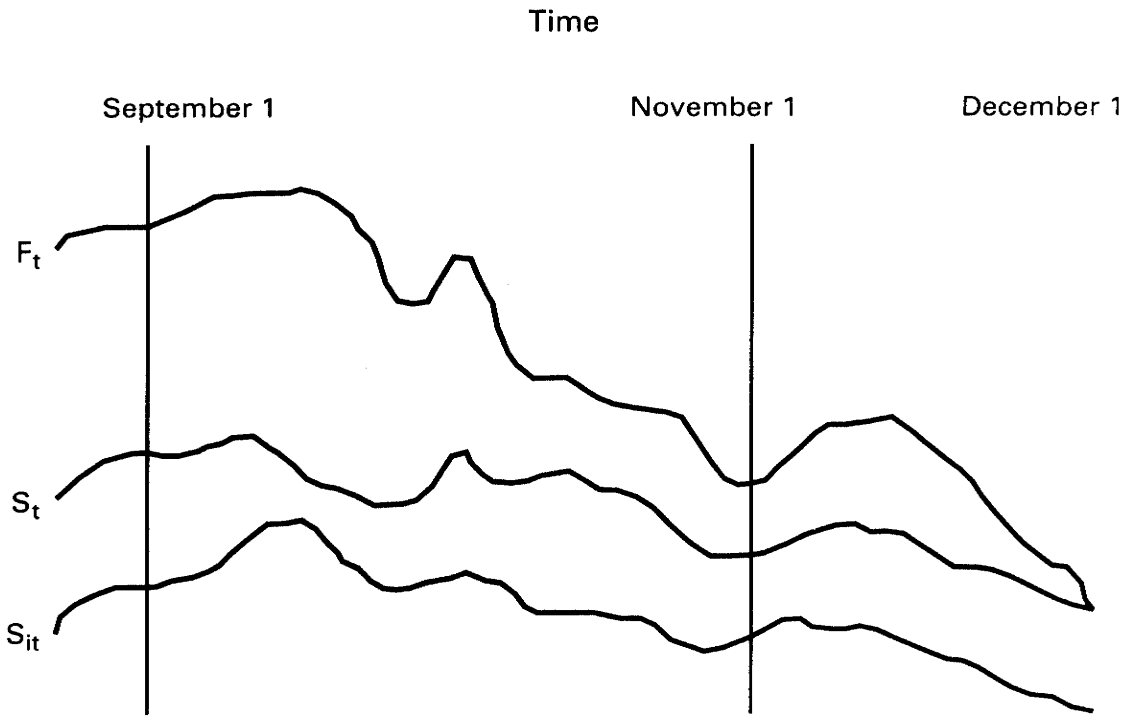
4 HEDGING WITH FUTURES CONTRACTS

A *hedger* uses the futures markets to reduce or eliminate the risk of adverse price fluctuations in the cash commodity. The *short hedger* sells short in the futures market against a long position in the underlying commodity. A typical short hedger is someone who, in the normal course of business, holds inventory of an underlying commodity—a stock portfolio manager, for example. The *long hedger* is long the futures contract and is short the underlying commodity. A short position in the underlying commodity means that the hedger has a fixed-price future commitment to deliver the underlying commodity or something highly correlated in price with the underlying commodity. An example of a long hedger is an exporter who has promised to deliver wheat at a fixed price but does not yet possess the wheat. The exporter buys a wheat futures contract to lock in the price at the time of delivery and then goes about acquiring the wheat for delivery. Detailed hedging examples are provided in later chapters when particular cash markets are examined. In this chapter, the basic principles of hedging are discussed.

4.1 A TRADITIONAL HEDGE

The hedger is concerned with unexpected changes in the price of the underlying commodity in which the hedger has a position. Figure 4.1 illustrates such unexpected changes in prices. Figure 4.1 is similar to Figure 3.1 except that a particular path of futures, spot, and cash prices is plotted. The path deviates from the expected path illustrated in Figure 3.1 because the futures price falls. As a result, spot and cash prices also fall. If traders had anticipated a fall in futures prices at time t, however, the futures price would have been lower at that time. Hedgers trade in

FIGURE 4.1 Paths of Spot and Futures Prices



The lines in this figure represent one possible path of *F*, *S*, and S_i through time.

the futures market to protect against losses from unexpected price movements in order that they may concentrate on their principal business activity, which may be, for example, processing a commodity or managing a security portfolio.

A hedger with a position in an underlying commodity wants to take a position in the futures market that guards against a price decline of the type shown in Figure 4.1. Hedging is effective if the cash price of the underlying commodity moves in the same way as the futures contract on the commodity. Another way of saying the same thing is that there is no unexpected change in the basis, $F_t - S_t$.

Table 4.1 contains a simple numerical illustration of a short hedge for someone in the business of storing wheat. In Table 4.1, we assume that the storer buys a bushel of wheat on September 1 and hedges it by selling a December 1 futures contract. On November 1, the inventory is sold and the hedge is lifted by an offsetting purchase of futures. Table 4.1 considers the value of this hedge portfolio at the two points in time illustrated by the two vertical lines in Figure 4.1.

On September 1, the bushel of wheat is acquired at \$3.00, and a December futures contract on the bushel of wheat is sold at \$3.09. We assume the futures price obeys the basis relation (3.2) developed in Chapter 3. We also assume that carrying costs (including a normal profit for the storer) are \$.03 per bushel per month at the time the hedge is established, or \$.09 for the three-month period.

TABLE 4.1 Profit results from a short hedge, assuming constant basis per month.

	Cash Market		December Futures	
Date	Transaction	Price	Transaction	Price
September 1	Buy 1 bushel at	3.00	Sell futures at	3.09
November 1	Sell 1 bushel at	2.70	Buy futures at	2.73
Gain		−0.30		0.36
Net gain				0.06
Net gain less storage costs of $.03 per month				0.00

After two months, on November 1, wheat prices have fallen dramatically and the wheat is sold at $2.70, causing a loss of $.30 in the cash market. Futures prices, however, have also fallen to $2.73, resulting in a gain of $.36. The difference between the futures price and the cash price has narrowed from $.09 to $.03, but the basis per month remains at $.03 since just one month remains to expiration of the futures contract. Since the hedger was short in the futures market, the gain there of $.36 more than offsets the loss of $.30 in the cash market. This gain reflects the fact that the total basis, $F_t - S_t$, narrows as one approaches maturity to reflect the reduced time over which the commodity must be carried. The hedger, however, carried the underlying commodity for two months and incurred storage costs, which we have assumed to be $.06 per bushel. As a result the $.06 net gain from changes in futures and cash prices is offset by the $.06 cost of carrying the underlying commodity. Table 4.1 illustrates the case in which the hedge is fully effective in the sense that the hedger suffers no losses and makes no gains.

4.2 BASIS RISK

The hedge is not fully effective if the difference between the futures price and the price of the underlying commodity does not converge smoothly at the carrying cost rate during the life of the futures. Basis risk arises if the difference, $F_t - S_t$, deviates from a constant basis per month. In general, if the basis unexpectedly widens (or "weakens"), the short hedger loses. If the basis unexpectedly narrows (or "strengthens"), the short hedger gains. The total gain depends on the carrying costs paid by the hedger. If carrying costs are not fixed, gains from the hedge may be offset by higher storage costs, and losses may be offset by lower storage costs.

These points are illustrated in Table 4.2 by assuming alternative futures prices on November 1. Suppose that all things are identical to Table 4.1, except that the futures price falls only to $2.75. In other words, by November 1, the basis has weakened to a level of $.05 per month. In that case the hedger gains only $.04,

TABLE 4.2 Profit results from a short hedge with basis risk.

	Cash Market		December Futures		
Date	Transaction	Price	Transaction	Alternative Prices	
September 1	Buy 1 bushel at	3.00	Sell futures at	3.09	3.09
November 1	Sell 1 bushel at	2.70	Buy futures at	2.75	2.71
Gain		–0.30		0.34	0.38
Net gain				0.04	0.08
Net gain less storage costs of $.03 per month				–0.02	0.02

which is not sufficient to cover storage costs for the preceding two months. If the hedger holds a commodity deliverable against the futures contract, however, the hedger can choose not to sell the commodity on November 1. Instead, the hedger can continue to hold the commodity until December 1 and deliver it against the futures contract. In the case of deliverable commodities, convergence of futures and cash prices guarantees a gain of $.09, which equals the basis when the hedge was established.

If storage costs for the remaining month remain at $.03 a bushel, an excess profit of $.02 will be earned in the last month, which is sufficient to offset the loss of $.02 in the first two months. On the other hand, if storage costs rise to $.05, as is implied by the basis on November 1, the loss on November 1 is unavoidable. The loss arises not from the ineffectiveness of the hedge, but rather from the hedger's failure to lock-in storage costs over the entire three-month period.

This is just another way of saying that the holder of inventory faces two sources of risk: (a) fluctuations in the price of the commodity and (b) fluctuations in the cost of holding the commodity. Futures markets may be perfectly effective in hedging the first risk, but ineffective in hedging the second.

The second case illustrated in Table 4.2 is a decline in the futures price to $2.71, which implies a narrowing or strengthening of the basis. In this case, there is a net gain of $.08 which more than covers the storage costs of $.06 in the first two months. The basis has changed in a favorable direction in this example. As long as the hedger has not contracted for storage facilities in the last month, this gain is real. However, if the hedger has borrowed funds and paid for a grain elevator to carry wheat for the entire three month period, a storage cost will be incurred in the third month whether or not wheat is stored. In that case the gain is not real. If the storage cost of $.03 will be incurred anyway, it pays to hold the wheat for an additional month to earn the basis of $.01, which offsets in part the storage cost of $.03.

The above illustrations and the implications listed below are predicated on the assumption that *the hedger has a position in the commodity underlying the futures contract.* Hedging non-deliverable commodities is discussed in the next section. A summary of the implications of the illustrations in this section (that is, examples where the hedger holds the deliverable commodity and hedges using the futures) follows under two headings:

Commodity Price Risk

Hedging with futures contracts can eliminate commodity price risk.

Carrying Cost Risk

a. Failing to lock in the carrying costs of the commodity over the life of the futures contract causes the short hedger to incur the risk that the costs will increase above the planned or expected amount. Conversely, the short hedger will gain if carrying costs fall below the expected amount.
b. Locking in the carrying costs eliminates the basis risk from the hedge. In terms of Tables 4.1 and 4.2, if the hedger can guarantee storage costs of $.03 per month, and if the underlying wheat is deliverable on December 1 at a futures price of $3.09, a net gain of $0.00 will be earned. There is no risk in this (unrealistic) example because all prices are preset on September 1.
c. Establishing the costs of carrying a commodity in advance may not be possible. As a result, holders of the commodity are subject to some amount of basis risk arising from uncertain carrying costs.
d. The only source of basis risk when the commodity is deliverable against the futures contract is carrying cost risk.

4.3 HEDGING NON-DELIVERABLE COMMODITIES

In addition to carrying cost risk, the hedger also faces commodity price risk when the commodity being hedged is not deliverable against the futures contract. If the commodity being hedged is non-deliverable, commodity price risk cannot be fully eliminated because the cash price and the futures price need not converge at the maturity of the futures contract. Figures 4.1 and 3.1 illustrate this point by the gap between S_i and F at maturity. In effect, on September 1, the hedger buys the underlying commodity and enters into a futures contract to sell another (albeit related) commodity. It is known on September 1 that there is no guarantee that the bushel of wheat can be sold for $3.09 if held to maturity.

The term *cross-hedging* is used to describe situations in which futures contracts are used to hedge non-deliverable commodities. In practice, most hedging is cross-hedging. A storer of wheat in Oklahoma may possess a deliverable grade of wheat, but the transportation costs of getting the wheat to a deliverable location are sufficiently high to make the wheat non-deliverable against the futures contract. This means that the price of wheat in Oklahoma and in Chicago do not converge. Furthermore, the difference between the price of wheat in Oklahoma and in Chicago may be uncertain because of uncertain demand conditions in the two markets. Other examples of cross-hedging include using (a) silver futures to hedge a position in platinum, (b) stock index futures to hedge a position in a single stock, (c) Treasury bond futures to hedge a position in corporate bonds, and (d) Chicago wheat futures to hedge wheat stored in France.

When cross-hedging is undertaken, it is unlikely that unexpected changes in the value of the underlying position will be matched in magnitude by changes in the futures price. For example, a 1-percent change in platinum prices might be accompanied by a 1.5-percent change in silver prices. To the extent that such relations are stable over time, one can adjust the size of the hedge to provide a better cross-hedge. We now turn to a method of determining the optimal amount to hedge and of assessing hedging effectiveness.

4.4 OPTIMAL HEDGING—MEAN/VARIANCE APPROACH

An individual wanting to reduce the price risk of a position in an underlying commodity must choose the size of the position to take in the futures contract. In the preceding examples, we assumed that the futures market position is of the same size as the cash market position, but this is not always optimal. In particular, when the futures price changes by less than the cash price, a larger futures position than cash position is optimal. On the other hand, if the futures price changes by more than the cash price, a smaller futures than cash position is optimal. To determine the optimal number of futures contracts to use, we develop a formal model.

Notation

The notation used in the development of the optimal hedge ratio is as follows:

S_0 = initial cash price. (This variable corresponds to the first entry in the cash market column of Table 4.1, that is, the value \$3.00.) We are omitting the subscript i indicating the cash price.

F_0 = initial futures price. (This value corresponds to the first entry in the futures market column of Table 4.1, that is, the value \$3.09.)

$\tilde{S}_T$ = uncertain cash price at future time T. (This variable corresponds to the value on November 1 in the cash column of Table 4.1, which is only one of many possible outcomes at that time.)

$\tilde{F}_T$ = uncertain futures price at future time T. (This variable corresponds to the value on November 1 in the futures column of Table 4.1, which is only one of many possible outcomes at that time.)

n_S = number of units of the cash commodity held. (n_S is positive for long positions and is negative for short positions.)

n_F = number of futures contracts held. (n_F is positive for long positions and is negative for short positions.)

Hedge Portfolio Profit

The uncertain gain, $\tilde{\pi}_h$, of the hedger who holds n_S units of the commodity and hedges using n_F futures contracts is

$$\tilde{\pi}_h = (\tilde{S}_T - S_0)n_S + (\tilde{F}_T - F_0)n_F. \tag{4.1}$$

The term $\tilde{S}_T - S_0$ is the (random) price change per unit of commodity over the life of the hedge, and the term $\tilde{F}_T - F_0$ is the (random) price change of the futures contract over the life of the hedge.

The expected gain to the hedger over the life of the hedge is

$$E(\tilde{\pi}_h) = [E(\tilde{S}_T) - S_0]n_S + [E(\tilde{F}_T) - F_0]n_F. \tag{4.2}$$

Expression (4.2) provides a convenient way of illustrating two ideas. First, if we assume that the current futures price is an unbiased predictor of the futures price at time T as implied by Figure 4.1, that is, if

$$F_0 = E(\tilde{F}_T), \tag{4.3}$$

the last term of equation (4.2) drops out. The expected profit on the hedge portfolio depends only on the expected change in the cash price. In a carrying cost market, the difference between the current cash price and the expected future cash price represents storage costs. Second, if the commodity is deliverable and is held to maturity, convergence ensures that

$$E(\tilde{S}_T) = E(\tilde{F}_T). \tag{4.4}$$

Combining (4.2), (4.3), and (4.4) implies

$$E(\tilde{\pi}_h) = (F_0 - S_0)n_S. \tag{4.5}$$

In other words, the expected profit on the hedge is directly proportional to the initial basis as indicated in Table 4.1. In general, the overall outcome depends on the carrying costs incurred by the hedger which are not incorporated in equation (4.1). Even if the commodity is not deliverable, the expected gain equals the basis, as long as the expected difference between the spot price and the cash price in Figure 3.1 remains constant.

Realized futures and cash prices deviate from expected prices; therefore, realized gains are greater than or less than expected gains. This is true, in particular, for cross-hedging in which case the opportunity to deliver the underlying commodity is not available. The optimal cross-hedge is the hedge that minimizes the deviation from the expected gain. We turn now to a derivation of the optimal hedge amount based on this criterion.

Optimal Hedge Ratio

To derive the optimal number of futures contracts to sell, first write equation (4.1) in terms of price changes per unit of the underlying commodity, that is,

$$\frac{\tilde{\pi}_h}{n_S} = (\tilde{S}_T - S_0) + (\tilde{F}_T - F_0)\frac{n_F}{n_S}. \tag{4.6}$$

Now define price change terms, $\tilde{\Delta}_S \equiv \tilde{S}_T - S_0$ and $\tilde{\Delta}_F \equiv \tilde{F}_T - F_0$, and substitute these terms into (4.6),

$$\frac{\tilde{\pi}_h}{n_S} = \tilde{\Delta}_S + h\tilde{\Delta}_F. \tag{4.7}$$

The hedge ratio, $h \equiv \frac{n_F}{n_S}$, is the number of futures contracts per unit of the underlying commodity.

Since a hedger is concerned with minimizing risk,[1] use (4.6) to write the variance of the hedge portfolio profit,

$$\sigma_h^2 = \sigma_S^2 + h^2\sigma_F^2 + 2h\sigma_{SF}, \tag{4.8}$$

where σ_h^2 is the variance of the hedge portfolio profit per unit of commodity; σ_S^2 is the variance of cash price change; σ_F^2 is the variance of futures price change; and σ_{SF} is the covariance between the cash and futures price changes. The value of h that minimizes σ_h^2 is found by taking the derivative of (4.8) with respect to h and setting it equal to zero:

$$\frac{d\sigma_h^2}{dh} = 2h^*\sigma_F^2 + 2\sigma_{SF} = 0.$$

Solving for h^*, the optimal hedge ratio is

$$h^* = -\frac{\sigma_{SF}}{\sigma_F^2}. \tag{4.9}$$

The optimal hedge ratio, h^*, thus depends on the covariance between the cash and futures price changes relative to the variance of the futures price change. It is interesting to note that the expression for the optimal hedge ratio, $\sigma_{S,F}/\sigma_F^2$, is the slope coefficient in an ordinary least squares (OLS) regression of the cash price change, $\tilde{\Delta}_S$, on the futures price change, $\tilde{\Delta}_F$. The OLS regression approach to estimating the optimal hedge ratio is described next.

---

[1]The mean-variance hedge ratio framework developed here is similar to that developed in Ederington (1979). In this framework, the hedger is assumed to be interested only in minimizing his risk exposure. In a more general framework, the hedger would be allowed to consider not only price change variance but also expected price change in determining the optimal hedge ratio.

4.5 OPTIMAL HEDGING—OLS REGRESSION APPROACH

Ordinary least squares regression[2] provides an alternative way to derive the optimal hedge ratio. Consider the following regression equation:

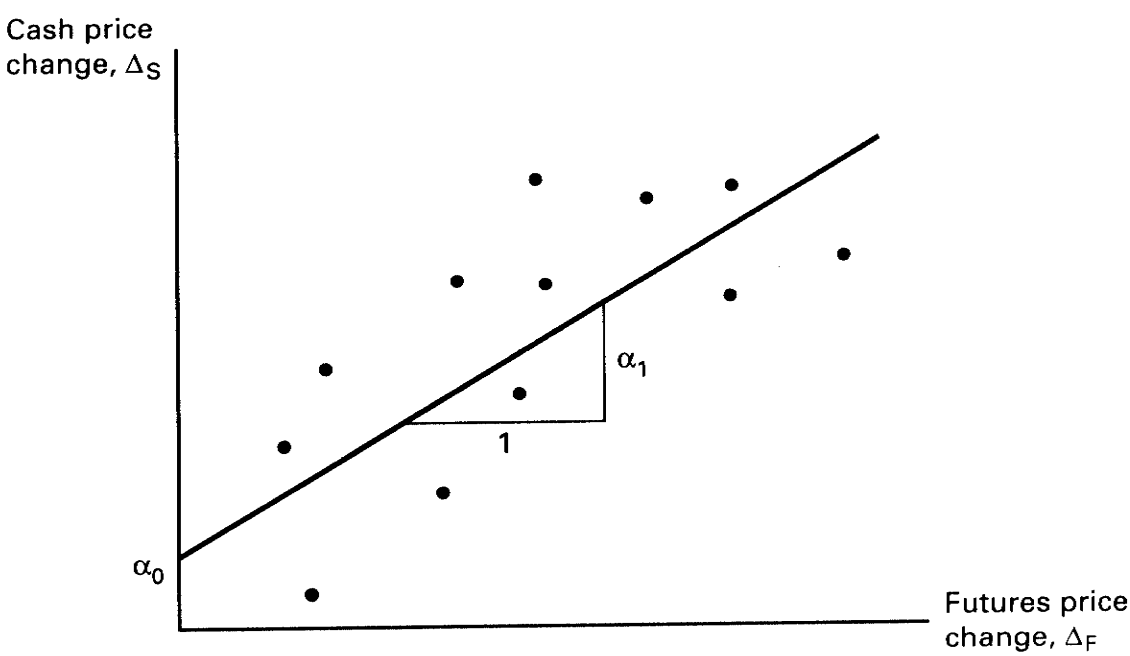
$$\tilde{\Delta}_S = \alpha_0 + \alpha_1 \tilde{\Delta}_F + \tilde{\epsilon}, \tag{4.10}$$

where α_0 and α_1 are regression parameters, $\tilde{\epsilon}$ is a random disturbance term with $E(\tilde{\epsilon}) = 0$. Equation (4.10), which is plotted in Figure 4.2, shows that the cash price change, $\tilde{\Delta}_S$, has three components: (a) a constant non-random price change component, α_0, the intercept in the figure; (b) a random component that is systematically related to the futures price change, $\alpha_1\tilde{\Delta}_F$, and (c) a unique random component, $\tilde{\epsilon}$, that is uncorrelated with the futures price change and is represented by the vertical distances between the line and points off the line in Figure 4.2.

Expected Return

The intercept term, α_0, captures any expected change in the cash price unaccompanied by an expected change in the futures price. From Figure 3.1, we know that, if $E(\tilde{\Delta}_F) = 0$, the expected cash price change equals the basis, that is, $E(\tilde{\Delta}_S) = E(\tilde{S}_T) - S_0 = F_0 - S_0$. The regression model states that the expected cash price

FIGURE 4.2 Cash Price Change Versus Futures Price Change



[2]An excellent treatment of ordinary least regression is provided in Pindyck and Rubinfeld (1981, Chs. 1–6).

change equals α_0 under the assumption that $E(\tilde{\Delta}_F) = 0$ (and that $E(\tilde{\epsilon}) = 0$), thus the intercept term in (4.10) represents the basis. The basis, in turn, reflects the storage costs which a storer of the assets must recover by price appreciation. The term, $\alpha_1\tilde{\Delta}_F$, reflects the fact that random changes in the futures price will be reflected in the cash price according to the slope coefficient, α_1. The term $\tilde{\epsilon}$ reflects basis risk, which arises from the fact that certain random changes in $\tilde{\Delta}_S$ are unique to the cash commodity and uncorrelated with the futures price change.

Optimal Hedge Ratio

The original expression for hedge portfolio profit (4.7) can be rewritten by substituting the regression equation (4.10) for $\tilde{\Delta}_S$, that is,

$$\frac{\tilde{\pi}_h}{n_S} = \alpha_0 + \alpha_1\tilde{\Delta}_F + \tilde{\epsilon} + h\tilde{\Delta}_F$$
$$= \alpha_0 + (\alpha_1 + h)\tilde{\Delta}_F + \tilde{\epsilon}. \quad (4.11)$$

Equation (4.11) shows clearly that the profit on the hedge portfolio, $\tilde{\Delta}_h$, can be made independent of movements in cash and futures prices by setting $h = -\alpha_1$. If $\alpha_1 = 1$, a one-dollar change in the cash price is matched by a one-dollar change in the futures price. This implies that futures and cash prices move in tandem. In this case, the optimal hedge is $h = -1$, or a 100 percent hedge. If $\alpha_1 = 0.0$, futures and cash prices are unrelated and there is no point in hedging and the optimal hedge ratio, h, is zero.

Hedging Effectiveness

A hedge is fully effective only if futures and cash price changes are perfectly correlated. This means that the random error term, $\tilde{\epsilon}$, in (4.10) is always zero. Although it may be optimal to hedge 100 percent, the hedge may not be fully effective because deviations from that average relation, represented by points lying off the line in Figure 4.1, may arise.

In order to measure the effectiveness of a hedge, first measure the ineffectiveness of a hedge. The ineffectiveness of a hedge is measured by the ratio of the variance of the price change of the hedged portfolio (σ_h^2) to the variance of the price change of the unhedged portfolio (σ_h^2 when $h = 0$). From equation (4.8), the variance of the price change without hedging is σ_S^2. From (4.11), the variance of the price change for the optimally hedged portfolio ($h = -\alpha_1$) is σ_ϵ^2, that is, the variance of the residual term, ϵ, in the regression model. The ratio measuring the ineffectiveness of the hedge is therefore $\sigma_\epsilon^2/\sigma_S^2$; this ratio can vary between 0 and 1. Since the effectiveness of the hedge is merely the complement of the ineffectiveness, the effectiveness of the hedge can be measured as

$$\rho^2 = 1 - \frac{\sigma_\epsilon^2}{\sigma_S^2}. \quad (4.12)$$

Coincidentally, expression (4.12) is the definition of the *adjusted R-squared* for the OLS regression (4.10). If the independent variable, $\tilde{\Delta}_F$ explains 100 percent of the variance in $\tilde{\Delta}_S$, $\sigma_\epsilon^2 = 0$ and $\rho^2 = 1$. In this case, the hedge is perfectly effective. If the independent variable $\tilde{\Delta}_F$ explains none of the variance in $\tilde{\Delta}_S$, $\rho^2 = 0$. In this case, $\sigma_\epsilon^2 = \sigma_S^2$, and the hedge is perfectly ineffective.

If $0 < \rho^2 < 1$, hedging is only partially effective in the sense that some of the movements in the cash price are reflected in futures prices, but some movements are unique. Hedging is effective in eliminating the systematic movements also reflected in futures price. Hedging is not effective in eliminating unique price risk that shows up in the random term $\tilde{\epsilon}$.

Hedging with Several Futures Contracts

The OLS regression approach can be generalized very easily to handle cases where two or more different futures contracts are used to hedge the cash position. The value of a corporate bond portfolio, for example, is sensitive to both interest rate risk and stock market risk. To hedge the value of such a portfolio, hedges against movements in both interest rates and the stock market level are required. Treasury bond futures and the S&P 500 index futures contracts are probably best suited for simultaneously hedging the two types of risk exposure of this portfolio.

To hedge against multiple sources of price risk, the cash price change is regressed on the price changes of several futures contracts in the form,

$$\tilde{\Delta}_S = \alpha_0 + \alpha_1 \tilde{\Delta}_{F,1} + \alpha_2 \tilde{\Delta}_{F,2} + \cdots + \alpha_n \tilde{\Delta}_{F,n} + \tilde{\epsilon}. \quad \textbf{(4.13)}$$

The estimated regression coefficients $\hat{\alpha}_1$ through $\hat{\alpha}_n$ are the respective hedge ratios for each of the n futures contracts. To hedge against all sources of risk, set $h_i = -\hat{\alpha}_i$ for all futures contracts. To hedge against only selected risk exposures, set $h_i = -\hat{\alpha}_i$ for the futures contracts corresponding to the selected risks.

4.6 ESTIMATING THE HEDGE RATIO

In this section, two illustrations of the optimal hedge ratio framework are provided. In the first, a stock portfolio manager uses stock index futures to hedge price risk, and, in the second, the value of a corporate bond is hedged using stock index and Treasury bond futures. Before describing the two applications, however, some subtle issues regarding price change measurement and regression estimation must be discussed.

Price Changes, Price Change Intervals, and Other Issues

An important decision facing the analyst who is attempting to estimate the optimal hedge ratio is the decision about the length of the time interval over which price changes should be measured. The time-series regression model,

$$\tilde{\Delta}_{S,t} = \alpha_0 + \alpha_1 \tilde{\Delta}_{F,t} + \tilde{\epsilon}_t, \quad \textbf{(4.14)}$$

gives some indirect guidance on this matter. The error term, ϵ_t, in (4.14) is governed by the following assumptions: $E(\epsilon_t) = 0$, $E(\epsilon_t \Delta_{F,t}) = 0$, $E(\epsilon_t^2) = \sigma_\epsilon^2$, and $E(\epsilon_t \epsilon_{t-1}) = 0$. The price change interval should be chosen such that none of these regression assumptions is violated.

The most frequently used price change intervals are daily, weekly, and biweekly. Holding the length of the estimation period constant (say, one year of data), one would imagine that the more frequent the price observations, the more information that is being gathered about the covariability of cash and futures price changes. But, very frequent price observations also give rise to other problems. For example, transaction prices are generally either at bid or ask levels. The shorter the time interval, the more the random movement between bid and ask prices contributes to price change variability and the less reliable the regression results become. This bid/ask price effect introduces negative serial correlation in the security price change series and an errors-in-the-variables problem in estimating (4.14).

A second problem with price changes measured over short time intervals is that if the cash commodity and the futures contract are not traded with equal frequency, the price changes of the two instruments may not reflect the same set of market information. This problem may manifest itself through non-zero serial correlation in the error term, that is, $E(\epsilon_t \epsilon_{t-1}) \neq 0$.

A third problem has to do with seasonality in security price changes. French (1980) and Gibbons and Hess (1981), for example, have documented a day-of-the-week effect in stock returns and returns of certain Treasury instruments. This day-of-the-week seasonality will cause the homoscedastic error term assumption, $E(\epsilon_t^2) = \sigma_\epsilon^2$, to be violated when daily price changes are used in the estimation.

To illustrate the effects of different price change interval measurement assumptions, we estimate the regression (4.14) using daily, weekly, and biweekly price changes for the S&P 500 index and the S&P 500 index futures contracts during the calendar year 1989. In 1989, there were 252 trading days, so 251 daily price changes are computed. The 251 days produce 51 weekly price changes, and 25 biweekly price changes. The weekly and biweekly price changes are measured from Wednesday to Wednesday because there are fewer Wednesday holidays than holidays on other days of the week. Partial weeks at the beginning and the end of the daily price change series are not used. The futures price changes are for the nearby futures contract. On the expiration of the nearby contract, the futures price change series is spliced from the nearby contract to the next nearby contract.[3] The regression results are reported in Table 4.3.

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[3]For 1989, the prices of the March 1989, June 1989, September 1989, December 1989, and March 1990 S&P 500 index futures contracts are used to generate the futures price change series. The March 1989 contract is used until its last day of trading on March 16, 1989. On that day, the prices of both the March 1989 and the June 1989 contract are recorded. The March contract price is used in combination with its previous day's price to compute the March 16 futures price change. The June contract price is used in combination with its price on the following day to compute the futures price change for March 17. This "splicing" procedure allows the futures price change series to be continuous throughout the year.

TABLE 4.3 Summary of hedge ratio coefficient estimates using daily, weekly, and biweekly S&P 500 index and S&P 500 index futures price changes[a] during the calendar year 1989.

$$\tilde{\Delta}_{S,t} = \alpha_0 + \alpha_1 \tilde{\Delta}_{F,t} + \tilde{\epsilon}_t$$

Interval	n	$\hat{\alpha}_1$	$s(\hat{\alpha}_1)^b$	95 Percent Confidence Interval Lower	Upper	Range	$\bar{R}^2$
Daily	251	0.8034	0.0226	0.7589	0.8478	0.0889	0.8352
Weekly	51	0.9914	0.0163	0.9586	1.0241	0.0673	0.9867
Biweekly	25	1.0013	0.0170	0.9662	1.0365	0.0703	0.9928

a. The price changes of the S&P 500 index and the S&P 500 futures contract are computed as $\Delta_{S,t} = S_t - S_{t-1}$ and $\Delta_{F,t} = F_t - F_{t-1}$, respectively.

b. s(·) is the estimated standard error of the regression coefficient.

A number of interesting results emerge from Table 4.3. The first is that the slope coefficient estimated using daily price changes, 0.8034, is dramatically different from those using weekly and biweekly price changes, 0.9914 and 1.0013, respectively. This difference results from the problems noted above. For example, the bid/ask price effect introduces an errors-in-the-variables problem in the daily price change regression, which tends to bias the estimated coefficients downward.

Another interesting comparison in Table 4.3 is of the confidence intervals. Holding other factors constant, the range of the confidence interval (the standard error) on α_1 *should* get larger as the price change interval increases from daily to weekly to biweekly because more and more information is being lost. However, that is not the pattern that appears in Table 4.3. The standard error and the confidence interval range is smaller for the weekly and biweekly price change regressions than for the daily price change regression. Again, this result reflects the problems associated with using daily price changes.

The comparison of the weekly and biweekly results also favors the use of weekly data in the estimation procedure. The coefficient magnitudes are very close, and yet the standard error and confidence interval range are smaller for the weekly regression. The lower standard error reflects the fact that twice as much price change information is impounded in the weekly regression as in the biweekly regression.

Related to the selection of the appropriate price change interval is the time horizon of the hedge. The hedge framework developed in this chapter is for a single period. In principle, when the hedge ratio is estimated through the regression analysis, the estimation of the hedge ratio is independent of the distance between price observations used in the regression. The intercept term, however, will grow larger as the length between price observations increases because it is an estimate of the basis between the futures and cash over the length of the observation interval.

Finally, to end the discussion, it is also important to note that the parameter estimate $\hat{\alpha}_1$ is produced from a regression on historical data and that the hedge that we are constructing is for a future period (i.e., estimates of α_1 are *ex post* but hedging decisions are *ex ante*). In applying this procedure, we are implicitly invoking an assumption of stationarity in the relation between cash and futures price changes. *A priori,* we must be comfortable that such an assumption is reasonable.

Hedging with a Single Futures Contract

To illustrate how the optimal hedge for a stock portfolio may be determined, consider a portfolio manager who has \$50 million in a stock portfolio similar in composition to the S&P 500 index at the end of 1989. Fearing that the stock market will fall over the next two months, the manager wants to hedge his price risk by selling S&P 500 futures contracts. How many futures contracts should be sold?

The regression results reported in Table 4.3 are useful in estimating the optimal number of contracts to sell. The estimated slope coefficient (hedge ratio) from the regression of weekly, S&P 500 index price changes on the price changes of the S&P 500 index futures contract is 0.9914. In other words, a one-dollar change in the futures price elicits a 0.9914 change in the price of the stock index portfolio, or, alternatively, the optimal number of futures contracts to sell for each unit invested in the stock portfolio is 0.9914. To determine the number of units invested in the stock portfolio, divide the portfolio value by the S&P 500 index level. At the end of 1989, the S&P 500 index was at 353.40. The number of units of the stock portfolio is therefore 141,482.74. In addition, the S&P 500 futures contract is denominated as 500 times the index level, so the number of units of the stock portfolio expressed in the denomination of the futures contract is 141,482.74/500 = 282.97. If the optimal hedge ratio were one, we should sell 282.97 futures contracts to hedge the \$50 million stock portfolio. Since the optimal hedge ratio is 0.9914, however, the optimal number of contracts to sell is (0.9914)(282.97) = 280.54.

It is worthwhile to emphasize that the regression produces only an *estimate* of the optimal hedge ratio. In this case, the relatively high coefficient of determination of 0.9867 indicates that the estimate and the effectiveness of the hedge appear to be quite good. Nevertheless, there is still potential error in the best guess that the optimal hedge ratio is 0.9914. The 95 percent confidence interval says that the true ratio is somewhere between

$$0.9586 \leq \alpha_1 \leq 1.0241.$$

This range implies that we are 95 percent confident that the optimal number of futures contracts to sell is between 271.26 and 289.79.

Hedging with Two Futures Contracts

To illustrate hedging with two futures contracts, consider hedging \$10,000,000 worth of Mobil Oil's 8½ percent coupon bond maturing in the year 2001. Corporate bonds have both long-term interest rate and stock market exposure, so the weekly price changes of Mobil's bond are regressed on the price changes of the CBT's T-bond futures contract and the CME's S&P 500 index futures contract for the year 1989. Again, we assume that the hedge is being formed at the end of December 1989. The regression results are reported in Table 4.4. Also reported are the regression results when the T-bond futures and the S&P 500 futures price changes are used separately as independent variables.

The results in Table 4.4 indicate that the T-bond futures contract explains more of the variance in the price changes of Mobil's bond than the S&P 500 futures

TABLE 4.4 Summary of hedge ratio coefficient estimates using weekly price changes[a] for Mobil Oil's 8 1/2 percent bond maturing in the year 2001 (MO), the CBT's Treasury bond futures contract (TBF), and the CME's S&P 500 futures contract (SPF) during the calendar year 1989.

$$\tilde{\Delta}_{MO,t} = \alpha_0 + \alpha_1 \tilde{\Delta}_{TBF,t} + \alpha_2 \tilde{\Delta}_{SPF,t} + \tilde{\epsilon}_t$$

n	$\hat{\alpha}_1$	$s(\hat{\alpha}_1)^b$	$t(\hat{\alpha}_1)^c$	$\hat{\alpha}_2$	$s(\hat{\alpha}_2)$	$t(\hat{\alpha}_2)$	$\bar{R}^2$
T-bond and S&P 500 futures:							
51	0.4473	0.0407	10.98	0.0159	0.0091	1.75	0.7651
T-bond futures only:							
51	0.4756	0.0382	12.46				0.7553
S&P 500 futures only:							
51				0.0554	0.0154	3.59	0.1920

a. Price changes are computed as $\Delta_{i,t} = I_t - I_{t-1}$, where I represents Mobil Oil's bond, the T-bond futures contract, or the S&P 500 futures contract.

b. s(·) is the estimated standard error of the regression coefficient.

c. t(·) is the t-value of the regression coefficent under the null hypothesis that $\alpha = 0$.

contract does. When the single variable regression results are considered, the hedging effectiveness of the T-bond futures contract is 0.7553 and the hedging effectiveness of the S&P 500 futures is 0.1920. But, if Mobil's bond has both interest rate and stock market exposures, the multiple regression results should be used when setting the hedge. In fact, when the multiple regression results are considered, the hedging effectiveness, 0.7651, is greater than either contract used by itself.

The optimal number of T-bond futures contracts to sell is 0.4473 and the optimal number of S&P 500 futures contracts to sell is 0.0159 for each dollar invested in the Mobil bonds. Again, the contract denominations must be taken into account. The T-bond futures has a denomination of $100,000, so the number of units of the spot commodity in terms of the T-bond futures is 100. The S&P 500 index level was at 353.40 at the end of 1989 and the denomination of the S&P 500 futures contract is 500 times the index, so the number of units of the spot commodity in terms of the S&P 500 futures is 56.59. Using the coefficient estimates reported in Table 4.4, the optimal number of T-bond futures to sell is $100 \times 0.4473 = 44.73$, and the optimal number of S&P 500 futures to sell is $56.59 \times 0.0159 = 0.90$.

4.7 SUMMARY

This chapter begins with an explanation of a traditional short hedge in which the owner of a commodity takes a short position in futures to guard against a decline in the commodity price. A short hedger locks in the basis, $F_t - S_t$. Short hedgers face the risk that the basis may change through time. In the case of hedgers of deliverable commodities, basis risk represents the risk of an increase in the costs of holding the commodity. In the case of hedgers of commodities not deliverable against the futures contract (cross-hedgers), basis risk also represents the risk of changing relative prices of the commodity and futures contract.

A framework for hedging under basis risk is developed in this chapter under standard mean/variance portfolio analysis and also within an ordinary least squares regression framework. The optimal hedge ratio is defined, and measurement of hedging effectiveness is discussed. The optimal hedge framework is then applied in the context of stock portfolio and bond portfolio risk management, with careful consideration given to the nature of security price data and regression estimation.