ESTIMATING THE EFFECTIVE BID/ASK SPREAD FROM TIME AND SALES DATA

TOM SMITH
ROBERT E. WHALEY

Security trading costs have been the focus of much controversy and debate in recent years. No doubt part of the heightened concern has resulted from the market crash in 1987, a time during which transactions became very costly to execute. Yet another part is attributable to increased competition among markets worldwide. Many U.S. exchanges that at one time had exclusive control of market making—in particular, securities—are now facing competition not only from other markets domestically but also from markets in Europe and the Pacific basin.

Whatever the motivation for the concern, understanding and accurately measuring trading costs is critically important for making reasoned business and regulatory decisions regarding market operations. The key trading cost for most market participants is the cost of immediate exchange or, more commonly, the market maker's bid/ask spread. Some securities markets such as the New York Stock Exchange (NYSE)

Research support was received from the Business Associates' Fund (Smith and Whaley) and the Isle Maligne Fund (Whaley). We thank Barbara Ostdick for carefully reviewing the final draft of the article.

- *Tom Smith is an Associate Professor of Finance at Duke University.*
- *Robert E. Whaley is the T. Austin Finch Foundation Professor of Business Administration and Director, Futures and Options Research Center at Duke University*

The Journal of Futures Markets Vol. 14 No. 4 437–455 (1994)

CCC 0270-7314/94/040437-19

and the Chicago Board Options Exchange (CBOE) maintain historical records of all transaction prices and bid/ask quotes. With such complete information, examining the size of the spread and its variation through time with different levels of market activity and volatility is relatively straightforward. Unlike securities markets, however, futures markets do not record the sequence of all transaction prices and bid/ask quotes during the trading day. Exchanges such as the Chicago Board of Trade (CBT) and the Chicago Mercantile Exchange (CME) record "times and sales data," which contain only the time and price of a transaction if the price is different from the previously recorded price. Bid and ask quotes appear in this file only if the bid quote exceeds or if the ask quote is below the previously recorded transaction price. Because the history of transaction prices is incomplete and bid/ask quotes are generally not recorded, estimating the spread in these markets is a difficult task.

This study focuses on estimating the bid/ask spread using time and sales data. Two approaches have been used in past research—the serial covariance estimator and the mean absolute price change estimator. Unfortunately, neither of these approaches is reliable. The serial covariance estimator depends on negative serial covariance between adjacent price changes, a condition which is frequently violated in practice. The mean absolute price change estimator is upward biased because it fails to distinguish between true price change volatility and volatility attributable to bid/ask price bounce. This study provides a new estimator of the bid/ask spread. This estimator is based on the moments of the absolute price change distribution and is shown to be robust to varying degrees of price-change serial correlation and volatility.

The following section describes the nature of the time and sales data and the serial covariance and mean absolute price change bid/ask spread estimators, and contains the development of the method of moments estimator. The second section contrasts the performance of the estimators using simulated price changes, and the third shows how the estimators perform using time and sales data for the S&P 500 futures contract.

SPREAD ESTIMATORS

The cost of immediate exchange is the market maker's effective (or realized) bid/ask quote spread. The effective spread differs from the quoted spread. The quoted spread is the difference between the market maker's bid and ask quotes. The effective spread is the difference between the price at which the market maker buys (sells) a security and the price at which he subsequently sells (buys) it.[1] The effective spread is the focus of this study's analysis since it is the economic cost incurred by the trading public.

Measuring the effective spread directly is generally not possible because the trading records of market makers are not publically available. Instead, the spread must be inferred from available intraday price data. For futures exchanges, the only available intraday price data are time and sales files. These files contain the time and the price of each futures contract transaction only when the price in the transaction is different from the price recorded previously. Bid (ask) quotes also appear on the time and sales data file if the bid (ask) quote exceeds (is below) the previously recorded transaction price. No trading volume figures are recorded. The middle column of Table I shows time and sales data for the September 1982 futures contract for a short interval on April 23, 1982.

Past Approaches

The two types of bid/ask spread estimators that have been used in past research are the serial covariance estimator and the mean absolute price-change estimator. The serial covariance estimator, the most commonly applied in stock market research, was developed by Roll (1984). In an informationally efficient market,[2] Roll showed that the effective bid/ask spread is defined as

$$s_A^* = 2\sqrt{-\mathrm{cov}(\Delta P_t^o, \Delta P_{t-1}^o)} \qquad (1)$$

where P_t^o is the observed stock price at time t. Underlying (1) is the assumption that if the observed price of the futures is at the bid (ask) the next price change is equally likely to be zero or plus (minus) the amount of the spread. In other words, there is a 50% chance that the next observed transaction price will be at its current level.

Time and sales data from the futures markets, however, generally do not include consecutive transactions at the same price. If the current

[1]To illustrate, assume that the quoted bid/ask spread of the S&P 500 futures contract is 0.05 (250.00 bid/250.05 ask). Further, assume that in the course of market making, a scalper is long 50 contracts at 250.00. Market buy orders might immediately absorb part of the scalper's current position, say, 40 contracts at 250.05. The remaining 10 contracts pose significant inventory risk for the scalper, so he may decide to speed the liquidation of his position by temporarily lowering his ask price to, say, 250.00. Since the 250.00 price is now the most attractive offer in the pit, subsequent market buy orders will automatically flow to this scalper, and his remaining position will be liquidated through "scratch sales". While the quoted spread in this illustration is 0.05, the effective spread is 0.04.

[2]An implication of the informationally efficient market assumption is that true price changes are serially uncorrelated.

TABLE I
Time and Sales Data for the September 1982 S&P 500 Index Futures Contract on April 23, 1982, and its Relation to Price Changes Included in SAMPLE 1 and SAMPLE 2

	SAMPLE 1 *Price Change*	*Time and Sales Data* *Price*	*SAMPLE 2* *Price Change*
		119.90	
	0.10	120.00	0.10
	−0.05	119.95	−0.05
	0.05	120.00	0.05
	0.05	120.05B	
	0.05	120.10B	
	0.05	120.15A	
	0.00	120.15	
	−0.05	120.10A	
	−0.05	120.05A	
	0.00	120.05	
	−0.05	120.00A	
	−0.05	119.95	
	−0.05	119.90	−0.05
	−0.05	119.85	−0.05
	0.05	119.90B	
	0.05	119.95	
	0.05	120.00B	
	0.00	120.00	
	−0.05	119.95	−0.05
Mean value	0.0026		−0.0083
Mean absolute value	0.0447		0.0583

transaction price is at a bid (ask) level, in all likelihood the next recorded price change *will be* plus (minus) the amount of the spread. Under this assumption regarding the observed price-change sequence, Followill and Helms (1990) showed that the serial covariance estimator is

$$s_A = \sqrt{-\mathrm{cov}(\Delta P_t^o, \Delta P_{t-1}^o)} \qquad (2)$$

The serial covariance estimator, while simple in theory, is troublesome in practice. The estimation of $\mathrm{cov}(\Delta P_t^o, \Delta P_{t-1}^o)$ using time and sales data frequently produces positive values. Followill and Rodriguez (1991), for example, found that, when serial covariance is estimated using intraday price changes of S&P 500 futures contracts for the period April 21, 1982 through February 10, 1983, the covariance estimate is positive in nearly 25% of the contract days considered. Using more recent S&P 500 futures data, this study shows that this phenomenon is even more pronounced today. (These results are discussed in the next section.) Unfortunately, when serial covariance is positive, the estimate of s_A produces an imaginary number and, hence, no estimate of the bid/ask spread can be obtained.

The second type of estimator that has been used to infer the effective spread is the mean absolute price-change. This estimator, first used by Thompson and Waller (1988),[3] is

$$s_B = \overline{|\Delta P^o|} = \frac{1}{T} \sum_{t=1}^{T} |\Delta P_t^o| \tag{3}$$

where T is the length of the futures price-change series. If the expected true futures price change and the variance of true price change are both zero, this estimator would capture the effective spread. Unfortunately, while it is reasonable to assume that the expected price change of the futures from transaction to transaction is zero, assuming that the variance of futures price changes is zero is unrealistic. The mean absolute price change, $\overline{|\Delta P^o|}$, therefore consists of two components—the bid/ask spread and the variance of true price changes. $\overline{|\Delta P^o|}$ is therefore an upward biased estimator of spread, with the magnitude of the bias depending on the variance of true price changes.

Method of Moments Estimator

The remaining part of this section outlines a new method of moments estimator that circumvents the problems noted above. The estimator uses all successive price changes in the time and sales price file. Observed futures transaction prices are assumed to occur either at a bid or an ask level, with equal probability. The expected observed transaction price therefore equals the true futures price—situated half way between the bid and ask. Assuming the bid/ask spread, s_C, is a constant, the observed transaction price P_t^o is therefore modeled as

$$P_t^o = P_t \pm \frac{1}{2} s_C \tag{4}$$

where P_t is the true price.

Under model (4), the observed price change is limited to three cases:

$$\Delta P_t^o = \epsilon_t + s_C \tag{5a}$$

[3]CFTC (1989) and Ma, Peterson, and Sears (1992) also used this approach.

$$\Delta P_t^o = \epsilon_t \tag{5b}$$

and

$$\Delta P_t^o = \epsilon_t - s_C \tag{5c}$$

where ϵ_t is the true price change of the futures. Assuming that the expected true price change of the futures, ϵ_t, is zero, case (5b) can be ruled out because zero price changes are generally not reported in time and sales data. It is, therefore, assumed that the observed price change can only be (5a) or (5c) with equal probability, that is,

$$\Delta P_t^o = \begin{cases} \epsilon_t + s_C, & \text{with probability } \frac{1}{2} \\ \epsilon_t - s_C, & \text{with probability } \frac{1}{2} \end{cases} \tag{6}$$

Under (6), the first two moments of the distribution of absolute price changes, $E(|\Delta P_t^o|)$ and $E(|\Delta P_t^o|^2)$, are given by:

$$E(|\Delta P_t^o|) = \frac{1}{2} E(|\epsilon_t + s_C|) + \frac{1}{2} E(|\epsilon_t - s_C|) \tag{7a}$$

and

$$E(|\Delta P_t^o|^2) = \frac{1}{2} E(|\epsilon_t + s_C|^2) + \frac{1}{2} E(|\epsilon_t - s_C|^2) \tag{7b}$$

To operationalize these relations to estimate the effective spread, it is assumed that the true price change, ϵ_t, is distributed normally with mean zero and variance σ^2. Under this assumption, the relations (7a) and (7b) may be written

$$E(|\Delta P_t^o|) = \sqrt{\frac{2}{\pi}}\, \sigma e^{-s_C^2/2\sigma^2} - s_C\left[1 - 2N\left(\frac{s_C}{\sigma}\right)\right] \tag{8a}$$

and

$$E(|\Delta P_t^o|^2) = \sigma^2 + s_C^2 \tag{8b}$$

where $N(d)$ is the cumulative unit normal distribution function with upper integral limit d.[4,5] To estimate the effective spread (and the

[4]Leone, Nelson, and Nottingham (1961) or Elandt (1961) showed that if x is normally distributed with mean μ and variance σ^2, $E(|x|) = \sqrt{2/\pi}\,\sigma e^{-\mu^2/2\sigma^2} + \mu[1 - 2N(-\mu/\sigma)]$ and $E(|x^2|) = \sigma^2 + \mu^2$, where $N(d)$ is the cumulative unit normal distribution with upper integral limit d. Since $\epsilon + s_C$ is distributed normally with mean s_C and variance σ^2 and $\epsilon - s_C$ is distributed normally with mean $-s_C$ and variance σ^2, eqs. (8a) and (8b) follow straightforwardly.

[5]Note that even if the bid/ask spread is zero, the expected absolute price change is positive. Substituting $s_C = 0$ into (8a) shows that the mean of the distribution of $|\Delta P^o|$ in the absence of a bid/ask spread is $E(|\Delta P^o|) = \sigma\sqrt{2/\pi}$.

variance of true price changes), $E(|\Delta P^o|)$ and $E(|\Delta P^o|^2)$ are replaced with the mean absolute price change and the mean squared price change from the observed futures price change distribution. These are then solved iteratively.

The method of moments estimator is superior to past approaches for two important reasons. First, unlike the serial covariance estimator of the effective bid/ask spread, the method of moments estimator is not sensitive to serial covariance in the true price change series. Suppose, for example, that the true futures price changes follow a standard $AR(1)$ process, $\epsilon_t = \rho\epsilon_{t-1} + \eta_t$, where η_t is distributed normally with mean zero and variance σ_η^2. The distribution for ϵ_t remains normal with mean zero and has variance $\sigma_\eta^2/1 - \rho^2$. The simultaneous solution of (8a) and (8b) using estimates of the mean absolute price change, $E(|\Delta P_t^o|)$, and the mean squared price change, $E(|\Delta P_t^o|^2)$, provides an unencumbered estimate of the effective spread, s_C. Second, unlike the mean absolute price change estimator, the method of moments estimator does not overstate the size of the effective spread by failing to purge the effect of the variance of true price changes. In fact, the estimation procedure provides an estimate of the true price change variance in a given interval. For the reader's convenience, the FORTRAN source code for the method of moments approach is given in the appendix.

SIMULATION EVIDENCE

To compare the effectiveness of the alternative bid/ask spread estimators, simulation analyses are performed. These analyses are designed to detect the robustness of the estimators to different levels of serial correlation and volatility of true price changes. The simulations are based on the assumption that observed price changes (ΔP_t^o) are generated by

$$\Delta P_t^o = \begin{cases} \epsilon_t + s & \text{with probability } \frac{1}{2} \\ \epsilon_t - s & \text{with probability } \frac{1}{2} \end{cases}$$

where s is the effective bid/ask spread and ϵ_t is the true price change. Throughout the simulations, the effective bid/ask spread (s) is held at a constant level of 0.05. True price changes, ϵ_t, are generated under the assumption that the changes are normally distributed with mean zero and volatility (standard deviation), σ. In addition, it is assumed that ϵ_t follows an $AR(1)$ process with autoregressive parameter

ρ. The simulations are performed using values of ρ ranging from -0.20 to $+0.20$ and values of volatility ranging from 0.03 to 0.05.[6] To make the analysis more representative of intraday time and sales data, the simulation is performed initially using 1440 price change observations (the number of 15-second intervals in a 6-hour trading day). One thousand repetitions are performed. The effectiveness of the spread estimators is evaluated based on the mean estimated values and the mean squared prediction errors (MSE). Table II contains the simulation results.

First, focusing on the performance of the serial covariance estimator relative to the method of moments estimator, note that the serial covariance estimator works well only in instances where the true price changes are serially independent ($\rho = 0$)—the mean spread estimate is near its simulation setting of 0.05 and the mean squared prediction error is relatively small independent of the level of true volatility being assumed. If the serial correlation is nonzero, however, the picture changes quickly. The serial covariance estimator becomes increasingly downward (upward) biased as the level of positive (negative) serial correlation increases. The size of the bias, however, is not nearly so alarming as the increase in the mean squared prediction error. At $\rho = 0.20$, for example, the MSE is about four times higher than when $\rho = 0$ where $\sigma = 0.03$, and is nearly ten times higher where $\sigma = 0.05$. In contrast, the method of moments estimator performs well at all levels of serial correlation. Its mean value is consistently at or near 0.05, and the MSE does not change meaningfully as ρ changes.

Second, as expected, the mean absolute price-change estimator is extremely sensitive to the true volatility of price changes. Table II shows that, as volatility increases, the mean absolute price change estimator becomes more and more upward biased. The bias ranges from 2.4% in the case of $\sigma_\epsilon = 0.03$ to 16.8% in the case of $\sigma_\epsilon = 0.05$. In addition, over this same range of volatility, the MSE increases by a factor of nearly 30. In contrast, the method of moments estimator does not increase as volatility increases and its MSE increases less than seven times. Overall, the simulation results reported in Table II indicate that the method of moments estimator is more robust than either of the previously used alternatives.

[6]Assuming that the simulation price changes correspond to 15-second intervals during the trading day, the assumed 0.03, 0.04, and 0.05 volatility parameters correspond to annualized volatilities (using a 360-minute trading day and 251 trading days a year) of 18%, 24%, and 30%, respectively.

TABLE II

Mean Estimate and Mean Square Prediction Error of the Effective Bid/Ask Spread Estimators Based on the First-Order Serial Covariance (s_A), the Mean Absolute Price Change (s_B), and the Method of Moments (s_C) Using Simulated Time and Sales Data. The Estimates are Based on 1440 Observed Price Changes Generated Using

$$\Delta P_t^o = \begin{cases} \epsilon_t + s & \text{with probability } \frac{1}{2} \\ \epsilon_t - s & \text{with probability } \frac{1}{2} \end{cases}$$

where s is the Effective Bid/Ask Spread and ϵ_t is the Innovation in True Price. The True Price Change Innovation is Assumed to be Normally Distributed with Zero Mean, Standard Deviation σ_ϵ, and First-Order Serial Correlation ρ. The True Spread s is Set Equal to 0.05. The Simulation Involves 1000 Repetitions, and the Mean Spread Estimates and Mean Squared Prediction Errors (times 100,000) are Computed. Prices Bounce Sequentially between Bid and Ask

True Volatility	*Bid/Ask Spread*		*Serial Correlation in True Price Changes* ρ								
σ_ϵ	*Estimator*	*Statistic*	*−0.20*	*−0.15*	*−0.10*	*−0.05*	*0.00*	*0.05*	*0.10*	*0.15*	*0.20*
0.03	s_A	Mean	0.0518	0.0514	0.0509	0.0505	0.0500	0.496	0.0491	0.0487	0.0482
		MSE	0.424	0.290	0.159	0.097	0.073	0.085	0.143	0.232	0.379
		n^a	1000	1000	1000	1000	1000	1000	1000	1000	1000
	s_B	Mean	0.0512	0.0512	0.0512	0.0512	0.0512	0.0512	0.0512	0.0512	0.0512
		MSE	0.226	0.230	0.205	0.207	0.204	0.206	0.190	0.185	0.181
	s_C	Mean	0.0500	0.0500	0.0501	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500
		MSE	0.102	0.093	0.082	0.075	0.070	0.065	0.060	0.052	0.050

continued

TABLE II continued

True Volatility σ_ϵ	Bid/Ask Spread Estimator	Statistic	Serial Correlation in True Price Changes ρ								
			−0.20	−0.15	−0.10	−0.05	0.00	0.05	0.10	0.15	0.20
0.04	s_A	Mean	0.0531	0.0523	0.0516	0.0508	0.0500	0.0492	0.0483	0.0475	0.0468
		MSE	1.141	0.674	0.388	0.196	0.129	0.177	0.387	0.717	1.156
		n^a	1000	1000	1000	1000	1000	1000	1000	1000	1000
	s_B	Mean	0.0541	0.0540	0.0540	0.0540	0.0540	0.0541	0.0540	0.0540	0.0540
		MSE	1.778	1.706	1.727	1.722	1.717	1.740	1.688	1.689	1.692
	s_C	Mean	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0499	0.0499	0.0500
		MSE	0.216	0.198	0.178	0.174	0.166	0.154	0.152	0.146	0.121
0.05	s_A	Mean	0.0548	0.0535	0.0524	0.0513	0.0499	0.0488	0.0475	0.0461	0.0449
		MSE	2.54 1	1.468	0.810	0.399	0.218	0.370	0.847	1.691	2.843
		n^a	1000	1000	1000	1000	1000	1000	1000	1000	1000
	s_B	Mean	0.0584	0.0582	0.0583	0.0584	0.0583	0.0584	0.0583	0.0584	0.0583
		MSE	7.129	6.915	7.043	7.172	7.019	7.085	7.047	7.065	7.048
	s_C	Mean	0.0499	0.0498	0.0499	0.0500	0.0499	0.0499	0.0499	0.0499	0.0499
		MSE	0.557	0.509	0.52 3	0.497	0.428	0.426	0.403	0.404	0.359

[a]The estimator s_A is based on the first order serial covariance of observed price changes, $s_A = \sqrt{-\text{cov}(\Delta P_t^o, \Delta P_{t-1}^o)}$, where ΔP_t^o is the observed price change in period t.

EMPIRICAL PERFORMANCE

The simulation evidence shows that the method of moments estimator is robust to varying degrees of serial correlation and volatility of true price changes. The following discussion focuses on applying the estimator to actual time and sales data. The illustrations are conducted using time and sales data for the four nearby S&P 500 futures[7] for the sample period April 21, 1982 through October 9, 1987.

Sample Construction

As was noted earlier, time and sales data contain the time and the price of each futures contract transaction only when the price in the transaction is different from the price recorded previously. Bid (ask) quotes also appear on the time and sales data file if the bid (ask) quote exceeds (is below) the previously recorded transaction price. Transaction prices after quote records appear independent of whether or not there has been a price change.

The time and sales price sequence provided in Table I illustrates these reporting rules. The first four reported prices are trade prices, with each successive trade price being different from the preceding price. The fifth entry in the sequence, "120.10B," is a bid quote and appears because it is higher than the preceding trade price. Apparently, the true price of the futures has moved upward with no supporting trade. The next price is also a bid and appears because it is higher than the preceding price. The "120.15A" then appears because the price level is higher than the preceding price. The "120.15" trade price then appears because the preceding record was a quote. And so on through the series. In general, the nearby futures contract is the most active, so the proportion of the time and sales records accounted for by quotes is smallest. On the other hand, quotes may well account for most of the records in a given day for distant contract maturities.

In terms of bid/ask spread estimation, the treatment of quotes in the time and sales data deserves consideration. Movement from a trade price to a quote is attributable more to new information than to the bid/ask spread. On the other hand, movement from a quote to a trade price contains little information at all. For these reasons, two samples

[7]For a short period of time in 1983 and 1984, the CME experimented with six quarterly contract maturities. The distant contracts did not trade actively and are not used in these analyses.

are created. The first sample, SAMPLE 1, treats *all* records, including records reporting bid and ask quotes, as if they are actual transactions.[8] The second sample, SAMPLE 2, excludes bid and ask quotes as well as prices of transactions adjacent to quotes. Adjacent transaction prices are eliminated because the price movement from the transaction record prior to a reported bid (ask) record to the transaction record after the bid (ask) record likely represents a price movement due to new information rather than bid/ask spread movement.

To clarify the differences between the two samples, consider the leftmost and rightmost columns of Table I. On the left are the price changes of SAMPLE 1. All prices are treated as if they are actual transactions; so, if n records are reported for the futures contract on a particular day, $n - 1$ prices changes are computed. Note also that zero price changes appear where a transaction price appears after a bid/ask quote at the same level. On the right are the price changes of SAMPLE 2. The number of price changes for a given day depends on the number of recorded bid/ask quotes. A greater number of bid/ask quotes as a proportion of total price records for the day results in fewer computed price changes. Table I, for example, shows only six computed price changes in a sequence of 20 recorded prices. On certain days, the number of bid/ask quotes for a distant maturity contract is so large that no SAMPLE 2 price changes appear.

The effects of censoring the time and sales data to generate SAMPLE 2 are perhaps best understood by comparing the frequency of price changes in the two samples. Table III contains a summary of the frequency of price changes by contract maturity and tick size for the time and sales data for the S&P 500 futures contracts during the overall sample period April 21, 1982 through October 9, 1987. The tick size for the S&P 500 futures contract is 0.05, and price changes are categorized by actual and absolute price value. Note that the nearby futures contract has by far the greatest number of recorded price changes. The SAMPLE 1 results indicate that there are more than 2.35 million price changes reported in the sample. Of these, the largest majority are one-tick price moves; however, price moves of two ticks and larger are not uncommon. For more distant contracts, two-tick price changes are more common than one-tick price changes. The SAMPLE 1 results show this for the third and fourth contract maturities.

[8] Also on the file are records for transactions that are cancelled or corrected. The cancelled records are eliminated because the transaction (price) was erroneously reported. The corrected (price) records are eliminated because the transactions may be out of chronological order.

TABLE III

Summary of Actual and Absolute Price Change Distributions by Contract Maturity and Number of Ticks for the S&P 500 Index Futures Contract for all Days in the Sample Period April 21, 1982, through October 9, 1987. The Tick Size for the S&P 500 Futures Contract is 0.05

		Number of Price Changes						
Contract Month	*Total*	*Less than −2 Ticks*	*−2 Ticks*	*−1 Ticks*	*0 Ticks*	*1 Tick*	*2 Ticks*	*More than 2 Ticks*
SAMPLE 1: Actual price changes								
1	2,350,030	1,404	28,427	1,129,993	28,604	1,131,158	29,108	1,336
2	910,771	3,072	38,562	365,415	93,863	368,526	38,404	2,929
3	127,460	1,366	39,425	13,234	17,827	14,803	39,706	1,099
4	19,231	247	7,203	741	2,328	961	7,496	255
SAMPLE 1: Absolute price changes								
1	2,350,030				28,604	2,261,151	57,535	2,740
2	910,771				93,863	733,941	76,966	6,001
3	127,460				17,827	28,037	79,131	2,465
4	19,231				2,328	1,702	14,699	502
SAMPLE 2: Actual price changes								
1	2,279,864	1,061	25,410	1,103,620	19,379	1,103,273	26,118	1,003
2	394,797	1,044	10,778	184,024	3,273	183,521	11,066	1,091
3	2,472	42	448	698	91	729	440	24
4	90	0	19	14	9	24	24	0
SAMPLE 2: Absolute price changes								
1	2,279,864				19,379	2,206,893	51,528	2,064
2	394,797				3,273	367,545	21,844	2,135
3	2,472				91	1,427	888	66
4	90				9	38	43	0

The SAMPLE 2 results, when contrasted with the SAMPLE 1 results, show that the data censoring reduces the number of price changes in the sample, with the relative size of reduction increasing by contract maturity. For example, the nearby contract has a total of 2.35 million price changes in SAMPLE 1 and 2.28 million transactions in SAMPLE 2. This represents about a 3% decrease. The second, third, and fourth contract maturities experience relative decreases of 56.6, 98.1, and 99.5%. The nearby contract has the most active and continuous market, so few bid/ask quotes appear in the time and sales data. On the other hand, distant maturities have few trades, so recorded time and sales data consist largely of bid/ask quote, reflecting market movements without accompanying trades. Since the goal is to estimate the effective bid/ask spread from the price movements attributable to the bid/ask spread and not to new information, the SAMPLE 2 results are probably the most reliable.

On another issue, Table III shows that the observed price change distributions are almost perfectly symmetric for both SAMPLE 1 and SAMPLE 2. This is reassuring considering that true price changes are assumed to be distributed normally in the development of the method of moments estimator in the last section.

Empirical Properties

On the basis of SAMPLE 1 and SAMPLE 2, the value of each estimator is computed for each S&P 500 futures contract each day during the sample period.[9] Table IV contains the mean and the standard deviation of the daily spread estimates as well as the estimates of true price change volatility obtained from the method of moments approach across days of the sample period. Also reported are the correlations between the daily estimates for the three spread estimation approaches.

The results reported in Table IV show that the serial covariance estimator is unreliable for time and sales data. First, the estimated serial covariance in price changes is negative in only 1202 of the 6103 contract days of SAMPLE 1 (19.7% of the time) and 1967 of the 3205 contract days of SAMPLE 2 (61.8% of the time). Recall that a negative estimate is necessary to ensure that the spread is not an imaginary number. Second, the mean of the estimated spread $\hat{s}_A$ is consistently lower than the other two approaches. In fact, the values are implausibly small. The

[9]Implicitly, it is assumed that the variance of true changes and the effective spread are constant throughout the trading day.

TABLE IV

Mean and Standard Deviation of and Correlation among Estimators of the Effective Spread for the S&P 500 Index Futures Contract for all Days in the Sample Period April 21, 1982, through October 9, 1987

		Spread Estimate, s_A[a]		*Spread Estimate, s_B*[b]	*Spread Estimate, s_C*[c]	*Volatility Estimate, σ*	*Correlation Matrix*		
Contract	*No. of Obs.*	*Mean/ Std. Dev.*	*No. of Obs.*	*Mean/ Std. Dev.*	*Mean/ Std. Dev.*	*Mean/ Std. Dev.*	*Estimator*	s_B	s_C
SAMPLE 1:									
Pooled	1202	0.0222	6103	0.0669	0.0636	0.0321	s_A	−0.100	−0.182
		0.0084		0.0198	0.0198	0.0229	s_B		0.809
Nearby	1016	0.0232	1385	0.0505	0.0503	0.0087	s_A	−0.153	−0.140
		0.0079		0.0035	0.0028	0.0084	s_B		0.478
2	140	0.0180	1385	0.0474	0.0459	0.0245	s_A	0.131	−0.127
		0.0074		0.0075	0.0067	0.0116	s_B		0.773
3	1	0.0415	1385	0.0757	0.0715	0.0440	s_A	n.a.	n.a.
		n.a.		0.0134	0.0167	0.0151	s_B		0.753
Distant	7	0.0136	1378	0.0850	0.0796	0.0457	s_A	0.756	−0.830
		0.0128		0.0123	0.0167	0.0235	s_B		0.304
SAMPLE 2:									
Pooled	1967	0.0245	3205	0.0556	0.0546	0.0129	s_A	0.588	0.434
		0.0163		0.0137	0.0130	0.0159	s_B		0.843
Nearby	1058	0.0229	1385	0.0509	0.0508	0.0080	s_A	−0.209	−0.208
		0.0080		0.0043	0.0036	0.0079	s_B		0.680
2	635	0.0182	1382	0.0552	0.0543	0.0154	s_A	0.382	0.142
		0.0098		0.0104	0.0096	0.0155	s_B		0.728
3	268	0.0456	430	0.0715	0.0681	0.0207	s_A	0.627	0.430
		0.0303		0.0257	0.0262	0.0275	s_B		0.850
Distant	6	0.0495	8	0.0703	0.0666	0.0196	s_A	−0.212	−0.138
		0.0231		0.0188	0.0252	0.0201	s_B		0.928

[a] The estimator s_A is based on the first-order serial covariance of observed price changes, $s_A = \sqrt{-\text{cov}(\Delta P_t^o, \Delta P_{t-1}^o)}$, where ΔP_t^o is the observed price change in period t. Note that, when the estimated first-order serial covariance is positive, the estimated spread is an imaginary number. Since these spread estimates are not usable, the number of observations used in computing the descriptive statistics is less than for the other estimators.

[b] The estimator s_B is the mean absolute value of the observed price changes, $s_B = \overline{|\Delta P^o|} = \frac{1}{T} \sum_{t=1}^{T} |\Delta P_t^o|$, where T is the length of the time series.

[c] The estimator s_C is determined simultaneously with σ^2 in the solution to the system of two equations, $\overline{|\Delta P^o|} = \sqrt{2/\pi}\,\sigma e^{-s_C^2/2\sigma^2} - s_C + 2 s_C N(s_C/\sigma)$ and $\overline{|\Delta P^o|^2} = \sigma^2 + s_C^2$, where $N(d)$ is the cumulative unit normal distribution function with upper integral limit d.

effective spread in the S&P 500 index futures market is probably very close to the minimum price movement (tick size), which is 0.05. Yet, the mean effective spread using (3) is 0.0222 in SAMPLE 1 and 0.0245 in SAMPLE 2, less than half of that amount. Reassuringly, the other two estimators are much nearer 0.05. Third, the correlation between the estimated spread measures should be positive and reasonably high. In the case of s_A, the correlations with the other spread measures, s_B and s_C, are frequently negative.

The descriptive statistics for the s_B and s_C estimates are much more plausible. First, the mean of the estimates of s_C is consistently less than s_B, which is anticipated since the mean absolute price change s_B contains not only the effective spread but also the variance of true price changes. Second, both the s_B and s_C estimates for the nearby S&P 500 futures contract are close to the minimum price movement (tick size). Considering that the nearby S&P 500 futures contract is widely perceived to be a single tick market, finding evidence to the contrary would be surprising. Third, the correlation between the estimates is extremely high, indicating that the proportion of the mean absolute price change attributable to the bid/ask spread is fairly stable from day-to-day, particularly when price changes are censored in the manner of SAMPLE 2. Finally, in SAMPLE 2 at least, the spreads tend to increase with contract maturity, reflecting the lower market liquidity in the distant contracts.

Table IV also contains summary statistics for the estimates of volatility of true price changes obtained from the method of moments estimation procedure. In SAMPLE 1, for example, the pooled results indicate that the standard deviation of true price changes is 0.0321, while the bid/ask spread estimate is 0.0636. This indicates that the dominant component of the variance of consecutively observed futures price changes is the bid/ask spread. Recall that eq. (8b) shows that the variance of observed price changes is the sum of the variance of true price changes plus the square of the bid/ask spread. The variance of true price changes is estimated to be $0.0321^2 = 0.001030$ and the variance attributable to the bid/ask spread is $0.0636^2 = 0.004045$, so the pooled results using SAMPLE 1 indicate that the bid/ask spread accounts for 79.7% of observed price change variance, on average.

The SAMPLE 2 results provide an interesting contrast. Recall that the censoring of the time and sales data to generate SAMPLE 2 deletes price changes that are deemed related to new information (i.e., price changes from a transaction price to either a bid or ask quote). If this censoring procedure is valid, the standard deviation of true price changes should be lower in SAMPLE 2 than it is in SAMPLE 1 since some of the true volatility is removed by construction. Table IV indicates that the mean volatility for the pooled sample is 0.0129, with a corresponding estimate of 0.0546 for the bid/ask spread. The pooled results using SAMPLE 2 indicate that the bid/ask spread accounts for 94.7% of observed price change variance, on average.

Precise interpretations of the values reported for volatility in Table IV are difficult because the volatility estimates apply to the time between successive price records used in the computation of price changes. The nearby contract has the greatest frequency of trading (least time between successive transactions), so it has the lowest estimated volatility on average, 0.0087 in SAMPLE 1 and 0.0080 in SAMPLE 2. The volatility estimates for the more distant contracts are considerably higher due to lower trading frequency.

CONCLUSIONS

This study presents a new method for estimating the effective bid/ask spread using time and sales data from futures markets. Past approaches, the mean absolute value price change and the first-order serial covariance of price changes, are problematic because they either assume that true price changes are serially dependent or that all price change variability is driven exclusively by bid/ask price bounce. A new estimator is proposed here that is based upon the first two moments of absolute price change distribution. This estimator circumvents the shortcomings of past approaches and reliably estimates the bid/ask spread under a range of assumptions regarding the serial correlation and volatility of true price changes. The performance of all estimators is gauged using both simulated and actual data.

APPENDIX

FORTRAN subroutine to compute effective spread and standard deviation of true price changes using the method of moments approach. PRUN(d) is the cumulative univariate normal density function with upper integral limit d.

```
      SUBROUTINE MMSPRD(S,V,ADP,SDP,N)
C ************************************************************************
C MMSPRD computes the spread S and the volatility (standard deviation)
C V given values for the mean absolute price change ADP and the mean
C squared price change SDP. Also returns number of iterations.
C ************************************************************************
C Set tolerance level.
C *****************************************************
      TOL = .00001
```

```
C ******************************************************
C Set highest and lowest possible spread values.
C ******************************************************
      SH = ADP
      SL = 0.0
C ***********************************
C Compute volatility counterparts.
C ***********************************
      VH = SQRT(SDP - SH*SH)
      VL = SQRT(SDP - SL*SL)
C ***********************************
C Compute absolute price change
C counterparts.
C ***********************************
      ADPH = ABSDP(SH,VH) - ADP
      ADPL = ABSDP(SL,VL) - ADP
C ******************************************************
C Set midpoint spread value.
C ******************************************************
      N = 1
    2 SM = (SL+SH) / 2.
C ***********************************
C Compute volatility counterpart.
C ***********************************
      VM = SQRT(SDP - SM*SM)
C ***********************************
C Compute absolute price change
C counterpart.
C ***********************************
      ADPM = ABSDP(SM,VM) - ADP
      IF(ABS(ADPM).LT.0.000001) GOTO 1
      IF(N.GT.100) GOTO 1
C ***********************************
C Replace higher or lower spread
C with midpoint.
C ***********************************
      IF(ADPM.GT.0.0) THEN
        SH = SM
        VH = VM
      ELSE
        SL = SM
        VL = VM
      ENDIF
      N = N + 1
      GOTO 2
C ********************************
C Return the value of the spread.
C ********************************
    1 CONTINUE
      S = SM
      V = SQRT(SDP - SM*SM)
      RETURN
      END
      FUNCTION ABSDP(S,V)
C **********************************************************************
C ABSDP computes the absolute price change ADP given values of the
C spread S and volatility (standard deviation) V.
C **********************************************************************
      PI = 3.141592
      ABSDP = SQRT(2./PI)*V*EXP(-(S*S)/(2*V*V)) - S + 2.*S*PRUN(S/V)
      RETURN
      END
```

BIBLIOGRAPHY

Brorsen, B.W. (1989): "Liquidity Costs and Scalping Returns in the Corn Futures Market," *The Journal of Futures Markets*, 9:225–236.

Commodity Futures Trading Commission (1989): *Economic Analysis of Dual Trading on Commodity Exchanges*, Report prepared by the Division of Economic Analysis with Technical Support from the Office of Information Resources Management.

Elandt, R.C. (1961): "The Folded Normal Distribution: Two Methods of Estimating Parameters from Moments," *Technometrics*, 3:551–562.

Followill, R.A., and Helms, B.P. (1990): "Put–Call–Futures Parity and Arbitrage Opportunities in the Market for Options on Gold Futures," *The Journal of Futures Markets*, 10:339–352.

Followill, R.A., and Rodriguez, A.J. (1991): "The Estimation and Determinants of Bid–Ask Spreads in Futures Markets," *Review of Futures Markets*, 10:339–352.

Leone, F.C., Nelson, L.S., and Nottingham, R.B. (1961): "The Folded Normal Distribution," *Technometrics*, 3:543–550.

Ma, C.K., Peterson, R.L., and Sears, R.S. (1992): "Trading Noise, Adverse Selection, and Intraday Bid–Ask Spreads in Futures Markets," *The Journal of Futures Markets*, 12:519–538.

Roll, R. (1984): "A Simple Implicit Measure of the Bid/Ask Spread in an Efficient Market," *Journal of Finance*, 39:1127–39.

Stoll, H.R. (1989): "Inferring the Components of the Bid/Ask Spread: Theory and Empirical Tests," *Journal of Finance*, 44:115–134.

Thompson, S.R., and Waller, M.L. (1988): "Determinants of Liquidity Costs in Commodity Futures Markets," *Review of Futures Markets*, 7:110–126.