting decision

- Define output variable.
  - @RiskOutput()+B19 plots distribution.



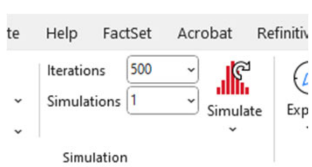
=@RiskOutput()+B19

A	B	C
	Evaluation of new equi	
Net future cash flows		
Expected NCF	140,000	
Standard deviation NCF	50,000	
Number of years	5	
Equipment cost today	-500,000	
Expected sale price in 5 years	40,000	
StDev of sale price	20,000	
Firm		
Cost of capital	12.00%	
		0
Net cash flow		-500,000
Equipment sale		
Discounted cash flows (DCFs)		-500,000
NPV (Sum of DCFs)	27,366	
Simulation output		
Description	*Amount*	*@Risk function*
Distribution output	27,366	
Mean	27,366	RiskMean
Standard deviation	0	RiskStdDev
Minimum	27,366	RiskMin
Maximum	27,366	RiskMaximum
Probability of loss	0.0000	RiskTarget
Probability of gain	1.0000	RiskTargetD

15

Capital budgeting decision

- Set Iterations (# of runs) and click Simulate.

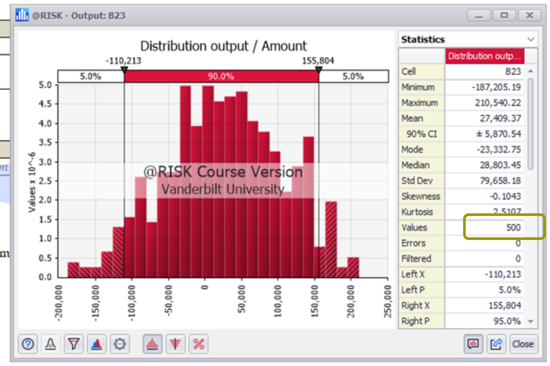


16

Capital budgeting decision

Firm		
Cost of capital	12.00%	
		0
Net cash flow		-500,000
Equipment sale		
Discounted cash flows (DCFs)		-500,000
NPV (Sum of DCFs)	27,366	

Simulation output		
Description	*Amount*	*@Risk function*
Distribution output	27,366	
Mean	27,409	RiskMean
Standard deviation	79,658	RiskStdDev
Minimum	-187,205	RiskMin
Maximum	210,540	RiskMaximum
Probability of loss	0.3720	RiskTarget
Probability of gain	0.6280	RiskTargetD



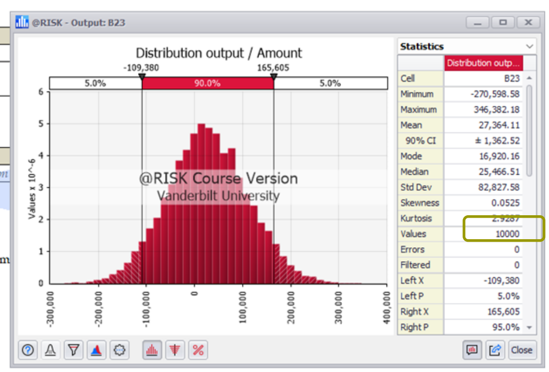
Results using 500 runs.

17

Capital budgeting decision

Firm		
Cost of capital	12.00%	
		0
Net cash flow		-500,000
Equipment sale		
Discounted cash flows (DCFs)		-500,000
NPV (Sum of DCFs)	27,366	

Simulation output		
Description	*Amount*	*@Risk function*
Distribution output	27,366	
Mean	27,364	RiskMean
Standard deviation	82,828	RiskStdDev
Minimum	-270,599	RiskMin
Maximum	346,382	RiskMaximum
Probability of loss	0.3696	RiskTarget
Probability of gain	0.6304	RiskTargetD



Results using 10,000 runs.

18

Capital budgeting decision

- Compute summary statistics (Set x = \$B\$19)
  - Mean: RiskMean(x)
  - Standard deviation: RiskStdDev(x)
  - Minimum: RiskMin (x)
  - Median: RiskPercentileD(x,.5)
  - Maximum: RiskMax(x)
  - Probability of loss: RiskTarget(x,0)
  - Probability of gain: RiskTargetD(x,0)

19

Capital budgeting decision

Simulation output		
Description	*Amount*	*@Risk function*
Distribution output	27,366	
Mean	27,366	RiskMean
Standard deviation	83,003	RiskStdDev
Minimum	-250,006	RiskMin
Median	26,851	RiskPercentile
Maximum	339,286	RiskMaximum
Probability of loss	0.3736	RiskTarget
Probability of gain	0.6264	RiskTargetD

Produces array of information about NPV distribution.

- NPV rule says accept because NPV > 0.
- Managers tend to be loss averse and may focus more on prob(loss).

20

Option valuation

- Black-Scholes/Merton (1973) option valuation model assumes:
  - Asset prices are log-normally distributed (to avoid negative prices).
    - Implies natural logarithm of price ratios is normally distributed.
  - Risk-free hedge can be formed between option and underlying asset.
  - If risk-free hedge can be formed between option and underlying asset, can use risk-neutral valuation.

21

Option valuation

- Normally distributed continuously compounded returns implies asset prices are lognormally distributed.

$$\tilde{S}_T = Se^{\mu T + \sigma \sqrt{T} \tilde{z}}$$

22

Option valuation

- Note subtle distinction between definitions.
  - *Mean of continuously compounded return* is

$$\mu = \frac{E\left[\ln\left(\tilde{S}_T / S\right)\right]}{T}$$

  - *Continuously compounded mean return* is

$$\alpha = \frac{\ln E\left[\tilde{S}_T / S\right]}{T}\left(= r \text{ in risk-neutral economy}\right)$$

  - Relation between parameters is

$$\mu = \alpha - .5\sigma^2 \left(= r - .5\sigma^2 \text{ in risk-neutral economy}\right)$$

23

Simulation run

- Draw z and compute asset price at time T.

$$\tilde{S}_T = Se^{\mu T + \sigma\sqrt{T}\tilde{z}}$$

- Compute and record option value at T.

$$\tilde{c}_T = \max(0, \tilde{S}_T - X) \text{ for call}$$

24

Summarize simulation runs

- After *n* simulation runs, compute mean terminal value, and then discount back to present.

$$c = e^{-rT} E(\tilde{c}_T) = e^{-rT} \left(\sum_{i=1}^{n} c_{i,T} / n \right)$$

- Accuracy improves as *n* is increased.

25

Option valuation

- Computes value of European-style call option.
  - Support file: MC option valuation.xlsx

26

Option valuation

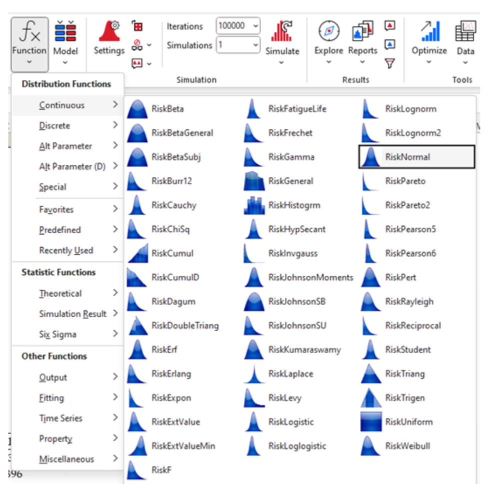
- Compute call value.
  - Set input parameters.

	A	B	C
1	**European-style option valuation**		
2	*Security*		
3	Price	50	
4	Volatility rate	20.00%	
5			
6	*Market parameters*		
7	Interest rate	5.00%	
8			
9	*Option contract*		
10	Exercise price	55	
11	Time to expiration	2.00	
12			

27

Option valuation

- Compute call value.
  - Generate drawing from unit normal distribution.



28

Option valuation

- Compute call value.
  - Generate drawing from unit normal distribution.

B14 fx =RiskNormal(0,1)

	A	B
1	**European-style option valuation**	
2	*Security*	
3	Price	50
4	Volatility rate	20.00%
5		
6	*Market parameters*	
7	Interest rate	5.00%
8		
9	*Option contract*	
10	Exercise price	55
11	Time to expiration	2.00
12		
13	*Generate security price and option prices at T.*	
14	Normal deviate	0.0000
15	Return of return	0.0600
16	Terminal asset price	53.09
17	Terminal call price	0.00

29

Option valuation

- Compute call value.
  - Compute return.

B15 fx =(B7-0.5*B4^2)*B11+B4*SQRT(B11)*B14

$$\tilde{R}_T = \left(r - .5\sigma^2\right)T + \sigma\sqrt{T}\tilde{z}$$

	A	B	C	D	E
1	**European-style option valuation**				
2	*Security*				
3	Price	50			
4	Volatility rate	20.00%			
5					
6	*Market parameters*				
7	Interest rate	5.00%			
8					
9	*Option contract*				
10	Exercise price	55			
11	Time to expiration	2.00			
12					
13	*Generate security price and option prices at T.*				
14	Normal deviate	0.0000			
15	Return of return	0.0600			
16	Terminal asset price	53.09			
17	Terminal call price	0.00			

30

Option valuation

- Compute call value.
  - Compute terminal asset price.

$$\tilde{S}_T = Se^{\tilde{R}_T}$$

B16 =B3*EXP(B15)

	A	B	C
1	European-style option valuation		
2	*Security*		
3	Price	50	
4	Volatility rate	20.00%	
5			
6	*Market parameters*		
7	Interest rate	5.00%	
8			
9	*Option contract*		
10	Exercise price	55	
11	Time to expiration	2.00	
12			
13	*Generate security price and option prices at T.*		
14	Normal deviate	0.0000	
15	Return of return	0.0600	
16	Terminal asset price	53.09	
17	Terminal call price	0.00	

31

Option valuation

- Compute call value.
  - Compute terminal call price.

$$\tilde{c}_T = \max\left(0, \tilde{S}_T - X\right)$$

B17 =MAX(0,B16-B10)

	A	B	C
1	European-style option valuation		
2	*Security*		
3	Price	50	
4	Volatility rate	20.00%	
5			
6	*Market parameters*		
7	Interest rate	5.00%	
8			
9	*Option contract*		
10	Exercise price	55	
11	Time to expiration	2.00	
12			
13	*Generate security price and option prices at T.*		
14	Normal deviate	0.0000	
15	Return of return	0.0600	
16	Terminal asset price	53.09	
17	Terminal call price	0.00	

32

Option valuation

- Compute call value.
  - Compute expected terminal call price using RiskMean.

C21 =@RiskMean(B17)

	A	B	C
1	**European-style option valuation**		
2	*Security*		
3	Price	50	
4	Volatility rate	20.00%	
5			
6	*Market parameters*		
7	Interest rate	5.00%	
8			
9	*Option contract*		
10	Exercise price	55	
11	Time to expiration	2.00	
12			
13	*Generate security price and option prices at T.*		
14	Normal deviate	0.0000	
15	Return of return	0.0600	
16	Terminal asset price	53.09	
17	Terminal call price	0.00	
18			
19	*Summarize terminal price distribution parameters.*		
20		Security	Call
21	Expected terminal price	55.259	6.330
22	Standard deviation	15.947	10.896
23	Percentile 5%	33.340	0.000
24	Percentile 95%	84.541	29.541

33

Option valuation

- Compute call value.
  - Discount expected terminal call price to present.

C26 =C21*EXP(-B7*B11)

	A	B	C
1	**European-style option valuation**		
2	*Security*		
3	Price	50	
4	Volatility rate	20.00%	
5			
6	*Market parameters*		
7	Interest rate	5.00%	
8			
9	*Option contract*		
10	Exercise price	55	
11	Time to expiration	2.00	
12			
13	*Generate security price and option prices at T.*		
14	Normal deviate	0.0000	
15	Return of return	0.0600	
16	Terminal asset price	53.09	
17	Terminal call price	0.00	
18			
19	*Summarize terminal price distribution parameters.*		
20		Security	Call
21	Expected terminal price	55.259	6.330
22	Standard deviation	15.947	10.896
23	Percentile 5%	33.340	0.000
24	Percentile 95%	84.541	29.541
25			
26	Simulated option value		5.728
27	Analytical (BSM) option value		5.728
28	Valuation error		0.000

34

Spread option valuation

- Compute value of spread options.
  - Spread option is option on difference between two prices.
    - Crack spread: Difference between gasoline futures price and crude oil futures price.
    - Cannot value analytically because if each asset price is lognormally distributed because difference is not.
    - Can be valued numerically using Monte Carlo simulation.
  - Support file: MC spread option valuation.xlsx
    - Spread option has $X = 0$ and $T = 0.5$ years.
    - Spread is difference between two asset prices.
      - Both asset prices are 50.
      - Volatility rates are 32% for asset 1 and 30% for asset 2.
    - Interest rate is 5% annually.

35

Spread option valuation

- Value spread option.
  - Generate drawing from unit normal for asset 1.

B16 =RiskNormal(0,1)

	A	B	C
1	Spread option valuation		
2	Assets	1	2
3	Price	50	50
4	Volatility rate	32%	30%
5	Correlation	0.50	
6			
7	Market parameters		
8	Risk-free interest rate	5%	
9			
10	Option contract		
11	Exercise price	0.00	
12	Time to expiration	0.50	
13			
14		Assets	
15	Simulation	1	2
16	Normal deviate	0	0
17	Terminal price	49.970	50.125

36

Spread option valuation

- Value spread option.
  - Generate drawing from unit normal for asset 2.
  - Provide no information about correlation.

C16 =RiskNormal(0,1)

	A	B	C
1	Spread option valuation		
2	Assets	1	2
3	Price	50	50
4	Volatility rate	32%	30%
5	Correlation	0.50	
6			
7	Market parameters		
8	Risk-free interest rate	5%	
9			
10	Option contract		
11	Exercise price	0.00	
12	Time to expiration	0.50	
13			
14		Assets	
15	Simulation	1	2
16	Normal deviate	0	0
17	Terminal price	49.970	50.125

37

Spread option valuation

- Value spread option.
  - Generate drawing from unit normal for asset 2.
  - Provide no information about correlation.
  - Call and put have values of 6.17.

Spread option valuation		
Assets	1	2
Price	50	50
Volatility rate	32%	30%
Correlation	0.50	
Market parameters		
Risk-free interest rate	5%	
Option contract		
Exercise price	0.00	
Time to expiration	0.50	
	Assets	
Simulation	1	2
Normal deviate	0	0
Terminal price	49.970	50.125
Average terminal spread	-0.155	
	Call	Put
Terminal option price	0.000	0.155
Expected option price	6.324	6.324
PV of expected price	6.168	6.168

38

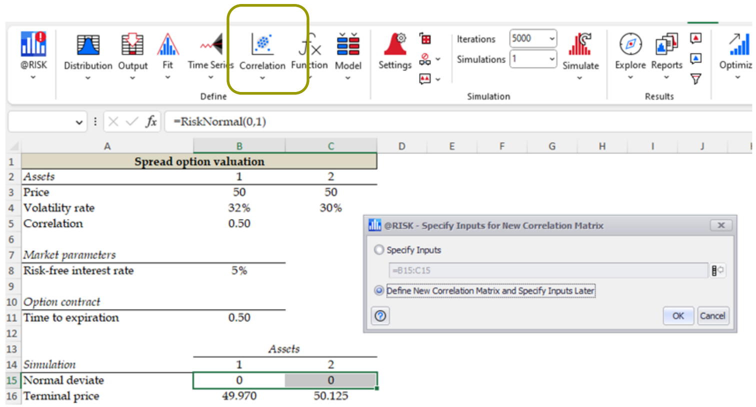
Spread option valuation

- Compute value of spread options.
  - Current assumptions:
    - Spread option has $X = 0$ and $T = 0.5$ years.
    - Spread is difference between two asset prices.
      - Both asset prices are 50.
      - Volatility rates are 32% for asset 1 and 30% for asset 2.
    - Interest rate is 5% annually.
  - New assumption:
    - Correlation between returns of assets is 0.5.
  - Will option values increase or decrease?

39

Spread option valuation

- Value spread options.
  - Highlight both drawings and set correlation.



40

Spread option valuation

- Value spread options.
  - Simulate.
  - Values are 4.38.

Spread option valuation		
Assets	1	2
Price	50	50
Volatility rate	32%	30%
Correlation	0.50	
Market parameters		
Risk-free interest rate	5%	
Option contract		
Exercise price	0.00	
Time to expiration	0.50	
	Assets	
Simulation	1	2
Normal deviate	0	0
Terminal price	49.970	50.125
Average terminal spread	-0.155	
	Call	Put
Terminal option price	0.000	0.155
Expected option price	4.490	4.490
PV of expected price	4.379	4.379

41

Spread option valuation

- Value spread options if correlation is –0.5.
  - Values are 7.54.

Spread option valuation		
Assets	1	2
Price	50	50
Volatility rate	32%	30%
Correlation	0.50	
Market parameters		
Risk-free interest rate	5%	
Option contract		
Exercise price	0.00	
Time to expiration	0.50	
	Assets	
Simulation	1	2
Normal deviate	0	0
Terminal price	49.970	50.125
Average terminal spread	-0.155	
	Call	Put
Terminal option price	0.000	0.155
Expected option price	7.728	7.728
PV of expected price	7.537	7.537

42

Lesson summary

- Monte Carlo simulation is most powerful tool in financial decision-making.
  - Largest obstacles in its application are:
    - No precise solution
    - Unfamiliarity and/or lack of ability to understand

43