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THE VALUATION OF AMERICAN CALL OPTIONS AND THE EXPECTED EX-DIVIDEND STOCK PRICE DECLINE*

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This study focuses on the ex-dividend stock price decline implicit within the valuation of American call options on dividend-paying stocks. The Roll (1977) American call option pricing formula and the observed structure of CBOE call option transaction prices are used to infer the *expected* ex-dividend stock price decline as a proportion of the amount of the dividend. The relative decline is shown to be not meaningfully different from one, confirming some recent evidence from studies which examined stock prices in the days surrounding ex-dividend.

1. Introduction

Since the introduction of organized stock option trading more than a decade ago, a number of advances in the theory of option pricing have been made. One such advance is the derivation of the valuation equation for an American call option on a dividend-paying stock by Roll (1977). Using the compound option valuation framework of Geske (1979a), Roll manages to price the American call option on a dividend-paying stock by assuming that the amount and the timing of the dividend paid during the option's life are known with certainty.

Prior to the development of the American call option formula, two approximation formulas were used. The first, the dividend-adjusted Black-Scholes (1973a) approximation, is computed by substituting the current stock price, less the present value of the dividend paid during the option's life, as the stock price parameter. The second, the Black (1975) approximation, is computed by

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taking the larger of the Black–Scholes value where the option is assumed to be exercised just prior to the ex-dividend instant and the dividend-adjusted Black–Scholes value. Whaley (1982) empirically examines the performance of these two approximation methods vis-à-vis the American call option formula and shows that the latter model reduces the systematic biases that had appeared for the approximations.[1] Later, Geske and Roll (1984) justify theoretically Whaley's results by showing that the early exercise premium of the American call option on a dividend-paying stock is systematically related both to the degree to which the option is in-the-money and to the option's time to expiration.

One issue related to American call option pricing which has not been satisfactorily resolved in the option pricing literature is whether the assumption that the stock decline at the ex-dividend instant is equal to the amount of the dividend is appropriate. The studies by Whaley (1982), Gultekin, Rogalski and Tinic (1982), Whaley and Cheung (1982) and Sterk (1983a) implicitly use this assumption by setting α, the proportion of the dividend that the stock price falls at the ex-dividend instant, equal to one. Beckers (1981) uses a value of 0.85, to reflect the preferential tax treatment of price appreciation versus dividend income. In this study, the value of the ex-dividend coefficient α is estimated implicity by allowing observed call option transaction prices to provide the 'best' estimate of the relative stock price decline. This idea was first suggested by Roll (1977, p. 256), and the procedure offers the advantage of providing the market's assessment of the expected relative stock price decline at the ex-dividend instant, rather than an ex-post estimate realized from the observation of stock prices in the days surrounding the ex-dividend day.[2]

The study proceeds as follows. In section 2, the theory of American call option valuation is briefly reviewed, and the Roll model is contrasted with the dividend-adjusted Black–Scholes approximation. In section 3, a description of the sample data is provided, and the frequency of the equivalence of the Roll and Black–Scholes call option model prices is reported and discussed. Section 4 contains the test design and results of the investigation of the ex-dividend stock price decline. The implied relative stock price decline is shown to be very near one in 1978 and 1979, confirming some recent stock market findings. A summary of the results and conclusions of the study is contained in section 3.

[1] Other research on the systematic biases present when using the Roll model includes Gultekin, Rogalski and Tinic (1982) and Sterk (1983b).

[2] The first published study of the observed relative ex-dividend stock price declines is by Campbell and Beranek (1955). Perhaps, the most frequently cited work in this area is Elton and Gruber (1970). A brief review of the results of some of such studies is contained at the beginning of section 4.

2. Theory of American call option valuation

Black and Scholes (1973a) derive the value of a European call option on a non-dividend-paying-stock, that is,

$$c(S, T; X) = SN_1(d_1) - Xe^{-rT}N_1(d_2), \tag{1}$$

where

$$d_1 = \left[\ln(S/X) + (r + 0.5\sigma^2)T\right]/\sigma\sqrt{T}, \qquad d_2 = d_1 - \sigma\sqrt{T},$$

S is the current stock price, r is the riskless rate of interest, and σ is the standard deviation of the instantaneous rate of return on the stock. The exercise price X and the time to expiration T are terms of the option contract. $N_1(d)$ is the cumulative univariate density function with upper integral limit d.

Although the Black–Scholes model is derived for European call options, Merton (1973) shows that is applies to American options as well as long as the underlying stock pays no dividends during the option's life. If a dividend is paid, however, the pricing problem becomes more complex because it may be profitable for the American option holder to exercise his option just prior to the ex-dividend instant.

Fig. 1 depicts the option holder's dilemma at the ex-dividend instant. Just prior to ex-dividend the call option may be exercised yielding proceeds $S_t + \alpha D - X$, where S_t is the ex-dividend stock price. An instant later, the option is left unexercised with value $c(S_t, T-t; X)$, where $c(\cdot)$ is the European

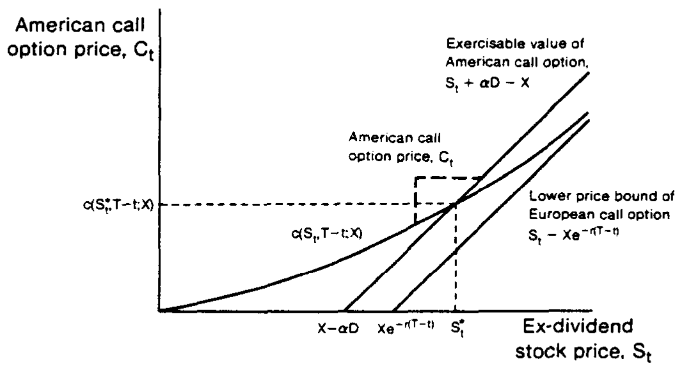

Fig. 1. American call option price immediately prior to the ex-dividend instant as a function of the ex-dividend stock price. The notation is as follows: $c(\cdot)$ is the value of a European call option, X is the exercise price of the option, T is the time to expiration of the option, r is the riskless rate of interest, D is the amount of the dividend, t is the time to ex-dividend, and α is the proportion of the dividend that the stock price falls at the ex-dividend instant.

call option formula (1). Thus, if the ex-dividend stock price, S_t, is above the critical ex-dividend stock price where the two functions intersect, S_t^*, the option holder will choose to exercise his option early just prior to the ex-dividend instant. On the other hand, if $S_t \leq S_t^*$, the option holder will choose to leave his position open until the option's expiration.

It is worthwhile to note that the stock price is assumed to drop by an amount αD rather than D at the ex-dividend instant. The coefficient, α, is the proportion of the dividend that the stock price falls at ex-dividend and reflects the fact that the stock price may fall by an amount other than the dividend amount. Occasionally, for example, it has been argued that the stock price drop is less than the dividend because dividend income is taxed at a higher rate than price appreciation.

It is also worthwhile to note that the functions, $S_t + \alpha D - X$ and $c(S_t, T-t; X)$, need not intersect. If the ex-dividend stock price decline is less than the present value of the interest income that would be earned by deferring exercise until expiration, that is, if

$$\alpha D < X[1 - e^{-r(T-t)}], \tag{2}$$

the lower boundary condition of the European call, $S_t - Xe^{-r(T-t)}$, lies to the left of the early exercise proceeds, $S_t + \alpha D - X$, in fig. 1, and hence the call option will not be exercised early. When condition (2) is met, the value of the American call is determined by substituting the current stock price less the percent value of the escrowed dividend, $S' = S - \alpha De^{-rt}$, as the stock price parameter in eq. (1), that is, $c(S', T; X)$.

The analytic formula for the American call option on a dividend-paying stock was developed in a piecewise fashion. Roll (1977) provides the framework for valuing analytically the American call option by recognizing that, when the dividend and the time to ex-dividend are known with certainty, the payoff contingencies of the call option are duplicated by a portfolio of European call options, one of which is a compound European option. Geske (1979b) later simplifies Roll's analytic solution by observing the American call option problem is simply a compound European option pricing problem which can be solved directly. Finally, Whaley (1981) provides the correct specification of the valuation formula, noting minor errors in the previous two author's work.

The valuation equation of an American call option with a single known dividend paid during the option's life (hereafter, the Roll model) is

$$\begin{aligned} C(S, T; X) = {} & S'\left[N_1(b_1) + N_2\left(a_1, -b_1; -\sqrt{t/T}\right)\right] \\ & - Xe^{-rT}\left[N_1(b_2)e^{r(T-t)} + N_2\left(a_2, -b_2; -\sqrt{t/T}\right)\right] \\ & + \alpha De^{-rt}N_1(b_2), \end{aligned} \tag{3}$$

where

$$a_1 = \left[\ln(S'/X) + (r + 0.5\sigma^2)T\right]/\sigma\sqrt{T}, \qquad a_2 = a_1 - \sigma\sqrt{T},$$

$$b_1 = \left[\ln(S'/S_t^*) + (r + 0.5\sigma^2)t\right]/\sigma\sqrt{t}, \qquad b_2 = b_1 - \sigma\sqrt{t},$$

and $N_2(a, b; \rho)$ is the cumulative bivariate normal density function with upper integral limits, a and b, and correlation coefficient, ρ. S_t^* is that ex-dividend stock price which satisfies

$$c(S_t^*, T - t; X) = S_t^* + \alpha D - X, \tag{3a}$$

as noted earlier in the discussion of fig. 1. In a risk-neutral economy,[3] the American call formula (3) may be thought of as the sum of two conditional expected values – the present value of the expected value of the early exercise of the option value conditional on early exercise, $S'N_1(b_1) - (X - \alpha D)e^{-rt}N_1(b_2)$, and the present value of the expected terminal exercise value of the call conditional on no early exercise, $S'N_2(a_1, -b_1; -\sqrt{t/T}) - Xe^{-rT}N_2(a_2, -b_2; -\sqrt{t/T})$. Note that as the amount of the dividend approaches the present value of the interest income that would be earned by deferring exercise until expiration, that is, $\alpha D \to X[1 - e^{-r(T-t)}]$, the value of S_t^* approaches $+\infty$, the values of $N_1(b_1)$ and $N_1(b_2)$ approach 0, the values of $N_2(a_1, -b_1; -\sqrt{t/T})$ and $N_2(a_2, -b_2; -\sqrt{t/T})$ approach $N_1(a_1)$ and $N_1(a_2)$, respectively, and the Roll model becomes the dividend-adjusted Black–Scholes European call option formula, $c(S', T; X)$.

3. Data

The data used in this study consist of transaction information for all CBOE call options traded during the first three months of the calendar years 1978 and 1979. This information, as well as T-bill rate and dividend information, is taken from the *Berkeley Options Data Base*. Only the first three months of data for each year are used because of the enormous number of transactions recorded in the options file.

Two exclusionary criteria are applied to the sample data. First, since the Roll and Black–Scholes call option pricing models are equivalent when no dividends are paid on the underlying stock and since there are relatively few options traded whose time to expiration includes two or more dividends on the underlying stock, only option transactions whose option's time to expiration includes exactly *one* dividend on the underlying stock are included in the

[3] Cox and Ross (1976) are the first to point out that the Black–Scholes valuation equation has a risk-neutral interpretation.

sample. Second, price quotations of less than or equal to 1/8th are eliminated because these options are so far out-of-the-money that they would not provide any meaningful, additional information in the empirical tests.[4]

After the exclusionary criteria are applied, the call option transaction information is summarized by the degree to which the option is in-the-money, as measured by the ratio of the stock price to the exercise price (S/X), and by the option's time to expiration measured in weeks (T). The results are reported in table 1.

Of the 697,733 option transactions in the sample, the greatest proportion are approximately at-the-money.[5] About 41 percent are within ± 5 percent of being exactly at-the-money and more than 67 percent are within ± 10 percent. This proportion is similar in the two years, even though the number of transactions during the first three months of the year increases by 44 percent from 1978 to 1979.

When the call option transactions are summarized by time to expiration, the greatest number of transactions fall within the 12-to-14 week classification. This result is not surprising since most of the firms pay dividends on a quarterly cycle, and exactly one dividend paid during the option's life is required for the transaction to be included in the sample. The 1978 and 1979 samples, however, differ in composition. In 1978, about 28 percent of the transactions are on options with less than 12 weeks to expiration, while in 1979 more than 45 percent are in this category. This phenomenon is, in part, attributable to the fact that between two subsample periods, a number of stocks had options listed for the first time, and these new options typically had 'near-month' maturities, that is, shorter times to expiration.

Although it is out of the mainstream of this analysis, each option transaction is checked to see if the amount of the dividend is less than the present value of the interest income that would be earned by deferring exercise until the option's expiration. Recall that in section 2 it is demonstrated that, when this condition is met, the American call will not be exercised early and the value of the American call is determined by the dividend-adjusted Black–Scholes model. In 1978, 58.4 percent of the transactions met this condition and, in 1979, 68.5 percent.[6] In light of this result, it is not surprising

[4] These options are so far out-of-the-money that the model prices are relatively insensitive to the parameter values α and σ. In addition, Phillips and Smith (1980, p. 197) point out that there are a number of restrictions that limit trading in such options.

[5] It is worthwhile to note that this distributional information influences the interpretation of the empirical results which have used closing price data in the assessment of the in-the-money/out-of-the-money bias. In these studies, one option pricing error at each exercise price is given an equal weight in assessing the bias. But, extreme in-the-money and out-of-the-money options are infrequently traded and, therefore, have greater non-simultaneity between the option and stock price quotations and larger pricing errors.

[6] The results are essentially the same using volume of contracts traded. In 1978, 57.8 percent of the call options with a single dividend paid during the option's life met condition (2), and, in 1979, 66.1 percent.

Table 1

Frequency distributions of the number of transactions by the degree to which the option is in-the-money (S/X) and by the time to expiration of the option (T) for CBOE call option transactions on individual stocks during the first three months of the calendar years 1978 and 1979.

Ratio of stock price to exercise price	Number of transactions 1978	1979	Both years	Time to expiration in weeks	Number of transactions 1978	1979	Both years
$S/X < 0.75$	2,163	2,845	5,008	$T < 2$	2,166	7,393	9,559
$0.75 \le S/X < 0.80$	4,663	7,151	11,814	$2 \le T < 4$	1,866	21,042	22,908
$0.80 \le S/X < 0.85$	15,358	24,943	40,301	$4 \le T < 6$	3,963	28,958	32,921
$0.85 \le S/X < 0.90$	29,629	43,122	72,751	$6 \le T < 8$	6,944	25,533	32,477
$0.90 \le S/X < 0.95$	53,068	74,061	127,129	$8 \le T < 10$	20,631	36,445	57,076
$0.95 \le S/X < 1.00$	70,082	97,212	167,294	$10 \le T < 12$	45,986	67,104	113,090
$1.00 \le S/X < 1.05$	56,417	59,454	115,871	$12 \le T < 14$	58,378	86,536	144,914
$1.05 \le S/X < 1.10$	19,132	41,427	60,559	$14 \le T < 16$	39,505	49,945	89,450
$1.10 \le S/X < 1.15$	15,292	26,997	42,289	$16 \le T < 18$	26,494	34,317	60,811
$1.15 \le S/X < 1.20$	9,054	10,879	19,933	$18 \le T < 20$	32,672	23,233	55,905
$1.20 \le S/X < 1.25$	4,820	7,534	12,354	$20 \le T < 22$	19,048	11,967	31,015
$1.25 \le S/X < 1.30$	2,198	7,208	9,406	$22 \le T < 24$	9,859	7,353	17,212
$1.30 \le S/X < 1.35$	1,485	3,646	5,131	$24 \le T < 26$	4,733	3,746	8,479
$1.35 \le S/X < 1.40$	1,046	1,455	2,501	$26 \le T < 28$	2,469	1,768	4,237
$1.40 \le S/X < 1.45$	452	1,094	1,546	$28 \le T < 30$	1,959	1,343	3,302
$1.45 \le S/X < 1.50$	521	912	1,433	$30 \le T < 32$	1,903	1,480	3,383
$1.50 \le S/X$	502	1,911	2,413	$32 \le T$	7,306	3,688	10,994
Total	285,882	411,851	697,733	Total	285,882	411,851	697,733

that Whaley (1982, p. 43) finds that there was only about a \$0.02 difference on average between the prices of the Roll and dividend-adjusted Black–Scholes models.

Finally, in one of the empirical tests to follow, the implied estimates of the ex-dividend stock price decline are compared with actual ex-dividend stock price changes, so the closing prices of the underlying stock on the day before and the day of ex-dividend are necessary. These values are compiled from various issues of the Standard and Poor's *Daily Stock Price Record*. Where the dividend reported in the *Record* does not correspond with the reported dividend in the Berkeley data file, the stock is not considered in the sample.

4. Estimation of the relative ex-dividend stock price decline

In this section, the focus of the study turns to estimating the value of α, the proportion of the dividend that the stock price is expected to fall at the ex-dividend instant. To date, the empirical literature has generally investigated this issue by computing the ratio of the ex-dividend stock price decline to the amount of the dividend directly, that is $(S_B - S_A)/D$, where S_B is the stock price just before the stock goes ex-dividend and S_A is the stock price just after. The first study along these lines is by Campbell and Beranek (1955). In their sample of 399 ex-dividend days for NYSE stocks in the early 1950's, they find that the stock price falls, on average, about 90 percent of the amount of the dividend on ex-dividend day. This they attribute to the preferential tax treatment of capital gains vis-à-vis dividend income. In the absence of transaction costs, the relative ex-dividend stock price decline, $(S_B - S_A)/D$, should be equal to $(1 - t_o)/(1 - t_g)$, where t_o is the ordinary income tax rate and t_g is the capital gains tax rate. If the capital gains rate is less than the ordinary tax rate, then the value of α should be less than one. Durand and May (1960) examine the ex-dividend stock price declines of ATT during the period 1948 to 1959 and find that the average value of α is 0.96. Using a sample of 4,148 observations of NYSE stocks during period April 1966 through March 1967, Elton and Gruber (1970) find that the value of α is, on average, 0.78 and, using an even more extensive sample of NYSE stocks for the period July 1962 through December 1970, Black and Scholes (1973b) realize an average α of 0.72. Both of these studies report that their estimates of α are significantly less than one. Kalay (1982) re-examines ex-dividend stock price behavior during Elton and Gruber's sample period using 2,540 cash dividends and finds that the average α is 0.73, but insignificantly different from one.

More recently, the ex-dividend day studies have focused on evaluating whether abnormal risk-adjusted return, after transaction costs, may be earned on the days surrounding ex-dividend. Using more recent stock price data, Eades, Hess and Kim (1984) and Lakonishok and Vermaelen (1985) infer that, since the introduction of negotiated commissions in 1975, the value of α has not been significantly different from one. They ascribe this phenomenon to the fact the transaction costs are so low that the once-present tax arbitrage opportunities now have disappeared.

The advent of organized secondary markets for stock options provides an opportunity to approach the α estimation problem in a new and different way. Instead of relying on observed stock price data to compute an ex-post estimate of α, reported option transaction prices for a particular stock can be used to infer the market's expectation of the relative ex-dividend stock price decline. In section 2 it is shown that an integral part of the valuation of call options on dividend-paying stocks is the proportion of the dividend that the stock price is expected to fall at the ex-dividend instant. If the market properly accounts for the ex-dividend stock price decline in the pricing of options and correctly prices options on average, then observed call option transaction prices and the structural form of the Roll model can be used to deduce the market's assessment of α.

This section is divided into three parts. In the first part, the estimation methodology is outlined and applied to the option transactions in the sample. Initially, estimates of α are computed for each stock on each day of the sample period and the hypothesis that $\alpha = 1$ is tested. Then, to develop more precise company-specific estimates of α, the daily estimates are aggregated within each three-month period, and the hypothesis that $\alpha = 1$ is retested. In the second part of this section, the implied α's is tested for predictive ability. It is shown that the implied estimate of α has more predictive ability than a historical estimate, but its predictive ability does not appear to be systematically better than a prediction of α equal to one for all stocks. The final subsection contains the global estimates of α for the sample period and shows that they appear to be stationary through time and not meaningfully different from one.

4.1. Estimates for individual stocks

To understand how the expected relative ex-dividend stock price decline for individual stocks is estimated, consider the call option transaction prices for a particular stock on a given day, C_j, to be equal to their respective Roll model values, $C_j(\alpha, \sigma)$, plus some random disturbance, ε_j, that is,

$$C_j = C_j(\alpha, \sigma) + \varepsilon_j. \tag{4}$$

In eq. (4), α, the relative ex-dividend stock price decline, and the σ, the standard deviation of the underlying stock returns, are the unknown parameters.[7] If the Roll model describes the observed structure of call option prices

[7]The remaining parameters of the Roll model are assumed to be known. S is the last observed stock price before the option transaction, X is the exercise price of the option contract, T is the time to expiration of the option in calendar days, r is the approximate yield on a T-bill of comparable maturity to the option, D is the amount of the dividend paid during the option's life, and t is the time to the ex-dividend instant measured in calendar days.

exactly, the relationship between C_j and $C_j(\alpha, \sigma)$ would be perfect and no disturbance term would appear. There are several reasons why the relationship may not be perfect, however.

One potential explanation is that the Roll model is misspecified in some unknown way. To the extent that it is, there will be disturbance in the relationship (4). Another explanation is that the stock and option prices used in the sample are transaction prices. As noted by Phillips and Smith (1980), a transaction price is either a bid price or an ask price, depending on whether the motivation for the transaction is to sell or to buy. To the extent that ask prices for options are regressed on model values computed using bid prices of the stock, and vice versa, there will be noise in (4). In a similar vein, there is the non-simultaneity issue first raised by Galai (1977). The stock price used in the computation of the model value is the price of the stock at the time of the last transaction. To the extent that the stock transaction and the option transaction take place at different points in time, there will be noise in the relationship (4).

To estimate the parameters of eq. (4), call option transaction prices are regressed on their respective Roll model prices,[8] that is,

$$\underset{\{\hat{\alpha}, \hat{\sigma}\}}{\text{minimize}} \sum_{j=1}^{n} \left[C_j - C_j(\hat{\alpha}, \hat{\sigma})\right]^2, \tag{5}$$

where n is the number of option transactions on a particular stock in a given day. Assuming the residuals from this regression are independent and normally distributed, the values of $\hat{\alpha}$ and $\hat{\sigma}$ are maximum likelihood estimates. If there are fewer than 30 transactions on a given day, no estimates of α and σ are computed because the sample size is too small to allow much confidence in the estimates and because non-synchronous price quotations of the option and the underlying stock are likely to introduce substantial autocorrelation in the residuals,[9] thereby violating the independence assumption required in the subsequent tests. If the number of transactions exceeds 100, only the first 100 observations are used in order to keep computational costs at a reasonable level. The non-linear regression (5) is estimated by using the subroutine

[8] Two other option pricing studies contain methodologies in which two or more parameters are estimated simultaneously. Manaster and Rendleman (1982) jointly estimate the stock price and the standard deviation of stock return of the Black–Scholes (1973a) formula, and Barone-Adesi (1986) jointly estimates the standard deviation of the return of the firm's assets, and the face value and the time to maturity of the firm's debt.

[9] The non-synchronous trading of the option and the stock introduces problems not unlike those discussed in a different context by Scholes and Williams (1977).

DSIM[10] developed by the Computer Center at the University of British Columbia.

Before proceeding further, two issues warrant investigation. First, although the maximum likelihood estimates obtained from the non-linear regression (4) are asympotically unbiased under the stated assumptions, they are biased in small samples.[11] That is, because the Roll model is non-linear in the parameters α and σ, the estimates of α and σ are not correct on average. To evaluate the severity of this bias, simulations are performed. The values of α and σ are set equal to 1.00 and 0.30, and then 10,000 call option prices are generated using the Roll model with typical sample parameter values (i.e., $S = 50$, $X = 50$, $r = 0.10$, $T = 0.25$, $D = 0.50$, $t = 0.20$) plus a normally distributed, random disturbance with mean zero and standard deviation 0.20.[12] The generated prices are then clustered into 100 groups of 100, and the non-linear estimation procedure described above is applied to each group. (Recall that 100 is the maximum number of observations used in each regression.) The mean and the standard deviation of the 100 maximum likelihood estimates of α are 1.0026 and 0.0037, respectively, and those of σ are 0.2998 and 0.0021. In other words, the bias induced by the non-linearity of the regression model appears to be very small. The estimate of σ in the simulations, for example, is upward biased only by the amount 0.0026, or about 0.26 percent of the original parameter value, one.

The second issue relates to the normality and independence assumptions underlying the maximum likelihood regression model. To examine the validity of these assumptions, a small sample of the regression results are drawn. The results reveal approximate normality but occasional significant autocorrelation up to and including the third lag. To investigate the effects of this problem, the regressions for the sample are re-run using only every fourth observation. The parameter estimates realized in this second set of regressions are virtually identical to those of the first.

The first empirical test involving the estimates of the expected relative ex-dividend stock price decline is for the null hypothesis that the coefficient α

[10] DSIM is based on the Nelder and Mean (1965) Simplex method for function maximization, which is generally efficient and more stable than alternative derivative-based methods when small numbers of variables are estimated. The Nelder and Mead technique employs a series of polyhedral expansions and interpolations and is not to be confused with the Simplex method used in linear programming.

[11] The problem investigated here is in the opposite order of causation of the bias issue addressed by Boyle and Anathanarayanan (1977) and Butler and Schacter (1986). They argue that if the variance estimate used in the Black–Scholes option pricing formula is unbiased, the call price computed using the Black–Scholes option pricing model is biased. Through numerical simulations, these authors show that this variance-induced bias is very small.

[12] The average standard error of the estimate of the 4,168 regressions is about 0.20, so that the disturbance term of the simulated prices is generated as a normally distributed variable with mean zero and standard deviation 0.20.

is equal to one for each stock on each day of the sample period. Two test statistics are computed. The first test statistic is

$$-n \ln R, \tag{6}$$

where R is the ratio of the sum of squared errors of the unconstrained (U) regression SSE_U [i.e., the sum of squared residuals from (5)] to the sum of squared errors of the constrained (C) model SSE_C (i.e., the sum of squared residuals from a non-linear regression where α is set equal to one and only σ is estimated). This statistic is asymptotically χ^2 distributed with one degree of freedom.

A problem with the test statistic (6) is that only its asymptotic properties are known. For finite normal samples, alternative tests are preferable. Gallant (1975, p. 79), for example, shows that for nested models the statistic

$$(n-p)(1-R)/q, \tag{7}$$

where q is the number of restrictions in the constrained regression and p is the number of parameters of the unconstrained model, follows approximately an F distribution with the number of constraints and the number of observations less the number of parameters estimated in the unconstrained model as the degrees of freedom. In the present context, the number of constraints is equal to one and the number of parameters is two. Thus, the test statistic

$$(1-R)(n-2) \tag{8}$$

is approximately distributed as $F_{1,n-2}$.

Table 2 contains a summary of the test results. The results of the two tests are not dissimilar, with the frequency of rejection slightly higher with the χ^2 test than the F test because the asymptotic test tends to reject the null too often in finite samples. Using the F test results, the null hypothesis that $\alpha = 1$ cannot be rejected in slightly more than one-half of the total number of cases. In a large proportion of the tests, however, the null hypothesis is rejected in favor of the alternative that $\alpha \neq 1$. The individual estimates of α must be more closely scrutinized to discover whether this result is economically meaningful or simply due to departures of the test statistic from Gallant's approximation.

To increase the precision of the company-specific estimates of α, the daily estimates for the individual stocks, $\hat{\alpha}_\tau$'s, in a given year are aggregated by weighting each estimate by the inverse of its asymptotic variance and summing, that is,

$$\hat{\alpha} = \sum_{t=1}^{m} h_\tau \hat{\alpha}_\tau \Big/ \sum_{t=1}^{m} h_\tau, \tag{9}$$

Table 2

Frequency of non-rejection/rejection of the null hypothesis that the relative ex-dividend stock price decline, α,[a] is equal to one for individual stocks on individual days during the first three months of the calendar years 1978 and 1979. The probability level used in the evaluation of the test statistics is five percent.

	Frequency of non-rejection/rejection					
	χ^2 test[b]			F test[c]		
Hypothesis	1978	1979	Both years	1978	1979	Both years
H_0: The ex-dividend stock price decline coefficient α is equal to one	1,049 (0.548)[d]	1,074 (0.476)	2,123 (0.509)	1,068 (0.558)	1,103 (0.489)	2,171 (0.521)
H_A: The ex-dividend stock price decline coefficient α is not equal to one	864 (0.452)	1,181 (0.524)	2,045 (0.491)	845 (0.442)	1,152 (0.511)	1,997 (0.479)
Total	1,913	2,255	4,168	1,913	2,225	4,168

[a] The estimate of α is determined by a non-linear regression of the observed call option prices on their respective Roll model values. The regression jointly estimates the parameters α, the ex-dividend stock price decline, and σ, the standard deviation of the stock return. All remaining parameters of the Roll model are assumed to be known. If there are fewer than 30 option transactions on a given stock in a given day, no estimates of α and σ are computed. If the number of transactions exceeds 100, only the first 100 observations are used in the regression.

[b] The test statistic, $-n \ln R$, where R is the ratio of the sums of squared errors of the unconstrained regression to the constrained regression (where $\alpha = 1$), is asymptotically χ^2 distributed with one degree of freedom.

[c] The test statistic, $(n-2)(1-R)$, is approximately distributed as $F_{1,n-2}$.

[d] The number within parentheses refers to the proportion of the total number of hypothesis tests accounted for by that cell.

where $h_\tau = 1/s^2(\hat{\alpha}_\tau)$ and m is the number of days in which there are a sufficient number of option transactions to permit α_τ to be estimated. (Recall that at least 30 transactions are necessary.) The estimate from (9) is optimal under the assumption that the daily estimates are constant for each stock and that the estimation errors are independent over time. The variance of this aggregated company-specific estimate is

$$s^2(\hat{\alpha}) = 1/\sum_{t=1}^{m} h_\tau. \tag{10}$$

The estimates are computed for both 1978 and 1979. In 1978, there are 70 estimates of α for the individual stocks, and, in 1979 there are 74.

These estimates and their respective standard errors are then used to test the hypothesis that the relative decline α for an individual stock is equal to one during the first three months of the two years of the sample period. In 1978 the null hypothesis is rejected in 14 of the 70 cases, and in 1979 the null hypothesis is rejected in 17 of 74 cases. Although the frequency of rejection falls to about 20 percent of the cases considered, it appears that some firms have expected ex-dividend stock price decline coefficients which are different from one. Whether this result is economically meaningful will be evaluated in terms of the ability of these implied α's to predict their subsequently realized values.

4.2. Predictive ability of individual stock estimates

To test the predictive power of the implied company-specific estimates of α, observed ex-dividend stock price declines are computed. The estimate of the realized value of α for individual firms is computed as the mean of the first two quarterly values of $(S_B - S_A)/D$ in each of the two years, where S_B is the closing stock price on the day before ex-dividend, S_A is the closing stock price on the day of ex-dividend, and D is the amount of the dividend. To create a benchmark by which to evaluate the predictive power of the implied estimates of α, a historical predictor is also computed. This predictor is based on the mean of $(S_B - S_A)/D$ of the four quarterly dividends in the year prior to the examination period. For example, to predict the relative ex-dividend stock price decline in 1978, the historical estimator is the average of the four quarterly relative ex-dividend stock price declines in 1977.

The mean absolute prediction error (*MAPE*) and the mean squared prediction error (*MSPE*) of the two models in each of the two three-month periods are reported in table 3. The results are tabulated for all observations, and then for two subsamples. The subsamples are included because the overall values of *MAPE* and *MSPE* are sensitive to outliers induced by trivially small div-

Table 3

Mean prediction errors of implied and historical estimates of the relative ex-dividend stock price decline from the estimated realized relative ex-dividend stock price decline[a] for individual stocks with call options listed on the CBOE during the first three months of the calendar years 1978 and 1979.

		Prediction errors					
		1978			1979		
Sample	Predictor	No. of obs.	*MAPE*[b]	*MSPE*[c]	No. of obs.	*MAPE*	*MSPE*
All observations	Implied[d]	70	1.2740	5.9912	74	2.1149	65.4354
	Historical[e]	70	1.3515	6.3219	74	2.2053	40.7554
Realized $D \geq 0.125$	Implied	66	0.9030	1.4223	69	1.1396	4.8149
	Historical	66	1.0484	2.2581	69	1.4699	6.8012
Realized $D \geq 0.250$	Implied	51	0.8084	1.0030	60	0.8855	1.6223
	Historical	51	0.8762	1.4488	60	1.1746	3.6182

[a] The estimated realized relative ex-dividend stock price decline is equal to the mean of the first two quarterly values of $(S_B - S_A)/D$ for each stock in each year, where S_B is the closing stock price on the day prior to ex-dividend, S_A is the closing stock price on the day of ex-dividend, and D is the amount of the dividend.

[b] *MAPE* is the mean absolute prediction error, that is, $MAPE = \Sigma_{j=1}^{n}|A_j - P_j|$, where A_j is the realized stock price decline estimate (see footnote a) for the jth stock, P_j is the predicted stock price decline estimate (see footnote d or e) for the jth stock, and n is the number of observations for the year.

[c] *MSPE* is the mean squared prediction error, that is, $MSPE = \Sigma_{j=1}^{n}(A_j - P_j)^2$, where all notation is defined as in footnote b.

[d] The implied relative ex-dividend stock price decline is an average of the daily company-specific estimates obtained from the non-linear regression of observed call option transaction prices on their respective Roll model values.

[e] The historical relative ex-dividend stock price decline estimate is equal to the mean of the four quarterly values of $(S_B - S_A)/D$ for each stock in the previous year.

idends. For example, the realized value of α for Bally Manufacturing in 1979 is -47.5.[13] This is not surprising considering that the average amount of the quarterly dividend for Bally during the first two quarters of 1979 is 0.025. To reduce the impact of outliers, *MAPE* and *MSPE* are also computed by enforcing the constraint that the average dividend during the year is greater than or equal to 0.125 and 0.25. These cutoffs are chosen because stock prices are quoted in increments of 1/8th. The second subsample probably provides the most accurate depiction of the results, however, since even with a minimum average dividend of 0.125 outliers such as Hewlett Packard with a realized α value of -11.04 in 1979 remained in the sample.

The results in table 3 indicate that the implied estimator of α outperforms the historical estimator. The *MAPE* of the implied estimator is less than the *MAPE* of the historical estimator in all of the cells of the table, and the *MSPE* of the implied estimator is lower for all cells except for the overall sample of 1979. The importance of this exception, however, is mitigated by the presence of the previously noted outliers in the overall sample.

The mean squared prediction errors reported in table 3 can be thought of as the sum of four components: (a) the mean deviation between the estimated realized and estimated predicted values: (b) the estimation error in the realized value; (c) the estimation error in the predicted value; and (d) for the historical predictor, the change in expectation from one period to the next. To quantify the first component of the *MSPE*'s, the mean value of the difference between the mean realized α and the mean predicted α are computed for both the implied historical predictions. In 1978, the mean deviations (and associated t-ratios) for the overall sample are -0.2903 (-0.99) and 0.0397 (0.13) for the implied and historical predictors, respectively, and in 1979 they are -1.5734 (-1.69) and -1.4303 (-1.96). While the magnitudes of the deviations may appear alarming in 1979, one must be reminded of the presence of outliers in the overall sample. In fact, almost half of the mean deviations for that year are attributable to one outlier – that of Bally Manufacturing. For the subsample where $D \geq 0.25$, the mean deviations (and t-ratios) are 0.1446 (1.03) and 0.3870 (2.40) for the implied and historical predictors in 1978, and they are -0.3197 (-1.99) and -0.5374 (-2.26) in 1979. Overall, in both years, they are -0.1064 (-0.97) and -0.1127 (-0.73). In other words, it does not appear that the predictors are systematically biased in the sense that the average deviations are not consistent in sign from year to year and are insignificantly different from zero at the five percent probability level overall.

[13] The magnitude of outliers such as that for Bally Manufacturing motivates a reconsideration of the test results of studies which have estimated directly the relative ex-dividend stock price decline. For the most part, this research has used unrestricted samples, and, although the sample sizes, in most cases, have been very large, a handful of outliers such as Bally's in 1979 may contribute dramatically to the overall estimate. In fact, the presence of outliers in these studies may well be the cause of the wide range of realized average α's that have been reported.

In addition, the average deviation for the implied predictor is similar to the historical predictor, and the historical predictor is known to be unbiased.

The differences between the mean realized and predicted values do not account for a meaningful proportion of the mean squared prediction errors reported in table 3. For example, the *MSPE* for the implied predictor for the second subsample in 1978 is 1.0030. The estimated bias component for this subsample is 0.1446 and its contribution to the *MSPE* is 0.0209 ($= 0.1446^2$). In other words, about two percent of the *MSPE* is attributable to bias. Furthermore, there does not appear to be a significant change in the expectation of α from one period to the next [i.e., component (d) of the *MSPE*] because the difference between the estimated realized and estimated predicted α's is approximately the same for the implied and the historical predictors in the overall sample. The majority of the prediction error, therefore, appears to come from components (b) and (c) of the *MSPE*.

On the basis of these results, it is reasonable to conclude that the implied estimator yields more precise estimates of future ex-dividend stock price declines than does a historical estimator. But, it is not clear that the implied estimator is the 'best' estimator in the sense that perhaps some other predictor, such as the constant, one, may predict future α's as well as the implied estimator. One way in which the issue may be investigated is by regressing the realized values of α on the implied values in the spirit of Chiras and Manaster (1978), that is,

$$\hat{\alpha}_{\text{realized}} = \gamma_0 + \gamma_1 \hat{\alpha}_{\text{implied}} + \varepsilon, \tag{11}$$

in each of the two years in the sample. If the implied estimate of α is a good predictor of the estimated realized α, the estimated coefficients $\hat{\gamma}_0$ and $\hat{\gamma}_1$ in (11) would be indistinguishable from zero and one, respectively. If most of the predictive power is contained by the mean of the implied estimates of α, the estimate of the slope coefficient would be insignificantly different from zero. The regression results are reported in table 4.

In table 4, there is little evidence that there are unique values of α for individual companies. In all of the regressions for 1978, the estimated slope coefficient γ_1 is not significantly different from one. In 1979, the slope coefficient is significant in the overall sample and in the subsample where the average dividend is in excess of 0.125, but the signs of the coefficients are wrong. Clearly, the presence of the outliers noted earlier nullifies the usefulness of these results. The slope coefficient in the remaining subsample is insignificant. On the basis of the second subsample results, it is reasonable to conclude that the implied estimates of α have little predictive power on a company-by-company basis.

Table 4

Summary of regression results of the estimated realized relative ex-dividend stock price decline[a] on the implied relative ex-dividend stock price decline[b] for individual stocks during the first three months of the calendar years 1978 and 1979. The regression equation is $\hat{\alpha}_{\text{realized}} = \gamma_0 + \gamma_1 \hat{\alpha}_{\text{implied}} + \varepsilon$.

	Parameter estimates							
	1978				1979			
Sample	No. of obs.	$\hat{\gamma}_0$	$\hat{\gamma}_1$	R^2	No. of obs.	$\hat{\gamma}_0$	$\hat{\gamma}_1$	R^2
All observations	70	0.7910	−0.0457	0.0003	74	2.7264	−2.5038	0.8972
		(1.85)[c]	(−0.14)			(10.83)	(−25.07)	
Realized $D \geq 0.125$	66	0.8059	0.2176	0.0130	69	1.5260	−1.2208	0.1330
		(3.02)	(0.92)			(3.68)	(−3.21)	
Realized $D \geq 0.250$	51	1.0044	0.1013	0.0040	60	0.0762	0.5594	0.0443
		(4.01)	(0.45)			(0.22)	(1.64)	

[a] The estimated realized relative ex-dividend stock price decline is equal to the mean of the first two quarterly values of $(S_B - S_A)/D$ for each stock in each year, where S_B is the closing stock price on the day prior to ex-dividend, S_A is the closing stock price on the day of ex-dividend, and D is the amount of the dividend.

[b] The implied relative ex-dividend stock price decline is an average of the daily company-specific estimates obtained from the non-linear regression of observed call option transaction prices on their respective Roll model values.

[c] The values in parentheses are t-ratios.

One final note about the regression results of the second subsample is worthwhile. Even though the slope coefficients in both years are insignificantly different from zero, both the intercept and slope estimates have dramatically different magnitudes from one year to the next. These changes could be attributable to changes in the nature of the sample from one year to the next, but table 1 shows that the samples have common characteristics. The most likely explanation lies in the behavior of the estimation errors induced by stock returns on the ex-dividend days in the two periods.

4.3. Global estimates of the relative ex-dividend stock price decline

Based on the results of the last subsection, the value of α does not appear to be unique for individual stocks. To develop the 'best' estimate of α for all stocks, the individual stock estimates are aggregated through time and across stocks in each year, again using the inverse of the asymptotic variances as estimate weights. In 1978, the 70 company-specific estimates produced an overall estimate of α of 0.9864 with a standard error of 0.0203, and in 1979 the 74 individual estimates produced an overall estimate of 0.9542 with a standard error of 0.0175.

The standard errors of these global estimates are computed under the additional assumption of cross-sectional independence of estimation errors across stocks. They are understated if there is positive dependence, biasing the t-tests against the null hypothesis that α is equal to one. While the 1978 estimate is indistinguishable from one, the 1979 estimate is significantly different from one. The difference, however, does not appear to be of economic significance, and its statistical significance may be partly due to a possible understatement of its standard error.

To test the null hypothesis that the implied α is constant from the first year to the second year, a t-test on the difference between the two yearly estimates is performed. The t-ratio is 1.21, well below the critical value at the five percent probability level, and the null hypothesis is not rejected. It appears that the implied α is constant, at least with respect to the sample period considered, and that it is not meaningfully different from one.

5. Summary and conclusions

The purpose of this study is to estimate the *expected* relative stock price decline implicit in the valuation of American call options on dividend-paying stocks. Previous work on measuring the ex-dividend decline generally focuses on *realized* stock market estimates and has mixed results. Here, the American call option pricing model of Roll (1977) is fitted to the observed structure of call option transaction data to develop ex-ante estimates of the relative stock price decline. The test results indicate that the expected relative stock price decline is not company-specific, is stationary from year to year, and is not meaningfully different from one. This finding confirms other recent evidence from studies on the behavior of stock prices on the ex-dividend day.

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