12 OPTION TRADING STRATEGIES

In Chapter 10, we developed no-arbitrage pricing relations between options and the underlying commodity. In Chapter 11, we developed option valuation equations. In this chapter, essential elements of the last two chapters are used to describe and analyze option/commodity portfolio positions. We begin by using arbitrage principles to examine in detail the six basic option/commodity terminal profit diagrams. Long and short positions in the call, the put, and the underlying commodity are considered. Combinations of these positions are then used to create a wide array of portfolios with different breakeven commodity prices, maximum losses, and maximum gains. Synthetic positions, spread strategies, write strategies, and speculative strategies are considered. In the second section, the assumption of a lognormal price distribution is used to extend the profit diagram analysis. Probabilities of maximum losses and maximum gains are computed, as are expected profits for the various strategies. The third section discusses a methodology for simulating long-term option positions using short-term options. This methodology is particularly useful in markets where long-term options are not actively traded. In the fourth section, we discuss dynamic return/risk management. First, we discuss the expected return/risk tradeoffs created using naked options and options in combination with investment in the underlying commodity. We then focus on portfolio risk management over short intervals of time and show how the risk exposures of individual option positions can be aggregated across options to find portfolio risk exposures. We show that judicious selection of investments in options, futures, and the underlying commodity can effectively manage these risks.

12.1 TRADING STRATEGIES AND PROFIT DIAGRAMS

This section focuses on strategies where the option/commodity positions are held until option expiration. After stating the assumptions underlying our analysis, we present the six basic profit diagrams upon which the trading strategies are built. The diagrams are first used to reconfirm the put-call parity relation using conversion and reverse conversion arbitrage. We then focus on spread strategies, where offsetting positions are taken to reduce commodity price risk. Following that, option writing strategies, together with certain speculative strategies, are presented. The specific trading strategies provided here are intended only to be illustrative, as the number of possible trading strategies is limitless. The framework of analysis is sufficiently general, however, that the reader can analyze more complex portfolios with the tools presented.

Assumptions

The only new assumption made in this section is that all positions, including any position in the underlying commodity, are held until the options' expiration, T. As in the previous chapters, the cost of carrying the underlying commodity occurs at the rate b. A long position implies that the holder pays the carry cost, and a short position implies that the holder receives the carry cost. Also, as in previous chapters, the carry cost includes not only interest but also additional charges (receipts) from holding the commodity. The cost of carrying an option contract is only the riskless rate of interest, r. The initial price of the commodity is denoted by S and the terminal price is denoted by S_T. The initial call and put option prices are denoted as c and p, respectively. If the option is purchased, the purchase price is financed at rate r, and, if the option is sold, the sales proceeds are invested at rate r until the option's expiration. Unless otherwise noted, the call and the put are assumed to have the same exercise price. A profit occurs when a position earns more than the interest cost of the funds tied up in the position.

The analysis of each strategy proceeds in a stepwise fashion. First, we present the profit function at maturity, π_T, and then show the profit function in diagram form. We then summarize the strategy by describing the breakeven terminal commodity prices (i.e., where the strategy has zero profit at the option expiration), the maximum loss, and the maximum gain. The diagrams plot the zero profit position as a solid horizontal line and plot the profit function as another solid line. The intersection of the solid lines represents the zero profit position.

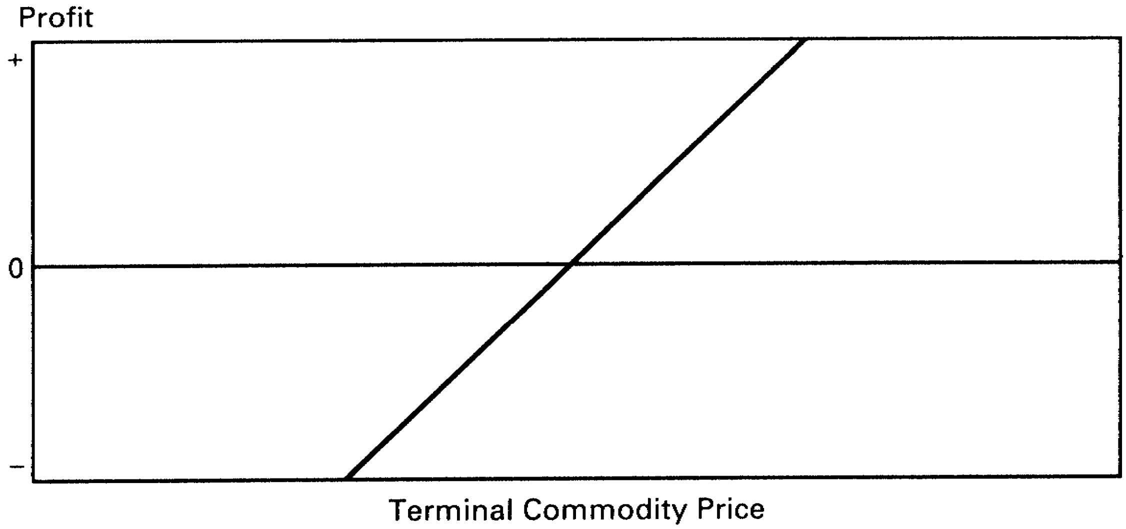
Six Basic Positions

The terminal profit functions of the six basic option/commodity positions are

1. *Long commodity:* Commodity is purchased and is carried at rate b until option expiration at time T.

$$\pi_T = S_T - Se^{bT} \tag{12.1a}$$

FIGURE 12.1a Trading Strategy: Long Commodity

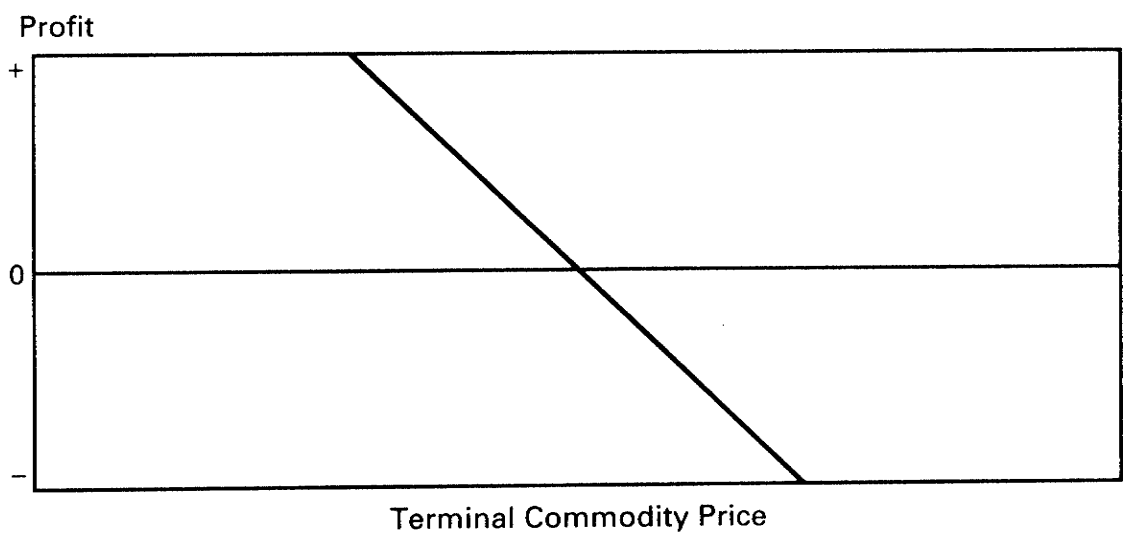


Breakeven point:	$S_T = Se^{bT}$
Maximum loss:	Se^{bT}, where S_T falls to zero
Maximum gain:	unlimited, where S_T rises without limit

2. *Short commodity:* Commodity is sold, and proceeds from sale earn rate b until option expiration at time T.

$$\pi_T = -S_T + Se^{bT} \tag{12.1b}$$

FIGURE 12.1b Trading Strategy: Short Commodity

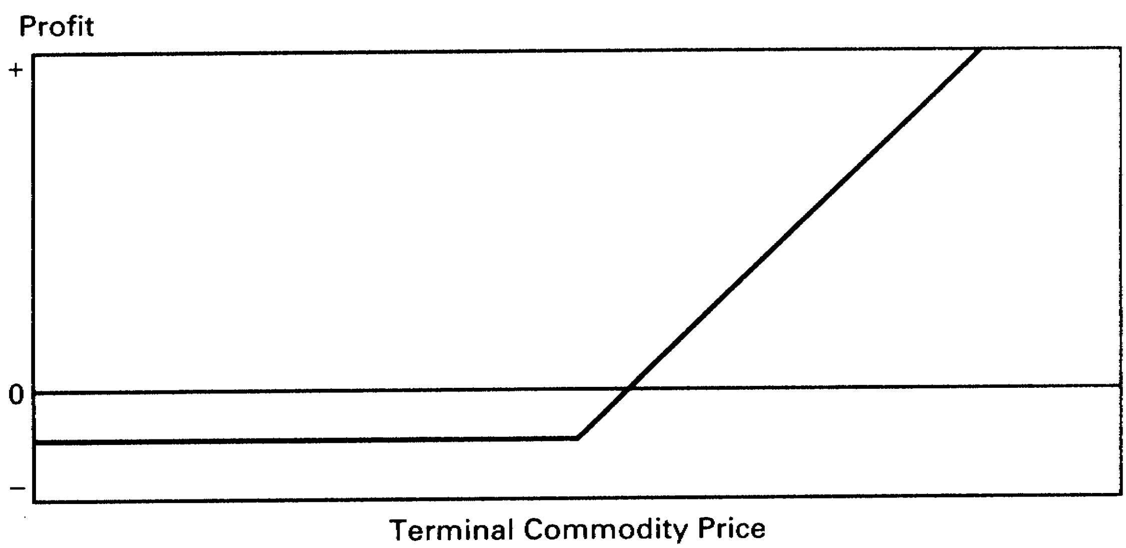


Breakeven point:	$S_T = Se^{bT}$
Maximum gain:	Se^{bT}, where S_T falls to zero
Maximum loss:	unlimited, where S_T rises without limit

3. *Long call:* Call option is purchased and carried at rate r until option expiration at time T.

$$\pi_T = \begin{cases} S_T - X - ce^{rT} & \text{if} \quad S_T > X \\ -ce^{rT} & \text{if} \quad S_T \leq X \end{cases} \tag{12.2a}$$

FIGURE 12.2a Trading Strategy: Long Call

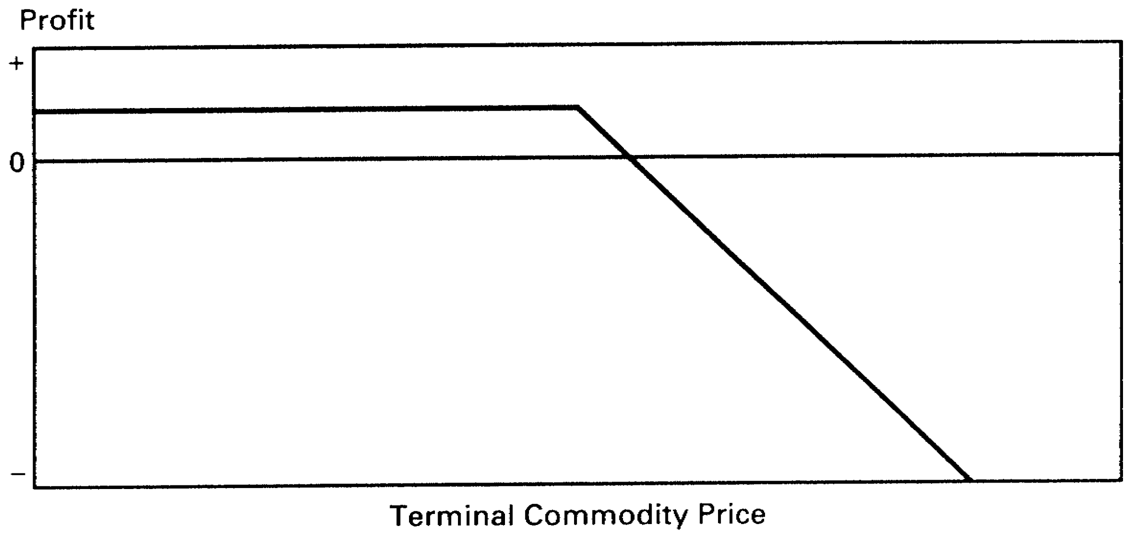


Breakeven point:	$S_T = X + ce^{rT}$
Maximum loss:	ce^{rT}, where $S_T < X$
Maximum gain:	unlimited, where S_T rises without limit

4. *Short call:* Call is sold, and proceeds from sale earn rate κ until option expiration at time T.

$$\pi_T = \begin{cases} -S_T + X + ce^{rT} & \text{if} \quad S_T > X \\ ce^{rT} & \text{if} \quad S_T \leq X \end{cases} \tag{12.2b}$$

FIGURE 12.2b Trading Strategy: Short Call

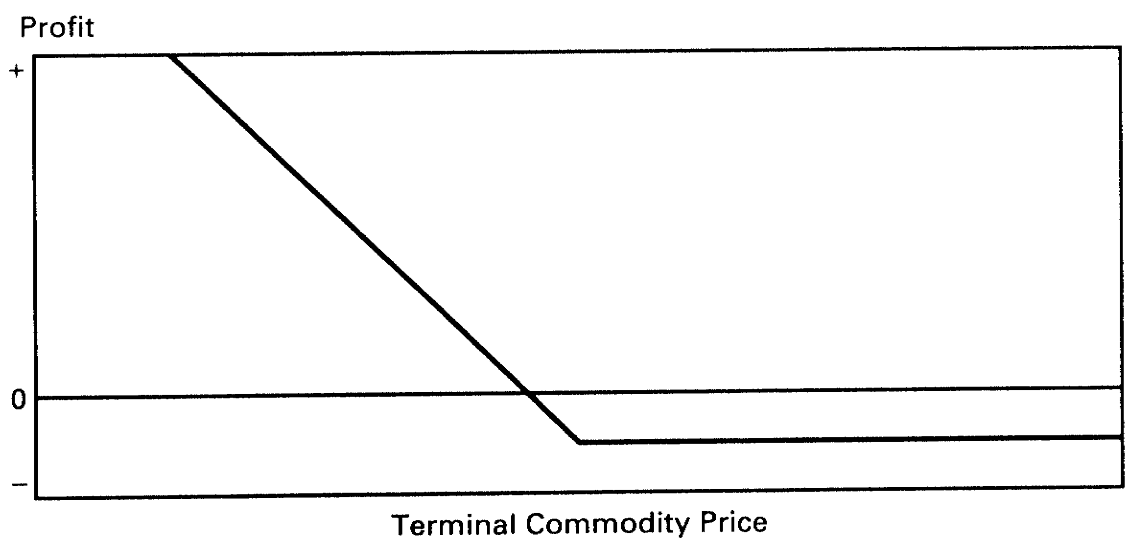


Breakeven point:	$S_T = X + ce^{rT}$
Maximum gain:	ce^{rT}, where $S_T < X$
Maximum loss:	unlimited, where S_T rises without limit

5. *Long put:* Put option is purchased and is carried at rate r to the option expiration at time T.

$$\pi_T = \begin{cases} -pe^{rT} & \text{if} \quad S_T > X \\ X - S_T - pe^{rT} & \text{if} \quad S_T \leq X \end{cases} \qquad \textbf{(12.3a)}$$

FIGURE 12.3a Trading Strategy: Long Put

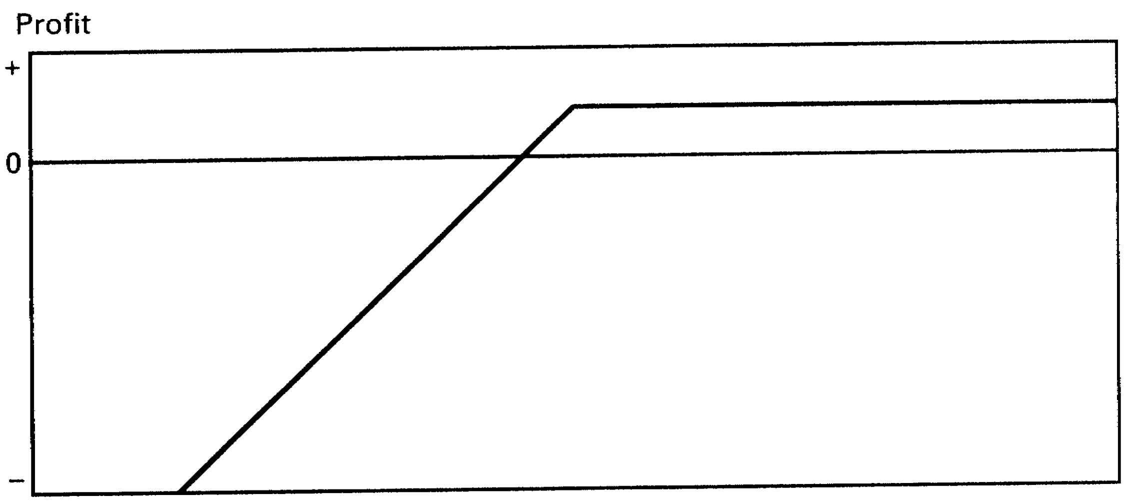


Breakeven point:	$S_T = X + pe^{rT}$
Maximum loss:	pe^{rT}, where $S_T > X$
Maximum gain:	$X - pe^{rT}$, where S_T falls to zero

6. *Short put:* Put is sold, and proceeds from sale earn rate r until option expiration at time T.

$$\pi_T = \begin{cases} pe^{rT} & \text{if} \quad S_T > X \\ -X + S_T + pe^{rT} & \text{if} \quad S_T \leq X \end{cases} \tag{12.3b}$$

FIGURE 12.3b Trading Strategy: Short Put



Breakeven point:	$S_T = X + pe^{rT}$
Maximum gain:	pe^{rT}, where $S_T > X$
Maximum loss:	$X - pe^{rT}$, where S_T falls to zero

In the option positions described above and later in the chapter, the premium paid by the buyer or received by the seller depends on the commodity price at the time the option contract was written. In plotting the profit diagrams, we make reasonable assumptions about the initial put or call premiums, but the shape of the profit diagrams is not affected by particular initial premiums paid. For positions involving a single exercise price, we generally assume options were at-the-money when first traded. For positions involving different exercise prices, we generally assume one option was at-the-money, while the other was not.

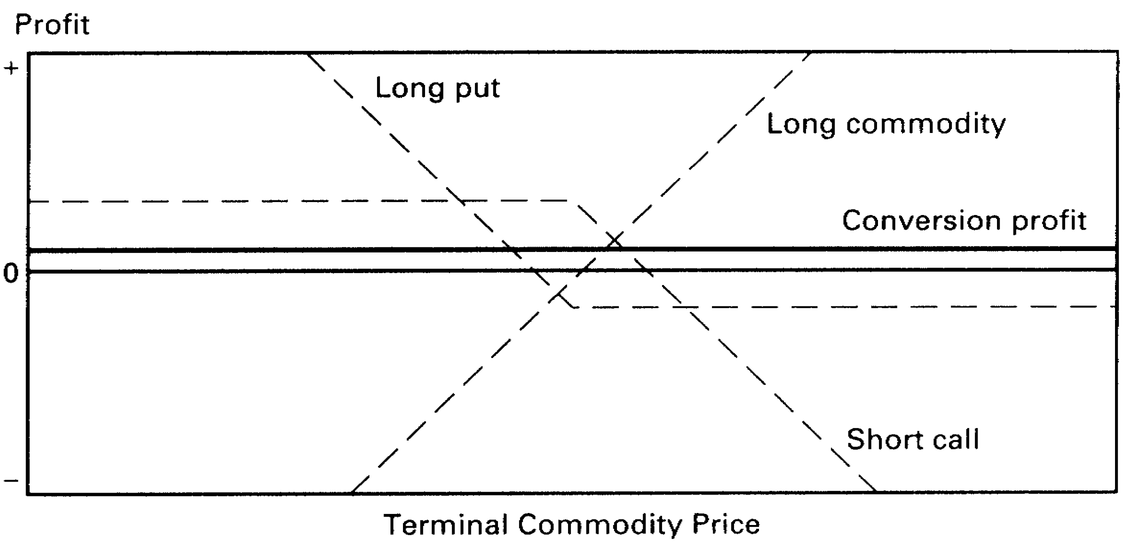
Riskless Arbitrage Strategies

The first two strategies, called *conversion* and *reverse conversion arbitrage*, were discussed in Chapter 10 in the put-call parity sections. These are riskless trading strategies designed to take advantage of temporary mispricings between the call, put, and underlying commodity. To determine the profit functions of these, as well as all subsequent trading strategies discussed in this section, we simply sum the corresponding profit functions of the component securities presented in equations (12.1) through (12.3).

7. *Conversion:* Buy commodity, buy put, and sell call.

$$\pi_T = -Se^{bT} + X + ce^{rT} - pe^{rT} \tag{12.4}$$

FIGURE 12.4 Trading Strategy: Conversion



Breakeven point:	none
Maximum loss:	$\pi_T = -Se^{bT} + X + ce^{rT} - pe^{rT}$
Maximum gain:	$\pi_T = -Se^{bT} + X + ce^{rT} - pe^{rT}$

The profit function (12.4) shows that the portfolio profit at option expiration is certain since the terminal commodity price, S_T, does not appear in the expression (i.e., the portfolio profit is insensitive to the future commodity price). This fact can also been seen in Figure 12.4, where the portfolio profit is depicted as the upper horizontal line. Since this line never crosses the zero profit line below it, the profit is certain for all values of the terminal commodity price.

The economic meaning of this certain-profit result is important. Recall that all positions in the commodity/options are entirely financed. It, therefore, stands to reason that if the options and commodity are properly priced in the marketplace, the terminal profit from this strategy is zero. And, if the terminal profit is zero, then the put-call parity relation,

$$c - p = Se^{(b-r)T} - Xe^{-rT}, \quad \textbf{(10.22)}$$

which was derived in Chapter 10, must hold. When put-call parity holds, the two solid lines in Figure 12.4 are coincident.

Conversion arbitrage comes into play when there are temporary mispricings. For example, an institutional trader might buy a large number of index call options in reaction to new information about the stock market. As a result of the excess buying pressure on calls, the call price might increase by more than is warranted relative to the prices of the put and the underlying stock index. As the market maker sells the calls to the institutional trader, she may simultaneously buy the puts and the index portfolio to lay off the commodity price risk of the short call position, thereby capturing the temporary violation of put-call parity. Such trading activity locks in a certain, positive profit.

8. *Reverse conversion or reversal:* Sell commodity, sell put, and buy call.

This strategy is simply the reverse of Trading Strategy 7 and is used when put-call parity is violated in the opposite direction.

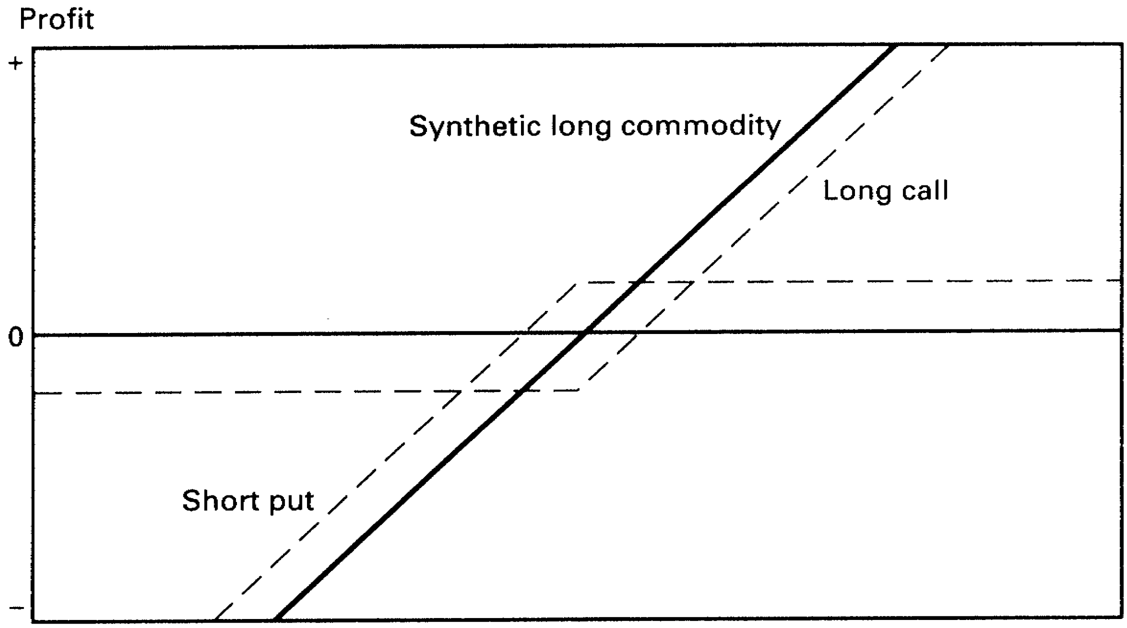
Synthetic Positions

Conversions and reversals exploit arbitrage opportunities by creating a synthetic position in any one security from judiciously selected positions in the other two securities. In a conversion, for example, a long position in a commodity, combined with a long put position, is equivalent to a long call position. When this long call position is combined with a short call position, a riskless portfolio results. To demonstrate this idea in greater detail, we now show how synthetic long and short positions in the underlying commodity can be created by using call and put options.

9. *Synthetic long commodity:* Buy call and sell put.

$$\pi_T = S_T - X - ce^{rT} + pe^{rT} \tag{12.5a}$$

FIGURE 12.5a Trading Strategy: Synthetic Long Commodity



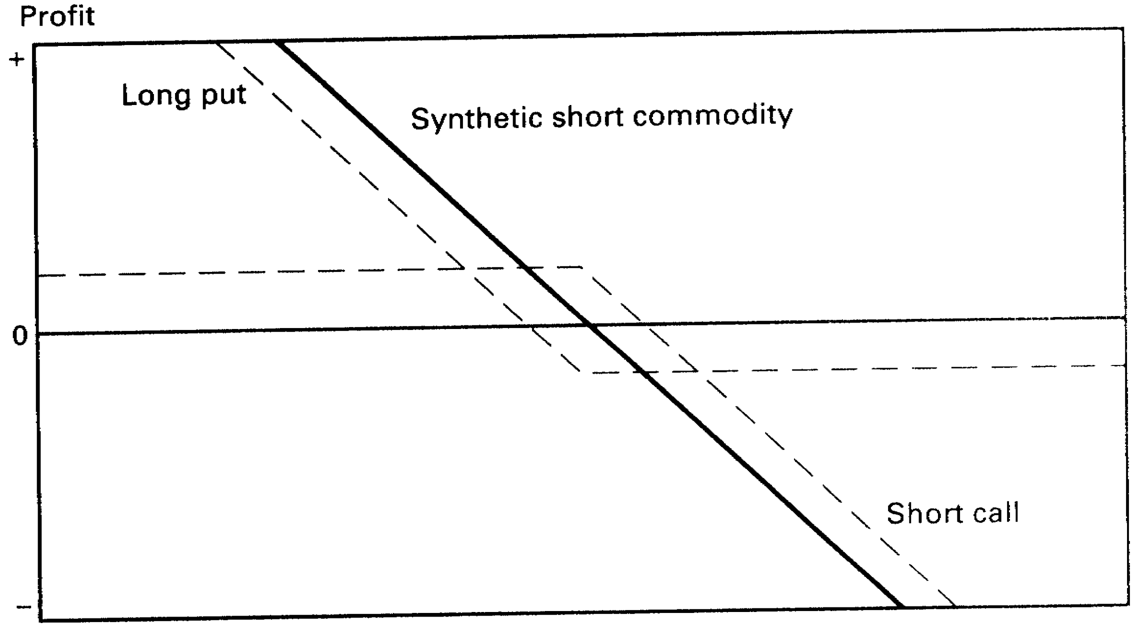
Breakeven point:	$S_T = X + ce^{rT} - pe^{rT}$
Maximum loss:	$X + ce^{rT} - pe^{rT}$, where S_T falls to zero
Maximum gain:	unlimited, where S_T rises without limit

Figure 12.5a shows that the profit function of the long call/short put position is virtually identical to the long commodity position shown in Figure 12.1a. The only difference between the profit functions (12.1a) and (12.5a) is that (12.1a) has the term Se^{bT}, where (12.5a) has the term $X + ce^{rT} - pe^{rT}$. But this is expected. If put-call parity (10.22) holds, these two values are equal. If, for whatever reason, $Se^{bT} > X + ce^{rT} - pe^{rT}$, it is cheaper to create a long commodity position synthetically than it is to buy the commodity itself.

10. *Synthetic short commodity:* Sell call and buy put.

$$\pi_T = -S_T + X + ce^{rT} - pe^{rT} \tag{12.5b}$$

FIGURE 12.5b Trading Strategy: Synthetic Short Commodity



Breakeven point:	$S_T = X + ce^{rT} - pe^{rT}$
Maximum gain:	$-X - ce^{rT} + pe^{rT}$, where S_T falls to zero
Maximum loss:	unlimited, where S_T rises without limit

The synthetic short commodity position is completely analogous to the short commodity position discussed earlier. This strategy is particularly well suited if short sale of the underlying commodity is difficult or expensive (e.g., short selling the S&P 500 index). In such cases, the short call/long put portfolio becomes a viable alternative.

Synthetic long and short option positions are also possible. Since the analyses of these positions are fairly straightforward, we only describe the portfolio compositions below:

11. *Synthetic long call:* Long commodity and long put.
12. *Synthetic short call:* Short commodity and short put.
13. *Synthetic long put:* Short commodity and long call.
14. *Synthetic short put:* Long commodity and short call.

Multiple Option/Commodity Positions

One last basic idea is needed before proceeding with the analyses of more complex option strategies. What happens when several options are purchased or sold? To understand this, recall the profit function for the long position. To generalize it for the purchase of n calls, we simply multiply (12.2a) by n, that is,

$$n\pi_T = \begin{cases} n(S_T - X - ce^{rT}) & \text{if} \quad S_T > X \\ -nce^{rT} & \text{if} \quad S_T \leq X \end{cases} \tag{12.6}$$

FIGURE 12.6 Trading Strategy: Long Multiple Calls

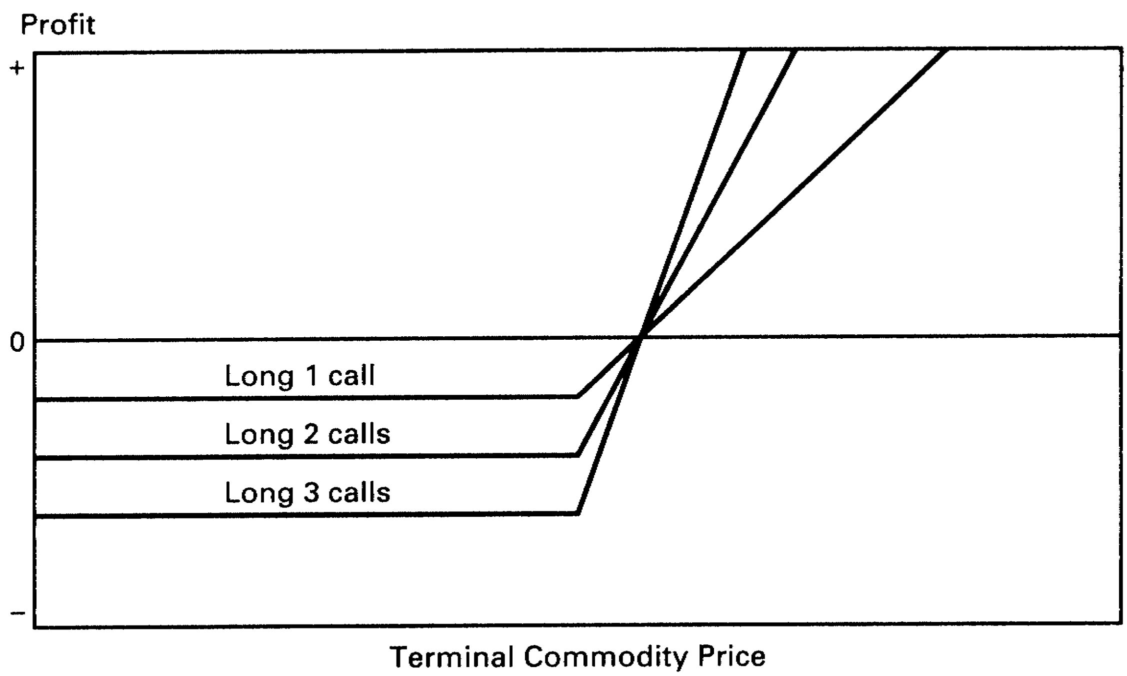


Figure 12.6 shows the profit diagrams for the purchase of one, two, and three calls. Note that the breakeven point is independent of the number of calls purchased. The important change is that more option premium is lost should the option expire out-of-the-money, and the rate of profit per dollar of commodity price is increased should the option expire in-the-money. This concept is used later in the chapter when various ratio spreading and ratio writing strategies are discussed.

Spread Strategies

Spread strategies are strategies in which the risk of one security position is offset, at least in some degree, by another security position; that is, one position benefits from a commodity price increase and the other loses. A strategy will usually wind up being *neutral*, *bullish*, or *bearish*. A neutral strategy is one in which the strategy is profitable when the commodity price does not move by very much after the position is taken. A bullish strategy is one that profits in the event that the underlying commodity price rises, and a bearish strategy profits when the commodity price falls.

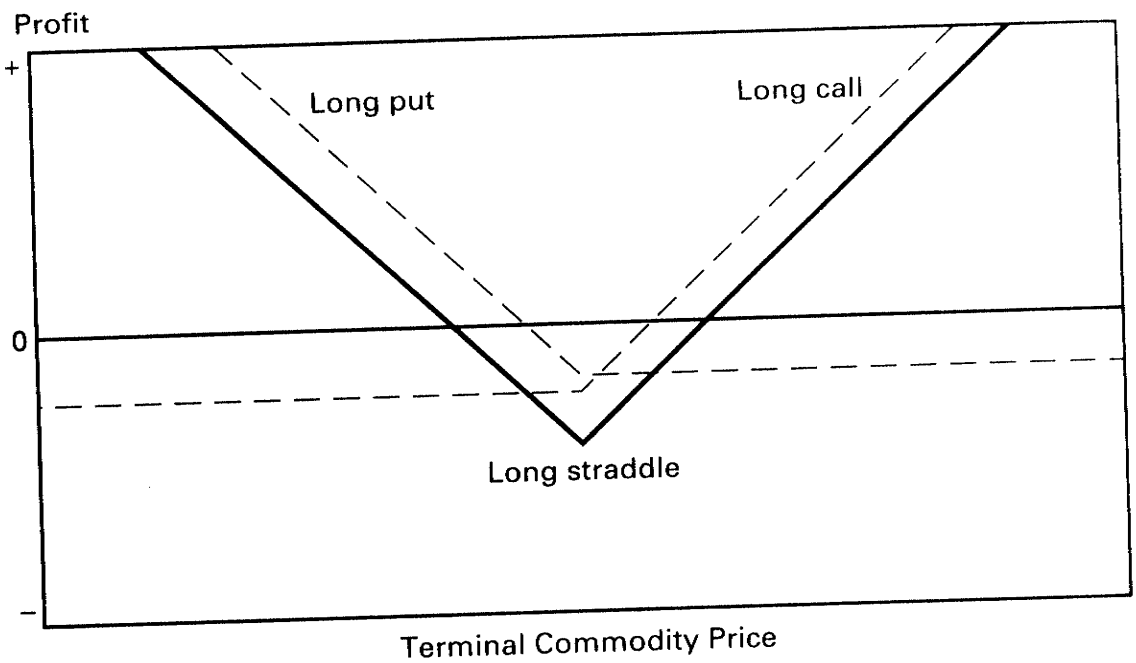
Spread strategies typically involve offsetting positions in options with different exercise prices or different maturities. One might buy a call option with one exercise price or one maturity and sell another call option with another exercise price or maturity. Some of the possible spread positions are now described under four categories: *volatility spreads*, *spreads based on differences in exercise prices*, *calendar spreads*, and *diagonal spreads*.

Volatility Spreads. Volatility spreads involve the purchase of a put and a call or the sale of a put and a call.

15. *Long straddle or long volatility spread:* Buy call and buy put.

$$\pi_T = \begin{cases} S_T - X - (ce^{rT} + pe^{rT}) & \text{if} \quad S_T > X \\ X - S_T - (pe^{rT} + ce^{rT}) & \text{if} \quad S_T \leq X \end{cases} \tag{12.7}$$

FIGURE 12.7 Trading Strategy: Long Straddle



Breakeven points:	(a) $S_T = X - (ce^{rT} + pe^{rT})$
	(b) $S_T = X + (ce^{rT} + pe^{rT})$
Maximum loss:	$ce^{rT} + pe^{rT}$, where $S_T = X$
Maximum gain:	(a) $X - (ce^{rT} + pe^{rT})$, where S_T falls to zero
	(b) unlimited, where S_T rises without limit

As the profit function (12.7) and Figure 12.7 show, the straddle produces positive profits where the underlying commodity price moves up *or* down by a sufficient amount. For this reason, buying a straddle is often referred to as *buying* *volatility.* The strategy loses money where the terminal commodity price, S_T, is within the band $X \pm (ce^{rT} + pe^{rT})$ at the options' expiration.

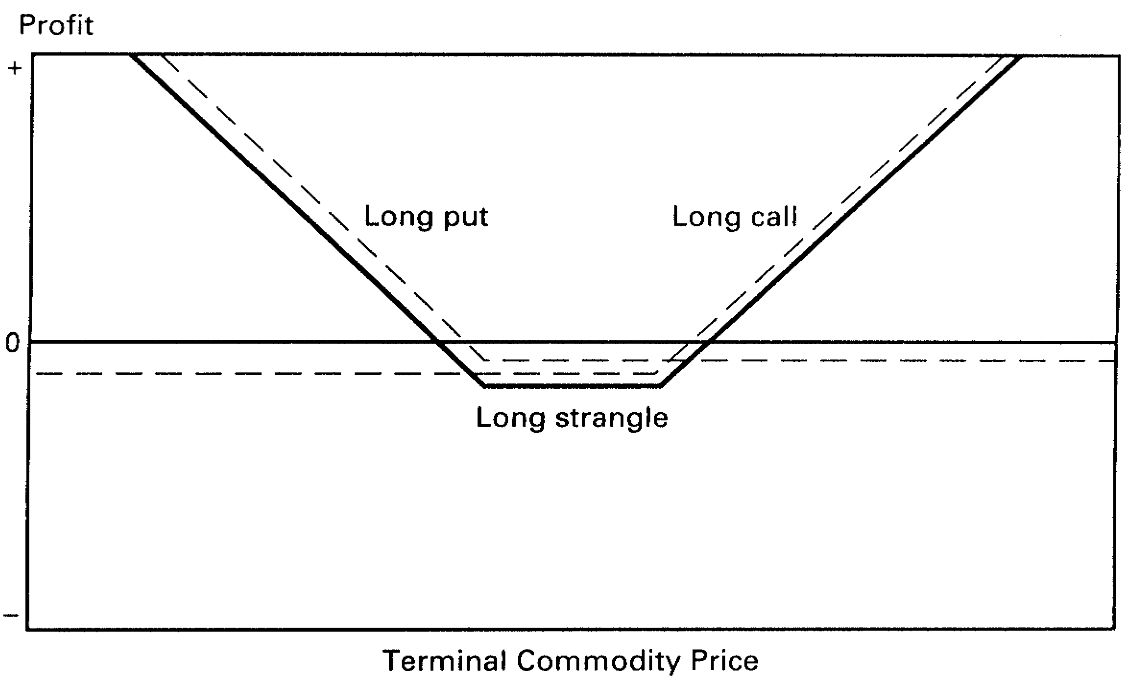
16. *Short straddle or short volatility spread:* Sell call and sell put.

This strategy is the reverse of a long straddle. Investors short straddles or *sell volatility* when they believe that the commodity price will not move by much before the options' expiration.

17. *Long strangle:* Buy call and buy put, where the exercise price of the put, X_p, is less than the exercise price of the call, X_c (i.e., $X_p < X_c$).

$$\pi_T = \begin{cases} S_T - X_c - (ce^{rT} + pe^{rT}) & \text{if} \quad S_T > X_c \\ -(ce^{rT} + pe^{rT}) & \text{if} \quad X_p < S_T \le X_c \\ X_p - S_T - (pe^{rT} + ce^{rT}) & \text{if} \quad S_T \le X_p \end{cases} \tag{12.8}$$

FIGURE 12.8 Trading Strategy: Long Strangle



Breakeven points:	(a) $S_T = X_p - (ce^{rT} + pe^{rT})$
	(b) $S_T = X_c + (ce^{rT} + pe^{rT})$
Maximum loss:	$ce^{rT} + pe^{rT}$, where $X_p < S_T \leq X_c$
Maximum gain:	(a) $X_p - (ce^{rT} + pe^{rT})$, where S_T falls to zero
	(b) unlimited, where S_T rises without limit

A long strangle has the same investment objective as a long straddle. The difference is that the strangle requires a lower investment since the call or the put is more out-of-the-money than with the straddle. With the lower investment cost, however, comes a wider region over which the strategy will be unprofitable, as Figure 12.8 shows. In other words, the commodity price must move by a greater amount than with the straddle in order for the strangle to be profitable at expiration. A variation on the long strangle is for the exercise price of the call to be below the exercise price of the put. Since both the put and call can be in the money under this strategy, the cost is greater and the profitability is greater.

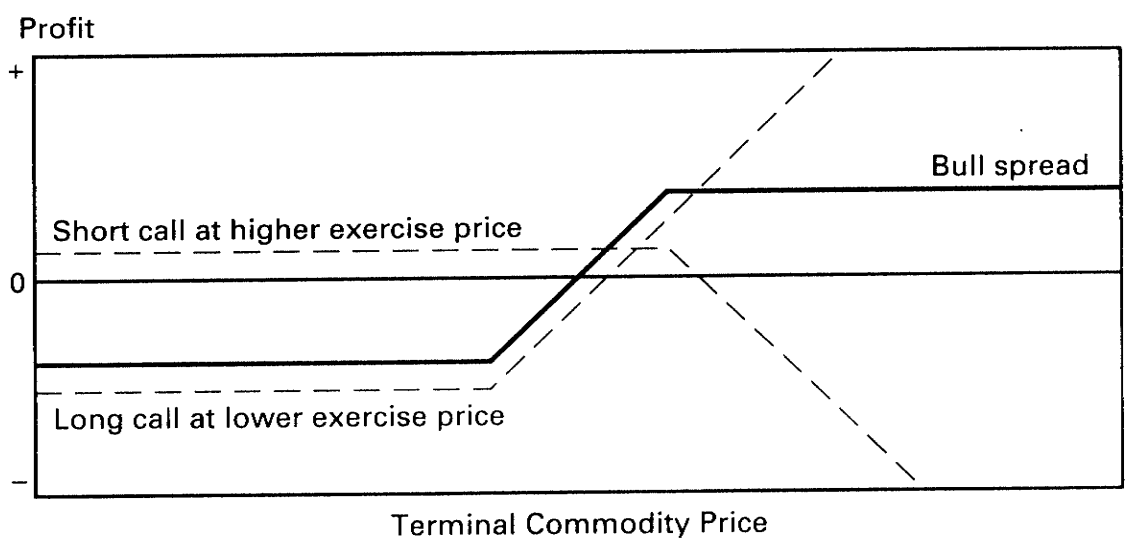
18. *Short strangle:* Sell call and sell put, where the exercise price of the put, X_p, is less than the exercise price of the call, X_c (i.e., $X_p < X_c$).

Spreading Options with Different Exercise Prices. Call or put options with different exercise prices can be combined to create bull or bear spreads, ratio or reverse ratio spreads, and long or short butterfly spreads.

19. *Bull spread – Call:* Buy call with a lower exercise price, X_l, and sell an otherwise identical call with a higher exercise price, X_h (i.e., $X_l < X_h$).

$$\pi_T = \begin{cases} X_h - X_l - (c_l - c_h)e^{rT} & \text{if} \quad S_T > X_h \\ S_T - X_l - (c_l - c_h)e^{rT} & \text{if} \quad X_l < S_T \le X_h \\ -(c_l - c_h)e^{rT} & \text{if} \quad S_T \le X_l \end{cases} \tag{12.9a}$$

FIGURE 12.9a Trading Strategy: Bull Spread – Call



Breakeven point:	$S_T = X_l + (c_l - c_h)e^{rT}$
Maximum loss:	$(c_l - c_h)e^{rT}$, where $S_T \leq X_l$
Maximum gain:	$X_h - X_l - (c_l - c_h)e^{rT}$, where $S_T > X_h$

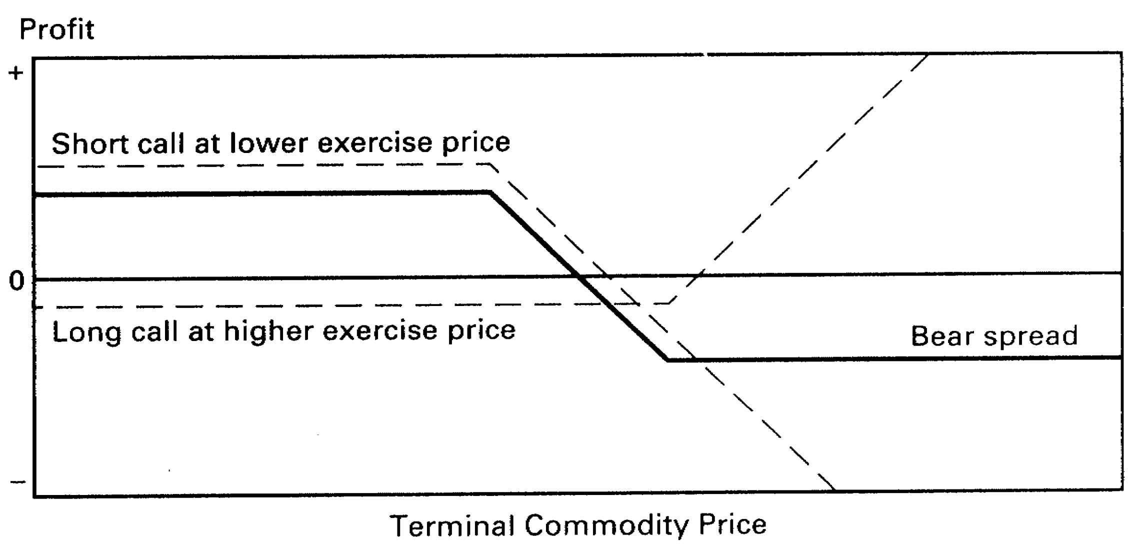
The term *bull* is used to describe the fact that this strategy profits when the underlying commodity price increases. The strategy is fairly conservative in the sense that if the investor were confident of a commodity price increase, a naked long call position would be more profitable. The benefit of buying the bull spread is that selling the out-of-the-money call provides income that offsets the cost of buying the other call. The cost is the loss of upside profit potential should the commodity price rise dramatically.

Prior to maturity, a bull spread takes advantage of the fact that the delta is higher for the in-the-money call option than it is for the out-of-the-money call option. As the commodity price increases, the gain on the long in-the-money call outstrips the loss on the short out-of-the-money call. As the commodity price rises further and both calls become deep in-the-money or as maturity is approached, the gain and the loss on the two positions become completely offsetting.

20. *Bear spread – Call:* Sell call with a lower exercise price, X_l, and buy an otherwise identical call with a higher exercise price, X_h (i.e., $X_l < X_h$).

$$\pi_T = \begin{cases} X_l - X_h + (c_l - c_h)e^{rT} & \text{if} \quad S_T > X_h \\ X_l - S_T + (c_l - c_h)e^{rT} & \text{if} \quad X_l < S_T \leq X_h \\ (c_l - c_h)e^{rT} & \text{if} \quad S_T \leq X_l \end{cases} \tag{12.9b}$$

FIGURE 12.9b Trading Strategy: Bear Spread – Call

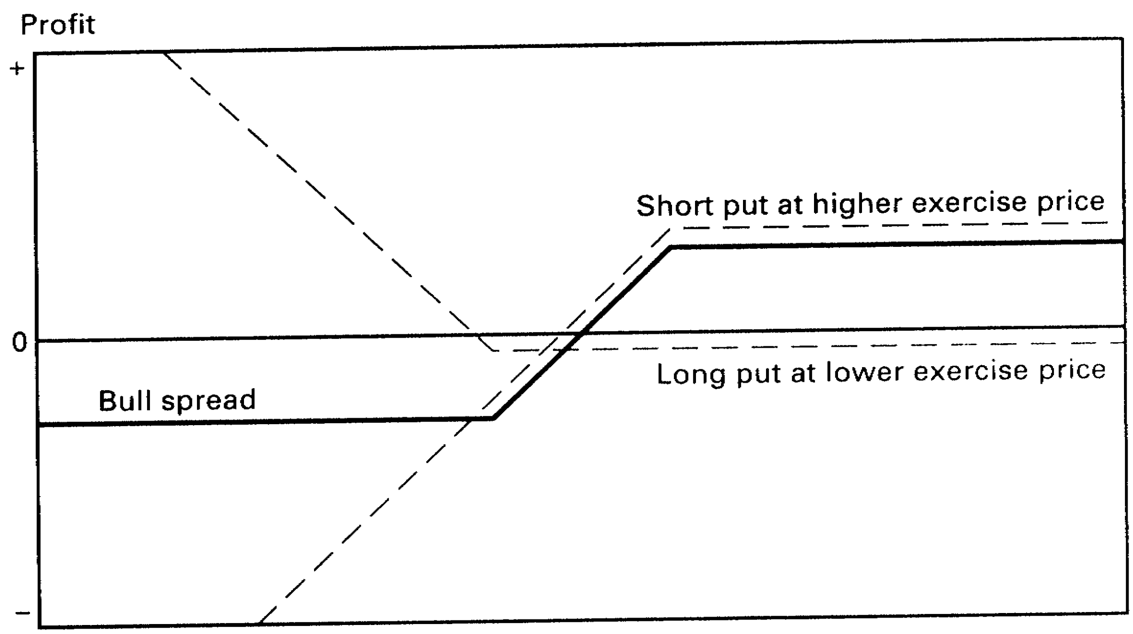


Breakeven point:	$S_T = X_l + (c_l - c_h)e^{rT}$
Maximum gain:	$(c_l - c_h)e^{rT}$, where $S_T \leq X_l$
Maximum loss:	$X_h - X_l - (c_l - c_h)e^{rT}$, where $S_T > X_h$

21. *Bull spread – Put:* Buy put with a lower exercise price, X_l, and sell an otherwise identical put with a higher exercise price, X_h (i.e., $X_l < X_h$).

$$\pi_T = \begin{cases} (p_h - p_l)e^{rT} & \text{if} \quad S_T > X_h \\ S_T - X_h + (p_h - p_l)e^{rT} & \text{if} \quad X_l < S_T \leq X_h \\ X_l - X_h + (p_h - p_l)e^{rT} & \text{if} \quad S_T \leq X_l \end{cases} \qquad \textbf{(12.10a)}$$

FIGURE 12.10a Trading Strategy: Bull Spread – Put



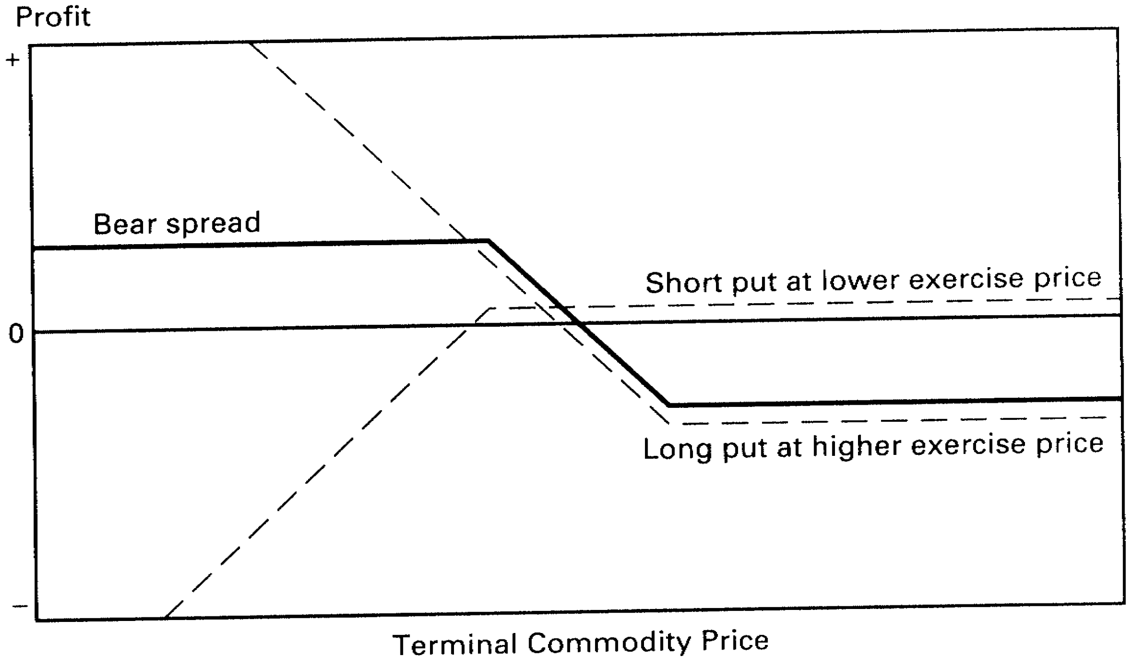
Breakeven point:	$S_T = X_h - (p_h - p_l)e^{rT}$
Maximum loss:	$X_l - X_h + (p_h - p_l)e^{rT}$, where $S_T \leq X_l$
Maximum gain:	$(p_h - p_l)e^{rT}$, where $S_T > X_h$

The bull spread using puts is equivalent to the bull spread using calls, although the sources of the outcomes are different. For the put bull spread, the profit, if the commodity price increases, comes from the premium on the put sold at the higher exercise price. For the call bull spread, the profit, if the commodity price increases, comes from the fact that the gain on the long call at the lower exercise price exceeds the loss on the short call at the higher exercise price.

22. *Bear spread – Put:* Sell put with a lower exercise price, X_l, and buy an otherwise identical put with a higher exercise price, X_h (i.e., $X_l < X_h$).

$$\pi_T = \begin{cases} -(p_h - p_l)e^{rT} & \text{if } S_T > X_h \\ X_h - S_T - (p_h - p_l)e^{rT} & \text{if } X_l < S_T \le X_h \\ X_h - X_l - (p_h - p_l)e^{rT} & \text{if } S_T \le X_l \end{cases} \quad \textbf{(12.10b)}$$

FIGURE 12.10b Trading Strategy: Bear Spread – Put

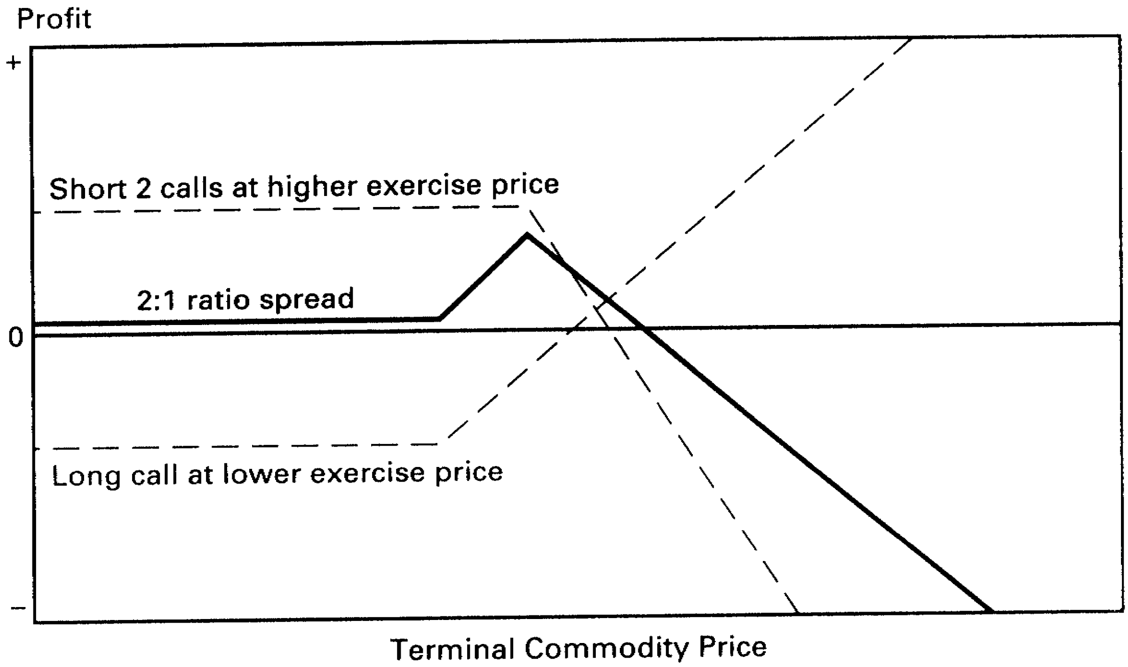


Breakeven point:	$S_T = X_h - (p_h - p_l)e^{rT}$
Maximum gain:	$X_l - X_h + (p_h - p_l)e^{rT}$, where $S_T \le X_l$
Maximum loss:	$-(p_h - p_l)e^{rT}$, where $S_T > X_h$

23. *Ratio spread – Call:* Buy call with a lower exercise price, X_l, and sell n otherwise identical calls with higher exercise price, X_h (i.e., $X_l < X_h$).

$$\pi_T = \begin{cases} nX_h - X_l - (n-1)S_T + (nc_h - c_l)e^{rT} & \text{if } S_T > X_h \\ S_T - X_l + (nc_h - c_l)e^{rT} & \text{if } X_l < S_T \le X_h \\ (nc_h - c_l)e^{rT} & \text{if } S_T \le X_l \end{cases} \quad \textbf{(12.11a)}$$

FIGURE 12.11a Trading Strategy: Ratio Spread – Call



Breakeven points:	(a) $S_T = \dfrac{nX_h - X_l + (nc_h - c_l)e^{rT}}{n - 1}$
	(b) $S_T = X_l - (nc_h - c_l)e^{rT}$, where $nc_h < c_l$
Maximum loss:	(a) unlimited, where S_T rises without limit
	(b) $(nc_h - c_l)e^{rT}$, where $nc_h < c_l$ and $S_T \leq X_l$
Maximum gain:	$X_h - X_l + (nc_h - c_l)e^{rT}$, where $S_T = X_h$

A call ratio spread is like a call bull spread except that several identical options are sold at the higher exercise price.

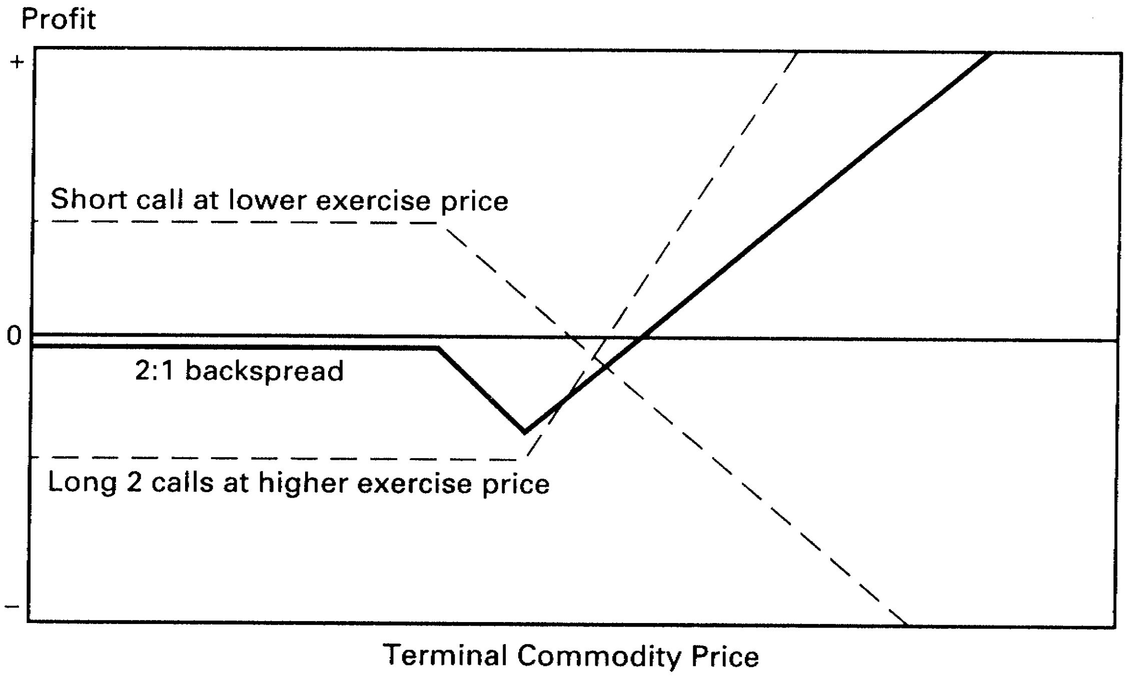
The "ratio" of the ratio spread is defined in terms of the quantity of calls sold, n, per call purchased, that is, an n:1 ratio spread. The ratio spread depicted in Figure 12.11a is a 2:1 ratio spread, which involves selling two calls with a higher exercise price against the purchase of one call with a lower exercise price.

The ratio spread is most profitable when the commodity price does not move by much. Its maximum profit is when the commodity price equals the exercise price of the shorted option. If the commodity price falls, the downside risk is fixed. If the commodity price falls below the lower exercise price, both options expire worthless. The initial investment can be either a net debit or net credit, depending upon the relative magnitudes of the premiums and the ratio of the spread. Figure 12.11a indicates that the 2:1 ratio spread shown has an initial net credit. If the commodity price rises without limit, the strategy loses money without limit.

24. *Backspread or reverse ratio spread — Call:* Sell call with a lower exercise price, X_l, and buy n otherwise identical calls with a higher exercise price, X_h (i.e., $X_l < X_h$).

$$\pi_T = \begin{cases} (n-1)S_T - nX_h + X_l - (nc_h - c_l)e^{rT} & \text{if } S_T > X_h \\ X_l - S_T - (nc_h - c_l)e^{rT} & \text{if } X_l < S_T \leq X_h \\ -(nc_h - c_l)e^{rT} & \text{if } S_T \leq X_l \end{cases} \quad \text{(12.11b)}$$

FIGURE 12.11b Trading Strategy: Reverse Ratio Spread — Call

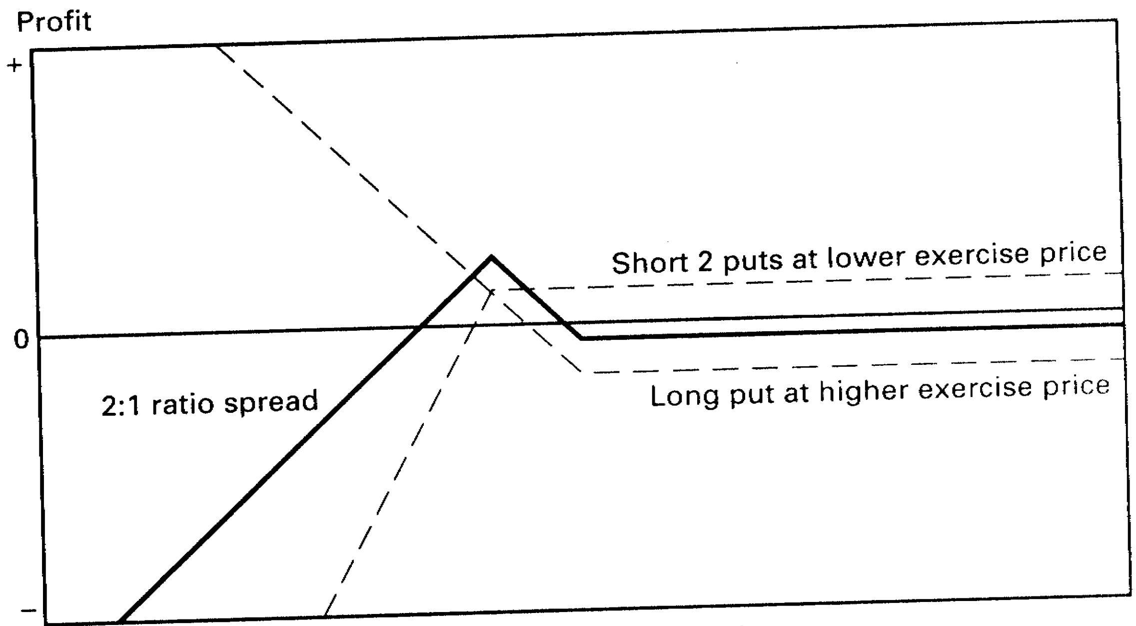


Breakeven points:	(a) $S_T = \dfrac{nX_h - X_l + (nc_h - c_l)e^{rT}}{n-1}$
	(b) $S_T = X_l - (nc_h - c_l)e^{rT}$, where $nc_h < c_l$
Maximum gain:	(a) unlimited, where S_T rises without limit
	(b) $(nc_h - c_l)e^{rT}$, where $nc_h < c_l$ and $S_T \leq X_l$
Maximum loss:	$X_h - X_l + (nc_h - c_l)e^{rT}$, where $S_T = X_h$

25. *Ratio spread — Put:* Buy put with a higher exercise price, X_h, and sell n otherwise identical puts with a lower exercise price, X_l (i.e., $X_l < X_h$).

$$\pi_T = \begin{cases} (np_l - p_h)e^{rT} & \text{if } S_T > X_h \\ X_h - S_T + (np_l - p_h)e^{rT} & \text{if } X_l < S_T \le X_h \\ X_h - nX_l + (n-1)S_T + (np_l - p_h)e^{rT} & \text{if } S_T \le X_l \end{cases} \tag{12.12a}$$

FIGURE 12.12a Trading Strategy: Ratio Spread – Put

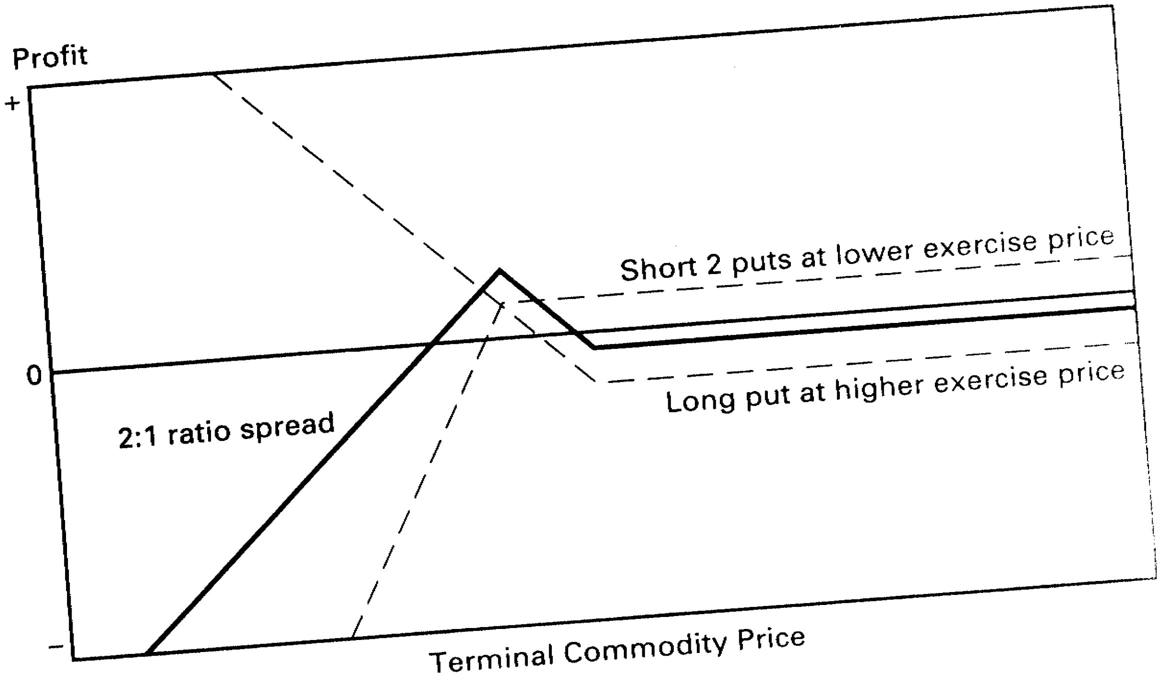


Breakeven points:	(a) $S_T = \dfrac{nX_l - X_h - (np_l - p_h)e^{rT}}{n-1}$
	(b) $S_T = X_h - (np_l - p_h)e^{rT}$, where $np_l < p_h$
Maximum loss:	(a) $X_h - nX_l + (np_l - p_h)e^{rT}$, where S_T falls to zero
	(b) $(np_l - p_h)e^{rT}$, where $np_l < p_h$ and $S_T \leq X_h$
Maximum gain:	$X_h - X_l + (np_l - p_h)e^{rT}$, where $S_T = X_l$

26. *Backspread or reverse ratio spread – Put:* Sell put with a higher exercise price, X_h, and buy n otherwise identical puts with a lower exercise price, X_l (i.e., $X_l < X_h$).

$$\pi_T = \begin{cases} (np_l - p_h)e^{rT} & \text{if} \quad S_T > X_h \\ X_h - S_T + (np_l - p_h)e^{rT} & \text{if} \quad X_l < S_T \le X_h \\ X_h - nX_l + (n-1)S_T + (np_l - p_h)e^{rT} & \text{if} \quad S_T \le X_l \end{cases} \tag{12.12a}$$

FIGURE 12.12a Trading Strategy: Ratio Spread – Put

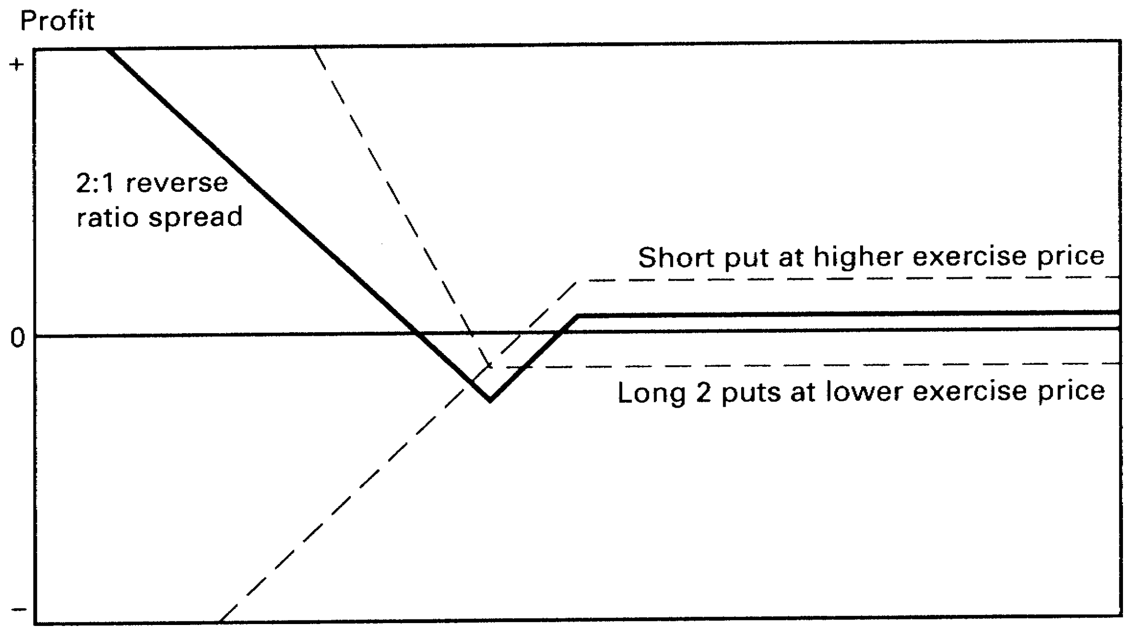


Breakeven points:	(a) $S_T = \dfrac{nX_l - X_h - (np_l - p_h)e^{rT}}{n-1}$
	(b) $S_T = X_h - (np_l - p_h)e^{rT}$, where $np_l < p_h$
Maximum loss:	(a) $X_h - nX_l + (np_l - p_h)e^{rT}$, where S_T falls to zero
	(b) $(np_l - p_h)e^{rT}$, where $np_l < p_h$ and $S_T \le X_h$
Maximum gain:	$X_h - X_l + (np_l - p_h)e^{rT}$, where $S_T = X_l$

26. *Backspread or reverse ratio spread – Put:* Sell put with a higher exercise price, X_h, and buy n otherwise identical puts with a lower exercise price X_l (i.e., $X_l < X_h$).

$$\pi_T = \begin{cases} -(np_l - p_h)e^{rT} & \text{if } S_T > X_h \\ S_T - X_h - (np_l - p_h)e^{rT} & \text{if } X_l < S_T \leq X_h \\ nX_l - X_h - (n-1)S_T - (np_l - p_h)e^{rT} & \text{if } S_T \leq X_l \end{cases} \tag{12.12b}$$

FIGURE 12.12b Trading Strategy: Reverse Ratio Spread – Put

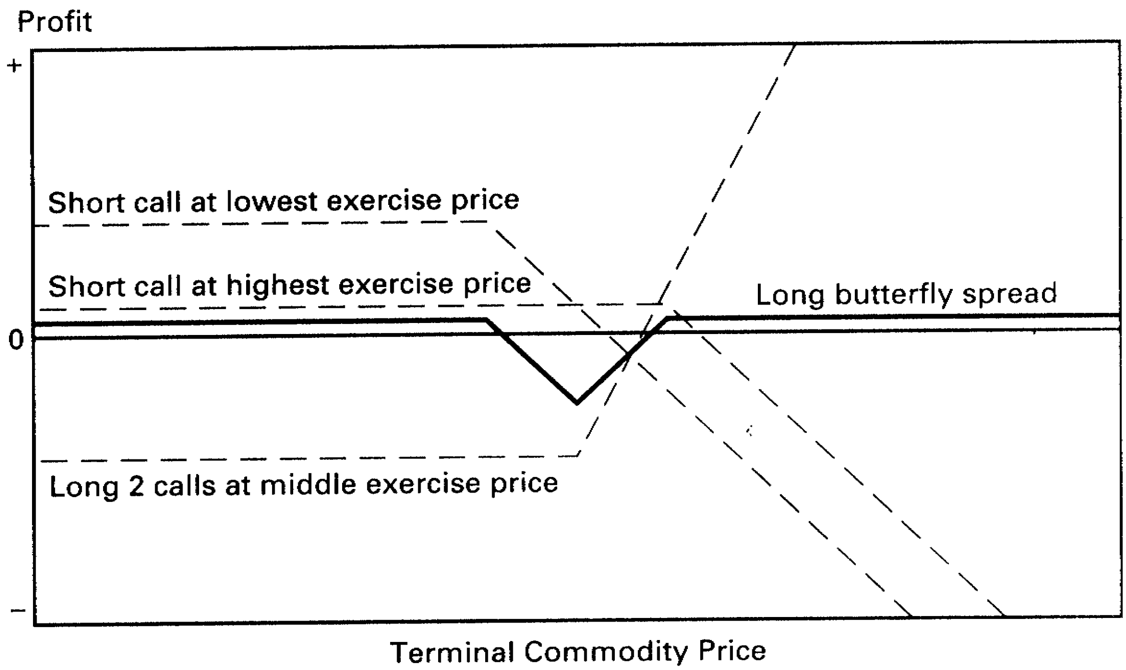


Breakeven points:	(a) $S_T = \dfrac{nX_l - X_h - (np_l - p_h)e^{rT}}{n-1}$
	(b) $S_T = X_h - (np_l - p_h)e^{rT}$, where $np_l < p_h$
Maximum gain:	(a) $X_h - nX_l + (np_l - p_h)e^{rT}$, where S_T falls to zero
	(b) $(np_l - p_h)e^{rT}$, where $np_l < p_h$ and $S_T \leqslant X_h$
Maximum loss:	$X_h - X_l + (np_l - p_h)e^{rT}$, where $S_T = X_l$

27. *Long butterfly spread—Call:* Sell call with a lower exercise price, X_l, buy two calls with a middle exercise price, X_m, and sell call with a higher exercise price, X_h.

$$\pi_T = \begin{cases} X_l - 2X_m + X_h + (c_l - 2c_m + c_h)e^{rT} & \text{if } S_T > X_h \\ S_T - (2X_m - X_l) + (c_l - 2c_m + c_h)e^{rT} & \text{if } X_m < S_T \leq X_h \\ X_l - S_T + (c_l - 2c_m + c_h)e^{rT} & \text{if } X_l < S_T \leq X_m \\ (c_l - 2c_m + c_h)e^{rT} & \text{if } S_T \leq X_l \end{cases} \tag{12.13}$$

FIGURE 12.13 Trading Strategy: Long Butterfly Spread – Call



Breakeven points:	(a) $S_T = 2X_m - X_l - (c_l - 2c_m + c_h)e^{rT}$
	(b) $S_T = X_l + (c_l - 2c_m + c_h)e^{rT}$
Maximum loss:	$X_l - X_m + (c_l - 2c_m + c_h)e^{rT}$, where $S_T = X_m$
Maximum gain:	(a) $X_l - 2X_m + X_h + (c_l - 2c_m + c_h)e^{rT}$, where $S_T > X_h$
	(b) $(c_l - 2c_m + c_h)e^{rT}$, where $S_T \leq X_l$

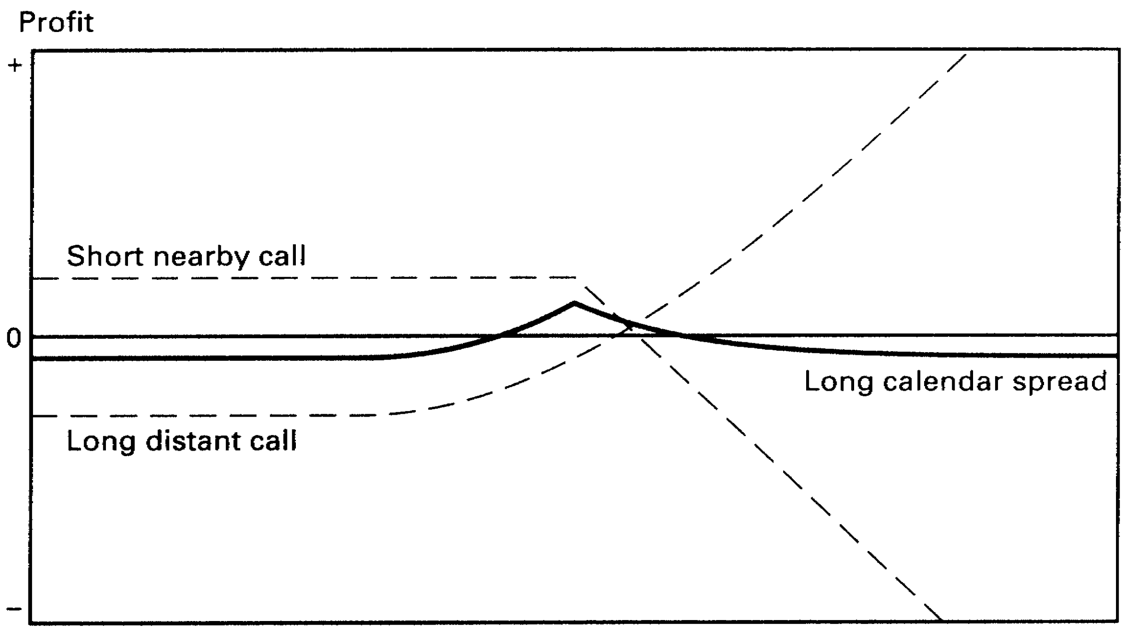
A long butterfly spread combines a bear spread and a bull spread. Profits are similar to those of a bull spread if the commodity price increases and to those of a bear spread if the commodity price falls. The investor loses money when the commodity price remains neutral. The resulting profit diagram (Figure 12.13) resembles a butterfly – hence, its name.

28. *Short butterfly spread – Call:* Buy call with a lower exercise price, X_l, sell two calls with a middle exercise price, X_m, and buy call with a higher exercise price, X_h.
29. *Long butterfly spread – Put:* Sell put with a lower exercise price, X_l, buy two puts with a middle exercise price, X_m, and sell put with a higher exercise price, X_h.
30. *Short butterfly spread – Put:* Buy put with a lower exercise price, X_l, sell two puts with a middle exercise price, X_m, and buy put with a higher exercise price, X_h.

Calendar Spreads. A calendar spread requires the purchase of a call or put of one maturity and the sale of an identical option with a different maturity. Presenting a profit function for a calendar spread is cumbersome since a pricing equation is required to show the value of the distant option at the nearby option expiration.[1] For this reason, we go immediately to the profit diagram.

31. *Long calendar spread – Call:* Buy call with a distant maturity, and sell identical call with a nearby maturity.

FIGURE 12.14 Trading Strategy: Long Calendar Spread – Call



At-the-money call options are used to generate the calendar spread in Figure 12.14, and the outcomes are plotted at the maturity of the nearby call. This spread is neutral since positive profits are earned as long as the commodity price does not move very much by the nearby option expiration.[2] Because the longer-term option has a higher price, this strategy has a net debit position (i.e., we *pay* the difference between the option prices when the position is formed). The maximum loss, however, is limited to the net debit amount. The maximum gain occurs where the commodity price equals the exercise price of the nearby option, but the amount is

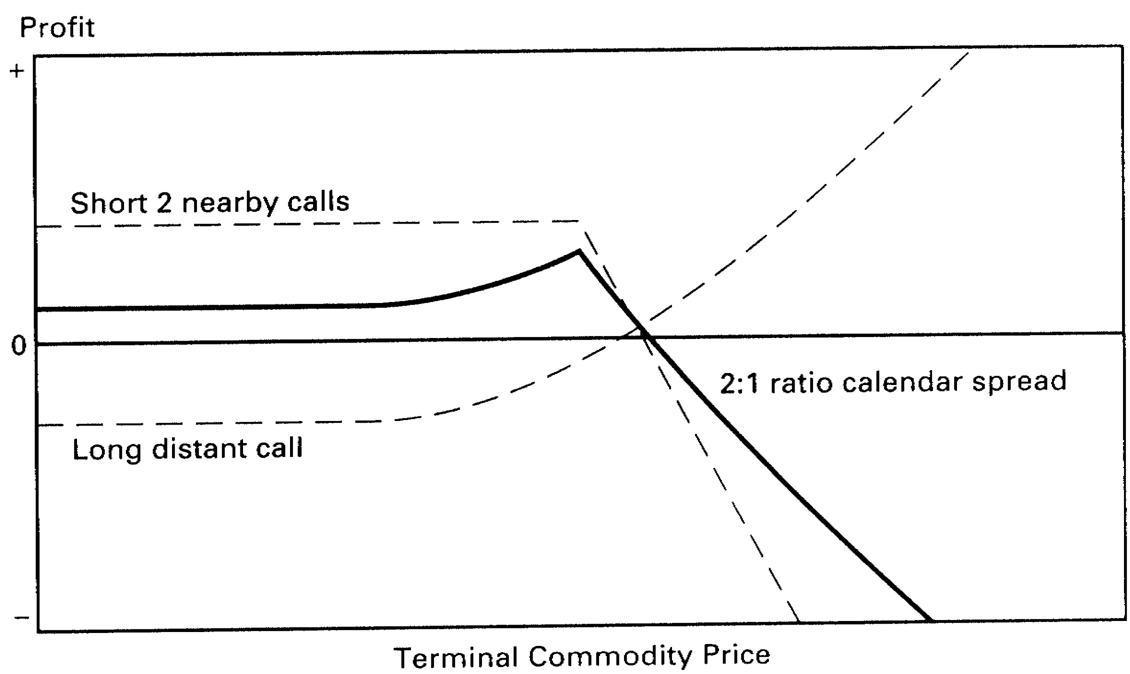
[1]The European call and put option valuation equations (11.25) and (11.28) are used to price the distant option at the nearby option's expiration.
[2]If out-of-the-money calls are used to form the calendar spread, the position is slightly bullish.

unclear since it depends on the remaining life of the distant option and the commodity's return volatility. The width of the profit range and the breakeven points are also functions of volatility and time to expiration.

Holding other factors constant, the profitability of a calendar spread is driven by the time decay of the option premiums. As we will show later in the chapter, the rate of time decay (i.e., the option's theta) is greater the shorter the option's time to expiration. In a long calendar spread, a short position is established in the nearby option in order to capture its time decay at the expense of the time decay in the distant option.

32. *Short calendar spread – Call:* Sell call with a distant maturity, and buy identical call with a nearby maturity.
33. *Long calendar spread – Put:* Buy put with a distant maturity, and sell identical put with a nearby maturity.
34. *Short calendar spread – Put:* Sell put with a distant maturity, and buy identical put with a nearby maturity.
35. *Long ratio calendar spread – Call:* Buy call with a distant maturity, and sell more than one identical calls with a nearby maturity.

FIGURE 12.15 Trading Strategy: Long Ratio Calendar Spread



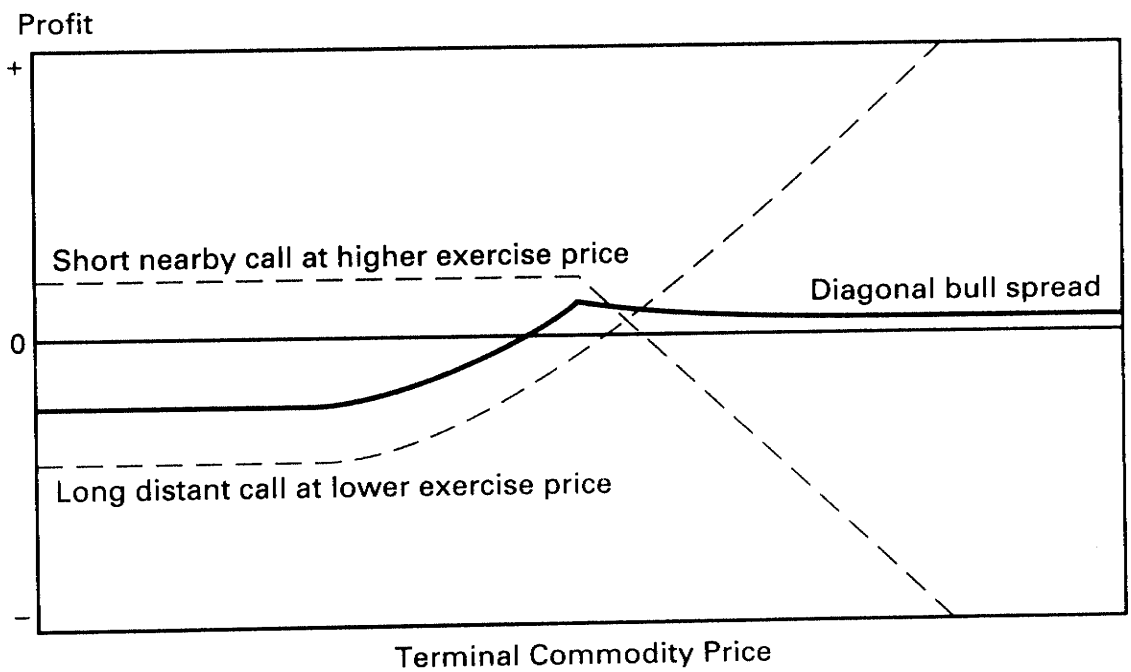
By writing more than one of the nearby calls, the calendar spreader usually receives an initial credit (i.e., he *receives* money when the position is formed). The net credit increases the profit if the commodity price falls below the exercise price prior to the expiration of the nearby option. Increases in the commodity price beyond the exercise price of the nearby option reduce the amount of the profit since, when both options are deep in-the-money, the spreader is synthetically short the commodity. Overall, the position is bearish. Figure 12.15 shows the profit diagram of a 2:1 ratio calendar spread.

36. *Short ratio calendar spread – Call:* Sell call with a distant maturity, and buy more than one identical calls with a nearby maturity.
37. *Long ratio calendar spread – Put:* Buy put with a distant maturity, and sell more than one identical puts with a nearby maturity.
38. *Short ratio calendar spread – Put:* Sell put with a distant maturity, and buy more than one identical puts with a nearby maturity.

Diagonal Spreads. In general, diagonal spreads are any spread positions consisting of different exercise prices and different expirations. A long diagonal spread requires that the distant option is purchased and the nearby option is shorted. If the ratio of the spread is 1:1, a diagonal bull (bear) spread results, depending upon whether the distant option has the lower (higher) exercise price. Long and short diagonalized spreads using other ratios produce a wide array of bullish and bearish positions. One possible diagonalized spread is described below.

39. *Diagonal bull spread – Call:* Buy call with a lower exercise price and distant maturity, and sell identical call with a higher exercise price and nearby maturity.

FIGURE 12.16 Trading Strategy: Long Diagonal Bull Spread



As Figure 12.6 shows, a diagonal bull spread is very similar to the bull spread described earlier in this section. The maximum loss is limited to the difference between the distant and nearby option prices (i.e., the net debit amount). The maximum gain occurs when the commodity price equals the nearby option exercise price at the nearby option's expiration. Beyond that level, increases in the commodity price reduce the profit level to the difference between the exercise prices and the net debit amount.

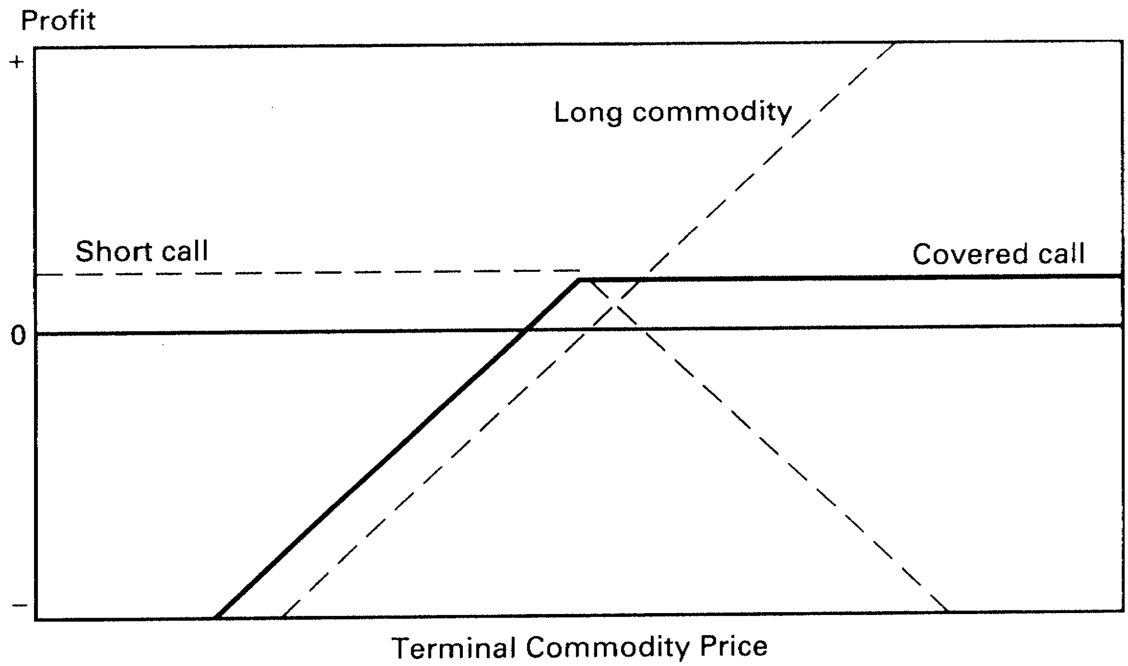
Writing/Speculative Strategies

In this section, we examine the effects of buying and selling options against a position in the underlying commodity. In general, we discuss strategies that reduce the risk of a long position in the commodity by writing calls or buying puts. But we also consider the effects of buying calls and writing puts in order to increase leverage.

40. *Covered call option writing:* Sell call against a long position in the underlying commodity.

$$\pi_T = \begin{cases} X - Se^{bT} + ce^{rT} & \text{if} \quad S_T > X \\ S_T - Se^{bT} + ce^{rT} & \text{if} \quad S_T \leq X \end{cases} \tag{12.14}$$

FIGURE 12.17 Trading Strategy: Covered Call Writing



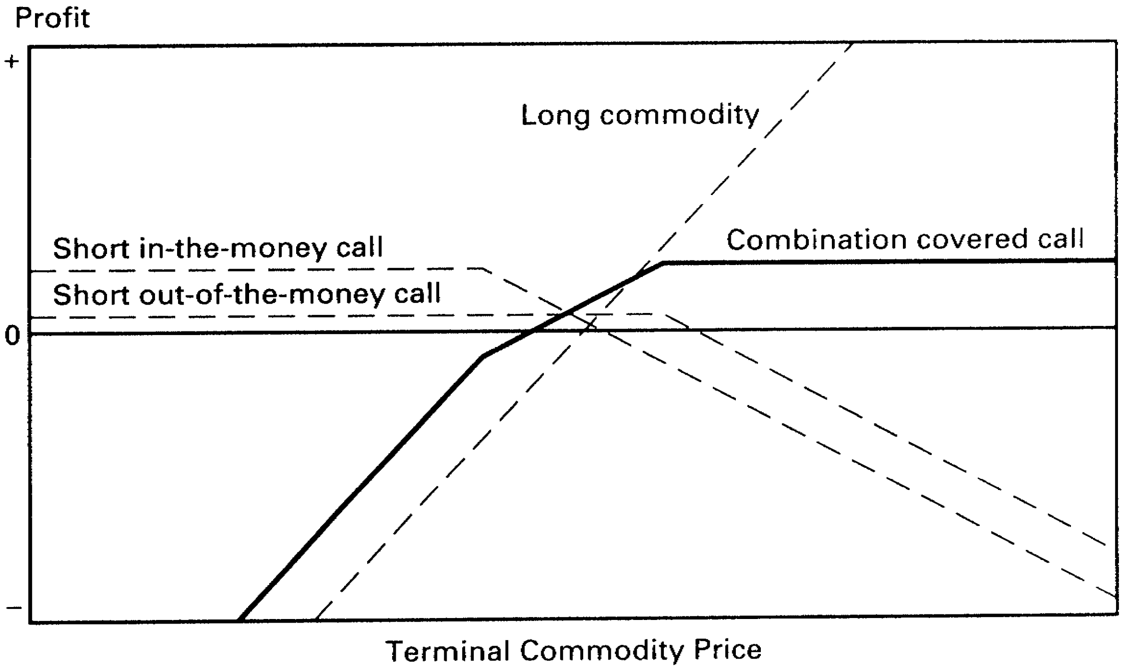
Breakeven point:	$S_T = Se^{bT} - ce^{rT}$
Maximum loss:	$Se^{bT} - ce^{rT}$, where S_T falls to zero
Maximum gain:	$X - Se^{bT} + ce^{rT}$, where $S_T > X$

Figure 12.17 shows that the covered call writer receives the option premium in exchange for the upside potential of the long commodity position. The position is equivalent to selling a naked put. Such a strategy makes sense only if an investor believes that the commodity price will not move much during the option's life. She does not benefit if the commodity price rises, and the option premium is little consolation if the commodity price falls dramatically.

Large stock funds often engage in a special form of covered call writing called *option overwriting*. In the usual case, the fund has separate stock and option portfolio managers. The stock portfolio manager handles the investment in stocks and advises the option overwriter on the current composition of the stock portfolio. The option overwriter then writes calls against the stocks. In the event that a call is exercised against the option overwriter, the overwriter must buy the stock for delivery on the option because she has no authority to deliver an existing stock position. The fund owner, however, should expect that some of her stocks will have to be liquidated, since writing call options against stocks is a risk-reducing strategy.

41. *Combination covered call option writing:* Sell in-the-money calls on half the commodity position, and sell out-of-the-money calls against the other half.

FIGURE 12.18 Trading Strategy: Combination Covered Call

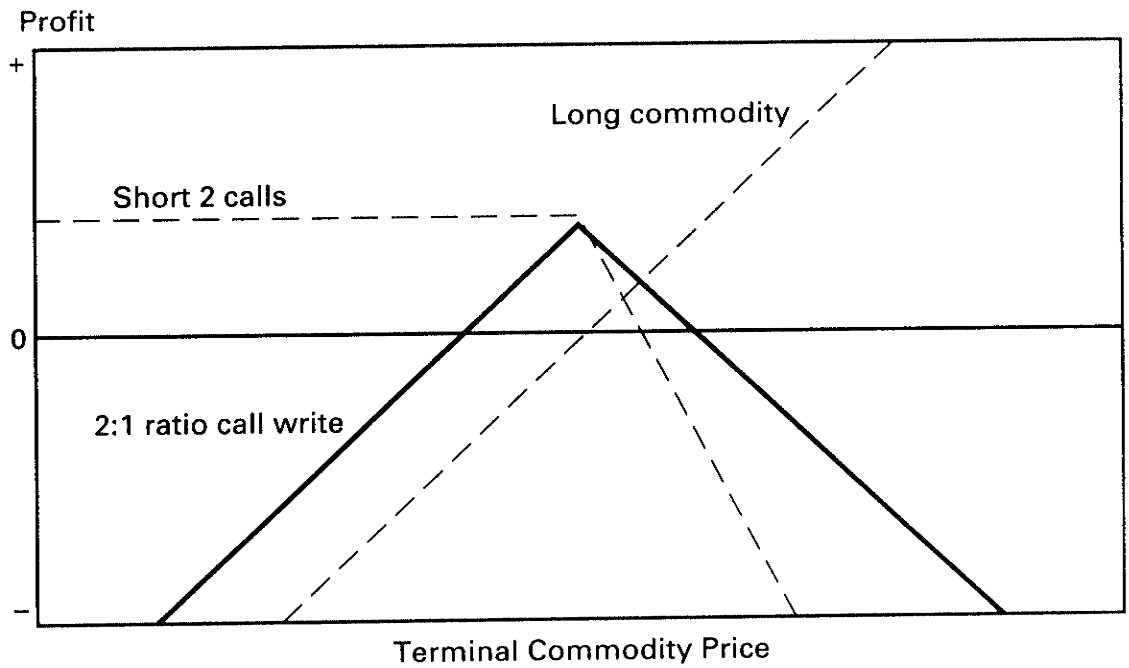


This strategy is, generally, the same as the covered call strategy. Figure 12.18 shows that the profit structure is just slightly different. Over the commodity price range between the exercise prices, the option writer shares in half of any gains made in the share price. However, like the previous covered call strategy, the upside potential of the long commodity position is completely negated once the commodity price exceeds a certain level, in this case the exercise price of the out-of-the-money option.

42. *Ratio call writing:* Sell more than one call against a long position in the underlying commodity.

$$\pi_T = \begin{cases} nX - (n-1)S_T + nce^{rT} & \text{if} \quad S_T > X \\ S_T - Se^{bT} + nce^{rT} & \text{if} \quad S_T \le X \end{cases} \tag{12.15}$$

FIGURE 12.19 Trading Strategy: Ratio Call Writing



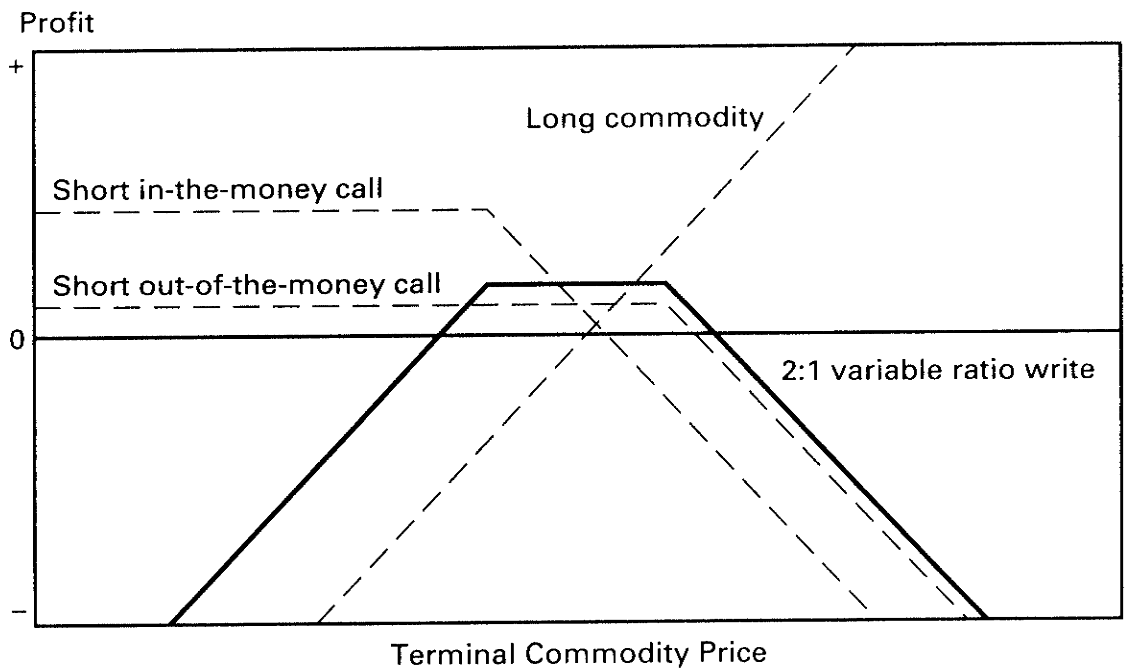
Breakeven point:	$S_T = Se^{bT} - nce^{rT}$
Maximum loss:	unlimited, where S_T rises without limit
Maximum gain:	nce^{rT}, where $S_T = X$ at expiration

Like ratio spreads, ratio writes are expressed in terms of the number of options sold, n, per unit of the underlying commodity. A 2:1 ratio write, therefore, refers to writing two call options against one unit of the commodity. In a 2:1 ratio write, half of the calls are covered while the other half are not. A 2:1 ratio write, such as that shown in Figure 12.19 creates a payoff diagram that looks exactly as if we have written a straddle. The maximum gain occurs when the commodity price equals the exercise price at the option expiration. Large commodity price swings in either direction, however, produce losses.

Ratio writing is usually pursued to earn premium income by those who expect that the commodity price will not move during the option life. The calls are written at the exercise price closest to the current commodity price. Profits are earned if the commodity price remains relatively unchanged. However, losses can be significant if the underlying commodity price changes significantly.

43. *Variable ratio writing:* Sell in-the-money calls and out-of-the-money calls against a long position in the commodity, such that the long position in the commodity is less than sufficient to cover delivery should the options be exercised.

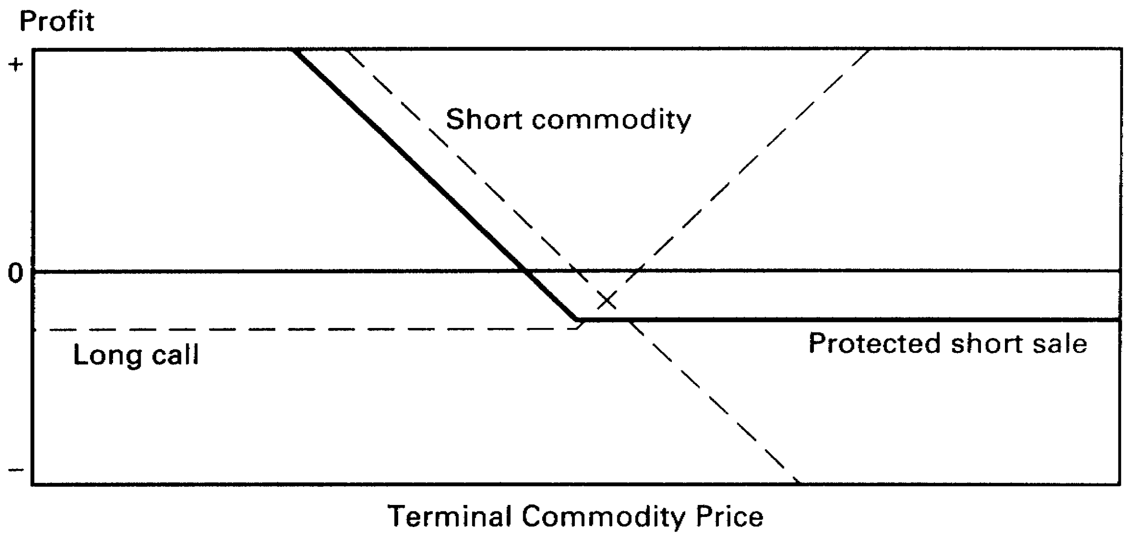
FIGURE 12.20 Trading Strategy: Variable Ratio Writing



A 2:1 variable ratio writing strategy is shown in Figure 12.20. As the illustration shows, variable ratio writing can produce a profit diagram that looks exactly like a short strangle position. Maximum profit is realized when the commodity price falls between the two exercise prices at the options' expiration. Large commodity price moves in either direction will produce losses.

44. *Protected short sale:* Buy call option against a short position in the underlying commodity.

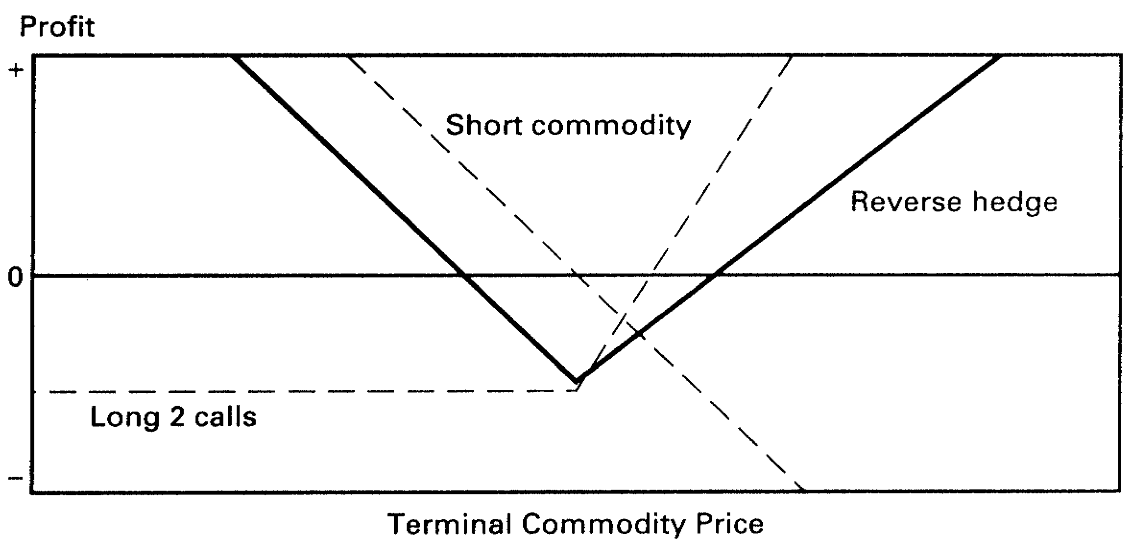
FIGURE 12.21 Trading Strategy: Protected Short Sale



Occasionally, an investor is short the commodity and wants to insure himself against possible large increases in the underlying commodity price. Buying a call option provides such insurance. As Figure 12.21 shows, buying a call option against a short position in the commodity produces a portfolio profit structure that looks exactly like a long put position. The position is also the opposite of the covered call. The maximum gain equals $Se^{bT} - ce^{rT}$, where the commodity price falls to zero. The maximum loss is $Se^{bT} - X - ce^{rT}$, which should be approximately equal to the value of a put with an exercise price of X and a time to expiration of T.

45. *Reverse hedge or simulated straddle:* Buy more than one call option against a short position in the underlying commodity.

FIGURE 12.22 Trading Strategy: Reverse Hedge

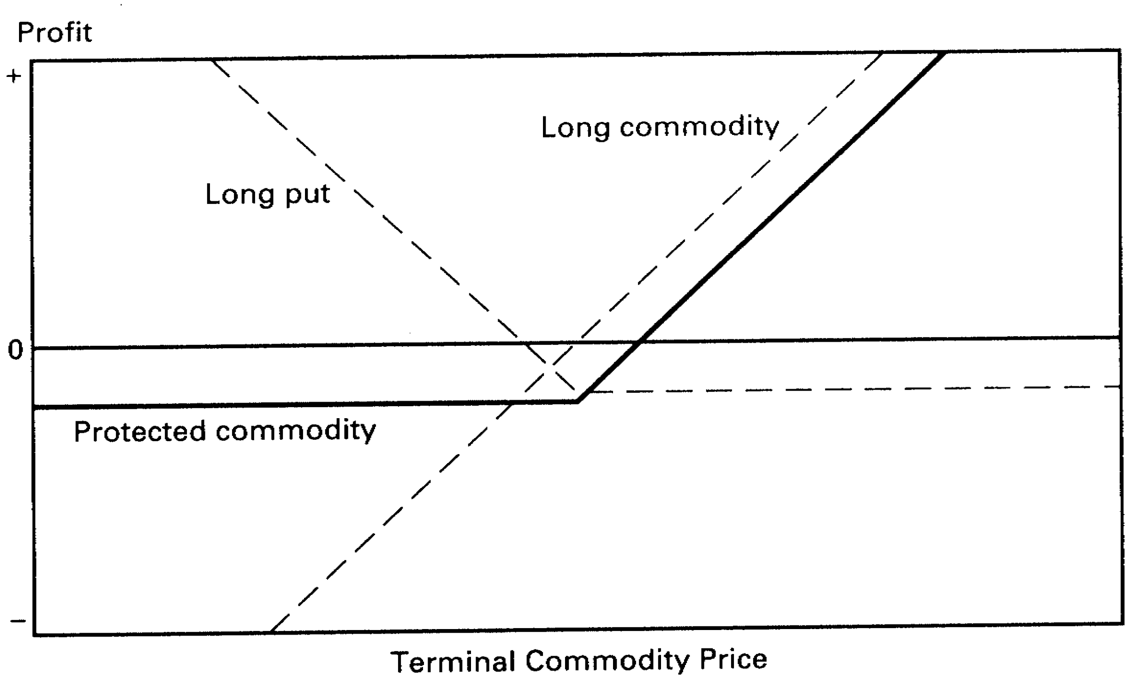


Buying two call options against a short position in the underlying commodity creates a profit diagram that looks exactly like a long straddle. For this reason, this strategy is sometimes referred to as a *simulated straddle.* The position is also the opposite of the ratio call writing position described earlier. The maximum loss is sustained when the commodity price equals the exercise price of the options at their expiration. The gain on the upside is unlimited, should the commodity price rise without limit. Downside commodity price movements are also beneficial, since the options expire worthless and the investor has a short commodity position.

46. *Protected commodity position:* Buy a put option against a long commodity position.

$$\pi_T = \begin{cases} S_T - Se^{bT} - pe^{rT} & \text{if} \quad S_T > X \\ X - Se^{bT} - pe^{rT} & \text{if} \quad S_T \leq X \end{cases} \tag{12.16}$$

FIGURE 12.23 Trading Strategy: Protected Commodity Position

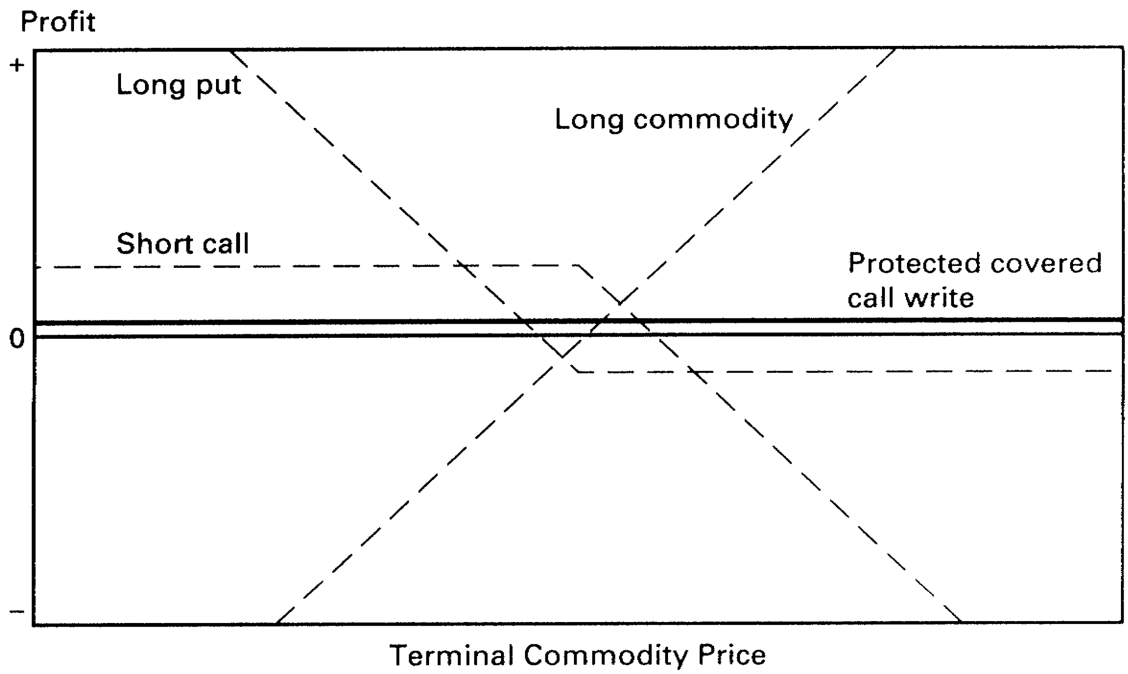


Breakeven point:	$S_T = Se^{bT} + pe^{rT}$
Maximum loss:	$X - Se^{bT} - pe^{rT}$, where $S_T \leq X$
Maximum gain:	unlimited, where S_T rises without limit

Buying protective puts is a favored form of commodity insurance. As Figure 12.23 indicates, an investor with a long commodity position is well protected in the event that the commodity price falls dramatically. The cost of such insurance is the put option premium. The resulting position is the same as buying a call.

47. *Protected covered call write:* Buy a put against a covered call write.

FIGURE 12.24 Trading Strategy: Protected Covered Call Write

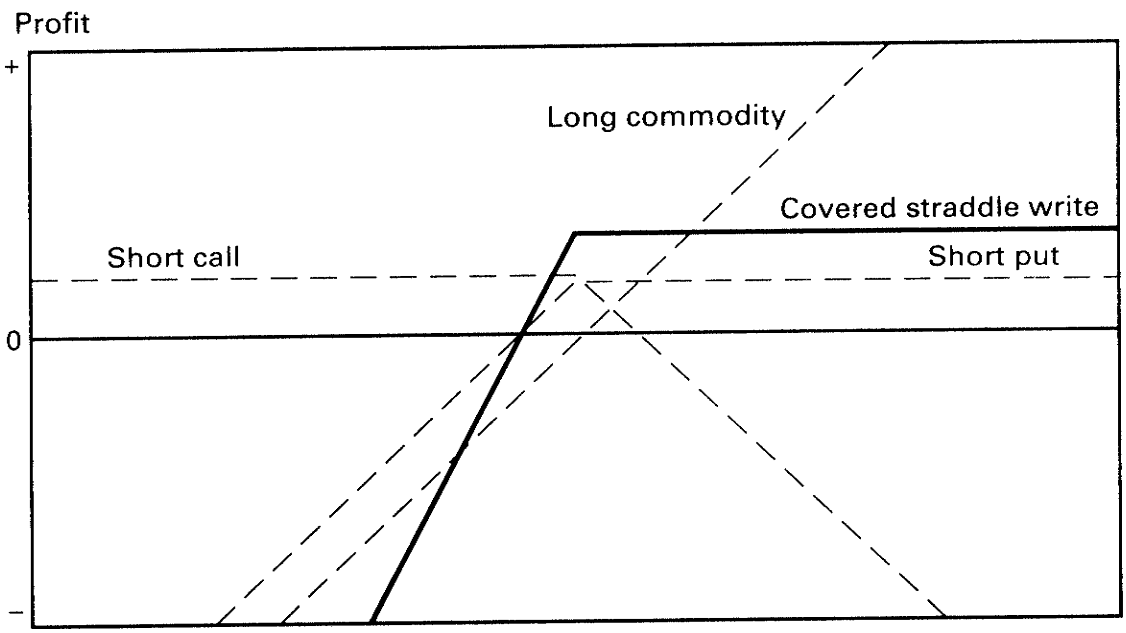


A covered call write means that the investor holds a long commodity/short call position. In the event that some time has elapsed since the covered call write was formed and the commodity price has not moved, the investor may want to lock in her profit from the time decay of the call by buying a put. When she does, she has, in effect, created a conversion arbitrage. Independent of which direction the commodity price moves subsequently, the portfolio profit is unchanged. Figure 12.24 demonstrates this clearly.

48. *Covered straddle write:* Buy commodity, sell call, and sell put.

$$\pi_T = \begin{cases} X - Se^{bT} + (c+p)e^{rT} & \text{if} \quad S_T > X \\ 2S_T - Se^{bT} - X + (c+p)e^{rT} & \text{if} \quad S_T \leq X \end{cases} \tag{12.17}$$

FIGURE 12.25 Trading Strategy: Covered Straddle Write

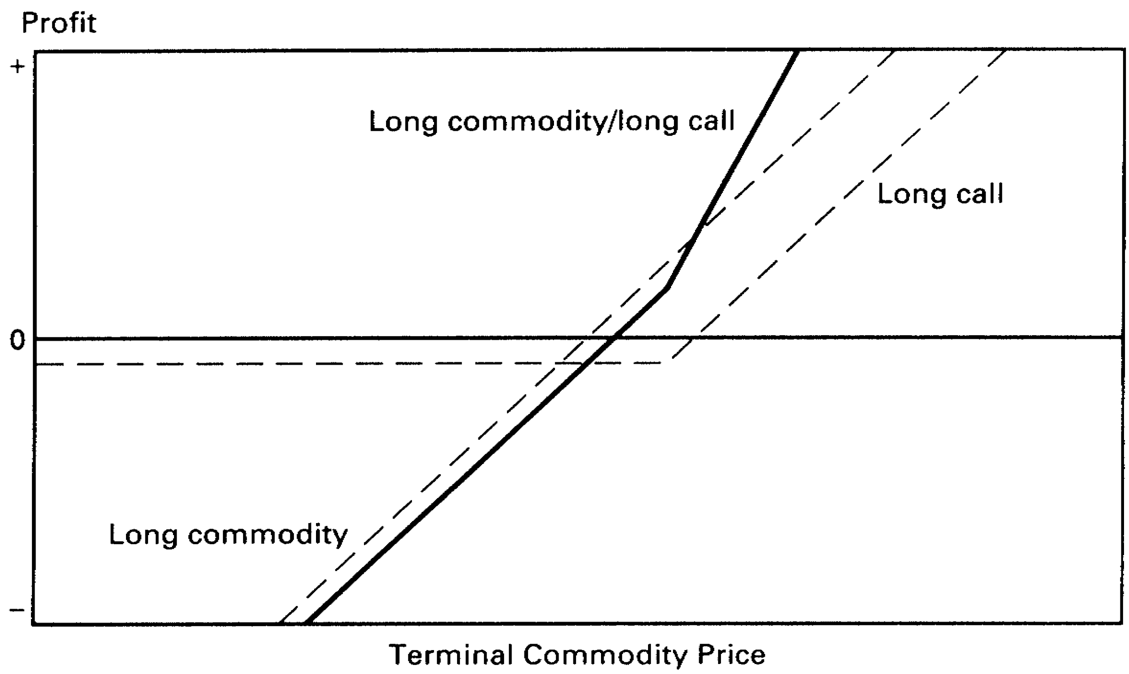


Breakeven point:	$S_T = \dfrac{Se^{bT} + X - (c+p)e^{rT}}{2}$
Maximum loss:	$-Se^{bT} - X + (c+p)e^{rT}$, where S_T falls to zero.
Maximum gain:	$X - Se^{bT} + (c+p)e^{rT}$, where $S_T > X$

In this case, the investor has written both a call and a put against a position in the underlying commodity. He has collected two option premiums, which equal the amount of the portfolio profit if the commodity price is above the option's exercise price at the option expiration. Should the commodity price fall, however, the portfolio profit drops by twice the amount, since the investor not only loses on the long commodity position, but also on the short put position.

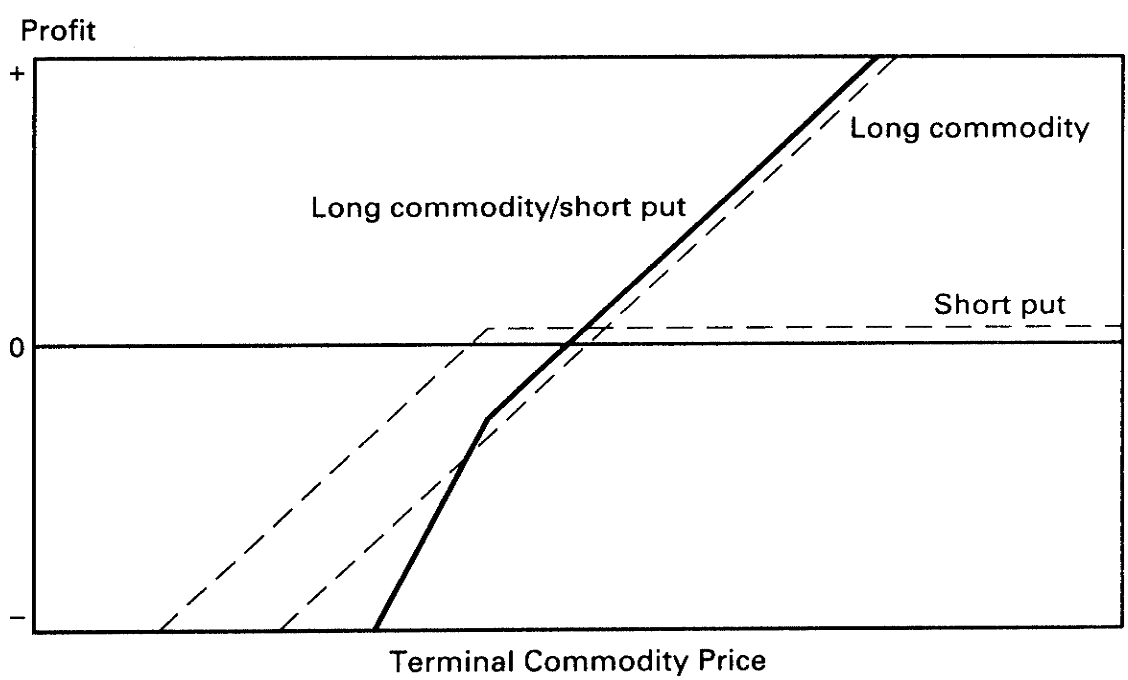
49. *Buy call against commodity:* Buy call against a long position in the commodity.

FIGURE 12.26a Trading Strategy: Long Commodity/Long Call Position



50. *Sell put against commodity:* Sell put against a long position in the commodity.

FIGURE 12.26b Trading Strategy: Long Commodity/Short Put Position



Buying a call against a long commodity position and selling a put against a long commodity position serve to leverage the rate at which portfolio profits are earned. The long commodity/long call position in Figure 12.26a, for example, shows that below a certain level of commodity price, the portfolio profit is less than the long commodity position, since the call option had to be purchased. Above a certain level of commodity price, however, portfolio profit increases at twice the rate that the commodity price does by itself. Hence, we have increased the leverage of the strategy.

Writing a put against a long position in the commodity has a similar effect. The proceeds from the sale of the option enhance portfolio profit on the upside. On the downside, if the commodity price drops, the investor loses both on the long commodity position and the short put position.

12.2 COMPUTING BREAKEVEN PROBABILITIES AND EXPECTED PROFITS

Two useful concepts for analyzing the commodity/option strategies just discussed are the probability that the portfolio will be profitable at the options' expiration and the expected profit from the trading strategy. Both of these concepts rely on the option pricing mechanics presented in Chapter 11. Since the expected profit concept itself relies on probability computations, the probability computations are reviewed first.[3]

Breakeven Probabilities

To compute the probability that a particular strategy will be profitable at expiration, we need to first establish the full set of breakeven points associated with the strategy. For example, for the long straddle position, Trading Strategy 15, two breakeven points exist. One breakeven point is where the terminal commodity price, S_T, equals the value $BE_a = X - (c + p)e^{rT}$, and the other is where S_T equals the value $BE_b = X + (c + p)e^{rT}$. Figure 12.7 shows that a long straddle has positive profit, where $S_T < BE_a$ or where $S_T > BE_b$. If we assume that the commodity price is log-normally distributed, as we did in Chapter 11, the risk-neutral probability that the straddle will be profitable at expiration, $\text{Prob}(\tilde{S}_T < BE_a \text{ or } \tilde{S}_T > BE_b)$, can be found by using the cumulative standard normal distribution function, that is,

$$\text{Prob}(\tilde{S}_T < BE_a \text{ or } \tilde{S}_T > BE_b) = N(-d_a) + N(d_b),$$

where $d_a = \dfrac{\ln(S/BE_a) + (b - .5\sigma^2)T}{\sigma\sqrt{T}}$ and $d_b = \dfrac{\ln(S/BE_b) + (b - .5\sigma^2)T}{\sigma\sqrt{T}}$. Recall that a minus sign on the argument d implies that the probability computation is for the region below a critical terminal commodity price, while a positive value implies that the probability is for the region above a critical terminal price. Recall also that

[3] In this section, we assume that we are in a risk-neutral world in which possible future commodity prices are brought forward at the riskless rate.

the expression for d transforms the lognormally distributed commodity price to a unit normal distribution.

EXAMPLE 12.1

Assume that the current commodity price is $50 and that the prices of at-the-money, three-month options are $3.35 for the call and $2.90 for the put. Compute the probability that a long straddle position using these options will be profitable at the end of three months. Assume that the cost-of-carry rate for the underlying commodity is 4 percent, the volatility rate of the underlying commodity is 32 percent, and the riskless rate of interest is 6 percent.

The first step is to compute the breakeven points:

$$BE_a = 50 - (3.35 + 2.90)e^{.06(.25)} = 43.656$$

and

$$BE_b = 50 + (3.35 + 2.90)e^{.06(.25)} = 56.344.$$

The second step is to transform the commodity price breakeven points to the breakeven points in terms of the unit normal distribution, that is,

$$d_a = \frac{\ln(50/43.656) + [.04 - .5(.32)^2](.25)}{.32\sqrt{.25}} = .8305$$

and

$$d_b = \frac{\ln(50/56.344) + [.04 - .5(.32)^2](.25)}{.32\sqrt{.25}} = -.7641.$$

Finally, the probability that the straddle will be profitable at the end of three months is

$$\text{Prob}(\tilde{S}_T < BE_a \text{ or } \tilde{S}_T > BE_b) = N(-.8305) + N(-.7641) = .4255$$

or 42.55 percent.

Expected Terminal Profit

The expected terminal profit from a commodity/option portfolio position is our best guess of what the portfolio profit will be at expiration. It can be computed by multiplying portfolio profit at each conceivable terminal commodity price by the probability of that commodity price occurring and then summing across all of these products. Conceptually, while the above procedure is straightforward, two important practical suggestions will make the procedure easier to implement.

First, given the assumption of lognormally distributed commodity prices, the range of future commodity prices is infinite. Computationally, however, we cannot use an infinite number of option profit positions. A practical alternative is to define the range of possible future commodity prices as ± 4 standard deviations from the expected commodity price, Se^{bT}, which, according to Appendix 11.3, should account for 99.994 percent of the commodity price distribution. The range of future commodity prices implied by ± 4 standard deviations around the expected price is defined by

$$+4 = \frac{\ln(S/S_{\min}) + (b - .5\sigma^2)T}{\sigma\sqrt{T}}$$

and

$$-4 = \frac{\ln(S/S_{\max}) + (b - .5\sigma^2)T}{\sigma\sqrt{T}}.$$

Rearranging, the expressions for the minimum and maximum of the commodity price range are

$$S_{\min} = Se^{(b-.5\sigma^2)T - 4\sigma\sqrt{T}} \quad \textbf{(12.18a)}$$

and

$$S_{\max} = Se^{(b-.5\sigma^2)T + 4\sigma\sqrt{T}}. \quad \textbf{(12.18b)}$$

A second consideration has to do with the computation of profit for a given probability. Even with a prespecified range of terminal commodity prices, S_T, there are an infinite number of commodity prices and, hence, an infinite number of portfolio profits and probabilities to evaluate. The computation is practical if we approximate the continuous distribution of terminal commodity prices with a discrete distribution. To do so, we partition the terminal commodity price distribution into n equal increments of S_{inc}, where

$$S_{\text{inc}} = \frac{S_{\max} - S_{\min}}{n - 1}. \quad \textbf{(12.19)}$$

We then begin at the lowest commodity price and assume that, over the first interval $S_{\min} - .5S_{\text{inc}}$ to $S_{\min} + .5S_{\text{inc}}$, the commodity price is $S_{\min}$. More generally, the commodity price is assumed to be S_i over the i-th interval, which has range $S_{i,T} \pm .5S_{\text{inc}}$, where

$$S_{i,T} = S_{\min} + (i-1)(S_{\text{inc}}). \tag{12.20}$$

The probability that the terminal commodity price will fall in this range is

$$\text{Prob}(S_{i,T} - .5S_{\text{inc}} < \tilde{S}_T < S_{i,T} + .5S_{\text{inc}}) = N(d_{l,i}) - N(d_{u,i}), \tag{12.21}$$

where

$$d_{l,i} = \frac{\ln[S/(S_i - .5S_{\text{inc}})] + (b - .5\sigma^2)T}{\sigma\sqrt{T}} \text{ and } d_{u,i} = \frac{\ln[S/(S_i + .5S_{\text{inc}})] + (b - .5\sigma^2)T}{\sigma\sqrt{T}}.$$

The expected terminal commodity price may, therefore, be computed as

$$E(\tilde{S}_T) = \sum_{i=1}^{n}[N(d_{l,i}) - N(d_{u,i})]S_{i,T}. \tag{12.22}$$

EXAMPLE 12.2

Compute the expected commodity price in three months, assuming the current commodity price is \$50, the cost-of-carry rate is 4 percent, and the volatility rate is 32 percent.

The first step is to compute the minimum and maximum of the commodity price range using (12.18):

$$S_{\min} = 50e^{[.04-.5(.32)^2].25-4(.32)\sqrt{.25}} = 26.2909$$

$$S_{\max} = 50e^{[.04-.5(.32)^2].25+4(.32)\sqrt{.25}} = 94.5589.$$

The next step is to divide the range of commodity prices into equal-spaced intervals. Choosing $n = 11$, the size of each interval is

$$S_{\text{inc}} = \frac{94.5589 - 26.2909}{11 - 1} = 6.8268.$$

The midpoint of each interval is assumed to be

$$S_{i,T} = 26.2909 + 6.8268(i-1),$$

the values of which are reported in the second column of Table 12.1.

The endpoints of each interval are then defined as

$$S_{l,i} = S_i - .5S_{\text{inc}}$$

TABLE 12.1 Estimation of expected terminal commodity price, using an equally spaced, discrete commodity price distribution approach: $S = 50$, $b = .04$, $T = .25$, and $\sigma = .32$.

Interval No.	(1) Commodity Price $S_{i,T}$[a]	Lower Integral Limit $d_{l,i}$	Upper Integral Limit $d_{u,i}$	(2) $N(d_{l,i}) - N(d_{u,i})$[b]	(1) Times (2)
1	26.2909	4.8692	3.2371	0.0006	0.0159
2	33.1177	3.2371	1.9441	0.0253	0.8391
3	39.9445	1.9441	0.8733	0.1653	6.6028
4	46.7713	0.8733	-0.0405	0.3249	15.1975
5	53.5981	-0.0406	-0.8377	0.2827	15.1536
6	60.4249	-0.8377	-1.5446	0.1399	8.4521
7	67.2517	-1.5446	-2.1796	0.0466	3.1326
8	74.0785	-2.1796	-2.7559	0.0117	0.8681
9	80.9053	-2.7559	-3.2836	0.0024	0.1953
10	87.7321	-3.2836	-3.7702	0.0004	0.0378
11	94.5589	-3.7702	-4.2216	0.0001	0.0066
				$E(\tilde{S}_T) =$	50.5013

a. $S_{i,T}$ is the terminal commodity price at the midpoint of the i-th interval.
b. $N(d_{l,i}) - N(d_{u,i})$ is the probability that the terminal commodity price will fall in the i-th interval.

and

$$S_{u,i} = S_i + .5S_{\text{inc}},$$

for $i = 1,\cdots,n$. Based on the interval endpoint values, the unit normal integral limits are computed and reported as the third and fourth columns of Table 12.1. Based on these limits, the probability that the terminal commodity price will fall in the i-th interval is computed using (12.21) and is reported in the fifth column.

The last column contains the product of the terminal commodity price and its respective probability. Summing across the values reported in the last column, we find that the expected terminal commodity price is

$$E(\tilde{S}_T) = \sum_{i=1}^{11}[N(d_{l,i}) - N(d_{u,i})]S_{i,T} = \$50.5013.$$

Note that this value corresponds closely to the true expected terminal commodity price, which we know from Chapter 11 to be

$$S = \$50e^{.04(.25)} = \$50.5025.$$

The slight discrepancy is due to the fact that this numerical method for computing the expected terminal commodity price is only an approximation, albeit a fairly accurate one in this illustration. Greater accuracy can be obtained by setting n to a higher value or by expanding the possible range of terminal commodity prices considered.

Extending this approach to compute expected terminal profit of an option portfolio is straightforward: simply replace the terminal commodity price $S_{i,T}$ in (12.22) with the option portfolio profit, given a commodity price of $S_{i,T}$, that is,

$$E(\tilde{\pi}_T) = \sum_{i=1}^{n}[N(d_{l,i}) - N(d_{u,i})]\pi(S_{i,T}). \tag{12.23}$$

The profit functions $\pi(\cdot)$ for a wide array of strategies were presented in the last section.

EXAMPLE 12.3

Compute the expected terminal profit of an at-the-money call option, assuming the current commodity price is \$50, the cost-of-carry rate is 4 percent, and the volatility rate is 32 percent. The riskless rate is assumed to be 6 percent, and the current call price is \$3.410.

All steps in this example are the same as those in Example 12.2, except that in place of multiplying the probability by the terminal commodity price in the interval, we multiply the probability by the call option profit conditional on the terminal commodity price, as shown in Table 12.2. The profit is the difference between the exercise value of the call and the initial price of the call adjusted for interest. Note that the expected terminal profit of the call option portfolio is approximately \$0.1937, which appears to indicate mispricing.

The theoretical value of this call using valuation (11.25) is \$3.410, the same as the initial value of the call, which means that there is no mispricing. The positive profit arises from the approximation implicit in Table 12.2 and the fact that the call option profit is a nonlinear function of the terminal commodity price. To rectify this problem, we should be careful to set n to a large value. With a larger number of steps, the discrepancy will be reduced. For example, where $n = 500$, the expected terminal profit is 0.0010—an approximation error of about one-tenth of one cent.

12.3 REPLICATING LONG-TERM OPTIONS

Portfolio managers occasionally want to buy or sell long-term options, but no such options are listed or the markets for the options are very inactive. In these cases, it is possible to replicate a long-term option with a portfolio that consists of short-

TABLE 12.2 Estimation of expected terminal profit of a long call position, whose current price is $c = 3.410$. The pricing parameters are: $S = 50$, $X = 50$, $b = .04$, $r = .06$, $T = .25$, and $\sigma = .32$.

Interval No.	Commodity Price $S_{i,T}$[a]	(1) $N(d_{l,i}) - N(d_{u,i})$[b]	(2) Profit $\pi_{i,T}$[c]	(1) Times (2)
1	26.2909	0.0006	–3.4615	–0.0021
2	33.1177	0.0253	–3.4615	–0.0877
3	39.9445	0.1653	–3.4615	–0.5722
4	46.7713	0.3249	–3.4615	–1.1248
5	53.5981	0.2827	0.1366	0.0386
6	60.4249	0.1399	6.9634	0.9740
7	67.2517	0.0466	13.7902	0.6424
8	74.0785	0.0117	20.6170	0.2416
9	80.9053	0.0024	27.4438	0.0662
10	87.7321	0.0004	34.2706	0.0148
11	94.5589	0.0001	41.0974	0.0028
			$E(\tilde{\pi}_T) =$	0.1937

a. $S_{i,T}$ is the terminal commodity price at the midpoint of the i-th interval.
b. $N(d_{l,i}) - N(d_{u,i})$ is the probability that the terminal commodity price will fall in the i-th interval.
c. $\pi_{i,T} = \max(0, S_{i,T} - X) - ce^{rT}$.

term options and a short-term riskless asset, such as T-bills.[4] The tools necessary to carry out this replication are the expected profit mechanics of the last section, together with multiple linear regression.

The approach is simple. First, as in the last section, find a range of plausible commodity prices at the end of the short-term options' life, t. Using expressions (12.18a) and (12.18b), identify a range that encompasses 99.994 percent of the probability distribution at t. Second, partition the range into n equal increments using (12.19), and identify the commodity prices, $S_{i,t}$, at the midpoint of each interval, using (12.20). Third, find the probability of the terminal commodity price falling within the i-th interval, using (12.21). So far, everything is as it was in the previous section.

On the basis of the commodity prices created in the second step of the last paragraph, $S_{i,t}$, $i = 1,\cdots,n$, compute the values of the long-term option value, $V_{LT}(S_{i,t})$, as well as the terminal values of all m short-term options, $V_{ST,j}(S_{i,t})$, that are assumed to be available, $j = 1,\cdots,m$. Use the values of the long-term option as

[4]Dynamic rebalancing of a portfolio that consists of the commodity and T-bills is another way of replicating a long-term option. We discuss this possibility in Chapter 14 under the heading "Dynamic Portfolio Insurance."

the dependent variable and the values of the short-term option as the independent variables, and perform a regression that minimizes the sum of squared errors,

$$\text{Min} \sum_{i=1}^{n} p_i [V_{LT}(S_{i,t}) - b_0 - \sum_{j=1}^{m} b_j V_{ST,j}(S_{i,t})]^2. \qquad \textbf{(12.24)}$$

The estimated regression coefficients, $\hat{b}_j$, $j = 1, \cdots, m$, are the amounts of the investments in the short-term options. The estimated intercept term, b_0 is the amount invested in the riskless asset. A check of how well the technique has performed can be made by comparing the current short-term option portfolio value to the theoretical value of the long-term option.[5]

EXAMPLE 12.4

Assume an investor owns a commodity portfolio and wants to buy a European put option with an exercise price of 100 and a maturity of one year. The current commodity price is 100, the cost-of-carry rate is 4 percent, and the volatility is 32 percent. The riskless rate of interest is 6 percent. The theoretical price of this option is $10.3887 on the basis of the European option valuation equation (11.28). However, no such long-term option exists.

Instead, the investor is considering buying a portfolio of three-month put options that can be used to replicate the performance of the one year option over the next three months. In three months, a new short-term position can be established to replicate the then nine-month option.[6] Seven three-month options are available:

Exercise Price	Option Price
85	1.0438
90	2.0681
95	3.6432
100	5.8302
105	8.6265
110	11.9752
115	15.7855

Setting the number of increments, n, to 300, the replication procedure described above is applied. First the values of the long-term option in three months for the possible values of the underlying commodity in three months are calculated. These

[5] Choie and Novomestky (1989) point out that if the terminal value of the short-term option portfolio corresponds to the long-term option value for all levels of commodity price at time t, then, in the absence of costless arbitrage opportunities in the marketplace, the current value of the short-term option portfolio should equal the current value of the long-term option.

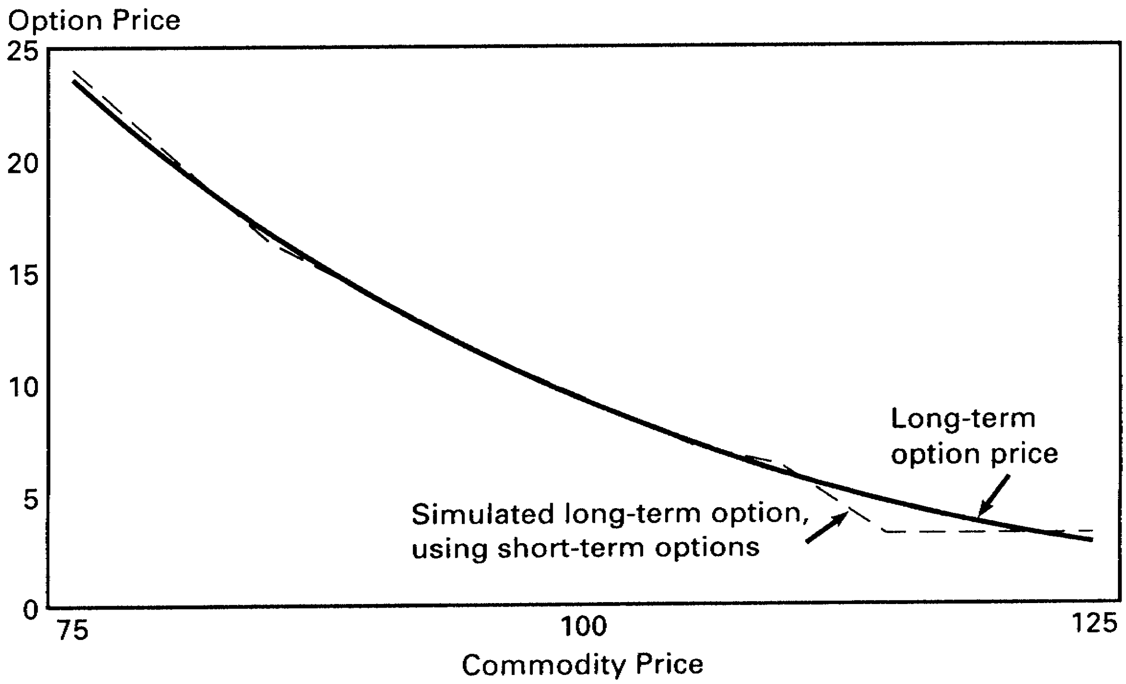
[6] In practice, using short-term options with more than one expiration date (e.g., three-month and six-month options) and/or rolling out of the short-term option positions prior to their expiration may provide a more effective replication of the long-term option position.

values are regressed against the possible values of the short-term options at maturity. Using the estimated regression coefficients, the portfolio composition is

Exercise Price	(1) Option Price	(2) Estimated Coefficient	(1) Times (2)
85	1.0438	.2286	.2386
90	2.0681	.0018	.0036
95	3.6432	.1004	.3657
100	5.8302	.0362	.2109
105	8.6265	.1730	1.4924
110	11.9752	−.4009	−4.8009
115	15.7855	.6136	9.6860
T-bill	.9851	3.2406	3.1923
Total			10.3887

With the exception of shorting the 110 put, all other puts are purchased. The sum of the portfolio weights times the security prices, 10.3887 equals exactly the long-term put option price. (The price of the T-bill is assumed to be $e^{-.06(.25)} = .9851$.) A comparison of the actual long-term put option with the simulated put option value is provided in Figure 12.27. Note how closely the values match until the commodity price becomes very high.

FIGURE 12.27 Simulated versus Long-Term Option Price



This procedure may be refined to account for non-negativity constraints, market liquidity, and observed option mispricings of short-term options. Changes in volatility through the life of the long-term option are also possible.[7] Our approach assumes that purchases and sales of short-term options are freely allowed in whatever quantity is demanded. We also assume that option prices conform with the European option valuation equations (11.25) and (11.28) and that the volatility rate is constant over the life of the long-term option.

Portfolio managers interested in long-dated options can either create them as indicated above or, as is more often the case, buy them from investment bankers in an over-the-counter transaction. The investment banker sells the option for her own account and hedges her position by taking an offsetting position in the replicating short-term option portfolio.

12.4 DYNAMIC PORTFOLIO RISK MANAGEMENT

Up to this point in the chapter, option positions have been held to maturity. In this section, we address the issue of dynamic risk management, that is, portfolio risk management that attempts to account for short-term price movements in the underlying commodity, short-term shifts in volatility, and the natural erosion of option premium as the time to option expiration is decreased. In this context, we rely particularly on the partial derivatives of the European option pricing formulas that we derived in Chapter 11. We will show that option deltas, gammas, etas, thetas, and vegas are invaluable aids in managing expected return and risk of a portfolio of options and the underlying commodity.

Expected Return and Risk

To begin, it is useful to have a clear understanding of the expected return/risk characteristics of option positions. In Chapter 11, we showed that the beta of an option equals the elasticity of the option price with respect to the commodity price times the beta of the underlying commodity, that is, $\beta_c = \eta_c \beta_S$ and $\beta_p = \eta_p \beta_S$. We also showed that the volatility of an option equals the absolute value of the elasticity of the option price with respect to the commodity price times the volatility of the underlying commodity, that is, $\sigma_c = |\eta_c| \sigma_S$ and $\sigma_p = |\eta_p| \sigma_S$. Recall that the elasticity depends on the commodity and option prices and on the other variables in the option pricing formula, such as volatility, time to expiration, and so on. As a result, the risk and return characteristics of option/commodity positions change through time as these variables change. If the risk of a position is to remain unchanged through time, the position must be appropriately rebalanced.

[7] See, for example, Jamshidian, and Zhu (1990).

We now examine the risk/return characteristics of a portfolio of options and the underlying commodity with the help of a simple numerical illustration. We assume that the commodity price is 50, the expected rate of return on the commodity is 16 percent, the commodity beta is 1.20, and the volatility of the commodity return is 40 percent. We assume a cost-of-carry rate of 4 percent and a riskless rate of 6 percent. Three-month European call and put options with exercise prices of 45, 50, and 55 are available, and all of these options have prices equal to their theoretical values, as determined by the pricing equations in the last chapter.

Focusing on the beta risk measure first, we can find the expected return/risk attributes of the options by first finding their respective betas, and then finding their equilibrium expected returns based on their betas. For example, the beta for the in-the-money call, which has an elasticity of $\eta_c = 5.289$, may be computed as

$$\begin{aligned}\beta_c &= \eta_c \beta_S \\ &= 5.289(1.20) = 6.35.\end{aligned}$$

Assuming the capital market is in equilibrium, the expected return on the commodity is

$$E(R_S) = r + [E(R_M) - r]\beta_S.$$

Substituting for $E(R_S)$ the expected return of the commodity and for r, the riskless rate of interest, we find the term $[E(R_M) - r]\beta_S$ equals .10. To find the expected return for the in-the-money call, we again use the security market line from the capital asset pricing model:

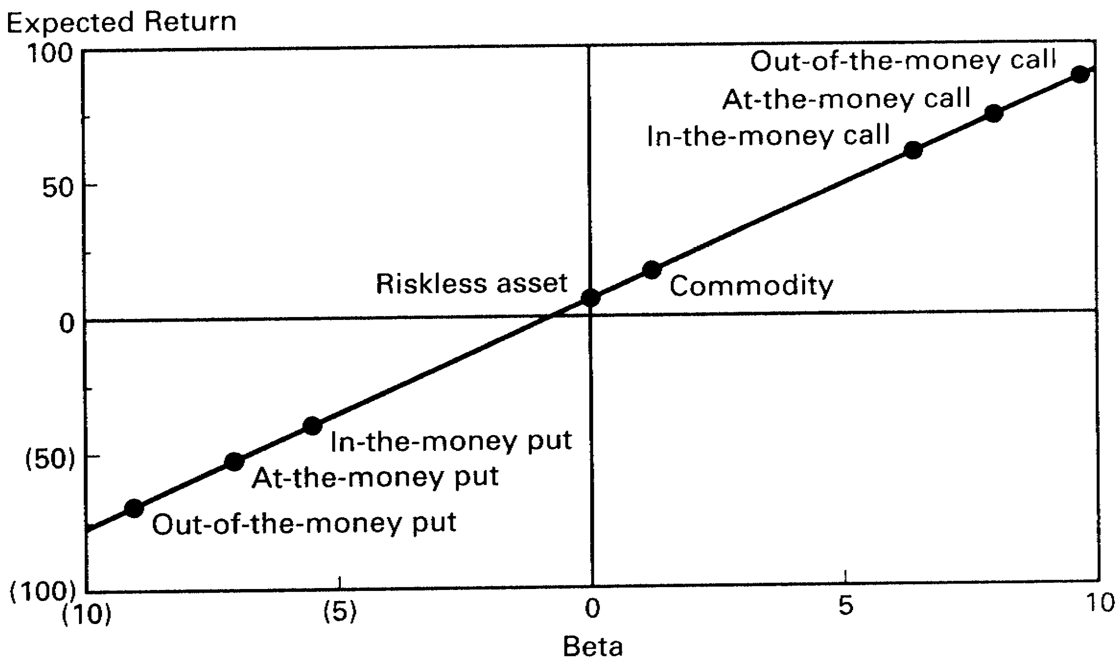
$$\begin{aligned}E(R_c) &= r + [E(R_M) - r]\beta_c \\ &= r + [E(R_M) - r]\beta_S \eta_c \\ &= .06 + .10\eta_c \\ &= .06 + .10(5.289) = 58.89\%.\end{aligned}$$

Using a similar procedure for the remaining options, we find the following expected returns and betas:

Option	Price	Delta	Elasticity	Expected Return	Beta
45 call	7.061	.747	5.289	58.89	6.35
50 call	4.196	.557	6.636	72.36	7.96
55 call	2.294	.370	8.069	86.69	9.68
45 put	1.640	–.248	–7.559	–69.59	–9.07
50 put	3.701	–.438	–5.920	–53.20	–7.10
55 put	6.724	–.625	–4.645	–40.45	–5.57
Commodity		1	1	16.00	1.20
Riskless Asset		0	0	6.00	0.00

Figure 12.28a illustrates these results.

FIGURE 12.28a Relation Between Expected Return and Beta



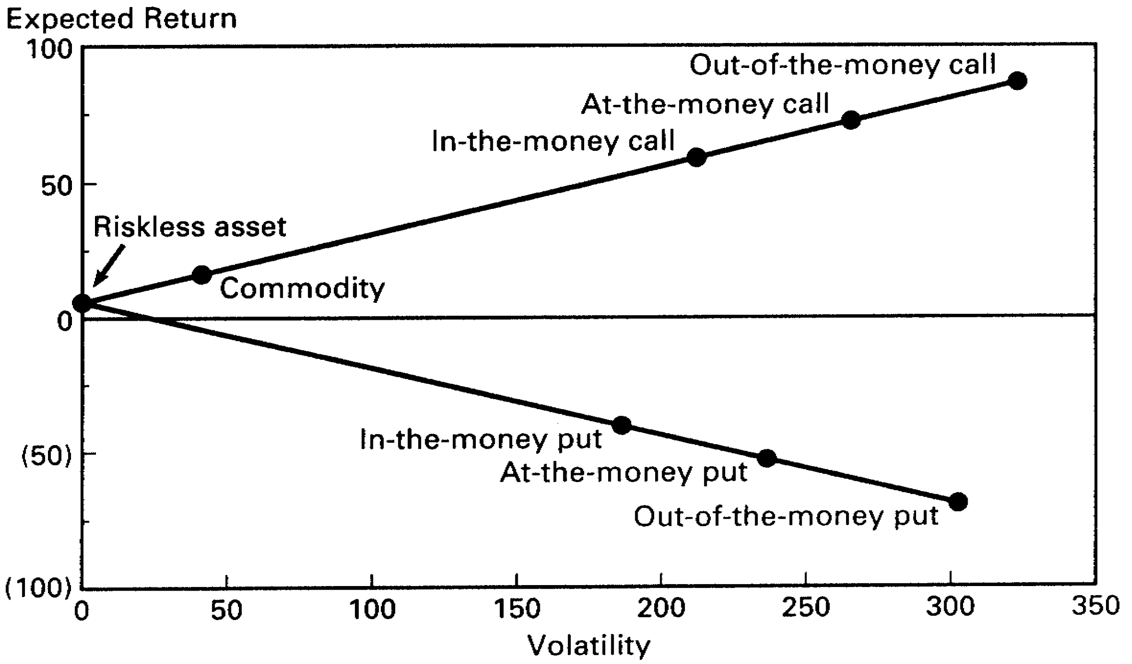
The expected return/beta relation depicted in Figure 12.28a is startling. The expected returns and betas of options are drastically different from the expected return and beta of the underlying commodity. Long call option positions, for example, have very high expected returns and betas—in fact, several times higher than the underlying commodity. The illustration also shows that the expected return and beta of the long call increase as the call goes deeper and deeper out-of-the-money. On the other hand, we see that put options generally have negative expected returns and negative betas, and the deeper the put is out-of-the-money, the lower (more negative) its expected return and its beta are.

Portfolio managers are also interested in knowing the level of return volatility. As noted above, option return volatility is simply the elasticity of the option price with respect to commodity price times the volatility of the underlying commodity return. In the illustration, the volatility of the commodity return is 40 percent. The option volatilities are, therefore,

Option	Elasticity	Return Volatility
45 call	5.289	211.56
50 call	6.636	265.44
55 call	8.069	322.76
45 put	−7.559	302.36
50 put	−5.920	236.80
55 put	−4.645	185.80
Commodity	1	40.00
Riskless Asset	0	0

Figure 12.28b illustrates these results.

FIGURE 12.28b Relation Between Expected Return and Volatility



The extreme riskiness of options is further confirmed by these values. Where the return volatility of the underlying commodity is 40 percent, the option return volatilities exceed, in some cases, several hundred percent.

Combining options with a position in the underlying commodity, however, can be risk-reducing. For example, a covered call strategy (i.e., writing a call against the underlying commodity) or a protective put strategy (i.e., buying a put against the underlying commodity) reduces the risk of the overall position. The expected return, beta, and return volatility of a portfolio that consists of an option and the underlying commodity may be computed using the following equations:

$$E(R_P) = X_S E(R_S) + (1 - X_S) E(R_o), \tag{12.25}$$

$$\beta_P = X_S \beta_S + (1 - X_S)\beta_o, \tag{12.26}$$

and

$$\sigma_P = \sqrt{X_S^2 \sigma_S^2 + (1 - X_S)^2 \sigma_o^2 + 2 d_o X_S (1 - X_S) \sigma_S \sigma_o}, \tag{12.27}$$

where the subscript o indicates option and the indicator variable d_o is $+1$ for calls and -1 for puts (i.e., call [put] option returns are perfectly positively [negatively] correlated with commodity returns). The weight X_S is the proportion of the S dollars invested directly in the commodity, that is,

$$X_S = \frac{S - n_o O_o}{S}, \tag{12.28}$$

where n_o is the number of options purchased (i.e., a positive value of n_o indicates the options are purchased, and a negative value indicates that the options are sold) and O_o is the market value of each option. Note that where n_o equals zero, all portfolio wealth is invested in the commodity. The value $1 - X_S$ is the proportion of the original investment in options.

To reinforce these mechanics, reconsider the above illustration and assume that a covered call write is created by selling the in-the-money call against a long position in the commodity. The proceeds from selling the call are invested in the commodity so the total investment in the commodity is

$$50 - (-1)(7.061) = 57.061.$$

The proportion of the original investment in the underlying commodity is therefore $X_S = 57.061/50 = 1.141$. The proportion of portfolio value invested in the call is $1 - X_S = -.141$.

With the weights known, we can compute the expected return, beta, and volatility of the covered call position using expressions (12.25), (12.26), and (12.27):

$$E(R_P) = 1.141(16.00) - .141(58.89) = 9.94\%;$$

$$\beta_P = 1.141(1.20) - .141(6.35) = .47,$$

and

$$\sigma_P = \sqrt{1.141^2(.40^2) + (-.141)^2(2.1156^2) + 2(1.141)(-.141)(.40)(2.1156)} \\ = 15.81\%.$$

In other words, writing the in-the-money call against the underlying commodity reduces the expected return and the risk of the underlying portfolio. In fact, all one-to-one covered call writes and all one-to-one protective put buys will share these attributes. For the illustration at hand, the characteristics of the commodity and the six different commodity/option portfolios are

Option	Commodity Investment	X_S	Expected Return	Beta	Return Volatility
no option	50.000	1.000	16.00	1.20	40.00
45 call write	57.061	1.141	9.94	.47	15.81
50 call write	54.196	1.084	11.27	.63	21.06
55 call write	52.294	1.046	12.75	.81	26.99
45 put buy	48.360	.967	13.18	.86	28.69
50 put buy	46.299	.926	10.88	.59	19.52
55 put buy	43.276	.866	8.44	.29	9.74

The above results are plotted in Figures 12.29a and 12.29b.

FIGURE 12.29a Relation Between Expected Return and Beta

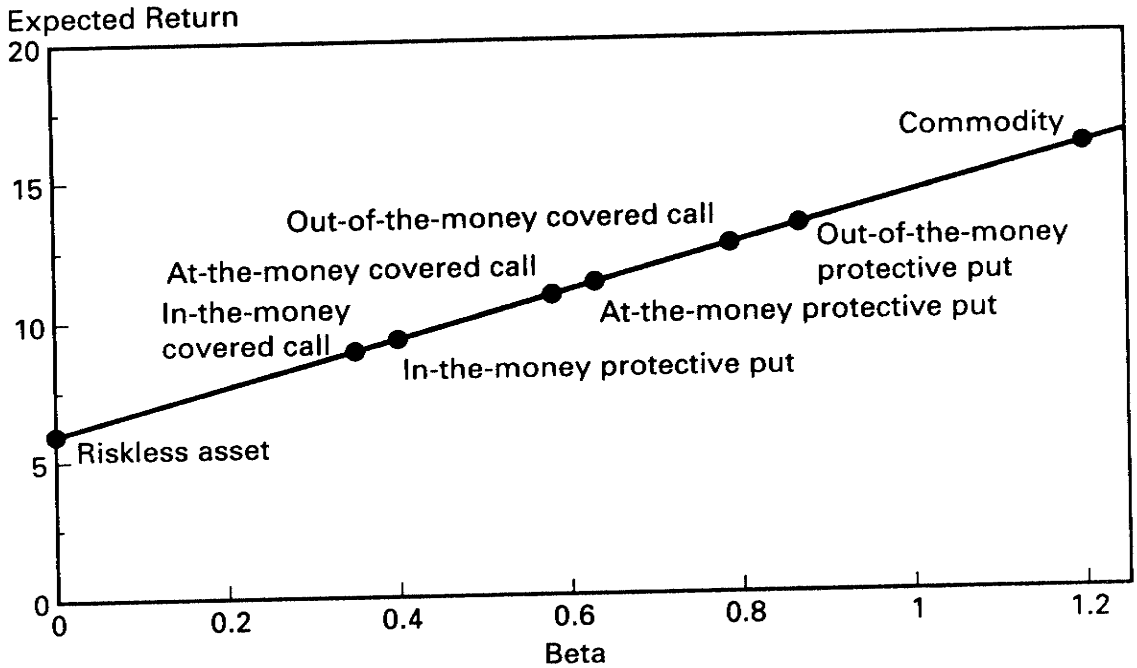
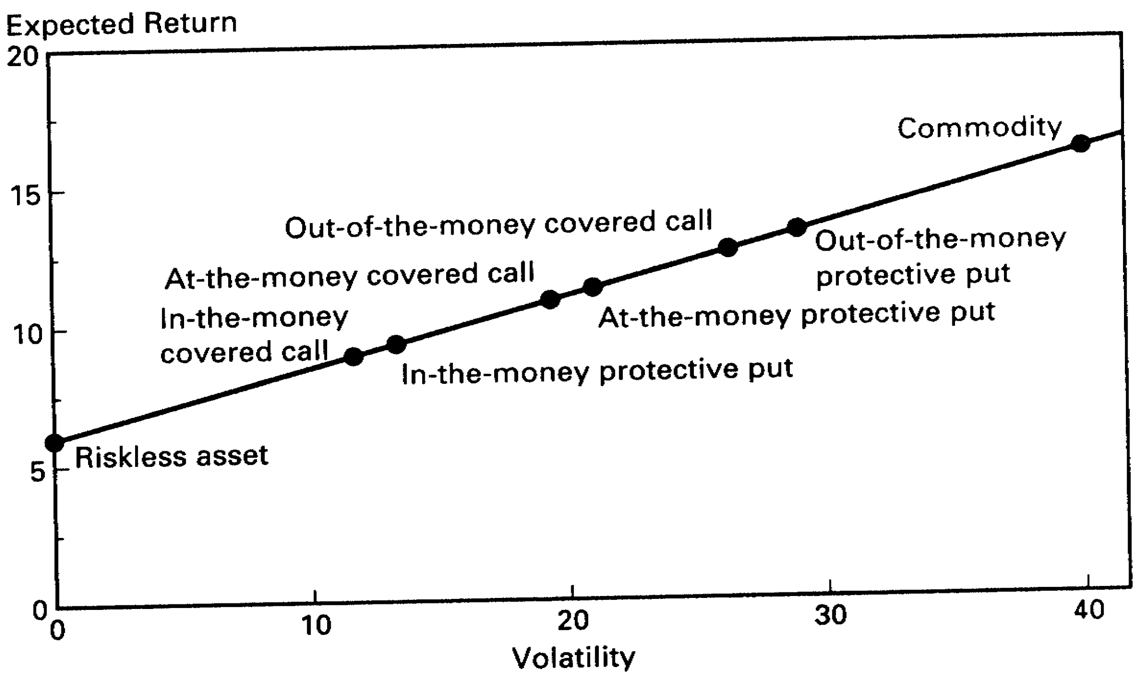


FIGURE 12.29b Relation Between Expected Return and Volatility



These figures show clearly that covered call writes and protective put buys serve to reduce portfolio expected return and risk.[8] For the covered calls, the return/risk reduction becomes larger the deeper in-the-money the call option is. This is simply because the option writer is willing to accept more cash (i.e., option premium) in exchange for the upside potential of commodity price movements. This activity is completely analogous to withdrawing investment from the commodity and investing in the riskless asset. In fact, given that the 45 call option has a delta of .747, the 45 covered call portfolio has a net delta value of $1.141 - .747 = .394$. If we create a portfolio that consists of .394 in the commodity and .606 in the riskless asset, the expected return, beta, and volatility of return are

$$E(R_P) = .394(16.00) + .606(6.00) = 9.94\%,$$

$$\beta_P = .394(1.20) + .606(0.00) = .47,$$

and

$$\sigma_P = \sqrt{.394^2(.40^2)} = 15.76\%,$$

the same return/risk attributes as the 45 covered call write.

That is not to say that writing call options against the underlying commodity is always risk-reducing. If too many calls are sold, portfolio risk may increase. Consider a 4:1 ratio call write. The total investment in the commodity is

$$50 + 4(7.061) = 78.244.$$

The proportion of the original investment in the underlying commodity is, therefore, $X_S = 78.244/50 = 1.565$. The proportion of portfolio value invested in the call is $1 - X_S = -.565$. The expected return on this portfolio is

$$E(R_P) = 1.565(16.00) - .565(58.90) = -8.24\%,$$

and the volatility is

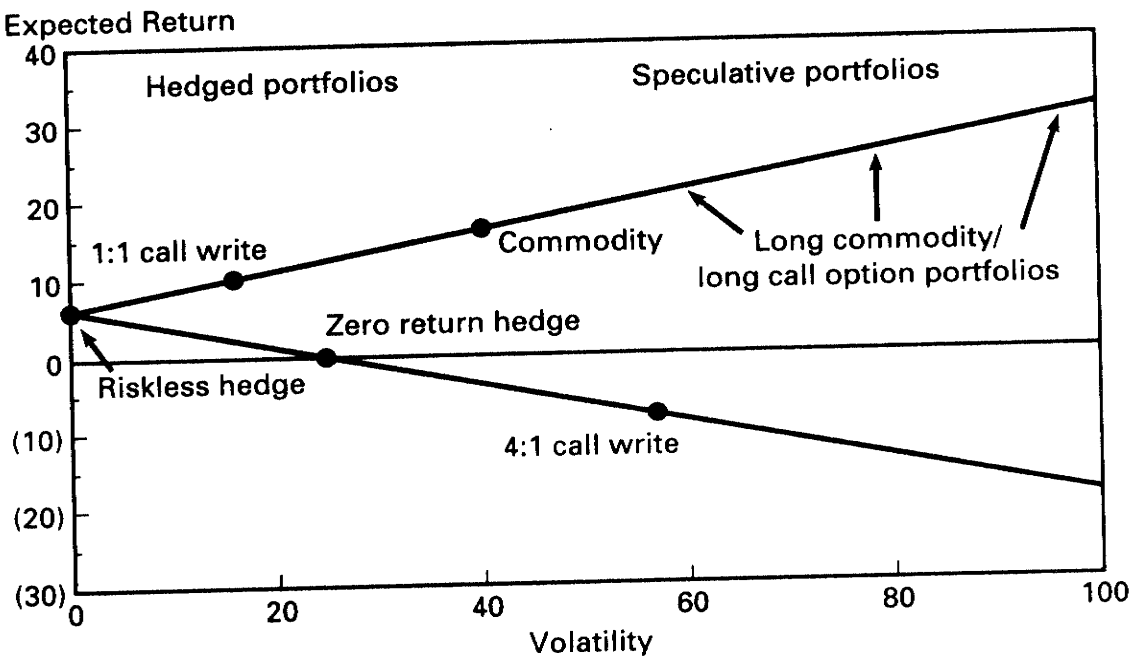
$$\sigma_P = \sqrt{1.565^2(.40^2) + .565^2(2.1156^2) - 2(1.565)(.565)(.40)(2.1156)} = 56.93\%.$$

[8] Recall that the expected return and beta of the option positions hold for the next instance over which the underlying variables don't change much.

Where the volatility of a portfolio that consists exclusively of the commodity is 40 percent, the volatility of a 4:1 ratio write is 56.93 percent. In this situation, we are overhedged.

Figure 12.30 helps clarify this point.

FIGURE 12.30 Relation Between Expected Return and Volatility



In the figure, we see the commodity plotted with an expected return of 16 percent and a volatility of 40 percent. The point labelled "1:1 call write" shows that the expected return and the volatility of the portfolio is reduced when a single 45 call is written against the commodity. As more calls are written, the expected return and volatility continue to decrease, until, finally, where 1.65 calls are written against the commodity, the portfolio is riskless.[9] Beyond this number, if more calls are written, expected return continues to decrease, but volatility increases. Eventually, the expected portfolio return becomes negative, and, if the number of calls written continues to grow, volatility begins to exceed 40 percent. The 4:1 call write portfolio, for example, has greater volatility than the commodity held in isolation. The critical number of calls written against the commodity to generate a 40 percent

[9] Earlier we illustrated that a covered call position is analogous to a portfolio that consists of somewealth in the commodity and some wealth in the riskless asset. The wealth invested in the commodity is $(S + n_c c - n_c S \Delta_c)$. If we set this value equal to zero (i.e., all wealth is invested in the riskless asset) and solve for n_c, we get $n_c = 1/(\Delta_c - c/S)$. Substituting the example values, $n_c = 1/(.747 - 7.061/50) = 1.65$.

volatility is 3.30, exactly double the number of calls that generated the riskless hedge.[10]

Managing Unexpected Changes in the Commodity Price

The risk of an option portfolio is subject to change as the price of the underlying commodity changes. Consequently, a portfolio manager needs not only to identify the current risk of the portfolio, as shown in the preceding section, but also to manage the portfolio to minimize the effects of unexpected changes in the commodity price. As a practical matter, knowledge of option deltas and gammas provide the tools necessary to immunize portfolios against adverse price movements in the underlying commodity.

In Chapter 11, we developed expressions for the partial derivatives of individual European options based on the valuation equations (11.25) and (11.28). The delta, for example, is the partial derivative of the option price with respect to a change in the underlying commodity price. The question that arises in option portfolio management is how the option portfolio value changes as a result of a change in the commodity price.

To understand the answer to this question, we first develop a simple, intuitive answer to the question that applies to all of the partial derivatives of the option price. First, write the expression for the value of the portfolio. Assume the portfolio consists of N different option positions, an underlying commodity position, and an investment in the riskless asset. Each option position has n_i contracts at current price O_i. Summing across positions and adding n_S units of the commodity at price S and the riskless asset, B, the value of the portfolio is

$$V = \sum_{i=1}^{N} n_i O_i + n_S S + B. \tag{12.29}$$

The partial derivative of the portfolio value with respect to a change in one of the option's determinants, k, is

$$\frac{\partial V}{\partial k} = \sum_{i=1}^{N} n_i \frac{\partial O_i}{\partial k} + n_S \frac{\partial S}{\partial k} + \frac{\partial B}{\partial k}. \tag{12.30}$$

[10]In the last footnote, we used the fact that a covered call is like a portfolio that consists of some wealth in the commodity and some wealth in the riskless asset in order to deduce the composition of the riskless hedge. The wealth invested in the commodity is $(S + n_c c - n_c S\Delta_c)$. If we set this value equal to $-S$ (i.e., a short sale of the underlying commodity) and solve for n_c, we get $n_c = 2/(\Delta_c - c/S)$. Substituting the example values, $n_c = 2/(.747 - 7.061/50) = 3.30$.

In other words, to find the change in the value of the portfolio resulting from a change in k, we simply compute how each option value changes from a change in k, multiply by the number of contracts of that option, and then sum across all option positions. The commodity and riskless asset positions may also affect the portfolio value. The same result holds when we examine the second partial derivative:

$$\frac{\partial^2 V}{\partial k^2} = \sum_{i=1}^{N} n_i \frac{\partial^2 O_i}{\partial k^2} + n_S \frac{\partial^2 S}{\partial k^2} + \frac{\partial^2 B}{\partial k^2}. \tag{12.31}$$

Now, let us return to the problem of managing changes in commodity price. The change in portfolio value with respect to a change in commodity price (i.e., the portfolio's delta) is

$$\Delta_V = \sum_{i=1}^{N} n_i \Delta_o + n_S. \tag{12.32}$$

Note that by assumption, the value of the riskless asset does not change as the commodity price changes. To immunize this portfolio from changes in the commodity price, we simply compute Δ_V and then take a position in options or the underlying commodity that makes the portfolio delta value zero.

EXAMPLE 12.5

Assume a futures option market maker has long positions of 150 calls with an exercise price of 45 and a time to expiration of two months, 200 puts with an exercise price of 50 and a time to expiration of three months, and 225 calls with an exercise price of 55 and a time to expiration of three months. Rather than face the risk that the underlying futures price may move significantly overnight, he decides to hedge the position using either the futures or calls with an exercise price of 50 and a time to expiration of three months. Compare the effectiveness of using the futures and the 50 call in creating a delta-neutral hedge. Assume the current futures price is \$50, the 50 call is priced at \$2.455 and has a delta of .5171, the riskless rate of interest is 6 percent, and the volatility rate is 25 percent.

Quantity	Option Type	Exercise Price	Time to Expiration	Option Price	Delta
150	Call	45	.16667	5.325	0.852
200	Put	50	.25	2.455	−0.468
225	Call	55	.25	0.828	0.238

Thus, the portfolio value is

$$V = 150(5.325) + 200(2.455) + 225(.828) = 1,476.05,$$

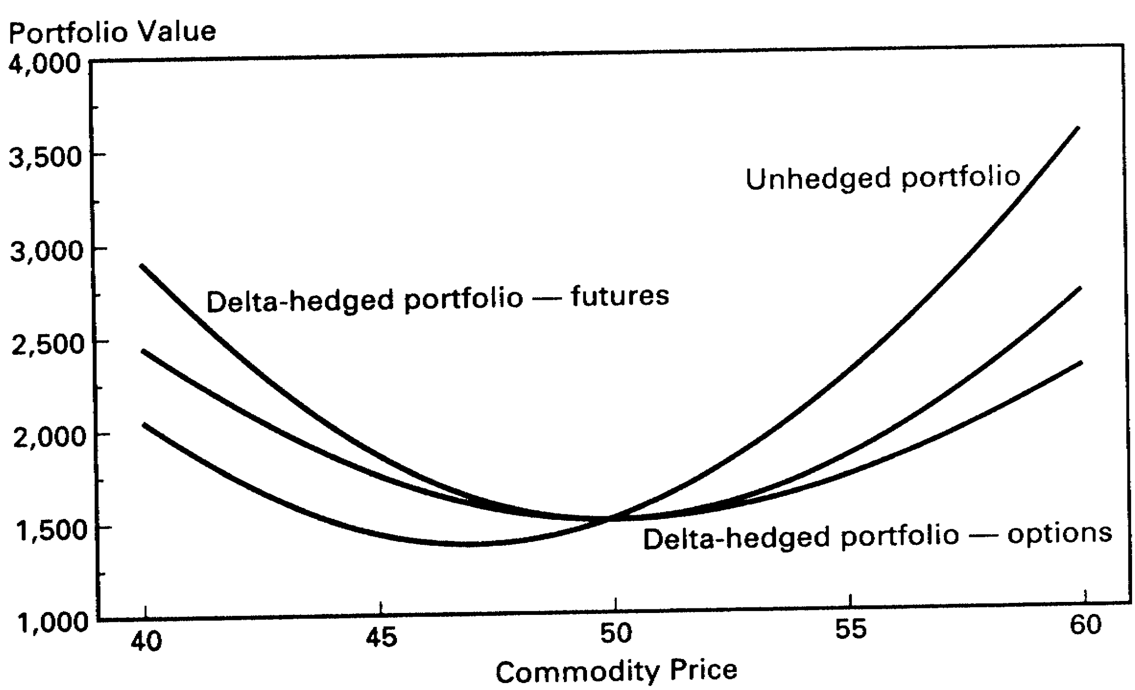
and the portfolio delta is

$$\Delta_V = 150(.852) + 200(-.468) + 225(.238) = 87.75.$$

The portfolio delta of 87.75 says that holding this portfolio is like holding a long position in 87.75 futures contracts. To create a delta neutral portfolio, we can either (a) sell 87.75 futures contracts or (b) sell 87.75/.5171 = 169.70 calls. Figure 12.31 shows the effectiveness of each hedge.

Figure 12.31 demonstrates that both the delta-neutral futures hedge and the delta-neutral option hedge reduce the range of possible portfolio values. The unhedged portfolio has a range of value from about 1400 to 3600 over the range of commodity prices—the futures hedge from about 1475 to 3000 and the options hedge from about 1475 to 2500. Clearly, the option hedge is the most effective.

FIGURE 12.31 Portfolio Value as a Function of Commodity Price



The reason that the option hedge winds up being the most effective has to do with gamma—the change in delta as the commodity price changes. As the commodity price changes, the option portfolio delta value changes. In fact, the option portfolio gamma is 30.08, as shown below. The futures contract has zero gamma, so the delta-neutral futures hedge still has a gamma of 30.08. On the other hand, the 169.70 calls that we sold have a gamma of $-169.70(.062758) = -10.65$, so the gamma of the delta-neutral option hedge is 19.43. Nonetheless, the gamma reduction with the option hedge is incidental in this case, so we will now illustrate how to account for both delta and gamma in hedging the portfolio against commodity price moves.

EXAMPLE 12.6

Again, we are considering the market maker described in Example 12.5. His portfolio position is

Quantity	Option Type	Exercise Price	Time to Expiration	Option Price	Delta	Gamma
150	Call	45	.16667	5.325	0.852	.04304
200	Put	50	.25	2.455	−0.468	.06276
225	Call	55	.25	0.828	0.238	.04922

As in the last example, the portfolio value is 1476.05, and the portfolio delta is 87.75. The portfolio gamma is

$$\gamma_V = 150(.04304) + 200(.06276) + 225(.04922) = 30.08.$$

To hedge both delta and gamma risk, two options are needed (i.e., the futures has zero gamma, so it is not effective at tailoring the gamma risk of the portfolio). In addition to the 50 call, which was available in Example 12.5, we will also assume that a three-month 55 put is available. Its price is \$7.754, its delta is $-.7468$, and its gamma is .04922. The 50 call has a gamma of .06276.

To compute the optimal delta-neutral/gamma-neutral hedge from these two options, we solve the following system of equations. We want the portfolio to be delta-neutral, so

$$n_c(.5171) + n_p(-.7468) = -87.75.$$

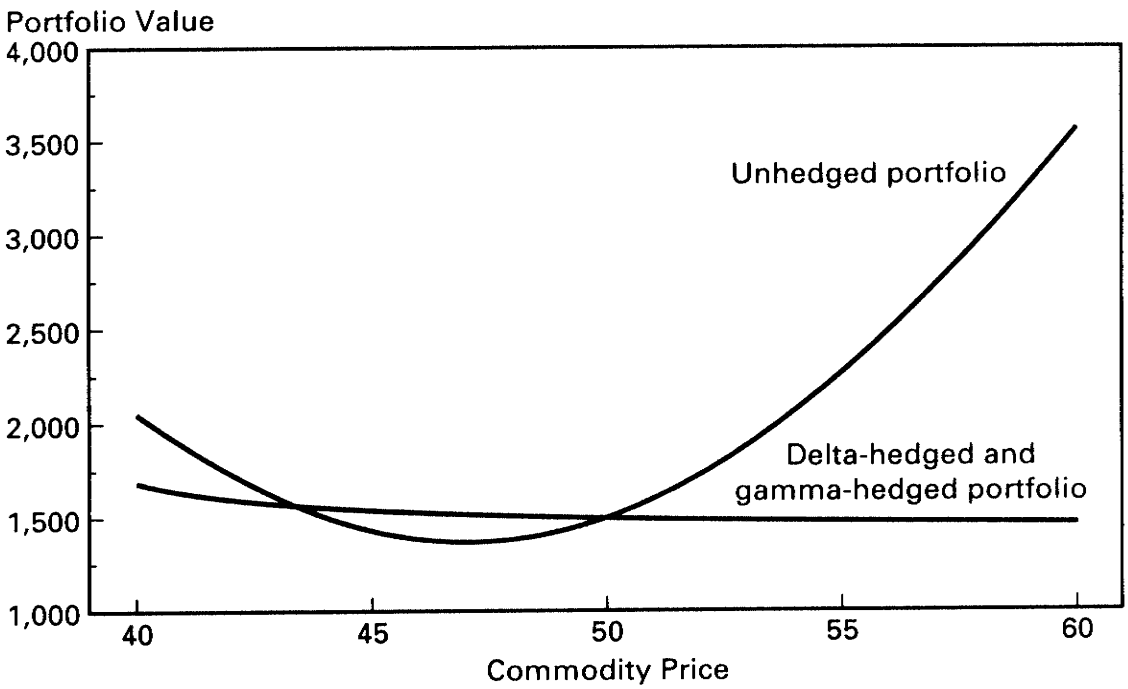
We also want the portfolio to be gamma-neutral, so

$$n_c(.06276) + n_p(.04922) = -30.08.$$

To solve, we can isolate n_c in the first equation, substitute into the second, and solve for n_p. The value of n_p is -138.93. We then substitute for n_p in the first equation and find that n_c is -370.34.

The value of the delta-neutral/gamma-neutral hedge portfolio is plotted in Figure 12.32, along with the unhedged portfolio value. Clearly, the hedge is effective. Where the unhedged portfolio varies by more than 2000 over the commodity price range shown, the hedged portfolio appears to vary by less than 200.

FIGURE 12.32 Portfolio Value as a Function of Commodity Price



Note that in Example 12.6, only two options are assumed to be available for setting the delta-neutral/gamma-neutral hedge. In practice, many options are available with which to set this hedge. At least two options are needed to execute the hedge, but more options can be used. Linear programming is sometimes used to find the minimum-cost set of options that will eliminate delta and gamma risk.

Managing Unexpected Changes in Volatility

Along with commodity price risk, traders also find themselves in a position where their option portfolios may suffer large losses if the volatility underlying the options shifts. For example, a market maker may be short calls and puts, and, while the position may be delta-neutral, a sudden increase in volatility will cause the market maker to incur significant losses. Like the delta-hedge shown above, the market maker can hedge volatility through vega-hedging.

EXAMPLE 12.7

Again, we are considering a market maker in the futures option contracts described in Example 12.5. This market maker's portfolio, however, is distinctly different. He is short 180 three-month 50 calls and 200 three-month 50 puts.

Quantity	Option Type	Exercise Price	Time to Expiration	Option Price	Delta	Vega
−180	Call	50	.25	2.455	0.517	9.806
−200	Put	50	.25	2.455	−0.468	9.806

Note that at-the-money futures options have the same price.

The portfolio is nearly delta-neutral, as is shown below:

$$\Delta_V = -180(.517) + -200(-.468) = .54.$$

Unfortunately, the vega-exposure is substantial:

$$\text{Vega}_V = -180(9.806) + -200(9.806) = -3726.28.$$

This means that if volatility increases from its current level of 25 percent to, say, 26 percent, the portfolio value will drop by 37.26 dollars.

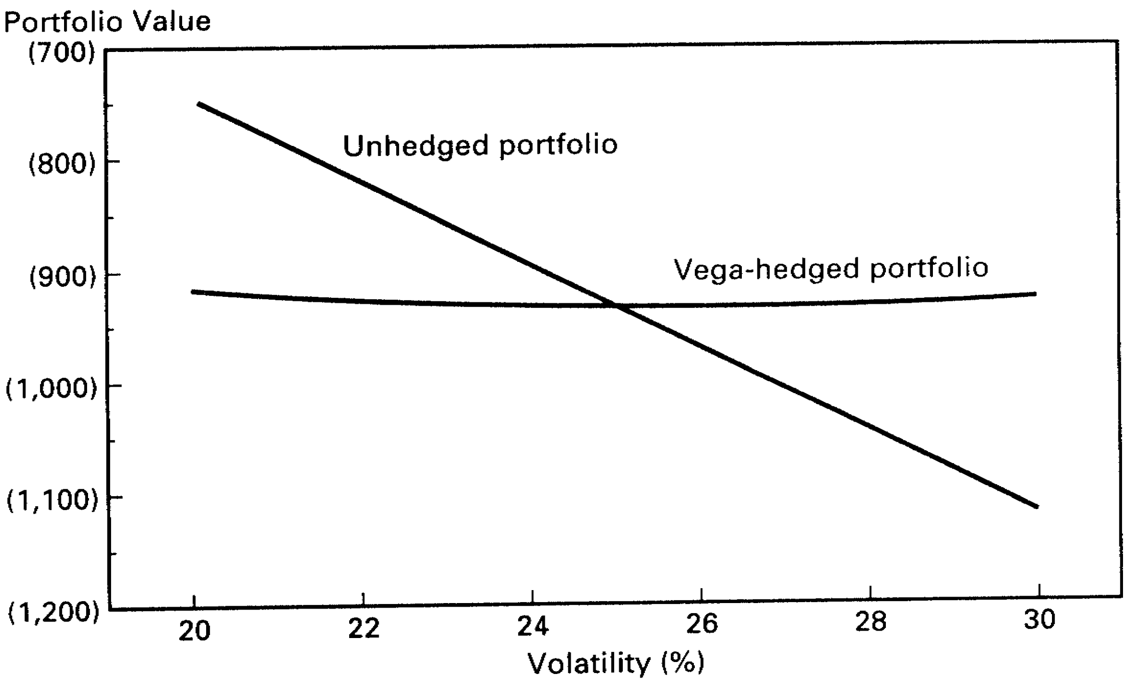
To hedge this risk, assume that the three-month 55 put from Example 12.6 is available. The 55 put has a vega of 7.69. In order to eliminate the vega risk of the portfolio, we should buy

$$n_p = \frac{3726.28}{7.69} = 484.56$$

puts. Figure 12.33 illustrates the effectiveness of this procedure.

Clearly, the vega-hedge is shown to be effective at eliminating the effects of shifts in volatility. Note, however, that the purchase of the 55 puts dramatically affected the portfolio's delta. It is now at a level of $484.56(-.7468) + .54 = -361.33$. This example is intended only to show how vega risk can be managed. Obviously, simultaneously considering delta, gamma, and vega may be sensible; this can be done with three or more available options.

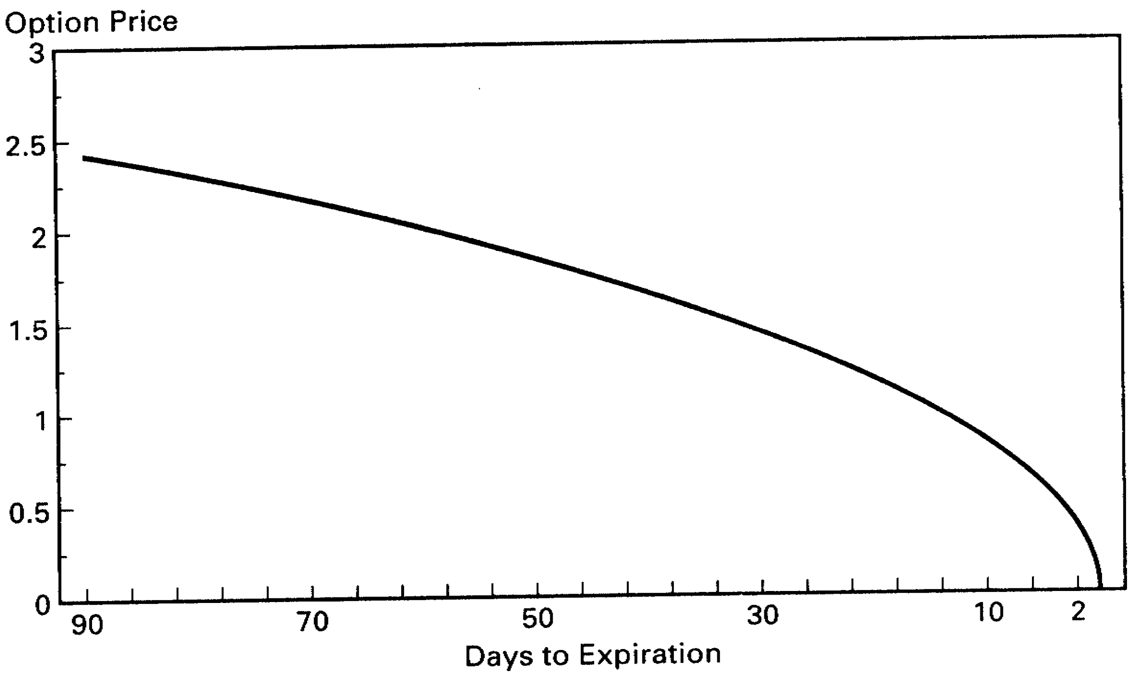
FIGURE 12.33 Portfolio Value as a Function of Volatility



Managing Time Decay

Our final discussion has to do with time decay. Long positions in options deteriorate in value through time as the prospect of a large commodity price move diminishes. Figure 12.34 illustrates how an at-the-money call option drops in value as its expiration date approaches.

FIGURE 12.34 Time Decay of an At-the-Money Call Option



The slope of the curve shown is the option's theta, that is, the change in option value as the time to expiration changes. At first the rate of decay is slow. In the final days before expiration, the rate is considerably faster. An implication of this figure is that managing time decay is more difficult as the time to option expiration is shortened.

To manage time decay, we use the same mechanics as we used for the other partial derivatives. In place of using delta, gamma, or vega, we find an option or options to negate the portfolio theta.

EXAMPLE 12.8

Consider the portfolio of the futures option market maker in Example 12.7. How can this person eliminate the time decay in his portfolio by using the 50 call option?

The portfolio position is

Quantity	Option Type	Exercise Price	Time to Expiration	Option Price	Theta
150	Call	45	.16667	5.325	3.043
200	Put	50	.25	2.455	4.756
225	Call	55	.25	0.828	7.690

The portfolio theta is, therefore,

$$\Theta = 150(3.043) + 200(4.756) + 225(7.690) = 3,137.90,$$

which means that, over the next day, the portfolio value will erode by 3,137.90 (1/365) = 8.597.

A theta-hedge can be created using the 50 call. Its theta is 4.756. To eliminate the time decay, we should sell 3,137.90/4.756 = 659.78 contracts.

In this section, we showed how portfolio delta, gamma, vega, and theta values may be used to effectively manage the value of an option portfolio. While the illustrations typically focused on one dimension at a time, it is clear that the daily man-

12.5 SUMMARY

In this chapter, we accomplished four things. First, we developed and analyzed more than fifty option trading strategies. Each strategy was put together from its basic security components and illustrated with a profit diagram. Breakeven commodity prices, maximum losses, and maximum gains were provided. Second, we showed that by using the lognormal commodity price distribution assumption from Chapter 11, we can compute probabilities of losses and gains, as well as the expected profit for each trading strategy. In the third section, we showed that a regression approach can be used to create long-term options from short-term options. Finally, we discussed the daily risk management of option portfolios.