DER 02.2

Return-risk estimation

1

Return-risk estimation

- Context:
  - Derivatives valuation and risk measurement involves characterizing statistical properties of underlying asset.
- Purpose:
  - Review basic statistical concepts.
  - Developing intuitive understanding of importance of continuously compounded returns.

2

Return measurement

- To measure return, consider stock price history.
  - Assume prices are equally-spaced (e.g., daily) through time.

$$S_0, S_1, S_2, ..., S_T$$

3

Holding period returns (HPRs)

- In practice, history of stock prices is used to compute *holding period* or *buy-and-hold* returns.
  - Defined as return earned over defined holding period (e.g., day, month, or year).

4

Holding period returns (HPRs)

- Daily holding period return
  - With no dividends:

$$R_t = \frac{S_t - S_{t-1}}{S_{t-1}}$$

  - With dividends:

$$R_t = \frac{S_t + D_t - S_{t-1}}{S_{t-1}} = \frac{S_t - S_{t-1}}{S_{t-1}} + \frac{D_t}{S_{t-1}}$$

5

Holding period returns (HPRs)

- Daily holding period return
  - With no dividends:

$$R_t = \frac{S_t - S_{t-1}}{S_{t-1}}$$

  - With dividends:

$$R_t = \frac{S_t + D_t - S_{t-1}}{S_{t-1}} = \boxed{\frac{S_t - S_{t-1}}{S_{t-1}}} + \frac{D_t}{S_{t-1}}$$

Price appreciation

6

Holding period returns (HPRs)

- Daily holding period return
  - With no dividends:

$$R_t = \frac{S_t - S_{t-1}}{S_{t-1}}$$

  - With dividends:

$$R_t = \frac{S_t + D_t - S_{t-1}}{S_{t-1}} = \frac{S_t - S_{t-1}}{S_{t-1}} + \boxed{\frac{D_t}{S_{t-1}}}$$

Dividend yield

7

Holding period returns (HPRs)

- Using HPRs can be misleading because they do not aggregate through time correctly.
  - E.g., monthly HPR is computed by adding up daily HPRs during month or by multiplying mean daily HPR by number of trading days in month.
  - E.g., annual HPR is computed by adding up monthly HPRs or by multiplying mean monthly HPR by 12.
  - This is wrong.

8

Holding period returns (HPRs)

- Illustration:
  - Assume price moves from 50 to 100 during first day, and then from 100 to 50 during second.
    - Two-day HPR is reported as

$$100\% + (-50\%) = 50\% \text{ or } 25\% \times 2 = 50\%$$

    - Actual return is 0%.

9

Continuous returns

- Correct approach is to use continuous returns.
  - Continuous return (or *ln* return) is defined as:

$$R_t = \ln\left(\frac{S_t}{S_{t-1}}\right)$$

10

Continuous returns

- Correct approach is to use continuous returns.
  - Continuous returns (or *ln* returns) can be summed through time (i.e., time aggregation property).
    - T-day return equals sum of daily returns.

$$\ln\left(S_T / S_0\right) = \sum_{t=1}^{T} \ln\left(S_t / S_{t-1}\right)$$

11

Continuous returns

- Illustration:
  - Assume price moves from 50 to 100 during first day, and then from 100 to 50 during second. What is return over two days?

$$\ln\left(100/50\right) + \ln\left(50/100\right) = \ln\left(100/100\right) = 0\%$$

12

Continuous returns

- Important properties:
  - Mean daily continuous return over T days is:

$$\mu \equiv \frac{\sum_{t=1}^{T} \ln\left(S_t / S_{t-1}\right)}{T} = \frac{\ln\left(S_T / S_0\right)}{T}$$

  - Variance of return is:

$$Var\left[\ln\left(S_T / S_0\right)\right] = \sum_{t=1}^{T} Var\left[\ln\left(S_t / S_{t-1}\right)\right] = \sigma^2 T$$

  - Standard deviation of return (i.e., "volatility") is:

$$\sqrt{Var\left[\ln\left(S_T / S_0\right)\right]} = \sigma\sqrt{T}$$

13

Measuring realized returns

- Many sources of data are available with which to measure *realized* or *historical* returns.
  - At Owen, have access to Bloomberg, Refinitiv (Datastream), Capital IQ, Morningstar, and FactSet.
    - http://researchguides.library.vanderbilt.edu/finance/home
  - On internet, http://finance.yahoo.com is free and easy-to-use for single security purposes.

14

Measuring realized returns

- All data sources report total return index (TRI) series.
  - Report for convenience.
    - Do not have to download prices and dividends.
  - Assume reinvestment of dividends.
- Level of *TRI* does not matter.

$$TRI_t = TRI_{t-1}\left(1+R_t\right),\ \text{where } R_t = \frac{S_t + D_t - S_{t-1}}{S_{t-1}}$$

Generate daily total HPR returns as $R_t = TRI_t / TRI_{t-1} - 1$

or daily ln returns as $R_t = \ln\left(TRI_t / TRI_{t-1}\right)$

15

Measuring realized returns

- Illustration: ICE (NYSE) Treasury futures indexes
  - Support files:
    - NYSE US Treasury futures index methodology.pdf
    - ICE UST.xlsx
      - Downloaded daily data from Bloomberg.
      - Chose 24-year period 19991231 through 20231229.
      - Chose price and total return indexes for UST.
        - USTTWO: 2-year
        - USTFIV: 5-year
        - USTTEN: 10-year
        - USTLBD: Long-term

16

U.S. trade dates

- Problem with Bloomberg and Refinitiv (Datastream) daily data.
  - Both are international data sources and have dates that are not US trading dates.
  - Including these dates in analyses of US markets is problematic.
    - Introduces error-in-the-variables problem
    - Should be eliminated.
  - Support file: US trading days 19271231-20231229.xlsx
    - Retain this file for future use.

17

Compute summary statistics

- Illustration: ICE (NYSE) Treasury futures indexes
  - Sheet: Purged data and analysis (BF)
    - With TRIs, first convert the indexes to ln returns before analyzing.
    - Long and tedious exercise. (See sheet.)

18

Digression: VBA functions

- Best way to handle repetitive computations in Excel is to create library of VBA functions.
  - Support file: VBAs.zip (VBAs.xlsm zipped)
    - Press ALT+F11. Insert module.
    - Program function to compute mean, standard deviation, and skewness.

```
n = ret.count
av = WorksheetFunction.Average(ret)
sd = WorksheetFunction.StDev(ret)
skew = WorksheetFunction.Skew(ret)
```

    - Return outputs in array.

```
aim_stat = Array(n,av,sd,skew)
```

19

Digression: VBA functions

- Developed series of statistical/financial functions that perform common and repetitive tasks in applied investment management.
  - Support files:
    - AIM installation instructions.pdf
    - AIM.xlam

20

Compute summary statistics

Summary								
Description	USTTWO	USTTWOT	USTFIV	USTFIVT	USTTEN	USTTENT	USTLBD	USTLBDT
No. of obs.	6,037	6,037	6,037	6,037	6,037	6,037	6,037	6,037
Mean (daily)	0.000032	0.000100	0.000079	0.000147	0.000112	0.000180	0.000135	0.000203
StDev (daily)	0.001012	0.001017	0.002510	0.002511	0.003808	0.003808	0.006556	0.006555
Skewness	0.356	0.453	0.038	0.048	0.017	0.016	-0.115	-0.121
Kurtosis	10.530	11.088	3.710	3.799	3.155	3.177	1.990	1.997
Autocorrelation	-0.0104	-0.0060	-0.0123	-0.0109	-0.0134	-0.0127	-0.0203	-0.0200
Minimum	-0.0093	-0.0093	-0.0192	-0.0192	-0.0243	-0.0243	-0.0419	-0.0419
Median	0.0000	0.0001	0.0001	0.0001	0.0001	0.0002	0.0003	0.0004
Maximum	0.0109	0.0115	0.0195	0.0196	0.0356	0.0357	0.0398	0.0398
Mean (annual)	0.81%	2.53%	1.98%	3.70%	2.82%	4.54%	3.39%	5.11%
StDev (annual)	1.61%	1.61%	3.98%	3.99%	6.05%	6.05%	10.41%	10.41%
CAGR	0.81%	2.56%	2.00%	3.77%	2.86%	4.65%	3.45%	5.25%
HPR	21.34%	83.23%	60.82%	142.84%	96.73%	197.05%	125.49%	240.50%

Columns are in pairs for 2-year, 5-year, 10-year, and LTB indexes.
- Price index
- TRI index

21

Returns

Summary								
Description	USTTWO	USTTWOT	USTFIV	USTFIVT	USTTEN	USTTENT	USTLBD	USTLBDT
No. of obs.	6,037	6,037	6,037	6,037	6,037	6,037	6,037	6,037
Mean (daily)	0.000032	0.000100	0.000079	0.000147	0.000112	0.000180	0.000135	0.000203
StDev (daily)	0.001012	0.001017	0.002510	0.002511	0.003808	0.003808	0.006556	0.006555
Skewness	0.356	0.453	0.038	0.048	0.017	0.016	-0.115	-0.121
Kurtosis	10.530	11.088	3.710	3.799	3.155	3.177	1.990	1.997
Autocorrelation	-0.0104	-0.0060	-0.0123	-0.0109	-0.0134	-0.0127	-0.0203	-0.0200
Minimum	-0.0093	-0.0093	-0.0192	-0.0192	-0.0243	-0.0243	-0.0419	-0.0419
Median	0.0000	0.0001	0.0001	0.0001	0.0001	0.0002	0.0003	0.0004
Maximum	0.0109	0.0115	0.0195	0.0196	0.0356	0.0357	0.0398	0.0398
Mean (annual)	0.81%	2.53%	1.98%	3.70%	2.82%	4.54%	3.39%	5.11%
StDev (annual)	1.61%	1.61%	3.98%	3.99%	6.05%	6.05%	10.41%	10.41%
CAGR	0.81%	2.56%	2.00%	3.77%	2.86%	4.65%	3.45%	5.25%
HPR	21.34%	83.23%	60.82%	142.84%	96.73%	197.05%	125.49%	240.50%

$$HPR = e^{0.00032\times 6{,}037} - 1 = 21.34\%$$

Return over 24-year period ignoring coupon payments.

22

Returns

Summary								
Description	USTTWO	USTTWOT	USTFIV	USTFIVT	USTTEN	USTTENT	USTLBD	USTLBDT
No. of obs.	6,037	6,037	6,037	6,037	6,037	6,037	6,037	6,037
Mean (daily)	0.000032	0.000100	0.000079	0.000147	0.000112	0.000180	0.000135	0.000203
StDev (daily)	0.001012	0.001017	0.002510	0.002511	0.003808	0.003808	0.006556	0.006555
Skewness	0.356	0.453	0.038	0.048	0.017	0.016	-0.115	-0.121
Kurtosis	10.530	11.088	3.710	3.799	3.155	3.177	1.990	1.997
Autocorrelation	-0.0104	-0.0060	-0.0123	-0.0109	-0.0134	-0.0127	-0.0203	-0.0200
Minimum	-0.0093	-0.0093	-0.0192	-0.0192	-0.0243	-0.0243	-0.0419	-0.0419
Median	0.0000	0.0001	0.0001	0.0001	0.0001	0.0002	0.0003	0.0004
Maximum	0.0109	0.0115	0.0195	0.0196	0.0356	0.0357	0.0398	0.0398
Mean (annual)	0.81%	2.53%	1.98%	3.70%	2.82%	4.54%	3.39%	5.11%
StDev (annual)	1.61%	1.61%	3.98%	3.99%	6.05%	6.05%	10.41%	10.41%
CAGR	0.81%	2.56%	2.00%	3.77%	2.86%	4.65%	3.45%	5.25%
HPR	21.34%	83.23%	60.82%	142.84%	96.73%	197.05%	125.49%	240.50%

$$CAGR = e^{0.000100 \times 252} - 1 = 2.56\%$$

Compound annual growth rate including coupon reinvestment.

23

Coupon yield

Summary								
Description	USTTWO	USTTWOT	USTFIV	USTFIVT	USTTEN	USTTENT	USTLBD	USTLBDT
No. of obs.	6,037	6,037	6,037	6,037	6,037	6,037	6,037	6,037
Mean (daily)	0.000032	0.000100	0.000079	0.000147	0.000112	0.000180	0.000135	0.000203
StDev (daily)	0.001012	0.001017	0.002510	0.002511	0.003808	0.003808	0.006556	0.006555
Skewness	0.356	0.453	0.038	0.048	0.017	0.016	-0.115	-0.121
Kurtosis	10.530	11.088	3.710	3.799	3.155	3.177	1.990	1.997
Autocorrelation	-0.0104	-0.0060	-0.0123	-0.0109	-0.0134	-0.0127	-0.0203	-0.0200
Minimum	-0.0093	-0.0093	-0.0192	-0.0192	-0.0243	-0.0243	-0.0419	-0.0419
Median	0.0000	0.0001	0.0001	0.0001	0.0001	0.0002	0.0003	0.0004
Maximum	0.0109	0.0115	0.0195	0.0196	0.0356	0.0357	0.0398	0.0398
Mean (annual)	0.81%	2.53%	1.98%	3.70%	2.82%	4.54%	3.39%	5.11%
StDev (annual)	1.61%	1.61%	3.98%	3.99%	6.05%	6.05%	10.41%	10.41%
CAGR	0.81%	2.56%	2.00%	3.77%	2.86%	4.65%	3.45%	5.25%
HPR	21.34%	83.23%	60.82%	142.84%	96.73%	197.05%	125.49%	240.50%

$$CAGR_{USTTWOT} - CAGR_{USTTWO} = 2.56\% - 0.81\% = 1.75\%$$

Annualized coupon yield for 2-year Treasuries.

24

Volatility

Summary				
Description	USTTWOT	USTFIVT	USTTENT	USTLBDT
No. of obs.	6,037	6,037	6,037	6,037
Mean (daily)	0.000100	0.000147	0.000180	0.000203
StDev (daily)	0.001017	0.002511	0.003808	0.006555
Skewness	0.453	0.048	0.016	-0.121
Kurtosis	11.088	3.799	3.177	1.997
Autocorrelation	-0.0060	-0.0109	-0.0127	-0.0200
Minimum	-0.0093	-0.0192	-0.0243	-0.0419
Median	0.0001	0.0001	0.0002	0.0004
Maximum	0.0115	0.0196	0.0357	0.0398
Mean (annual)	2.53%	3.70%	4.54%	5.11%
StDev (annual)	1.61%	3.99%	6.05%	10.41%
CAGR	2.56%	3.77%	4.65%	5.25%
HPR	83.23%	142.84%	197.05%	240.50%

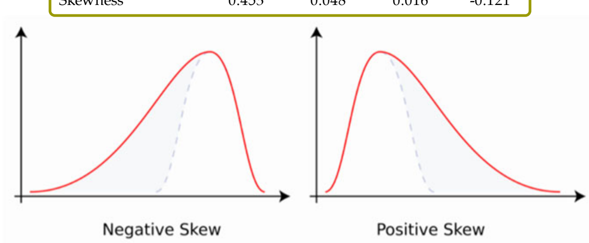
Eliminated price indexes.

Annualized volatility rates are monotonically increasing.

25

Skewness

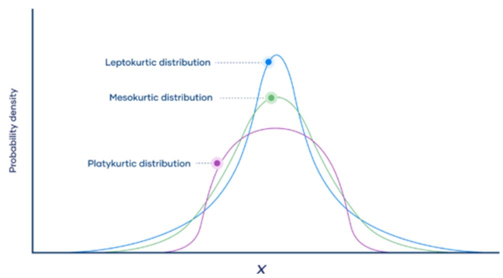
Summary				
Description	USTTWOT	USTFIVT	USTTENT	USTLBDT
No. of obs.	6,037	6,037	6,037	6,037
Mean (daily)	0.000100	0.000147	0.000180	0.000203
StDev (daily)	0.001017	0.002511	0.003808	0.006555
Skewness	0.453	0.048	0.016	-0.121



26

Kurtosis

Summary				
Description	USTTWOT	USTFIVT	USTTENT	USTLBDT
No. of obs.	6,037	6,037	6,037	6,037
Mean (daily)	0.000100	0.000147	0.000180	0.000203
StDev (daily)	0.001017	0.002511	0.003808	0.006555
Skewness	0.453	0.048	0.016	-0.121
Kurtosis	11.088	3.799	3.177	1.997



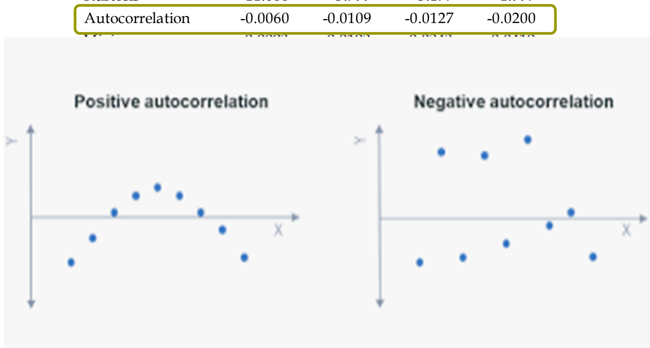
	Category		
	Mesokurtic	Platykurtic	Leptokurtic
Tailedness	Medium-tailed	Thin-tailed	Fat-tailed
Outlier frequency	Medium	Low	High
Kurtosis	Moderate (3)	Low (< 3)	High (> 3)
Excess kurtosis	0	Negative	Positive
Example distribution	Normal	Uniform	Laplace

Source of figure and table: ttps://www.scribbr.com/statistics/kurtosis/

27

Autocorrelation

Summary				
Description	USTTWOT	USTFIVT	USTTENT	USTLBDT
No. of obs.	6,037	6,037	6,037	6,037
Mean (daily)	0.000100	0.000147	0.000180	0.000203
StDev (daily)	0.001017	0.002511	0.003808	0.006555
Skewness	0.453	0.048	0.016	-0.121
Kurtosis	11.088	3.799	3.177	1.997
Autocorrelation	-0.0060	-0.0109	-0.0127	-0.0200



Positive autocorrelation: If yesterday's return is positive, today's return is likely to be positive, and vice versa.

Negative autocorrelation: If yesterday's return is positive, today's return is likely to be negative, and vice versa.

28

Correlation

Correlations	USTTWOT	USTFIVT	USTTENT	USTLBDT
USTTWOT	1	0.912	0.823	0.662
USTFIVT	0.912	1	0.965	0.842
USTTENT	0.823	0.965	1	0.926
USTLBDT	0.662	0.842	0.926	1

Contemporaneous correlation:
Assesses degree to which returns move together.

29

Lesson summary

- Understanding derivatives valuation and risk measurement requires knowledge of underlying asset return distribution.
  - Return distributions are characterized by statistical properties.
    - Used ICE Treasury futures indexes to demonstrate key properties.

30