Received: 9 June 2022 | Accepted: 22 August 2022

DOI: 10.1002/fut.22377

RESEARCH ARTICLE

WILEY

Effects of nondiscretionary trading on futures prices

Michael J. O'Neill[1] | **Robert E. Whaley**[2]

[1]Department of Finance, Bond Business School, Bond University, Robina, Queensland, Australia

[2]Financial Markets Research Center, Owen Graduate School of Management, Vanderbilt University, Nashville, Tennessee, USA

Correspondence
Robert E. Whaley, Financial Markets Research Center, Owen Graduate School of Management, Vanderbilt University, 401 21st Ave South, Nashville, TN 37203, USA.
Email: whaley@vanderbilt.edu

Abstract

This paper examines the effects of the nondiscretionary trading demands of volatility index (VIX) exchange-traded products (ETPs) issuers on the prices and volumes in the VIX futures. We find that the ETPs' informationless, mechanical rebalancing of futures positions to maintain the constant maturity of the index and the promised leverage ratios of the VIX ETPs have significantly positive predictive power for end-of-day futures returns. We also show that the impact on price has diminished through time from increased liquidity provided by hedge funds, and the "natural" hedging of the issuers' inverse products.

KEYWORDS

contango, replication strategy, VIX ETPs, VIX futures

JEL CLASSIFICATION

G12, G13, G14

1 | INTRODUCTION

The effect of demand on prices of financial assets has been subject to no small number of empirical investigations. Most studies focus on discretionary demand. Important new private information about corporate actions, such as impending mergers, acquisitions, or major restructurings, or regulatory interventions, such as a short sales ban, can and do affect prices. The classic studies defining this line of investigation are Ball and Brown (1968) and Fama et al. (1969). But, what if the motivation for trading is informationless and nondiscretionary? Modern-day finance theory presupposes such trading has no effect on price.[1] Basic economic intuition, however, suggests that supply cannot be perfectly elastic, at least in the short run.

Natural laboratories for investigating the price effects of nondiscretionary trading are few. Ideally, demand/supply shocks must convey no new private information, and the size of the trading imbalance must be known in advance. In such an environment, the long-term or equilibrium price of the security will not change. In the short-term, however, the trading imbalance will cause a temporary price increase or decrease depending on the direction and size of the trade, and the willingness of liquidity providers to supply capital for this type of trading activity. Duffie (2010) argues that the supply of capital available to absorb these shocks increases through time as investors learn about trading opportunities. With increased supply comes reduced price impact and quicker price reversals.

Standard and Poor's (S&P) announcements of changes in the composition of the S&P 500 index have become the best-known laboratory environment for assessing the effects of informationless, nondiscretionary trading. The reasons

[1]Key contributions include the corporate financial models of Modigliani and Miller (1958) and Miller and Modigliani (1961), the option pricing models of Black and Scholes (1973) and Merton (1973), and the arbitrage pricing theory of Ross (1976).

are straightforward. First, they are sterile. Additions to and deletions from the S&P 500 are made using known, mechanical rules based on public information. The announcement conveys no new information about the prospects of the firm. Second, the trading imbalance created by the index revision is large relative to the available supply. As of the end of December 2019, 16.5% of the total market capitalization of the S&P 500 portfolio was accounted for by index funds pegged to the S&P 500.[2] This means that 16.5% of the shares outstanding of a newly added index stock must be purchased in a handful of days before the index change becomes effective. To ease order imbalances, S&P began preannouncing changes to the index in October 1989. The effect of the policy change was to bring new liquidity providers to the market. Index funds continued to rebalance their holdings on the effective day to minimize tracking error, however, risk arbitragers such as hedge funds began to step in to acquire shares to turn a quick profit—an activity called the "S&P game." The effect was to increase liquidity in the market on the effective day when the index funds were bought and the risk arbitragers sold.[3] The empirical evidence regarding the price effects of S&P 500 revisions has evolved through time in a manner consistent with Duffie's (2010) prediction.[4]

In recent years, investor demand for nontraditional index products has engendered new opportunities for investigating the effects of nondiscretionary trading in pristine research environments. Consider, for example, the growth in the AUM of unlevered and levered ETPs benchmarked to the prices of commodities and volatility. While such exposures were possible in the past using in futures contracts, many institutions are (and continue to be) barred by charter from trading futures, and many retail customers have neither the financial wherewithal nor the trading sophistication. To accommodate these demands, banks and other financial entities have stepped in to fill the void by repackaging collateralized futures positions as stock-like securities. Since the futures trading demand of the ETP issuers is driven only by the need for replication (in a formulaic manner), futures prices can be used to investigate the effects of informationless, nondiscretionary trading.

This study explores the effects of informationless, nondiscretionary trading by volatility index (VIX) ETP issuers in the VIX futures market. This environment offers six important and distinct advantages over past work. First, like the S&P 500 revision studies, the quantity and the timing of the trades are known in advance. Unlike S&P 500 studies, however, trading demand can be measured precisely. While S&P 500 replacement demand is well-defined, it depends on the aggregate value of S&P 500 index funds, many of which are held privately. The AUM of privately held funds cannot be measured accurately. Second, unlike past futures studies, we can ex ante measure trading demands of specific futures contracts expiration months precisely. No proxies are necessary.[5] VIX ETPs are created using collateralized futures positions that are benchmarked to S&P 500 VIX Short-Term Futures Index (SPVXSP), and the index is, in turn, is a precisely defined weighted average of the prices of the nearby and second nearby VIX futures contracts. Third, relative to past futures studies, the sample size is large. The sample period is November 2010–December 2017, and the replication demands are daily. Hence, we have 1784 daily demand measurements for both the nearby and second nearby VIX futures with which to measure price and volume effects.[6] Fourth, we have two measures of futures demand each day—the roll demand arising from having a constant maturity futures index, and the leverage demand arising from the daily rebalancing required to provide for the following day's levered benchmark index return. Roll demand is relatively smooth and occurs once a day. The number of nearby futures to be sold and the

[2]The assets under management (AUM) of mutual funds, institutional funds, separately managed accounts, and insurance products pegged to the S&P 500 was $3774.2 billion as of the end of December 2019. The AUM of S&P 500 exchange-traded products (ETPs) accounted for another $816.2 billion, bringing the total AUM of S&P 500 indexed funds to $4590.5 billion. On December 31, 2019, the market capitalization of all stocks in the S&P 500 portfolio was $27,854.2 billion, which means that about 16.5% of the market value of the S&P 500 belongs to index funds.

[3]This practice was dubbed "sunshine trading" by Admati and Pfleiderer (1991). Sunshine trading refers to preannouncing the size and timing of the informationless trades to attract new liquidity suppliers as well as natural counterparties.

[4]Using a sample of S&P changes through 1983, Shleifer (1986) documents a price impact of about 3% for S&P 500 additions with no price reversal; using a sample ending in June 1994, Beneish and Whaley (1996) find that the price impact reverses partially with 10 days after the change becomes affective new announcement policy; and, using a sample ending in December 2014, Patel and Welch (2017) show that S&P 500 index additions experience a 2% price impact that almost fully reverses within 2 days.

[5]In their study of credit-linked notes (CLNs), Henderson et al. (2015) simply assume the benchmark is the nearby commodity futures contract month. While this is understandable in the sense that they are using 1905 CLNs with a broad array of underlying benchmarks, it undermines the interpretation of the results.

[6]Bessembinder et al. (2016) focused on the roll demands of the United States Oil Fund (USO) exchange-traded fund (ETF) during the period March 2008–February 2009. USO is created from a fully collateralized crude oil futures position. At the time USO rolled once a month on a preannounced date from the nearby to the second nearby crude oil futures contract. Based upon a sample size of 12, the replication demand costs USO about 25 basis points per roll on average. To ease order imbalance, USO initiated the practice of executing the roll activity uniformly over a 4-day period shortly thereafter.

number of second nearby contracts to be bought are known at the beginning of the trading day, so the timing of the trades is not crucial. The leverage demand, on the other hand, varies widely from day to day, depends on the intraday movement of the futures index, and must be executed at the end of the day. Fifth, the length of the sample period allows us to track the evolution of trading by different market participants through time. Consistent with Duffie (2010), we see hedge funds and natural counterparties step in once the size and understanding of the VIX ETP market are better known. Finally, the evolution of the markets is documented in yet another way. As the AUM of VIX ETPs grew, the Chicago Board Options Exchange (CBOE) chose to launch a trade-at-settlement (TAS) VIX futures market to reduce the execution risk of VIX ETP issuers and VIX futures market liquidity providers. Historically, TAS markets had been used only in conjunction with the unwinding of intermarket commodity futures positions.

The paper is organized as follows. Section 2 provides a brief history of ETPs and describes where the VIX ETPs fit within the overall product landscape. Section 3 shows how VIX ETP issuers replicate their promised benchmark returns using risk-free bonds and VIX futures, and how the replication portfolio changes each day with movements in the benchmark index and any creations or redemptions. With the VIX ETP replication demands for each VIX futures clearly defined, Section 4 measures the demands and examines the relationship between demand and VIX futures trading volume and open interest. As part of the analysis, we examine the inter- and intraday market activity of the VIX futures TAS contracts, a market developed specifically for accommodating VIX ETP issuer replication needs. Section 5 focuses on VIX futures demand and end-of-day (EOD) returns. We show that the calendar roll and leverage demands have significantly positive predictive power for EOD returns. We also show how the market's ability to absorb the replication trades has improved through time from the increased participation of hedge funds and demand for inverse products. Section 6 discusses some recent controversies in the ETP markets. While these controversies do not affect the conclusions of the study, they are important in identifying key issues in the development of ETPS—product design, market integrity, and operations management. Section 7 provides a summary of the main conclusions.

2 | VIX EXCHANGE-TRADED PRODUCTS

VIX ETPs are the focus of our analysis. VIX ETPs come in two forms—exchange-traded notes (ETNs) and ETFs. For our purpose, the distinction between VIX ETNs and VIX ETFs is not critical since both products, either implicitly or explicitly rely on collateralized futures positions to generate benchmark index returns. Technically, VIX ETFs fall under the Investment Act of 1940 and report daily futures holdings. VIX ETNs, on the other hand, fall under the Securities Exchange Act of 1934. They are not required to report specific holdings. While some VIX ETN issuers may use volatility swaps to generate benchmark returns, swap dealers hedge their counterparty volatility risk exposures using VIX futures to minimize basis risk. In other words, all VIX ETP volatility risk exposures can be expressed in terms of specific VIX futures contract positions.

The six most active VIX ETPs are benchmarked to the S&P 500 VIX Short-Term Futures Index. They are listed in Table 1. VIX ETPs come with a range of leverage ratios or "gears," A single factor differentiates them at a conceptual level—Lx, where L is the leverage ratio. VXX and VIXY are $1x$ products. They are created using a fully collateralized futures position. In place of holding portfolios of the constituent securities like traditional ETFs, these funds hold T-bills (i.e., the collateral) and an equal notional amount of long futures. Credit Suisse launched $2x$ (TVIX) and $-1x$ (XIV) products in November 2010. The inverse product, XIV, is also a fully collateralized futures position, however, the fund is long T-bills and short an equal notional amount of futures. The promised return is −100% of the benchmark index, SPVXSP. TVIX is also a collateralized futures position, but it is not fully collateralized. It is long T-bills (i.e., the collateral) and long twice the notional amount of futures. The promised return is 200% of the SPVXSP. ProShares launched competing products, SVXY ($-1x$) and UVXY ($2x$) in October 2011. The only difference is that the ProShares products have an ETF structure. With respect to the benchmark index, all funds except VXX have the notation XR, which stands for the S&P 500 VIX Short-Term Futures Index excess return index, SPVXSP. VXX's benchmark is the total return index SPVXSTR, which is the return of SPVXSP plus the daily rate of interest on a 90-day T-bill.[7] The distinction is unimportant in our analysis. The variability in index returns is driven by SPVXSP.

[7]A detailed description of the numerical computation of SPVXSP from actual VIX futures prices is provided in the Appendix A. These mechanics are outlined in S&P 500 VIX Futures Indices: Methodology (2018) and are essential in understanding our daily measurements of VIX ETP replication demands.

TABLE 1 Selected characteristics of VIX ETPs benchmarked to the S&P 500 VIX Short-Term Futures Index on December 29, 2017

Ticker	Name	Issuer	Launch date	*L*	Return index	*ER* (%)	*AUM*	*DTV*	Turnover	Holding period	*SPRD* (%)
VXX	iPath S&P 500 VIX Short-Term Futures ETN Class A	Barclays Bank	20090129	1	TR	0.89	939.69	1053.25	1.006	0.995	0.012
VIXY	ProShares VIX Short-Term Futures ETF	ProShares	20110104	1	XR	0.85	137.76	31.86	0.220	4.550	0.025
TVIX	VelocityShares Daily 2x VIX Short-Term ETN	Credit Suisse	20101129	2	XR	1.65	221.26	102.89	0.417	2.399	0.056
UVXY	ProShares Ultra VIX Short-Term Futures ETF	ProShares	20111004	2	XR	0.95	393.57	341.93	0.794	1.260	0.037
XIV	VelocityShares Daily Inverse VIX Short-Term ETN	Credit Suisse	20101129	−1	XR	1.35	1207.71	704.82	0.539	1.854	0.020
SVXY	ProShares Short VIX Short-Term Futures ETF	ProShares	20111004	−1	XR	0.95	802.12	589.53	0.566	1.768	0.021
Total							3702.11				

Note: The first three columns are the ticker, name, issuer, and launch date of the six different ETPs in our sample. The leverage factor, *L*, is the multiplier applied to the daily return of the benchmark index. A multiplier of 1 implies that the ETP promises the daily rate of return on the underlying futures index. A multiplier of 2 implies two times the return, and −1 implies minus the daily return of the index. The return index column indicates whether the return benchmark is the total return (TR) or excess return (XR) of the short-term futures index, and the expense ratio (*ER*) column is the annual expense ratio expressed in percent. The *AUM* column is the dollar amount of assets under management on December 29, 2017, expressed in millions of dollars, and *DTV* is the average daily dollar volume over the 22 trading days (30 calendar days) up to and including December 29, 2017 expressed in millions of dollars. Turnover is defined as the average daily dollar trading volume divided by the average daily dollar assets under management. The holding period is the inverse of the turnover rate and is expressed in days. *SPRD* is the average of the average daily quoted relative spread.

Abbreviations: ETF, exchange-traded fund; ETN, exchange-traded note; ETP, exchange-traded product; S&P, Standard and Poor; VIX, volatility index.

Table 1 summarizes the attributes of the VIX ETPs in our sample. The summary statistics are based mostly on daily market data downloaded from Bloomberg. The data collected for each VIX ETP begins on its launch date and extends through December 29, 2017. Bid/ask quote data were obtained from NYSE's TAQ files. Note that, for each leverage ratio, there is an ETN/ETF pairing. In all cases, the ETN was launched first (e.g., VXX on 20090129 vs. VIXY on 20100103). From a historical perspective, the first-mover advantage usually results in greater *AUM* and trading volume. VXX has *AUM* of \$940M and average daily dollar trading volume of \$1053M. In contrast, VIXY has an *AUM* of \$138M and an average volume of \$32M. The first-mover advantage is also apparent for the inverse products, XIV and SVXY, although the differences are less extreme. XIV has an *AUM* of \$1208M and a volume of \$705M, while SVXY has an *AUM* of \$802M and a volume of \$590M. The two times products, TVIX and UVXY are an exception to the rule. TVIX has an *AUM* (dollar volume) of \$221M (\$103M) compared with \$394M (\$342M) for UVXY. Part of the explanation may be the reputational damage TVIX suffered on February 21, 2012 when Credit Suisse suspended creations and redemptions of the ETN. In doing so, excess demand to buy TVIX drove the share price to a premium of nearly 90% more than NAV per share on March 21, 2012. When the creation/redemption activity resumed on March 22, 2012, the share price dropped by 30%, followed by another 30% drop on March 23, 2012.

Table 1 also lists other attributes. "Turnover" is defined as the average daily dollar trading volume of the ETP over the 22 trading days (30 calendar days) up to and including December 29, 2017, divided by the average daily dollar AUM. "Holding period" is the inverse of the turnover rate. The final column is the relative bid/ask spread (*SPRD*) across the 22 trading days in December 2017. The daily estimates were computed by averaging all pairs of quotes appearing in the NYSE TAQ file. At least two results are noteworthy. First, high turnover rates indicate that none of the VIX ETPs are suitable buy-and-hold products. VXX, for example, has a daily turnover rate more than 100%, implying that a typical share is held for less than a day. The longest holding period is for VIXY, and it is 4.55 days. Collectively, this evidence suggests that VIX ETPs are used primarily as speculative short-term bets on the direction of market volatility. Second, the results reported for the relative spreads are intuitive. VXX, VIXY, XIV, and SVXY have the same expected return volatility. They are all 1*x* ETPs benchmarked to the VIX short-term futures index. Hence, holding trading volume constant, we would expect the relative spreads of these products to be close to one another. XIV and SVXY are very similar, at 0.020% and 0.021%, respectively. The slightly lower level for XIV is probably due to greater average trading volume. VIXY is the highest, at 0.025%. Its trading volume is the lowest, at \$31.86M a day. The most inexpensive product to trade is VXX at 0.012%. It has an average trading volume exceeding \$1B a day. Part of its dominance is likely attributable to its first-mover advantage. The relative spreads for TVIX and UVXY are 0.056% and 0.037%, respectively. These values are higher than the others because they have twice the level of return volatility, exposing market makers (MMs) to twice the level of inventory holding costs. The spread for TVIX is higher than that of SVXY. Here, the first-mover advantage argument does not appear to work. Daily trading volume is lower, as is AUM. Again, this may have resulted from the 2012 creation/redemption suspension controversy.

3 | VIX ETP STRUCTURE AND VIX FUTURES REPLICATION STRATEGY

Using futures contracts and cash to create the VIX ETPs promised returns has two distinct elements. First, we need to identify the replication strategy demand of a geared fund promising L times futures-based index return in terms of futures index units. Second, we need to identify individual futures contract demands, as SPVXSP is comprised of the two nearby VIX futures contracts weighted in a manner to preserve a constant 1 month to expiration.

3.1 | Futures index demand

To replicate the daily benchmark return using futures, the ETP issuer's replication strategy must satisfy two important constraints. The first is the *value constraint*—the dollar investment in each strategy must be identical. The AUM of the ETP must equal the value of the collateralized futures position,

$$AUM_t(\equiv n_{S,t} NAV_t) = FH_t + C_t, \tag{1}$$

where $n_{S,t}$ is the number of shares outstanding, with each share being valued at NAV_t, FH_t is the aggregate notional amount of the futures position, and C_t is the cash collateral. Since there is no initial cash outlay when the futures position is entered,

$FH_t = 0$. The direct investment in the ETP equals the value of the 91-day US T-bills serving as cash collateral. The second is the *delta constraint*—the change in value of the ETP investment must equal the change in the value of the collateralized futures position over the investment horizon. For VIX ETPs, the investment horizon is 1 day. From the close on day $t-1$ to the close on day t, the delta of the futures hedge must equal the delta of the ETP investment, that is,

$$\begin{aligned}\Delta AUM_t &= \Delta FH_t \\ &= n_{F,t}F_t - n_{F,t-1}F_{t-1} \\ &= n_{F,t-1}(F_t - F_{t-1}) + (n_{F,t} - n_{F,t-1})F_t + \Delta C_t \\ &= n_{F,t-1}(F_t - F_{t-1}) + (n_{F,t} - n_{F,t-1})F_t,\end{aligned} \tag{2}$$

where $n_{F,t-1}$ is the number of futures contracts bought or sold at the close of day $t-1$, and F_{t-1} is the futures settlement price. The term ΔC_t is the daily interest income generated from the cash deposited in the ETP.[8] For the purpose of demonstrating the hedging mechanics in this section, we assume that the interest income is negligible ($\Delta C_t \approx 0$). In the Appendix A, we show the level of interest income historically, that is, the difference in return performance of the SPVXSP and SPVXSTR futures index benchmarks.

The final expression for ΔAUM_t in (2) has two parts. The first is the change in value of the futures position. To determine the exact number of futures index units to use in the replication, we use the relation between the return on AUM, ΔAUM_t, and the ETP's promised daily return, $R_{S,t} = LR_{F,t}$, where L is the leverage factor and $R_{F,t}$ is the daily benchmark futures index return. Where L is 2, the index exposure is twice the level of total net asset value because of the promised return of 200% of the index return, and, where L is –1, the promised return is −100% of the index return. Incorporating the promised return into the left-hand side of (2),

$$\begin{aligned}\Delta AUM_t = AUM_{t-1}R_{S,t} &= AUM_{t-1}LR_{F,t} \\ &= AUM_{t-1}L\left(\frac{F_t - F_{t-1}}{F_{t-1}}\right).\end{aligned} \tag{3}$$

Setting (3) equal to $n_{F,t-1}(F_t - F_{t-1})$ in (2), the number of futures contracts required to replicate the investment in the ETP as of the close of day $t-1$ is

$$n_{F,t-1} = L\left(\frac{AUM_{t-1}}{F_{t-1}}\right). \tag{4}$$

The second term on the right-hand side of (2) has no monetary value. It simply reflects the number of futures contracts bought or sold to account for the (a) changes in the number of shares outstanding (i.e., creations/redemptions) and/or (b) leverage adjustments due to the daily return of the futures index benchmark. As a practical matter, the creation/redemption adjustment is straightforward. Relying on the value constraint, the incremental number of futures required due to creations/redemptions is

$$n_{F,t}^{CR} = L\left(\frac{\Delta n_{S,t} NAV_t}{F_t}\right), \tag{5}$$

where $\Delta n_{S,t}$ is the number of shares created or redeemed on day t. Creations/redemptions do not occur at settlement per se. The effects of information-based demand, which creations and redemptions potentially represent, occur throughout the day and are immediately subsumed in the prices of the VIX ETPs. Suppose, for example, an unexpected announcement causes a spike in volatility midday and a rush to buy 1*x* and 2*x* VIX ETPs. On the other side of the trade is a liquidity provider, such as an MM,[9] who immediately, after consummating the sale, hedges by buying appropriate numbers of nearby and second nearby futures contracts. In effect, the creation is completed when the ETP trade is executed and hedged. As a matter of bookkeeping, the MM may choose to reverse his/her futures position at the end of the day by buying units of the ETP from the issuer either in kind or in cash. Alternatively, the MM may try to avoid paying the creation fee by postponing the

[8]For the moment, we assume that there are no daily creations/redemptions.

[9]Technically speaking, only an Authorized Participant (AP) engages in the creation/redemption process with the ETP issuer. APs are designated by the issuer and may be market makers, specialists, or other large financial players.

creation for a day or two to see if his/her short position in the ETP unwinds itself naturally in the course of trading. From purely a risk perspective, the MM is indifferent.[10] The creation/redemption term in Equation (5) matters only insofar as it contributes to the number of contracts being rolled at EOD settlement.

The leverage adjustment is more subtle. At settlement on day t, futures contracts are bought or sold to "re-lever" the ETP's futures hedge position to a level that ensures the delivery of the promised return on day $t+1$. To gather intuition regarding this adjustment, note that, in the absence of creations/redemptions, the change in the number of futures contracts due to the leverage adjustment is

$$\begin{aligned} n_{F,t}^{LEV} &= \left(\frac{AUM_t}{F_t} - \frac{AUM_{t-1}}{F_{t-1}}\right)L \\ &= \left(\frac{AUM_{t-1}(1+R_{S,t})}{F_t} - \frac{AUM_{t-1}(1+R_{F,t})}{F_t}\right)L \\ &= \left(\frac{AUM_{t-1}}{F_t}\right)(L^2 - L)R_{F,t}. \end{aligned} \tag{6}$$

Equation (6) is the futures-based equivalent of the change in the notional amount of total return swaps demand expression presented in Cheng and Madhavan's (2009, p. 47, eq. 11).

Equation (6) is informative in a number of ways. First, if the ETP is unlevered ($L = 1$), the incremental futures hedging demand due to the leverage adjustment is 0. Daily rehedging is unique to levered and inverse funds. Leverage demand is also 0 if the futures index return is zero ($R_{F,t} = 0$). In the absence of benchmark index movement, leverage is irrelevant. Second, if $L \neq 1$ and $R_{F,t} \neq 0$, the incremental futures hedging demand has the same sign as the daily futures benchmark return, $R_{F,t}$, and is directly proportional to its magnitude. At first blush, this may seem counterintuitive. One might expect that the rebalancing activities of levered and inverse products are in opposite directions. This is not the case.[11] The sign depends on the quantity, $(L^2 - L)$. For a 2x ETP, the quantity is 2. For a –1x ETP, it is also 2. If the futures index benchmark rises during the day, levered (inverse) ETPs must buy more futures to account for the fact that net asset value per share increased (decreased) at a faster rate than the futures price. In other words, levered and inverse products "pile on," buying (selling) at the settlement if the benchmark index has risen (fallen) over the day. Cheng and Madhavan (2009) note that this positive feedback loop can destabilize markets, a research issue worthy of investigation in its own right given the proliferation of levered and inverse funds in the marketplace.

Combining all three components, the total number of futures contracts incorporated in the replication strategy at the settlement on day t is

$$\begin{aligned} n_{F,t} &= L\left(\frac{AUM_{t-1}}{F_{t-1}}\right) + L\left(\frac{\Delta n_{S,t} S_t}{F_t}\right) + \left(\frac{AUM_{t-1}}{F_t}\right)(L^2 - L)R_{F,t} \\ &= n_{F,t-1} + n_{F,t}^{CR} + n_{F,t}^{LEV}, \end{aligned} \tag{7}$$

where $n_{F,t-1}$ is the number of futures contracts in the replication strategy at the close of day $t-1$, $n_{F,t}^{CR}$ is the number of futures contracts bought or sold during day t as a result of creations/redemptions (i.e., changes in the number of shares outstanding), and $n_{F,t}^{LEV}$ is the number of futures contracts bought or sold at the close of day t to provide for the promised leverage rate L on day $t+1$.

3.2 | Individual VIX futures contract demands

Equation (7) expresses futures trading demands in index units. The first term, $n_{F,t-1}$, is the number of units in place from the previous day. It stays in place and, therefore, does not directly contribute to EOD demand. It does,

[10] To this end, the daily reported creations/redemptions may not accurately reflect the actual level of activity that has gone on throughout the day. Nonetheless, it is the only number we have available.

[11] Technically speaking, this is true only where $L < 0$ or $L > 1$. Since all VIX ETP products satisfy one of these conditions, the issue is moot.

however, have an indirect effect. Because the calendar roll is nested in the futures index construction to maintain a constant 1-month time to expiration, about 5% of the nearby futures position must be sold and 5% of the second nearby bought at their settlement prices. Similarly, the creation/redemption component, $n_{F,t}^{CR}$, has no direct effect. Its direct demand was absorbed into futures prices during the day as new information was disseminated into the marketplace. Creations/redemptions do have an indirect effect through the calendar roll.

The contract-specific calendar roll amounts are defined by the construction of the S&P 500 VIX Short-Term Futures Index (SPVXSP). Specific details are provided in the Appendix A. Here, we provide only the essential concepts. The SPVXSP (heretofore represented as F_t) is a weighted average of the prices of the nearby and second nearby VIX futures contracts whose days to maturity straddle 1 month, that is,

$$F_t = w_t F_{1,t} + (1 - w_t) F_{2,t}. \tag{8}$$

The weight given to the price of the nearby futures contract is $w_t = ndaysr_t / ndayst_t$, where $ndayst_t$ is the total number of business days in the current roll period beginning with and including the starting VIX futures settlement date and ending with but excluding the following VIX settlement date and $ndaysr_t$ is the total number of business days within the roll period beginning with and including the following business day and ending with but excluding the following VIX futures settlement date.[12] The weight on the price of the second nearby contracts is $1 - w_t$.

To understand the consequences of the daily calendar roll, move forward 1 day. To maintain the constant 1-month maturity of the S&P 500 VIX Short-Term Futures Index, the nearby VIX futures position must be reduced and the second nearby VIX futures must be increased. By definition, $w_t < w_{t-1}$. The calendar roll demand is

$$n_{F_1,t}^{CAL} = (w_t - w_{t-1})\left(n_{F,t-1} + n_{F,t}^{CR}\right) \tag{9a}$$

for the nearby VIX futures and is

$$n_{F_2,t}^{CAL} = (w_{t-1} - w_t)\left(n_{F,t-1} + n_{F,t}^{CR}\right). \tag{9b}$$

for the second nearby. Note that (9a) and (9b) are equal amounts but opposite sign. Both these trades are assumed to be executed at the settlement prices on day t, $F_{1,t}$ and $F_{2,t}$, respectively. VIX futures spread trades are cost-effective in this context since the quantities are identical but opposite sign. Quantities are mechanical (informationless) and required by the issuer to deliver on the promised daily levered benchmark return on day $t + 1$ (nondiscretionary). Note also that, if $F_{2,t} > F_{1,t}$, as is usually the case with VIX futures, the marking-to-market of the futures position will cause the VIX ST futures index (and, hence the value of the replication portfolio) to fall deterministically through time.

The contract-specific leverage demands at settlement are straightforward:

$$n_{F_1,t}^{LEV} = w_t n_{F,t}^{LEV} \tag{10a}$$

for the nearby contract and

$$n_{F_2,t}^{LEV} = (1 - w_t) n_{F,t}^{LEV} \tag{10b}$$

for the second nearby. Since the weights in (10a) and (10b) are the post-calendar roll weights, no further adjustment is necessary. Note also that these weights are unique to levered and inverse funds, $L \neq 1$. No leverage rebalancing is required for 1x funds.

[12]Under these definitions, all of the weight is assigned to the nearby contract maturity at the close on the day before settlement.

4 | EFFECTS OF VIX ETP HEDGING DEMAND ON TRADING VOLUME

With the links between VIX ETP replication demand and the VIX futures market established, we now turn to examining the relation between the nondiscretionary trading demand by VIX ETP issuers at settlement and EOD trading activity in the underlying VIX futures contracts. This section has five parts. First, we explain the contango trap that plays a central role in VIX ETP return performance. In particular, the advent of levered and inverse funds has increased the amount of rolling of VIX futures, causing an increase in the slope of the VIX futures price curve. Indeed, we show that the single largest component of VIX futures daily trading demand is the calendar roll. Second, we compare the daily dollar trading volume of the VIX ETPs with that of the VIX futures contracts that underlie the VIX futures index. Since there is a well-defined arbitrage relation between the prices of VIX ETPs and the prices of VIX futures, daily trading volumes should be closely integrated. Third, we measure nearby and second nearby VIX futures replication demands in numbers of contracts each day. The two demands are the calendar roll demand defined by Equations (9a) and (9b), and the leverage adjustment demands defined by Equations (10a) and (10b). The demand magnitudes figure prominently into our analyses of EOD returns in Section 4.1 of the paper. Fourth, we examine the magnitude of the net demand in relation to the daily changes in open interest of the VIX futures contracts. Finally, we analyze data from the VIX futures TAS) market.[13] This market operates in parallel with the original VIX futures market and was launched to accommodate the sizable replication needs of VIX ETP issuers. The main distinction of the TAS market is that the trade price is not known when the trade is consummated during the trading day. The buyer and seller simply agree on quantity. The execution price will be the settlement price at the close.

4.1 | Measuring the effects of contango in the VIX futures market

Contango arises in the VIX futures markets when the demands of long hedgers exceed those of short hedgers. Speculators step in to absorb the hedging imbalance, but only when the futures price is high enough to earn a satisfactory risk premium.[14] In such futures markets, equilibrium expected futures-based benchmark returns are negative. The persistence of contango in the VIX futures market has been documented in past empirical work. Whaley (2013), for example, reports that the VIX futures price curve is upward-sloping 81% of the days in his sample that begins with the introduction of SPVXSP on December 20, 2005–March 31, 2012. He shows that a long position in a 30-day VIX futures typically drops 0.0304 index points a day. Because the demand for VIX ETPs has increased substantially in recent years, it is important that we re-examine this issue. It affects roll demand directly.

Table 2 provides summary statistics of the daily price changes and returns of the 1-month-weighted average of the prices of the two nearby VIX futures contracts from which SPVXSP is computed. Daily levels of the index are from Bloomberg. The price change is the number of index points that futures index moves in a given day. Index points are the preferred unit of measurement by traders. Researchers prefer returns, so the natural logarithm of the daily futures index ratio is also included. The table contains three different types of price change and return measures. The settlement-to-settlement return, for example, is the SPVXSP benchmark return. The average daily return is –0.26% for the full sample period August 21, 2006[15]–December 29, 2017. The holding period return over the entire sample period was –99.95%. Long VIX ETPs (1*x* and 2*x*) are not sensible buy-and-hold investments. To understand the source of the poor performance, we need to consider the two other price changes and return measures—the "calendar roll" and "futures market." The calendar roll is *not* driven by market movement. It is driven by the calendar rebalancing of index weights that occurs at the end of the day after the futures index settlement level is computed. Recall Equations (10a) and (10b). Of the average daily settlement-to-settlement index price movement of –0.0397 during the overall sample, –0.0390 is attributable to the calendar roll. This represents a windfall loss to holders of 1*x* or 2*x* VIX ETPs. The VIX

[13]To our knowledge, the only other empirical study to use VIX futures data from the TAS market is Huskaj and Norden (2015).

[14]Unlike the commodity futures markets such as crude oil, VIX futures prices, and market liquidity are not confounded by storage costs or the cost of the required physical delivery.

[15]S&P's reports daily futures index levels beginning December 20, 2005. In attempting to reconstruct SPVXSP, however, it was discovered that in some of the early data, it is impossible because the second nearby futures had not yet been listed, and hence price data were not available in the CBOE records. The first date on which both futures prices are available to provide a continuous daily series is August 21, 2006. Being able to measure the weighted-average futures price each day is critical to measuring the calendar rebalance and futures market components of the settlement-to-settlement return.

TABLE 2 Summary statistics for daily S&P 500 VIX Short-Term Futures Index (SPVXSP) price changes and returns for the sample period August 21, 2006–December 29, 2017 ($n = 2680$ days)

Panel A: Full period						
Begins:	20060821			Number of contango days:		2353
Ends:	20171229			Number of total days:		2859
				Percent of total:		82.3%
	Daily price changes			**Daily log returns**		
	Settlement-to-settlement	**Calendar roll**	**Futures market**	**Settlement-to-settlement (%)**	**Calendar roll (%)**	**Futures market (%)**
Mean	−0.0397	−0.0390	−0.0007	−0.26	−0.26	−0.01
Standard deviation	0.9937	0.0892	0.9915	4.05	0.33	4.05
Skewness	0.9026	2.6187	0.5823	65.50	−5.46	53.90
Volatility				64.23	5.25	64.30
HPR				−99.95	−99.94	−14.98
CAGR				−48.58	−47.84	−1.42
Panel B: First subperiod						
Begins:	20060821			Number of contango days:		791
Ends:	20101126			Number of total days:		1075
				Percent of total:		73.6%
	Daily price changes			**Daily log returns**		
	Settlement-to-settlement	**Calendar roll**	**Futures market**	**Settlement-to-settlement (%)**	**Calendar roll (%)**	**Futures market (%)**
Mean	−0.0187	−0.0263	0.0075	−0.14	−0.18	0.04
Standard deviation	1.1622	0.1203	1.1538	3.70	0.36	3.69
Skewness	0.9031	2.8737	0.5208	73.84	121.59	61.05
Volatility				58.79	5.71	58.52
HPR				−78.02	−86.10	58.15
CAGR				−29.89	−37.03	11.34
Panel C: Second subperiod						
Begins:	20101126			Number of contango days:		1562
Ends:	20171229			Number of total days:		1784
				Percent of total:		87.6%
	Daily price changes			**Daily log returns**		
	Settlement-to-settlement	**Calendar roll**	**Futures market**	**Settlement-to-settlement (%)**	**Calendar roll (%)**	**Futures market (%)**
Mean	−0.0524	−0.0467	−0.0057	−0.34	−0.30	−0.03
Standard deviation	0.8768	0.0622	0.8797	4.24	0.30	4.26
Skewness	0.8055	−1.1764	0.6159	62.88	−153.34	51.08
Volatility				67.29	4.81	67.56
HPR				−99.76	−99.55	−46.24
CAGR				−57.34	−53.43	−8.39

Note: Each panel is divided into two parts: price changes and returns. Price changes are changes in the weighted-average price of the two nearby VIX futures that comprise SPVXSP. Returns are the daily log returns. Volatility is the annualized daily standard deviation, HPR is the holding period return, and CAGR is the cumulative average growth rate.

Abbreviations: S&P, Standard and Poor; VIX, volatility index.

futures market price movement is –0.0007 on average. This is expected. Absent the contango effect, the VIX futures index should behave more like the cash VIX, and the cash VIX follows a mean reverting process with a long-term mean of zero. Note that the sum of the calendar roll and futures market price changes (log returns) is equal to the settlement-to-settlement price change (log return). This is true by definition. Any discrepancies shown in the table (e.g., the mean daily log returns of the full period) are due to rounding errors.

Now, turning to the subperiod results, we find that the contango effect has grown through time. The average calendar roll is –0.0263 index points (–0.18%) in the first subperiod, and –0.0467 (–0.30%) in the second. In addition, the frequency with which the VIX futures market is in contango has increased. In the first subperiod, contango appears in 791 of the 1075 total days or 73.6% of the time, while, in the second, 1562 of the 1785 total days or 87.6% of the time. The greater the VIX ETP AUM, the greater the VIX futures replication need, and the greater the daily roll demand. The calendar roll demand involves selling the nearby futures and buying the second nearby futures, actions which may steepen the VIX futures price curve. The steeper the futures price curve, the greater the contango effect on the SPVXSP.

Other aspects of the results in Table 2 are also interesting. Not only has the contango effect increased, but also its effects have become more certain. The standard deviation of the price change (return) from the first subperiod to the second fell from 0.1203 to 0.0622 (0.36%–0.30%). Comparing relative magnitudes of the standard deviations of the different return components also provides insight. The standard deviations reported for the calendar roll and futures market price changes (returns) are markedly different. In the second subperiod, for example, the levels are 0.0622 (0.30%) versus 0.8797 (4.26%). Shocks to futures market volatility completely swamp the variation associated with calendar rebalancing. In other words, the VIX ETP benchmark index return can be thought of as the sum of two components: (a) a calendar roll (or "contango trap") return whose expected value is negative with a high degree of certainty, and (b) a highly risky futures market return whose expected value is zero.

4.2 | Comparing dollar trading volume of VIX ETPs with VIX futures

The inextricable link between the VIX ETP and the VIX futures market activity manifests itself in a number of ways in the data. One is dollar trading volume. Daily trading data is gathered from two sources. Daily price, AUM, and trading volume information for the VIX ETPs is from Bloomberg. Daily price, volume, and open interest data for the VIX futures are from the CBOE Futures Exchange (CFE) website. For VIX ETPs, we compute the daily sum of the dollar trading volumes of the six VIX ETPs that comprise our sample. For the VIX futures, we compute: (a) the dollar trading volume of the nearby VIX futures, (b) the dollar trading volume of the second nearby, and (c) the total demand across contracts. Figure 1 shows

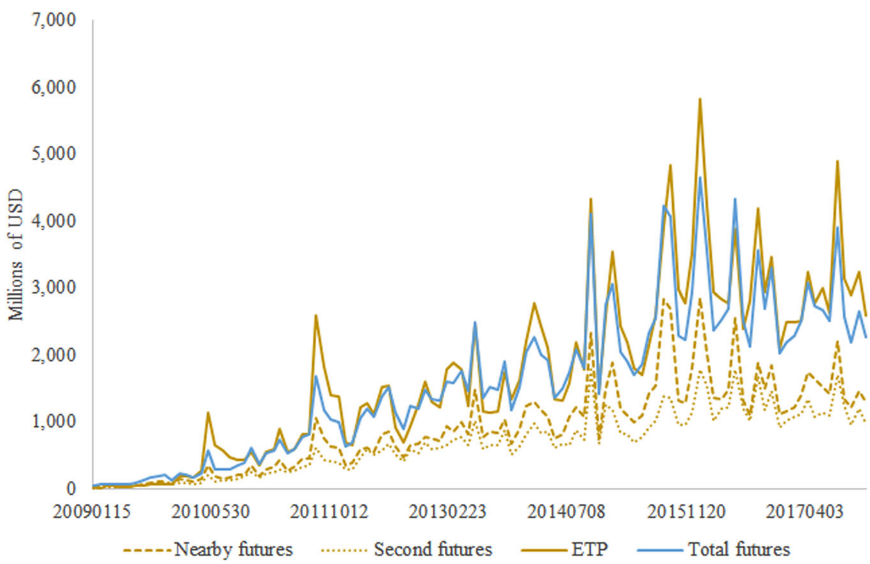

FIGURE 1 Average daily dollar trading volume by month for two nearby VIX futures contracts and VIX ETPs during the sample period January 2009–December 2017. ETP, exchange-traded product; VIX, volatility index.

TABLE 3 Summary statistics for daily VIX ETP demands for VX futures contracts for the sample period November 29, 2010–December 29, 2017 ($n = 1784$ days)

Trading demand							Leverage demand	
Type	Symbol	Mean	Standard deviation	Minimum	Median	Maximum	Opposite sign (%)	Opposite sign and greater (%)
Panel A: Summary statistics based on actual values								
Calendar	$n_{F_1,t}^{CAL}$	−4,480	2,270	−10,875	−4,400	748		
	$n_{F_2,t}^{CAL}$	4,480	2,270	−748	4,400	10,875		
Leverage	$n_{F_1,t}^{LEV}$	−315	4,761	−36,728	−74	55,616		
	$n_{F_2,t}^{LEV}$	−400	4,941	−49,414	−156	43,821		
Net demand	$n_{F_1,t}^{Net}$	−4,794	5,403	−39,400	−4,728	50,512	45.0	9.1
	$n_{F_2,t}^{Net}$	4,080	5,383	−45,335	4,078	50,876	59.1	12.3
Panel B: Summary statistics based on absolute values								
Calendar	$n_{F_1,t}^{CAL}$	4,484	2,262	15	4,400	10,875		
	$n_{F_2,t}^{CAL}$	4,484	2,262	15	4,400	10,875		
Leverage	$n_{F_1,t}^{LEV}$	2,425	4,109	0	1,079	55,616		
	$n_{F_2,t}^{LEV}$	2,641	4,194	0	1,166	49,414		
Net demand	$n_{F_1,t}^{Net}$	5,683	4,458	7	4,924	50,512		
	$n_{F_2,t}^{Net}$	5,207	4,301	1	4,386	50,876		

Note: Demands are expressed in numbers of futures contracts. Each pair of rows represents nearby and second nearby contract demands, respectively. Calendar roll demand results from the fact that the futures position in the benchmark index is rebalanced each day between nearby and second nearly futures to maintain a constant 1-month time to maturity. Leverage demand arises from adjusting the degree of leverage to ensure the ETP delivers the promised levered return on the next day. Last two columns under the heading "Leverage demand" are percent of days in which leverage demand is the opposite sign of calendar demand and percent of days leverage demand is the opposite sign and greater than or equal to calendar demand.

Abbreviations: ETP, exchange-traded product; VIX, volatility index.

average daily volumes by month, and the sample period extends from the launch of the first VIX ETP, VXX, in January 2009–December 2017. The figure confirms that the trading activity in the VIX ETP and the VIX futures markets are closely integrated. Where VIX ETP activity spikes, VIX futures activity spikes, confirming the complementarity between the two markets. Where the nearby VIX futures contract volume spikes, the second nearby spikes, confirming that the trading volume of the nearby and second nearby VIX futures is driven in large part by the simultaneous trading demands for both contracts underlying the VIX ETPs and not by idiosyncratic demands for the specific contract expirations. Nonetheless, differences do appear. The nearby VIX futures is clearly the more active contract.

4.3 | Measuring components of demand

The close relation in the trading activity of the VIX ETP and VIX futures market is, no doubt, driven by VIX ETP replication demands. The nondiscretionary demands are the calendar roll (i.e., Equations 9a and 9b) and the re-leveraging (i.e., Equations 10a and 10b). In theory, they must be executed at daily settlement prices. In measuring these demands, we use November 29, 2010 as the start date of the sample period. Recall this was the launch date of levered and inverse products (see Table 1).

Table 3 contains the summary statistics for daily trading demand components. Panel A has actual demands, while Panel B has absolute values of demands. Absolute values are included since it is the size of the trade, not the sign, that matters when measuring trading activity. The first row of Panel A provides the summary statistics for the calendar roll demand for the nearby VIX futures contract. The average number of nearby futures

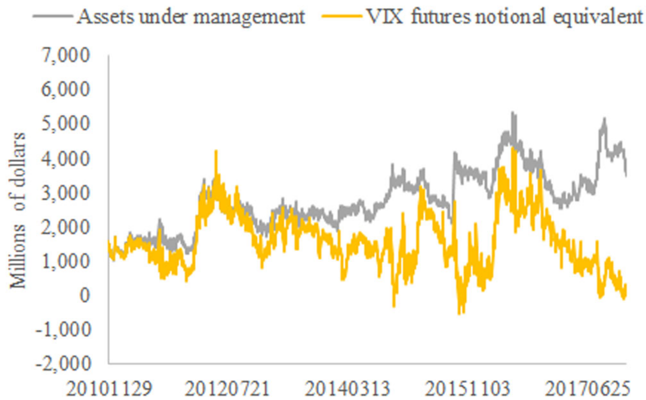

FIGURE 2 VIX ETP assets under management and notional amount of VIX futures required to replicate VIX ETPs during the period November 29, 2010–December 29, 2017. ETP, exchange-traded product; VIX, volatility index.

contracts sold and second nearby contracts bought each day is 4480.[16] The median is 4400. These numbers reflect the fact that about 1/20th or 5% of the nearby futures position is closed and rolled into the second contract month each trading day during the month.

The daily calendar roll demands for the nearby contract range from −10,875 to 748. The fact that the calendar demand for the nearby futures can be positive may seem odd. Occasionally during the sample period, however, the sum of the daily AUMs across the two −1*x* ETPs exceeded the sum of the 1*x* and 2*x* products. Figure 2 shows the total value of the VIX ETP AUM and the total notional value of the VIX futures required to replicate the VIX ETPs on a daily basis throughout the investigation period. Early in the period, the two values are close to one another, indicating that the market dominance of the 1*x* products. In moving to the right across the figure, the futures notional falls below AUM as the inverse products begin to show their increasing popularity. The short hedging demand of −1*x* products offsets, at least in part, the long hedging demands of 1*x* and 2*x* products. By the end of 2014, the demand for the inverse products dominates the VIX ETP market and the net replication demand results in a short VIX futures position. The pendulum swings back to net long until the end of 2015, at which time the demand for inverse products again dominates. During 2017, there was a steady increase in the popularity of inverse products, no doubt driven by the lack of volatility in the stock market and the ever-present contango pattern in the VIX futures market. When inverse products dominate in terms of replication demand, issuers are net short both the nearby and second nearby contracts, in which case the daily rebalance involves buying the nearby to reduce the short position and selling the second nearby to increase the short position.

Comparing the magnitudes of the leverage (*LEV*) demand to the calendar (*CAL*) demand in Panel A of Table 3 does not make much sense since the *LEV* demand is randomly positive or negative each day, and *CAL* demand is generally negative for the nearby contract and positive for the second nearby. A fairer way to evaluate the relative trading demands is to use absolute numbers of contracts. Panel B reports summary statistics for the absolute values. On average, 4484 nearby contracts are traded each day due to *CAL* demand, and 2425 due to *LEV* demand. *LEV* demand is slightly more than half the *CAL* demand each day on average. In the case of *LEV* demand, the median absolute value, 1079, is below the mean absolute value, 2425. This implies that absolute *LEV* demand is skewed to the right. Recall that *LEV* demand is a direct function of the daily benchmark index return (see Equation 6). On occasion, the daily index return is extreme one way or another. On these days, *LEV* demand can be extraordinary. On June 28, 2016, for example, the settlement-to-settlement VIX short-term futures index return was −18.59%. The *LEV* demand was −32,090 contracts for the nearby futures and −22,921 for the second nearby. The *CAL* demand on the same day was −7420 for the nearby and 7420 for the second nearby. But, even this particular day's daily leverage demand was far from the extreme. Panel A shows that the leverage demand ranges from −36,728 to 55,516 for the nearby contract and from −49,414 to 43,821 for the second nearby contract. Panel A also shows that leverage demand frequently offsets and, sometimes, dominates calendar roll demand. For the nearby (second nearby) futures, *LEV* has the opposite sign of *CAL* on 45.0% (59.1%) of the days in the sample period. On 9.1% (12.3%) of the days, it

[16]Recall that we showed analytically in Section 3 that, for the calendar rolls, the number of second nearby contracts are equal and opposite the nearby.

has the opposite sign and is greater in absolute magnitude than *CAL*. Clearly, the *LEV* demand has the potential of being a disruptive force in the VIX futures market.

4.4 | Comparing VIX ETP trading demand with change in VIX futures open interest

Another way of gauging the VIX ETP futures demands is to compare the net daily replication demand (i.e., the sum of the *CAL* and *LEV* demands) to the daily change in open interest of the underlying VIX futures contracts. Table 4 summarizes the results. The summary statistics for the net demand are the same as those reported in Panel A of Table 3. The net demand for the nearby VIX futures is –4794 contracts and the net demand for the second nearby is 4048. The change in open interest is –6798 for the nearby and 6239 for the second nearby contracts. It seems that the VIX futures market is comfortably absorbing the demand by VIX ETPs on a typical trading day. Note also that the ratio of the standard deviations of the nearby contract to the second nearby is higher for open interest (1.244) than it is for net demand (1.004). This reflects the fact that there is more non-VIX ETP trading activity in the nearby contract than that in the second nearby. Recall that Figure 1 showed that the nearby contract had a higher average daily trading volume throughout the sample period.

4.5 | Reducing execution risk through trades at settlement

Before the advent of TAS trading in the VIX futures market, VIX ETP issuers faced a number of execution costs/risks when the futures position was rebalanced each day. Since the ETPs are benchmarked to the settlement price of SPVXSP, the issuers needed to execute their calendar roll and leverage trade orders at the VIX futures settlement prices. As a practical matter, this was impossible. First, settlement prices of VIX futures were averages of the best bid and best ask quotes prevailing when regular trading hours (RTHs) ceased—not prices at which the futures could be traded. Second, the day's replication quantities were not yet fully resolved. Recall the EOD leverage demand Equation (6). The number of contracts bought or sold at the end of the day is a function of the benchmark SPVXSP return for the day, and the return for the day is not fully known until settlement. Third, the issuer cannot wait until the second before the market closes to execute his order. The order may not get fully consummated, and, if it does not, the issuer will be unable to match the following day's promised levered benchmark return. On the other hand, executing the trade earlier in the day means incurring the basis risk between the trade price and the settlement price.

The VIX futures TAS market offered a solution to the VIX ETP issuer's execution risk problem. To understand why, consider traditional TAS markets. They have existed for commodity futures such as crude oil and natural gas for decades. They were designed to facilitate the unwinding of intermarket commodity futures cross-market arbitrage positions where the daily settlement price is the same but the times of settlement/delivery are different. Consider US crude oil futures, for example. Each month, the Intercontinental Exchange (ICE) West Texas Intermediate (WTI) contract is cash settled on the day before delivery is made on the New York Mercantile Exchange (NYMEX) WTI contract. Now, consider a trader who went long the ICE contract and short the NYMEX contract when the spread became much too wide for whatever reason. Assume prices have returned to normal levels, and the trader wants to unwind. Absent a TAS market, this means placing simultaneously executing market orders in both markets, with no guarantee of price convergence. With a TAS market, the

TABLE 4 Summary statistics for total daily net VIX ETP futures demand and daily change in open interest of VIX futures contracts for interval November 29, 2010–December 29, 2017 ($n = 1784$ days)

Trading demand Type	Symbol	Mean	Standard deviation	Ratio
Net demand	$n_{F_1,t}$	4,794	5,403	1.004
	$n_{F_2,t}$	4,080	5,383	
Change in open interest	$\Delta OI_{F_1,t}$	−6,798	7,158	1.244
	$\Delta OI_{F_2,t}$	6,239	5,755	

Note: Total daily net VIX ETP futures contract demand is the sum of calendar roll and leverage demands for each of the two nearby VIX futures contracts. Abbreviations: ETP, exchange-traded product; VIX, volatility index.

position can be unwound risklessly at the same prices. On the day before delivery of the NYMEX contract, its price is settled at the same price as the ICE contract is cash settled, that is, the settlement price for NYMEX WTI crude. All risk is removed if the trader buys the NYMEX TAS on the day before delivery. The marking-to-market in the NYMEX market will be at exactly the same price as the cash settlement in the ICE market. The TAS market for VIX futures arose from the realization that TAS trading could be used in financial futures markets for a nontraditional purpose.

The CBOE Futures Exchange launched a TAS market for VIX futures on November 4, 2011. The move was designed to help VIX ETP issuers with their replication trades.[17] In the VIX futures TAS market (ticker symbol "VXT"), a trader can place an order to buy or sell at any time during the day, specifying only the number of contracts to be bought or sold. The settlement price of the benchmark index, SPVXSP, is determined at the close of the day based on the settlement prices of the two nearby regular VIX futures (ticker symbol "VX") contracts. Each VX futures settlement price, in turn, is based on the average of the prevailing bid/ask quotes when the market closes. Since the VX futures prices have a minimum bid/ask spread of 0.05, the settlement price is necessarily reported to at least three decimal places (e.g., if the closing bid/ask price quotes are 20.00/20.05, the settlement price is 20.025). When the VXT trade is consummated during the trading day, the price may have a small premium over or discount from the VX futures settlement price. While the amount of the premium or discount is specified at the VXT trades, the settlement price is not determined until trading stops. Premiums and discounts are in 0.01 index point ($10) increments, with the maximum permissible price range of ± 0.10 index points.[18]

To underscore how the contracts work, consider an actual trade. On December 28, 2017, a VXT trade of 100 January 2018 (Jan/18) futures at 0.00 index points was executed 4 s before the VXT market close. At the time of the trade, the prevailing bid/ask price quotes of the Jan/18 VXT futures contract were 0.00/0.01, so, in all likelihood, the trade was the result of a sell order. At the end of the day, the position was marked-to-market. The short position of 100 Jan/18 VXT futures became a short position of 100 Jan/18 VX futures. The settlement price on that day was 11.225, so the mark-to-market price was also 11.225.

Turning to VXT market operation and history, we provide the daily trading volumes of VX and VXT futures. Because the VXT futures market was developed in response to the risk management needs of VIX ETP issuers benchmarked to the SPVXSP and the SPVXSP is based only on the prices of the nearby and second nearby futures contracts, the trading volume of the nearby and second nearby VXT contracts should be about the same, and the trading volume of the distant contract months to be inconsequential. Figure 3 shows the average daily volumes of VXT by contract month and confirms this proposition. The average daily trading volumes of the nearby and second nearby contracts are about the same, and the average daily volume for the more distant futures is barely noticeable as it ripples along the horizontal axis. While not reported in a table, the average daily volume over the sample period was 8597 for the nearby VXT contract and 8035 for the second nearby. In contrast, the average daily volume of all other VXT contract expirations is 211. The figure also shows that the average daily trading volume per month in the VXT market has grown through time. The rate of growth is consistent with the growth of VIX ETP AUM shown in Figure 2. The VIX ETP and VXT markets appear closely integrated as a result of VXT becoming the "go-to" market for managing the return/risk properties of VIX ETPs. This also can be seen in comparing the daily trading volumes of VXT versus VX futures. Figure 4 shows that VXT futures trading volume pales in comparison to VX futures trading volume and that the nearby VX futures contract trades more heavily than the second nearby VX futures. The fact that the nearby VX futures trading activity differed from the second nearby (77,621 vs. 55,587 a day on average) implies that the motives for VX futures trading go well beyond the demands of VIX ETP issuers.

The intraday timing of the execution of VIX ETP replication trades is also interesting, but not unexpected. Although the *CAL* demand is known at the previous day's close, trading VXT futures early in the day is suboptimal. The full amount of the *LEV* demand remains uncertain until market close. Moreover, Panel B of Table 3 shows that *LEV* demand is typically the half size of *CAL* demand each day. Panel A of the same table shows that *LEV* demand partially offsets *CAL* demand on 45.0% of the days for the nearby contract and 59.1% for the second nearby and fully offsets it in 9.1% and 12.3% of the days, respectively. Since nothing is lost by deferring the *CAL* trades until the end of the day, we should expect that VXT trading is concentrated at the end of the day when net quantities are better known.

[17]The CBOE launch of the VIX futures TAS market in November 2011 coincided with the launch of the ProShares UVXY ($2x$) and SVXY ($-1x$) ETFs.
[18]On the launch date, the VX TAS trades (ticker symbol VXT) were in increments of 0.05 index points. It became evident quickly that the increment was too large and that competition would reduce bid/ask spreads if the minimum price increment was reduced. On February 21, 2012, VXT began trading in 0.01 index point increments. By April 2012, quoted VXT bid/ask spreads were down to the minimum allowable level of 0.01 index points (effective spreads were 0.005 index points), and they remained at that level through the sample period.

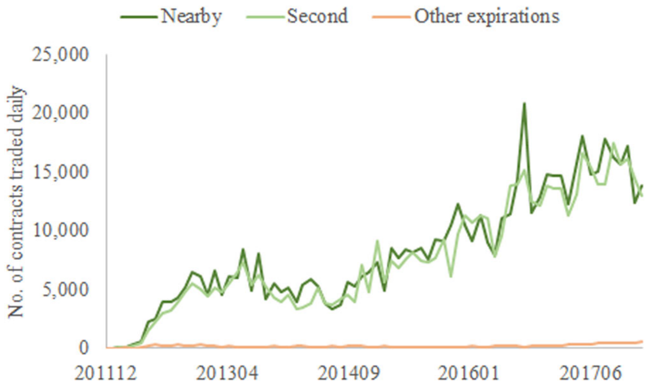

FIGURE 3 Average daily dollar trading volume by month for all VXT futures contract expirations during the period November 2011–December 2017

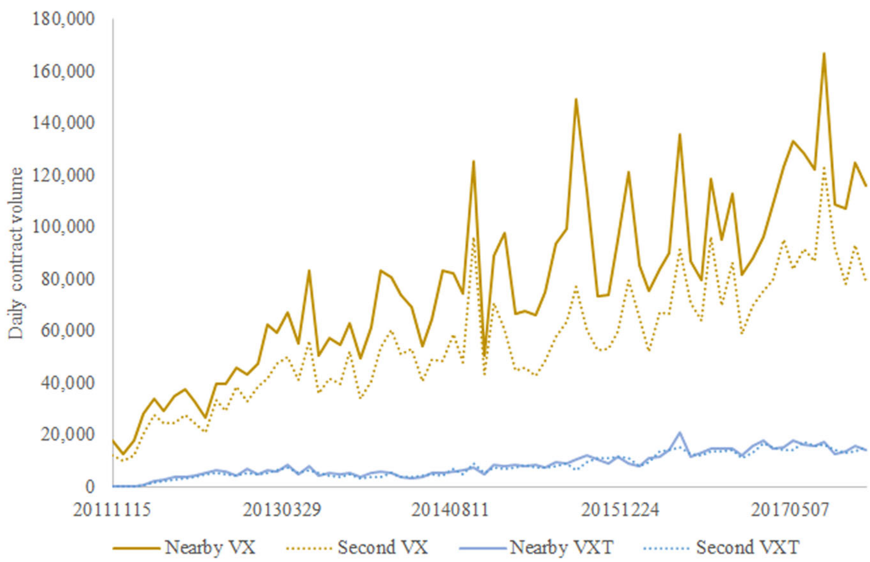

FIGURE 4 Average daily dollar trading volume by month for the two nearby VX futures and the two nearby VXT futures during the period November 2011–December 2017

To examine this proposition, we collected intraday trade and quote data for the period November 4, 2011–December 29, 2017. The VX futures data are from the CBOE DataShop, and the VXT futures data are from Thompson-Reuters. RTHs in the VX futures market are from 8:30 a.m. to 3:15 p.m. We partition this 6.75-h period into 81 five-min intervals. For each interval, we compute the sum of the contracts traded and express it as a proportion of the total daily volume. The proportions are then averaged across the days in the sample period, and the results are shown in Figure 5. Perhaps the most striking feature in the figure is the spike in VXT trading volume at the end of the day. Where the nearby and second nearby VXT contracts are dormant throughout the trading day, their trading activity is rekindled at the end of the day. About 55% of the daily VXT trading volume occurs after 3:00 p.m. (25% after 3:10 p.m.). Trading volume near the close is not nearly as concentrated for the VX futures. About 13% of daily VX futures trading occurs after 3:00 p.m. on average.

5 | EFFECTS OF NONDISCRETIONARY VIX FUTURES DEMAND ON EOD PRICES

Thus far, we have established both theoretically and empirically that VIX futures trading demand is driven in large part by the trading demands of the most active VIX ETPs. This demand is nondiscretionary and informationless. Calendar roll demand (*CAL*) arises from the ETP issuers need to mimic the constant 1-month to the expiration of the short-term VIX futures index, SPVXSP. Leverage demand (*LEV*) arises from rebalancing their VIX futures position at settlement to

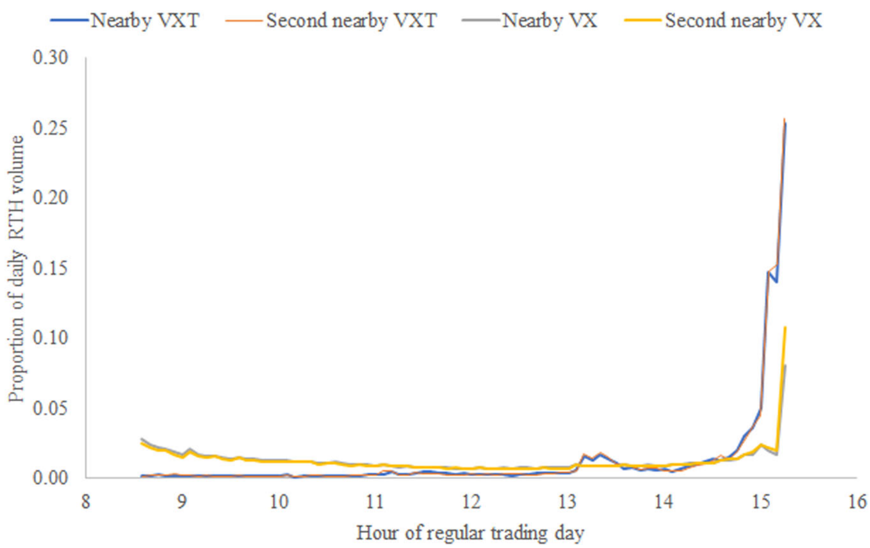

FIGURE 5 Average proportion of total regular hours trading volume occurring in each 5-min interval throughout regular trading hours (RTH) for the two nearby VX and VXT futures contracts during the sample period November 2011–December 2017

provide the promised levered return for the next day. In this section, we assess the effects of these nondiscretionary tradings on the behavior of inter- and intraday VIX futures returns.

The section has three parts. In the first, we examine EOD VIX futures returns. Since VIX ETP futures demands are likely to be executed near the close to avoid tracking error, we can expect short-term price movements in the VIX futures at the end of the day in direct proportion to demand size. We examine the effects of *CAL* and *LEV* demands separately and together, and show that, while both demands affect returns, their relative effects are indistinguishable. In the second, we show that the nature of the VX futures price response to VIX ETP demand has evolved through time. Where VIX ETP demand had a significantly positive effect on the EOD VX futures returns in the early years, the effect has become muted by the increased participation by hedge funds and the increased demand for inverse VIX products. In the third, we examine the robustness of our test results.

5.1 | Assessing effects of trading demand on futures returns

In studies of the effects of nondiscretionary trading, knowing the size of the impending order is important. The greater the demand, the greater the potential effect on price. Equations (9a) and (9b) and (10a) and (10b) specify the *CAL* and *LEV* futures demands each day. In the case of *CAL* demand, the quantity is known at the time of VIX futures settlement on the previous day. *CAL* depends on the AUM at yesterday's settlement, and, therefore, includes yesterday's creations and/or redemptions, if any. *LEV* demand, on the other hand, depends on the settlement-to-settlement futures index return and is not fully revealed until 3:15 p.m. CT, when the VIX futures market settlement prices are computed. But, while the exact quantity of the *LEV* demand cannot be determined until 3:15 p.m., it can be accurately estimated before that time. By 3:00 p.m., for example, 23.75 h of the 24-h settlement-to-settlement return has been realized. Consequently, the *LEV* demand computed as of 3:00 p.m., based on the intraday level of SPVXSP, is likely to be a close proxy for *LEV* demand at 3:15 p.m.

Exactly how VIX ETP trading demands affect EOD returns is an empirical question. It depends on the nature of the trading game that the liquidity provider (i.e., risk arbitrageur) chooses in an attempt to make short-term profits. The underlying trading strategy is based on the knowledge that VIX ETP replication trades will cause a price aberration in the VIX futures market at the end of the day. The size of the aberration will depend on trade size, and the trade size is known. The trader's game choices are twofold—front-running before the close or reversion-trading at the close. Recall these were the same choices in the S&P 500 index revision studies. The main difference is that the "VIX ETP game" is played daily, whereas the S&P game is played quarterly.

Holding the size of the price aberration constant, the front-running strategy is dominant. The arbitrager's VIX futures price risk exposure is only a small fraction of what it is for the revision-trading strategy (i.e., a handful of

TABLE 5 Results of regressions of end-of-day VIX return on components of VIX ETP futures replication demand for interval November 29, 2010–December 29, 2017 ($n = 1784$ days)

			Parameter estimates/t ratios				
Model	**Contract**	**Adjusted R^2**	β_0	β_1	β_2	β_2^-	β_2^+
1	Nearby	0.0178	0.0013	0.2595			
			4.03	5.77			
	Second	0.0409	−0.0006	0.3364			
			−2.55	8.77			
2	Nearby	0.0181	0.0018	0.3783	−0.1516		
			3.49	3.68	−1.29		
	Second	0.0405	−0.0008	0.3820	−0.0561		
			−1.96	4.43	−0.59		
3	Nearby	0.0183	0.0020	0.3845		−0.0874	−0.2223
			3.67	3.74		−0.67	−1.67
	Second	0.0400	−0.0008	0.3815		−0.0438	−0.0688
			−1.80	4.42		−0.41	−0.64
4	Nearby	0.0183	0.0020	0.3804		−0.2178	−0.0850
			3.67	3.70		−1.65	−0.64
	Second	0.0402	−0.0008	0.3828		−0.0261	−0.0929
			−1.73	4.44		−0.25	−0.85

Note: End-of-day VX futures return $R_{F,t}$ is defined as the VIX futures return from the first trade after 3:00 p.m. until VIX futures settlement at 3:15 p.m. Demands in Panel A are in millions of VX futures contracts. Calendar roll demand arises from rebalancing the nearby and second nearby futures positions each day to maintain a constant 1-month time to expiration. Leverage demand arises from adjusting the degree of leverage to ensure the ETP delivers the promised levered return on the next day. Leverage demand is measured as of 3:00 p.m. each day based upon the SPVXSP benchmark index.

$$\text{Model 1:}\quad R_{F,t} = \beta_0 + \beta_1\left(n_{F,t}^{CAL} + n_{F,t}^{LEV}\right) + \varepsilon_t,$$

$$\text{Model 2:}\quad R_{F,t} = \beta_0 + \beta_1\left(n_{F,t}^{CAL} + n_{F,t}^{LEV}\right) + \beta_2 n_{F,t}^{LEV} + \varepsilon_t,$$

$$\text{Model 3:}\quad R_{F,t} = \beta_0 + \beta_1\left(n_{F,t}^{CAL} + n_{F,t}^{LEV}\right) + \left(\beta_2^- I_{LEV<0} + \beta_2^+ I_{LEV\geq 0}\right)n_{F,t}^{LEV} + \varepsilon_t,$$

$$\text{Model 4:}\quad R_{F,t} = \beta_0 + \beta_1\left(n_{F,t}^{CAL} + n_{F,t}^{LEV}\right) + \left(\beta_2^- I_{sign(LEV)\neq sign(CAL)} + \beta_2^+ I_{sign(LEV)=sign(CAL)}\right)n_{F,t}^{LEV} + \varepsilon_t.$$

Abbreviations: ETP, exchange-traded product; VIX, volatility index.

minutes during the day as opposed overnight). Consequently, this section focuses exclusively on front-running. The implementation of the trading strategy is straightforward. If intraday net demand is positive, the trader buys before the close and then reverses at the close. Exactly when the trader makes his initial trade is not unknown. In this initial round of testing, we assume it occurs 15 min before the market close. In other words, the dependent variable is the 15-min (3:00–3:15 p.m.) EOD futures return.[19]

Next, we consider different model specifications for how the different demands affect returns. The models are listed in the header of Table 5. Model 1 combines the two demands, *CAL* and *LEV*, as a single regressor measured in a number of contracts. Demands are combined since they may be naturally offsetting, as we discussed in the last section. The results for Model 1 show that the estimated slope coefficient is positive and statistically significant for both each of the VIX futures contract months. To interpret the nearby futures coefficient estimate is 0.2595, consider that demand is

[19]On the day before a US holiday, the stock market closes at 12:00 p.m. CT, and the futures market close at 12:15 p.m. On these days, the EOD futures return is computed over the interval 12:00–12:15 p.m.

measured in millions of contracts. From Table 3, we know that the net replication demand for the nearby futures was –4794 contracts per day on average. Combining these estimates, the expected return of the nearby futures contract on a typical day is

$$0.2595 \times \left(\frac{-4,794}{1,000,000}\right) = 0.00125 \text{ or } -0.125\,\text{bps}.$$

In other words, based on the average net VIX futures contract daily demand of VIX ETPs, the expected return from playing the VIX ETP game is a paltry –0.125% a day. A rough proxy for round-trip trading costs in this market is 0.25%. That is not to say, however, that the strategy cannot be profitable. Earlier we used contract demands on June 28, 2016 to illustrate the variation in daily demand. *LEV* was –32,090 for the nearby futures contracts and –22,291 for the second nearby. The *CAL* demands were –7420 and 7420. The expected returns from playing the game on this day were –1.025% for the nearby contract and –0.521% for the second nearby, both returns sufficiently large to cover typical trading costs.

In Table 5, the Model 1 slope coefficients are positive for both the nearby and second nearby futures contracts. Positive (negative) net demand in a given day causes the futures price to get pushed up (down) during the last 15 min of the trading day. Interestingly, the coefficient for the nearby contract is smaller than it is for the second nearby. This is consistent with the fact that the nearby VIX futures have more market activity unrelated to the VIX ETP replication needs. (Recall Figure 3, where we showed the trading volume is higher for the nearby contract.) As a result, its price is more elastic.

Model 2 evaluates whether *LEV* demand has a differential effect. *LEV* demand can change wildly from day-to-day depending on the close-to-close settlement return. The *CAL* demand, on the other hand, is predictable, and its level has been known since the previous day's settlement. The results of Model 2 indicate that, while *LEV* demand has less impact on EOD returns than does *CAL* demand (i.e., both LEV coefficients are negative), the difference is not significant. The adjusted R^2 increases by a small amount for the nearby contract and falls by a similar amount for the second nearby. Note that the coefficients of the net demand variable for the two futures are now similar in magnitude, 0.3783 and 0.3820, as the effects of *CAL* and *LEV*, although insignificant, have now been separated.

One possible reason for not seeing a clear distinction between the effects of *CAL* and *LEV* demands is an asymmetry in how EOD VX futures returns behave depending on whether the intraday SPVXSP return is positive or negative (or, equivalently, *LEV* is positive or negative). Model 3 addresses this potential asymmetry. The Model 3 results are directly comparable to those of Model 2. With the leverage demand split by positive and negative return days, Model 3 has marginally greater explanatory power for the nearby contract and marginally less for the second nearby. Like Model 2, the coefficients of the net demand are significantly positive, and of similar magnitudes for the two futures contracts. The results for the *LEV* demand variables suggest that *LEV* has a stronger effect on EOD return when the intraday benchmark index rises (a larger negative coefficient) and futures contract sales ensue than when the benchmark falls. But, also like Model 2, the *LEV* coefficients are not different from 0 in the statistical sense.

Another potential reason for not seeing a clear distinction between the effects of *CAL* and *LEV* demands is that there is an asymmetry between days on which the signs of *CAL* and *LEV* demands are the same and the days on which they are different. Recall from Table 2 that this occurs about half of the time. For daily *LEV* demand, the sign for both contracts is the same, but the levels are not equal. For *CAL* demand, on the other hand, it is almost uniformly negative for the nearby contract and positive for the second nearby, and the levels are the same. Model 4 examines this possible asymmetry. The results, again, indicate that *LEV* has no different effects of EOD return than those of does *CAL*. The effects of the *LEV* variables on EOD return are not different from 0 in a statistical sense, and the adjusted R^2 values show no real improvement over Model 2.

5.2 | Assessing effects of evolving supply

The results of Table 5 clearly show that the net replication demand of VIX ETP issuers affects EOD futures returns. The analysis, however, implicitly assumes that the character of the supply of liquidity remains the same over the sample period. Recall that Duffie (2010) predicts that the price impact of nondiscretionary shocks will be reduced over time as investors learn about trading opportunities. Without question, the tripling of the AUM of VIX ETPs (see Figure 2) drew the attention of a broader group of more sophisticated investors, some of whom became intrigued by the nature of the investment products and their attendant daily hedging requirements.

5.2.1 | Source of trader identities

Oftentimes financial markets research cannot evaluate the effects of certain market behaviors because they are uninformed about the specific roles of the market participants standing on each side of a trade. The data is simply not made publicly available. The study of US futures markets is an exception. The CFTC publishes two weekly reports that are useful in terms of developing an understanding of supply and demand in the VX futures market. The first is its legacy Commitment of Traders (COT) report. For each US futures markets,[20] the CFTC reports the positions of "large traders," and categorizes them as being "commercial" (hedgers) and "noncommercial" (speculators), with each category broken down by the numbers of contracts long and short. In the context of VX futures, VIX ETP issuers fall into the noncommercial category, and liquidity providers such as MMs and hedge funds fall into the commercial category. The second weekly report is the Traders in Financial Futures (TFF) report.[21] It separates open interest into four, more refined categories. The "Dealer/Intermediary" (henceforth dubbed "Dealer") category are traders who are primarily on the sell-side, designing and selling various types of financial products for clients, and earning commissions or capturing bid/offer spreads. The traders include large banks and dealers in securities, swaps, and other derivatives, who price off and hedge using VX futures contracts. VIX ETN issuer positions fall into this category. The "Asset manager/Institutional" (dubbed "Asset manager") category are buy-side traders and are clients of the sell-side participants. This category contains institutional investors, including pension funds, endowments, insurance companies, mutual funds, and portfolio/investment managers who use futures to manage risk. VIX ETF issuer positions fall into this category. The "Levered fund" (dubbed "Hedge fund") category traders are also on the buy-side. They are typically hedge funds and various types of money managers, including registered commodities trading advisors, registered commodity pool operators, or unregistered funds identified by the CFTC. Their strategies may involve taking outright positions or arbitrage positions within and across markets. The traders may be engaged in managing and conducting proprietary futures trading and trading on behalf of speculative clients. VIX ETP liquidity providers like hedge funds fall into this category. Finally, the "Other reportable" category contains reportable buy-side traders who do not fall into one of the above three categories. Corporate treasuries, central banks, smaller banks, mortgage originators, among others, are included. We develop our insights regarding the relation between VIX ETP issuers replication demand and VX futures contract supply from the TFF reports.

5.2.2 | Evolving participation

The TFF reports for the CFE's VIX futures contract market reveal several interesting facts regarding VIX ETP replication trading.[22] First, early after the launch of the VIX ETP market in January 2009, VIX ETP replication demand almost fully accounted for VX futures open interest. From January 2009 through November 2010, the only VIX ETP that traded was Barclay's VXX (1*x*). Figure 6 displays the number of futures contracts required to replicate VXX's daily AUM over the period, together with the number of VX futures outstanding.[23] The solid blue line is the VXX replication need and the solid gold line is the total open interest of the VX futures. The dotted blue line shows VXX replication demand as a proportion of total open interest. During the examination period, VXX replication demand grew quickly, and, occasionally, exceeded VX futures open interest. Presumably, on these days, the issuer found other hedging vehicles meet the excess demand.[24] Second, the sum of the dealer and asset manager open interest (demand) is virtually the mirror image of the open interest of hedge funds (supply). Figure 7 shows the data over the sample period. Third, except for the first 2 years, the replication demand of VIX ETP issuers appears to be largely provided by hedge funds. In Figure 8, the solid blue line is the daily replication demand of VIX ETP issuers. In January 2009, the VIX ETP market was a brand new concept, and only a single product was traded. While hedge funds appear to have

[20]The COT reports are published each Friday based on the previous Tuesday's open interest. The reports are prepared for each market in which 20 or more traders hold positions equal to or above the reporting level established by the CFTC. Currently, a large trader in the VIX futures market is someone with 200 or more contracts outstanding.

[21]See https://www.cftc.gov/sites/default/files/idc/groups/public/@commitmentsoftraders/documents/file/tfmexplanatorynotes.pdf.

[22]Few studies have recognized the usefulness of TFF reports in general, or the reports of VIX futures positions in particular. A notable exception is Chen and Wang (2021), who focus on the role of VIX futures in the trading activities of dealers and leveraged fund managers during low-VIX and high-VIX periods during the period September 2009–June 2016.

[23]The start date of the series was June rather than January 2009 because the CFTC, apparently, lost a 5-month block of TFF data.

[24]There is no mandate for the VIX ETN's to hedge using VX futures. Indeed, they do not have to hedge at all.

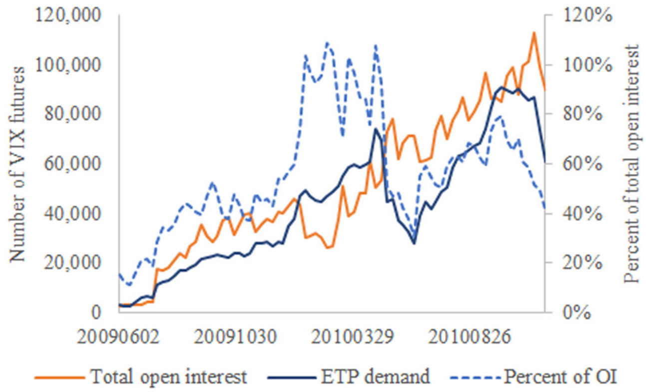

FIGURE 6 VXX replication demand and VX futures open interest in number of futures contracts during the period June 2, 2009–November 29, 2010 when VXX was only VIX ETP contract. ETP, exchange-traded product; VIX, volatility index.

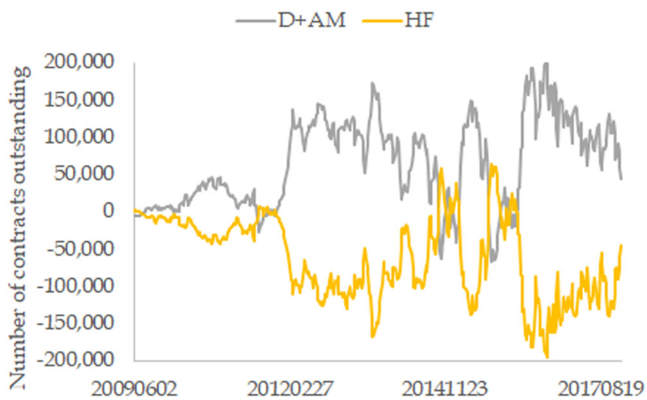

FIGURE 7 Net VX futures open interest of dealers plus asset managers (D + AM) and hedge funds (HF) during the period June 2, 2009–December 26, 2017.

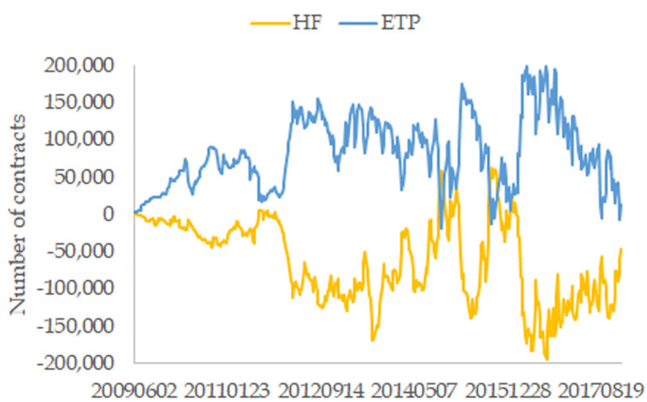

FIGURE 8 Replication demand by VIX ETPs (ETP) and VX futures open interest of hedge funds (HF) during the period June 2, 2009–December 26, 2017.

accommodated some of the demand, other smaller (nonreportable), liquidity traders such as MMs provided the rest. As the familiarity with and demand for volatility products grew, different unlevered and levered products were launched. Increased VIX ETP replication demands drew the interest of potential counterparties. Halfway through the time-series shown in Figure 8, the demand by VIX ETPs appears to be equally offset by new supply from hedge funds.

Table 6 summarizes the same AUM and TFF information differently using correlation levels. The correlation between D + AM and HF in Panel A is −0.989. This should not be surprising in the sense that net hedger demand

TABLE 6 Correlation coefficients between numbers of contracts held by dealers (D), asset managers (AM), and hedge funds (HF), and replication demands by exchange-traded products (ETPs, ETNs, and ETFs) during the period June 2, 2009–December 26, 2017

	D	AM	D + AM	HF	ETP	ETN	ETF
D	1						
AM	0.481	1					
D + AM	0.953	0.723	1				
HF	−0.952	−0.696	−0.989	1			
ETP	0.731	0.810	0.854	−0.817	1		
ETN	0.711	0.535	0.744	−0.702	0.883	1	
ETF	0.551	0.890	0.740	−0.719	0.854	0.509	1

Abbreviations: ETF, exchange-traded fund; ETN, exchange-traded note; ETP, exchange-traded product.

should equal speculator supply in all futures markets. The correlation should be −1. Any noise is attributable to nonreportable positions. When hedging demand is segmented so as to isolate the replication demands of VIX ETP issuers, however, the correlation between ETP replication demand and HF open interest is −0.817. Standing on the other side of VIX ETP demand appears to be hedge funds. Also of interest are the correlation between D and ETN, 0.711, and the correlation between AM and ETF, 0.890. Earlier in this section, we described the TFF categorizations of open interest. These categories are assigned by CFTC staff based on their historical knowledge of large traders and their activities. For the VX futures, VIX ETN replication demand is assigned to the dealer category. Table 6 shows the correlation between ETN and D is 0.711, while it is only 0.535 for ETN and AM. Similarly, ETF replication demand is assigned the AM category. The correlation between ETF and AM is 0.890, while it is only 0.551 for ETF and D. The CFTC category assignments appear consistent with the VIX ETP types.

5.2.3 | Changepoint analysis

We now return to the evolving nature of the VIX ETP trading game. In Table 5, we showed that the sum of the *CAL* and *LEV* demand is a significant, positive predictor of the EOD VX futures return. The regression implicitly assumed, however, that the elasticity of VX futures price with respect to a change in demand was constant over the sample period. On the basis of the TFF analyses, we have reason to believe that this may not be the case. To test for this possibility, we break the sample into three separate subperiods. The two changepoints are chosen based on certain stylized facts regarding VIX ETP and VIX futures markets.

The first changepoint is April 17, 2012. The events underlying this decision were threefold. First, we know that, while the TAS market for VX futures was launched on November 4, 2011, the trading volumes of the nearby and second nearby VXT futures did not exceed 2000 contracts a day until April 2012. See Figure 4. Second, while ProShares launched trading in their levered and inverse VIX ETFs in November 2011, their AUM did not gain traction until April 2012. See Figure 9. Third, we know from the TFF report data that the number of larger hedge fund traders who shorted VX futures contracts to provide market liquidity for VIX ETP issuers increased from 17 on November 8, 2011 to 48 on April 17, 2012 (about 182%). On the basis of this confluence of events, mid-April 2012 seemed a reasonable choice.

The second changepoint is April 21, 2016. Here, we relied on Figures 2 and 8, which show that hedge fund participation ebbs and flows, depending upon market demand for inverse VIX ETP products. In Figure 2, for example, the demand for direct VIX ETPs (1*x* and 2*x*) products completely dominates inverse products on days when VIX ETP AUM meets VIX futures notional (i.e., the gray line and the gold line meet). In those days, there is no natural hedge between the demand for direct and inverse products, and the counterparties to the replication demands are hedge funds. As we move right from such a point and see the gold line falling relative to the gray line, the replication demand of the inverse products begins to naturally offset the replication demand of the direct products, thereby reducing the need for the liquidity provision by hedge funds. Indeed, when the gold line reaches 0, as it did several times in the latter half of the sample, VIX ETPs are naturally hedged. Figure 8 shows that VIX ETP replication demands and hedge fund open interest, and the same phenomenon appears on similar dates. The second changepoint is chosen from the dates on which replication demand begins to drop steadily over a prolonged period. On April 21, 2016, the AUM of VIX ETPs

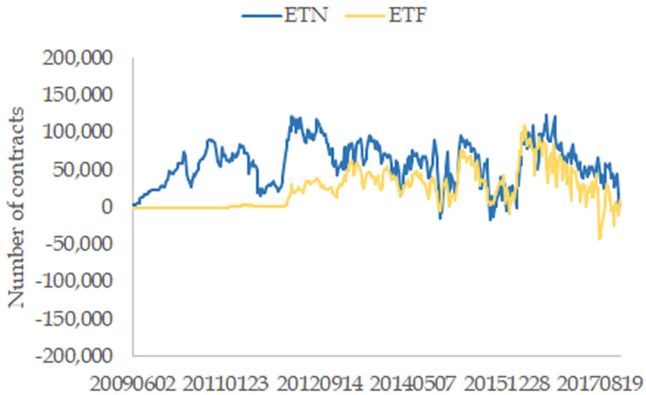

FIGURE 9 VIX ETN and ETF replication demand number of VX futures contracts assets during the period June 2009–December 2017. ETF, exchange-traded fund; ETN, exchange-traded note; VIX, volatility index.

was $4 billion. From there, it decreased steadily to less than $0 by August 22, 2017. Thus, we estimate Model 1 with two changepoints—20120417 and 20160421. The results are reported in Table 7.

In Table 7, we provide two sets of regression results: one for the nearby futures and one for the second nearby. Each set has three regressions. The first is Model 1 with no changepoint points. The results are the same as those reported in Table 5 and are included only for comparison purposes. The second regression has a single changepoint. We estimate the model assuming the intercept and slope change on that day. The indicator variable is 0 for all days before 20120417, and 1 thereafter. The increment to the slope coefficient is −1.1915 and is significantly less than 0. This implies that the response rate of a unit increase in demand was 1.4107, and 0.2192 thereafter. One possible explanation for the drop is increased competition. By April 17, 2012, the market identified a new trading opportunity based on the daily replication demands of VIX ETPs. Additional liquidity providers (i.e., hedge funds) stepped in, and competition drove the price of liquidity down. Another possibility is that the advent of the TAS market made the VIX ETP game less risky. Where, beforehand, position unwinding involved market orders to buy or sell as near as possible to the close, afterward, liquidity providers could simply place an offsetting VXT order at 3:00 p.m. when entering the VX trade to open the short-term profit trade. The third regression has two changepoints. With the additional break at 20160421, the response rate is further reduced to a level of 0.0573. As the demand for inverse VIX ETPs increased after 20160421, the issuers, themselves, became their own counterparties. By mid-August 2017, the nondiscretionary demand from VIX ETP issuers was near 0, and the price of liquidity was not different from 0 in a statistical sense. From an economic perspective, the case is even more compelling. The expected return for the nearby futures over the 15-min interval is −0.027% on an average day during the third subperiod. The results for the second nearby contract are qualitatively similar. The EOD return response rate to incremental replication demand decreases monotonically through time, from 1.2225 before 20120417, to 0.4620 from 20120417 to 20160421, to 0.1282 thereafter. While the coefficients are significant in a statistical sense, the economic effects of nondiscretionary futures demand by VIX ETP issuers are slowly disappearing.

5.3 | Assessing the robustness of the results

The implications of the results of Table 7 are straightforward. The VIX futures market and its various actors appear to have adapted to the replication demands of VIX ETP issuers in an orderly manner through time. In the analysis, however, we made several assumptions. We now examine the robustness of the two-changepoint estimation results in four different ways. The results are reported in Table 8. For ease of interpretation, the two-changepoint results of Table 7 are repeated in Panel A in Table 8.

5.3.1 | Changepoint selection

Our first robustness test focused on our choice of changepoints. While these are well-reasoned, they are subjective. They are based on anecdotal evidence regarding the trading demands of VIX ETP issuers and the open interest and number of traders in the short hedge fund category of the VX futures contracts reported in the CFTC's TFF reports. An alternative to our approach

TABLE 7 Results of regressions of end-of-day VIX return on VIX ETP futures replication demand for the period November 29, 2010–December 29, 2017 ($n = 1784$ days) with breaks in time-series

			Parameter estimates/t ratios						Response rates by # of breaks		
Contract	**Number of breaks**	**Adjusted R^2**	β_0^0	β_1^0	β_0^1	β_1^1	β_0^2	β_1^2	**0**	**1**	**2**
Nearby	0	0.0178	0.0013	0.2595					0.2595		
			4.03	5.77					5.77		
	1	0.0285	0.0048	1.4107	−0.0038	−1.1915			1.4107	0.2192	
			5.25	5.59	−3.85	−4.64			5.59	4.55	
	2	0.0379	0.0048	1.4107	−0.0026	−1.0443	−0.0033	−0.3090	1.4107	0.3663	0.0573
			5.28	5.62	−2.55	−4.02	−4.38	−3.35	5.62	5.21	0.90
Second	0	0.0409	−0.0006	0.3364					0.3364		
			−2.5500	8.77					8.77		
	1	0.0493	−0.0035	1.2225	0.0031	−0.9175			1.2225	0.3050	
			−4.5000	5.64	3.71	−4.17			5.64	7.50	
	2	0.0592	−0.0035	1.2225	0.0026	−0.7605	0.0007	−0.3338	1.2225	0.4620	0.1282
			−4.5300	5.67	3.06	−3.42	1.28	−4.29	5.67	8.03	2.40

Note: End-of-day VX futures return $R_{F,t}$ is defined as the VIX futures return from the first trade after 3:00 p.m. until VIX futures settlement at 3:15 p.m. Demands in Panel A are in thousands of VX futures contracts. Calendar roll demand arises from rebalancing the nearby and second nearby futures positions each day to maintain a constant 1-month time to expiration. Leverage demand arises from adjusting the degree of leverage to ensure the ETP delivers the promised levered return on the next day. Leverage demand is measured as of 3:00 p.m. each day based on the SPVXSP benchmark index. The two breaks are set at 20120417 and 20160421. Response rates are the sum of the estimated slope coefficients.

$$\text{No breaks:}\quad R_{F,t} = \beta_0^0 + \beta_1^0\left(n_{F,t}^{CAL} + n_{F,t}^{LEV}\right) + \varepsilon_t,$$

$$N\text{ breaks:}\quad R_{F,t} = \beta_0^0 + \beta_1^0\left(n_{F,t}^{CAL} + n_{F,t}^{LEV}\right) + \sum_{i=1}^{N} I_{t\geq NBRK_i}\left[\beta_0^i + \beta_1^i\left(n_{F,t}^{CAL} + n_{F,t}^{LEV}\right)\right] + \varepsilon_t.$$

Abbreviations: ETP, exchange-traded product; VIX, volatility index.

is to allow the time-series data to identify the changepoints statistically using the changepoint analysis of Andrews et al. (1996, hereafter ALP). In applying the technique, the user specifies the number of changepoints, and the time-series of data identify when the changepoints occur. The relevant time-series are the net daily replication demands of VIX ETPs (demand) and the net hedge fund open interest (supply). Absolute EOD returns should be higher when hedge funds must step in. Setting the number of changepoints to 2, the ALP procedure identifies the changepoints as 20140805 and 20160412. To gain some insight into why or why not these dates are reasonable, recall Figure 8. On 20140805, the net open interest of hedge funds is near zero. On 20160412, the net open interest of hedge futures is short more than 130,000 VIX futures contracts. From an intuitive standpoint, this does not seem to make sense, nonetheless, we use the ALP changepoints to redo the analysis of Table 8. The results are reported in Panel B of Table 8 for the two ALP changepoint specifications. The new changepoints result in a reduction in the explanatory power of the EOD returns for the nearby contract, and an increase for the second nearby. The most important result is, however, that interpretation of the response rates is the same as they were for the preset changepoints, even though the intervention dates are different. The response rates for both the nearby and second nearby contracts are monotonically decreasing. By the last subperiod, the rates are 0.0618 and 0.1196, respectively, for the nearby and second nearby contracts, both insignificant from an economic perspective. Only the second nearby rate is significantly different from 0.

5.3.2 | Return window

The second robustness test focuses on the 15-min EOD return window as the dependent variable. In essence, what this window tries to capture is the price of liquidity. In this case, the liquidity begins to build inventory at the beginning of

TABLE 8 Results of regressions of end-of-day VIX return on VIX ETP futures replication demand for the period November 29, 2010–December 29, 2017 ($n = 1784$ days) with breaks in time-series

Contract	Adjusted R^2	Parameter estimates/t ratios β_0^0	β_1^0	β_0^1	β_1^1	β_0^2	β_1^2	Response rates by # of breaks 0	1	2
Panel A: Preset changepoints: 20120417 and 20160421. 3:00-3:15 EOD returns										
Nearby	0.0379	0.0048	1.4107	−0.0026	−1.0443	−0.0033	−0.309	1.4107	0.3663	0.0573
		5.28	5.62	−2.55	−4.02	−4.38	−3.35	5.62	5.21	0.90
Second	0.0592	−0.0035	1.2225	0.0026	−0.7605	0.0007	−0.3338	1.2225	0.4620	0.1282
		−4.53	5.67	3.06	−3.42	1.28	−4.29	5.67	8.03	2.40
Panel B: ALP changepoints: 20140805 and 20160412. 3:00-3:15 EOD returns										
Nearby	0.0315	0.0031	0.4939	−0.0017	−0.1028	−0.0026	−0.3293	0.4939	0.3910	0.0618
		5.10	4.43	−2.02	−0.76	−3.08	−3.28	4.43	4.45	0.97
Second	0.0619	−0.0022	0.8178	0.0009	−0.4055	0.0013	−0.2927	0.8178	0.4123	0.1196
		−4.4100	8.00	1.28	−3.42	1.94	−3.57	8.00	5.64	2.27
Panel C: Preset changepoints: 20120417 and 20160421. 2:45-3:15 EOD returns										
Nearby	0.0215	0.0061	1.724	−0.004	−1.3412	−0.0028	−0.3463	1.724	0.3828	0.0365
		4.77	4.90	−2.82	−3.68	−2.59	−2.62	4.90	3.84	0.44
Second	0.0449	−0.0042	1.5125	0.0029	−0.9475	0.0015	−0.4480	1.5125	0.5650	0.1170
		−3.98	5.12	2.48	−3.11	1.87	−4.21	5.12	7.09	1.81
Panel D: Preset changepoints: 20120417 and 20160421. 3:00-3:15 returns. LEV demand is at open										
Nearby	0.0006	0.0043	1.5644	−0.0066	−1.8491	0.0033	0.9177	1.5644	−0.2848	0.633
		1.03	1.28	−1.42	−1.45	0.96	1.80	1.28	−0.76	2.06
Second	0.0031	0.0033	−1.3566	−0.0013	0.7117	−0.0052	0.9386	−1.3566	−0.6450	0.2937
		1.22	−1.72	−0.43	0.86	−2.23	2.48	−1.72	−2.34	1.37
Panel E: Preset changepoints: 20120417 and 20160421. 3:00-3:15 returns. LEV demand is at settlement										
Nearby	−0.0019	−0.0552	−2.4487	0.001	1.5437	−0.0076	−1.4899	−2.4487	−0.905	−2.3949
		−2.34	−0.35	0.04	0.21	−0.39	−0.51	−0.35	−0.44	−1.17
Second	0.0046	0.0020	−1.3246	−0.0016	0.7764	−0.0041	0.7713	−1.3246	−0.5482	0.2231
		0.96	−2.17	−0.65	1.21	−2.24	2.62	−2.17	−2.74	1.24

Note: End-of-day VX futures return $R_{F,t}$ is defined as the VIX futures return from the first trade after 3:00 p.m. until VIX futures settlement at 3:15 p.m. Demands in Panel A are in thousands of VX futures contracts. Calendar roll demand arises from rebalancing the nearby and second nearby futures positions each day to maintain a constant 1-month time to expiration. Leverage demand arises from adjusting the degree of leverage to ensure the ETP delivers the promised levered return on the next day. Leverage demand is measured as of 3:00 p.m. each day based on the SPVXSP benchmark index. The two breaks are set at 20120407 and 20160421. Response rates are the sum of the estimated slope coefficients.

$$N = 2 \text{ breaks:} \quad R_{F,t} = \beta_0^0 + \beta_1^0\left(n_{F,t}^{CAL} + n_{F,t}^{LEV}\right) + \sum_{i=1}^{N} I_{t \geq NBRK_i}\left[\beta_0^i + \beta_1^i\left(n_{F,t}^{CAL} + n_{F,t}^{LEV}\right)\right] + \varepsilon_t.$$

Abbreviations: ALP, Andrews, Lee, and Ploberger; EOD, end-of-day; ETP, exchange-traded product; LEV, leverage; VIX, volatility index.

the interval (3:00 p.m.) which he plans to unwind at the close (3:15 p.m.) when VIX ETPs will rebalance their replication positions. In principle, there is no reason why 3:00 p.m. is ideal. It is simply when the VIX ETP market closes. While the replication needs are more uncertain, liquidity providers could have begun to take VX futures positions earlier in the day, say, 2:45 p.m. To assess the robustness of the results to this assumption, we rerun the regression using 2:45–3:15 p.m. VIX futures returns. *LEV* demand is measured as of 2:45 p.m. The results are reported in Panel C. The size and significance of the response rates are consistent with those in Panel C, and lead to the same conclusions—the price of liquidity became increasingly small over the sample period. The adjusted R^2 levels are lower in Panel C than in Panel A. Apparently, playing the VIX ETP game is safer if played over the last 15 min of the day.

The third robustness test follows along the same line, extending the window to the maximum possible. Specifically, it assumes the replication trades occur at the beginning of the day at the opening prices of the nearby and second nearby contracts. As noted earlier, the *CAL* demand is fully known as of the settlement on the previous day. The *LEV* demand at the open, however, is based only on the overnight return, and contains no information about what lies ahead in the trading day. The results are reported in Panel D of Table 8. The explanatory power of the preset two-changepoint model drops to near 0, and makes little sense. Intraday futures index return, and its effect on leverage demand, is too uncertain to gauge the direction of the day's price movements in the VIX futures contracts.

5.3.3 | Return reversal

The final robustness test takes a different tact. Suppose replication trading takes place right up until the time of settlement at 3:15 p.m. Both *CAL* demand and *LEV* demand are known precisely. EOD return is no longer an option as the dependent variable since the trading day is over. Overnight return must be used. Here, we define overnight return as the return on the VX futures from the settlement at 3:15 p.m. on day t until open at 8:30 a.m. on day $t + 1$. In our investigation period, the "overnight" return has two different regimes. From November 29, 2010 until October 25, 2013, the VIX futures market had similar trading hours as the stock market. RTHs in the VIX futures market were, and continue to be, 8:30 a.m.–3:15 p.m. each business day. The return reversal period is, therefore, from 3:15 p.m. until 8:30 a.m. on the following morning (17¼ h).[25] On October 28, 2013, the CFE introduced an extended trading hour session from 3:30–4:15 p.m.[26] While this reduced the reversal period to 15 min, trading volumes were meager in the after-hours session and did not resume in force until the following RTH opening. The regression results from the two-changepoint regression using reversal returns are reported in Panel E of Table 8. The results are weak. The adjusted R^2 level is near 0 for both futures contracts, and the response rates are generally insignificant, and, in some cases, the wrong sign.

6 | CONTRACT DESIGN AND MARKET INTEGRITY

The well-defined VIX futures replication strategy underlying the VIX ETP structure made it ideal for examining informationless, nondiscretionary trading. To this end, we chose a sample consisting of all ETPs the six benchmarked to the VIX short-term futures index in a period during which there were few extraneous events to undermine the investigation. On the basis of an 8-year sample period ending December 2017, our results supported the hypothesis that liquidity provision improved through time as the VIX ETP market as investors learned about the trading opportunity. In the ensuing years, ETP markets experienced disarray for a variety of reasons including poor product design, waning market integrity, and sheer incompetence. While not germane to the purpose of our analysis, it is important to place these product-related experiences (or, equivalently, costs of financial innovation) in perspective.

The first has been dubbed "Volmageddon." On February 5, 2018, VIX spiked upward by a whopping 115%, the largest single-day move in its history. Naturally, XIV and SVXY, the two −1*x* VIX ETPs traded at the time, should have experienced extreme losses. They did not. Credit Suisse's XIV, for example, opened at about $110 in the morning and closed at 4:00 p.m. at $99, a drop of a mere 10%. As it turns out, the reported levels of VIX short-term futures index upon which VIX ETPs enact their replication strategies were wrong due to an "Auto Hold" feature that S&P's used in its index computation. Under an Auto Hold, the computation of the index (a tick) is stopped when it moves from its previous tick by more than predetermined percentage threshold level.[27] The index continues to be disseminated at 5-s intervals, however, albeit at the same level. From the Securities Exchange Commission (SEC, 2021, p. 2) order,

[25]While RTHs remain continued to start at 8:30 a.m., the CFE introduced extended hours trading from 7:20 a.m. to 8:30 a.m. (reducing the reversal period from 17¼ to 15.92 h) on December 10, 2010, and then from 7:00 to 8:30 a.m. (15¼ h) on September 26, 2011. While we incorporate these earlier opening times in the reversal analyses in this section, we refer to the sample period as having only two reversal regimes.
[26]From RG13-031 (20131013).
[27]The threshold levels are determined by an S&P index committee. Details of this incident and its consequences are provided in SEC (2021).

"… investors did not know that they have been purchasing and/or holding a product that had an economic value that was substantially less than what XIV's calculation agent had publicly reported and that was at risk of being accelerated by its issuer." On February 6, Credit Suisse exercised its right to accelerate, or call, all outstanding XIV notes, causing investors to lose 96% over a single day. Lawsuits against Credit Suisse claiming fraud were dismissed.[28] Without admitting or denying the SEC findings, S&P paid a $9 million civil penalty. The factors involved in this unique event include product design (choice of benchmark and calculation agent), market integrity (ability to deliver what was promised), and managerial competence (more immediate reaction to the triggered Auto Hold).

The extreme volatility shown during Volmageddon also had delayed effects. After the market close on February 26, 2018, ProShares unexpectedly announced that they were lowering the leverage ratios of UVXY from $2x$ to $1.5x$ and SVXY from $-1x$ to $-0.5x$ effective as of the close on business on February 27. This deleveraging had two important consequences. First, UVXY and SVXY shareholders had the character of their investments changed unalterably. They were now holding securities with 25% and 50% less risk, not by choice, but by fiat. Second, and more dramatically perhaps, UVXY and SVXY long option holders suffered massive windfall losses because the Options Clearing Corporation (OCC) made no adjustments to the terms of the outstanding UVXY and SVXY option contracts. Tengulov and Whaley (2020) estimated that more than $100 million was transferred from long option holders to short option holders from the OCC's (in)action. Again, the issues of product design and market integrity are at play, but perhaps at a more subtle level. The terms of the prospectus allowed the issuer to change the leverage ratio, but were investors aware of this possibility?[29] Was it reasonable to expect that the OCC would make no adjustment to the terms of outstanding options when such a fundamental change in the underlying security had occurred?

Levered and inverse ETPs have also come under fire. The issue here is largely one of market integrity. There is ample evidence suggesting that these products are not well understood by a large portion of the investment community. Otherwise, AUM would disappear. Cheng and Madhavan (2009) showed theoretically that, under plausible assumptions regarding the return/risk of the underlying asset, the long-term expected values are 0. Pessina and Whaley (2021) use Monte Carlo simulation to illustrate these predictions using return/risk properties derived from the historical behaviors of a variety of asset classes including stocks, bonds, commodities, and volatility. They show clearly that these products should not be considered as long-term investment opportunities. Indeed, for many products, their expected lives are a matter of years not decades. The problem is particularly egregious for commodity futures-based products in which futures prices are persistently in contango. Equilibrium expected benchmark returns are negative because the demands of long hedgers often exceed those of short hedgers. Speculators step in to absorb the hedging imbalance only when the futures price is high enough to earn a satisfactory risk premium. Levering these futures-based benchmarks merely exacerbates the performance problem and accelerates a fund's demise. Is the fact that their expected values of these ETPs are zero hidden? No. Product issuers say so in their prospectuses. The anomaly is that many of these products continue to have levels of AUM in the billions of dollars. Why? Many investors are attracted by the leverage and oblivious to expected outcomes. In November 2019, the SEC proposed requiring broker-dealers and investment advisors to vet individual investors before approving them to trade the products. In October 2020, the SEC shelved the measure explaining that more than 6000 comment letters had been received from people concerned about losing access to the funds.[30]

Closely intertwined with the levered and inverse fund controversy is the ETN versus ETF product structure. As noted earlier, the distinction between VIX ETNs and VIX ETFs is not critical since both products either implicitly or explicitly rely on collateralized futures positions to generate benchmark index returns. The primary difference between ETNs and ETFs is that ETFs fall under the Investment Act of 1940 are required to report daily futures holdings. ETNs, on the other hand, fall under the Securities Exchange Act of 1934 are merely promissory notes backed by the issuer. The means by which the issuer generates the promised benchmark return need not be reported. This difference in transparency, together with an increased awareness of the potential effects of leverage, led to a rash of recent delistings. In 2020, Credit Suisse delisted TVIX, its popular $2x$ ETN on the VIX short-term futures index (and part of our sample), together with nine other ETNs on volatility, gold and silver, and natural gas.[31] Interestingly, with both Credit Suisse's −1x (XIV) and 2x (TVIX) ETNs on the VIX short-term futures index gone, two new products with the same leverage

[28] See Stempel (2019).
[29] ProShare's decision to cut leverage ratios was followed by Direxion in March 2020 when it cut the leverage ratios of 10 of its $3x$ ETFs to $2x$. See Ballentine (2020).
[30] See Keinan (2020).
[31] See Greifeld (2020).

ratios, UVIX (2*x*) and SVIX (−1*x*), were launched by Volatility Shares LLC in March 2022 as replacements, but this time using an ETF wrapper.

Finally, the growing disenchantment with ETNs was further evidenced by the publicity surrounding a management fiasco involving two of their popular ETNs in early 2022. In what has been uncharitably called a "blunder," Barclay's Bank had to suspend the issuance of VXX and OIL on March 14, 2022 because it had sold millions of dollars more than it had permission for. The suspension of share issuance meant undermined the arbitrage between the ETN and futures prices, and ETN prices soared in relation to futures-based indicative values. In the weeks thereafter, Barclay's discovered that their mismanagement was much more pervasive than originally thought, forcing the bank to suspend sales in another 30 ETNs.[32]

All of this is to say, financial innovation does not come without cost. As new products emerge, the market takes time to learn and develop. As familiarity with new product markets grow, so does the commitment of capital for liquidity provision. We document this behavior for VIX ETPs during the first 8 years of their existence. Recently, several isolated incidents identifying problems with ETP product design, market integrity, and mismanagement have occurred. These are the growing pains (costs) of financial innovation.

7 | SUMMARY AND CONCLUSIONS

Nondiscretionary trades are based on a known, mechanical trading rule which specifies the size and the timing of the trades. They reflect no new private information and/or no judgment. Perhaps the best-known example of nondiscretionary trading involves revisions to the S&P 500 index. When S&P's announces that a new stock will be added to the index, the market knows that index funds will go about acquiring a fixed percent of the firm's outstanding shares within a specified period of time. The research question associated with such trading activity is "How does the market react?" Duffie (2010) contends that the market's ability to absorb these trades depends on the supply of capital, and that the supply of capital increases through time as investors learn about the trading opportunity. With increased supply comes reduced price impact and quicker price reversals. In the case of S&P 500 revisions, stock market reactions are consistent with Duffie's predictions. In this study, we identify a new research setting, equally, if not more, ideal than the stock index revision studies. VIX ETPs are created from collateralized VIX futures positions. Each day, VIX ETP issuers must rebalance their futures holdings as a result of the dynamic nature of the VIX futures index to which they are benchmarked and the leverage ratio that the fund promises. We show how each of these daily futures demands can be specified precisely in terms of the numbers of contracts based on the dollars of AUM, the leverage ratio, the prices of the two nearby VIX futures, and the daily return of the VIX futures index.

The empirical analyses conducted in this study focus on the daily trading activity and inter- and intraday price behavior of the VIX ETP and VIX futures markets during the period January 2009–December 2017. The relation between dollar trading demand of VIX ETP issuers and dollar trading volume and open interest of VIX futures show that the two markets are closely intertwined. First, we find that the futures demand of VIX ETP issuers, as reflected by VIX ETP trading volume, is the major driver of the trading volume in VIX futures. The total daily dollar trading volume of VIX ETPs follows in lockstep with the total dollar volume of the two nearby VIX futures which comprise the benchmark index ever since VIX ETP inception. Second, we find that the daily calendar demand generally exceeds the re-leveraging demand by about 2–1. At the same time, daily leverage demand by VIX ETP issuers on large market movement days can sometimes be 4–5 times higher than calendar roll demand. Note that leverage demand is not known until the end of the trading day when the daily return of the futures index is known. This implies that both the calendar roll and leverage orders should be placed near settlement since the two types of demands may offsetting, thereby reducing trading costs which are generally quoted on a per contract basis. Third, the sum of the calendar roll and leverage trading demands, that is, the "net hedging demand," is about two-thirds of the change in open interest from 1 day to the next on average, indicating that VIX ETPs have an important presence in the trading of VIX futures contracts and that the VIX futures market is large enough to comfortably absorb the VIX ETP demands. Fourth, the advent of the VIX futures TAS market altered the way VIX ETPs implemented their replication strategies. Indeed, the VIX futures TAS market was created explicitly to help VIX ETP issuers. Since the TAS market allows the issuer to trade at the settlement price, the execution risk of trying to mimic the settlement price is eliminated. Fifth, TAS trading

[32]See Dakers (2022).

volume is concentrated in the last few minutes of the trading day, when the net trading demand becomes virtually fully known. VIX ETP issuers appear to manage the timing of their trades by minimizing trading costs. A comparison between VIX ETP issuer net daily replication demand and VIX futures trading volume and open interest highlights that the VIX futures market comfortably absorbs the demand by VIX ETPs on a typical day. We show the close integration of the VIX futures TAS market with the VIX ETP market—the TAS market being the "go-to" market for VIX ETP issuers for managing settlement price and volume risk.

The return analyses provide two key insights with respect to how trading demand affects EOD returns and how liquidity provision in the VIX futures market has evolved through time. First, regression analysis of EOD VIX futures returns on VIX futures calendar roll and leverage adjustment demands shows that, while both sources of demand have significantly positive predictive power for EOD returns, their relative effects are indistinguishable from one another. This stands to reason. While calendar demand is known early in the day, leverage demand is not fully revealed until the end of the day. In an effort to minimize trading costs, issuers delay their replication orders until the end of the day when net daily futures demands are known. Second, the elasticity of VIX futures prices to changes in VIX ETP demand has become smaller over time. Early in the sample period, the VIX ETP market and its attendant hedging needs were relatively unknown. VIX futures prices reacted strongly and significantly to the inordinate demands at the end of the day. As time passed, the awareness of the VIX ETP market in general and the VIX ETP trading game in particular grew. By the end of the sample period, the price of liquidity with respect to VIX ETP rebalancing had fallen to near 0 because of the increased competition in the provision of liquidity by hedge funds, as well as the offsetting demands from the increase in popularity of inverse products.

ACKNOWLEDGMENTS

Discussions with and/or comments by Jesse Blocher, Nick Bollen, Maneesh Deshpande, Cam Harvey, John Hiatt, Blair Hull, David LaValle, Peter Layton, Scott Mollner, Steve Oh, Neil Ramsey, Jacob Sagi, Berk Sensoy, Tom Smith, Bill Speth, Hans Stoll, and Damon Walvoord and research assistance by Kelly LaVoice are gratefully acknowledged. The Financial Markets Research Center at Vanderbilt University provided data support.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are from six sources: (1) Daily VIX ETP market data are from Bloomberg, along with daily market data on VIX futures indices SPVXSP and SPVXSTR and discount rates USB3TMA; (2) Intraday VIX ETP trade and quote data are from NYSE TAQ; (3) VIX futures Daily open-high-low-close prices, volume, and open interest data are from CBOE Futures Exchange (CFE) website; (4) Intraday VX futures data are from CBOE DataShop; (5) Intraday trade and quote data for VIX TAS futures (VXT) are from Thompson-Reuters; and (6) Commitment of Traders (COT) and Traders in Financial Futures (TFF) reports are from Commodity Futures Trading Commission (CFTC). Restrictions apply to the availability of data from Bloomberg, NYSE TAQ, CBOE DataShop, and Thompson-Reuters, which were used under license for this study, and hence are not available to share.

ORCID

Michael J. O'Neill http://orcid.org/0000-0002-2824-2883
Robert E. Whaley http://orcid.org/0000-0001-9456-7734

REFERENCES

Admati, A., & Pfleiderer, P. (1991). Sunshine trading and financial market equilibrium. *Review of Financial Studies*, *4*, 443–482.

Andrews, D., Lee, I., & Ploberger, W. (1996). Optimal changepoint tests for normal linear regression. *Journal of Econometrics*, *70*(1), 9–38.

Ball, R., & Brown, P. (1968). An empirical evaluation of accounting income numbers. *Journal of Accounting Research*, *6*, 159–178.

Ballentine, C. (2020), Direxion cuts leverage on 10 funds to 200%, close eight others. Bloomberg (Electronic version: March 23, 2020).

Beneish, M., & Whaley, R. (1996). An anatomy of the S&P game: The effects of changing the rules. *Journal of Finance*, *51*, 1909–1930.

Bessembinder, H., Carrion, A., Tuttle, L., & Venkataraman, K. (2016). Liquidity, resiliency, and market quality around predictable trades: Theory and evidence. *Journal of Financial Economics*, *121*, 142–166.

Black, F., & Scholes, M. (1973). The pricing of options and corporate liabilities. *Journal of Political Economy*, *81*, 637–659.

Chen, Y. L., & Wang, J. J. (2021). Trader positions in VIX futures. *Journal of Empirical Finance*, *61*, 1–17.

Cheng, M., & Madhavan, A. (2009). The dynamics of levered and inverse exchange-traded funds. *Journal of Investment Management*, *7*, 1–20.

Daily data for VIX Exchange Traded Products (ETPs), VIX futures indices SPVXSP and SPVXSTR, and discount rates USB3TMA; August 21, 2006–December 29; not publicly available; data can be obtained from a Bloomberg terminal on payment of a subscription fee; https://bba.bloomberg.net/

Daily data for VIX futures open-high-low-close prices, volume, and open interest reported by CBOE Futures Exchange (CFE); 2009–2017; https://www.cboe.com/us/futures/

Daker, M. (2022). Barclays halts sales of 30 more ETNs after paperwork blunder. Bloomberg (Electronic edition: April 28, 2022).

Duffie, D. (2010). Presidential address: Asset price dynamics with slow-moving capital. *Journal of Finance*, *65*, 1237–1267.

Fama, E., Fisher, L., Jensen, M., & Roll, R. (1969). The adjustment of stock prices to new information. *International Economic Review*, *10*, 1–21.

Greifeld, K. (2020). Credit Suisse is delisting an ETN that's made 200% this year. The Wall Street Journal (Electronic version: June 22, 2020).

Henderson, B., Pearson, N., & Wang, L. (2015). New evidence of the financialization of commodity markets. *Review of Financial Studies*, *28*, 1285–1311.

Huskaj, B., & Norden, L. (2015). Two order books are better than one? Trading at settlement (TAS) in VIX futures. *Journal of Futures Markets*, *35*, 506–215.

Intraday VIX Exchange Traded Products (ETPs) trade and bid/ask quote data; January 29, 2009–December 29, 2017; not publicly available; data can be obtained from NYSE Daily TAQ database for a subscription fee; https://www.nyse.com/market-data/historical/daily-taq/

Intraday VIX futures trade and quote data; August 21, 2006–December 29, 2017; not publicly available; data can be obtained from CBOE DataShop for a subscription fee; https://datashop.cboe.com/

Intraday VIX TAS futures (VXT) trade and quote data; November 4, 2011–December 29, 2017; not publicly available; data can be obtained from Thompson-Reuters for a subscription fee; https://www.thomsonreuters.com/

Keinan, P. (2020). SEC shelves measure to protect investors in leveraged ETFs. The Wall Street Journal (Electronic version: October 28, 2020).

Merton, R. (1973). The theory of rational option pricing. *Bell Journal of Economics and Management Science*, *4*, 141–183.

Miller, M., & Modigliani, F. (1961). Dividend policy, growth and the valuation of shares. *Journal of Business*, *34*, 411–433.

Modigliani, F., & Miller, M. (1958). The cost of capital, corporation finance, and the theory of investment. *American Economic Review*, *48*, 261–297.

Patel, N., & Welch, I. (2017). Extended stock returns in response to S&P 500 index changes. *Review of Asset Pricing Studies*, *7*, 172–208.

Pessina, C. J., & Whaley, R. E. (2021). Levered and inverse funds: Blessing or curse? *Financial Analysts Journal*, *77*, 10–29.

Ross, S. (1976). The arbitrage theory of capital asset pricing. *Journal of Economic Theory*, *13*, 341–360.

Securities Exchange Commission. (2021). *Order instituting cease and desist proceedings, pursuant to Section 8A of the Securities Act of 1933, making findings, and imposing a cease-and-desist order*. File No. 3-20310/May 17, 2021.

S&P Dow Jones Indices. (2018). *S&P 500 VIX futures indices: Methodology*. S&P Dow Jones Indices LLC, a division of S&P Global.

Shleifer, A. (1986). Do demand curves for stocks slope down? *Journal of Finance*, *41*, 579–590.

Stempel, J. (2019). *Credit Suisse defeats lawsuits over huge U.S. 'volatility' crash*. Reuters. (Electronic version: September 25, 2019).

Tengulov, A., & Whaley, R. E. (2020). Levered and inverse VIX ETF option contract adjustment: No harm, no foul? *Accounting and Finance*, *60*, 3253–3277.

Weekly data categorizing open interest in U.S. futures markets into four sub-categories reported by Commodity Futures Trading Commission (CFTC) in its Traders in Financial Futures (TFF) report; June 2, 2009–December 26, 2017; Whaley, R. (2013). Volatility trading: At what cost? *Journal of Portfolio Management*, *40*, 95–108. https://www.cftc.gov/MarketReports/CommitmentsofTraders/

Weekly data categorizing positions of "large traders" in U.S. futures markets reported by Commodity Futures Trading Commission (CFTC) in its Commitment of Traders (COT) report; June 2, 2009–December 26, 2017; https://www.cftc.gov/MarketReports/CommitmentsofTraders/

How to cite this article: O'Neill, M. J., & Whaley, R. E. (2023). Effects of nondiscretionary trading on futures prices. *The Journal of Futures Markets*, 43, 33–68. https://doi.org/10.1002/fut.22377

APPENDIX A: CONSTRUCTION AND INTERPRETATION OF THE S&P 500 VIX SHORT-TERM FUTURES INDEX

The S&P 500 VIX Short-Term Futures Index is computed and disseminated by S&P's. It has a base level of 100,000 on the day it was first reported, December 20, 2005. The excess return version of the index (Bloomberg symbol SPVXSP) simulates the realized return from a long nearby/long second nearby VIX futures position that is rebalanced each day to maintain constant 1-month time to expiration. The total return version of the index (Bloomberg symbol SPVXSTR) includes interest on the notional value of the index. The interest accrues based on the 3-month US Treasury rate (Bloomberg symbol USB3TMA). Because the index is the key benchmark index used for VIX ETPs, the mechanics of the index composition are critically important. The purpose of this appendix is to provide these numerical mechanics, and, as result, an intuitive understanding of the index movements through time.

The appendix is divided into four parts. In the first, we describe how S&P sets the weights of the two nearby VIX contracts each day. In the second, we show the simplest way to compute the index level. The third part also performs the index calculation, but in a manner that develops an economic intuition for the daily erosion of index value from the persistent contango that appears in the VIX futures market. The final section contains a summary of SPVXSP behavior through time.

A.1 | Contract weights

The first step in creating the index is identifying the investment weights that are applied to the two nearby VIX futures contract prices to generate a constant 1 month to maturity. Unlike most other constant maturity indexes, the SPVXSP and SPVXSTR are based on trading days rather than calendar days. Since trade dates are used, holidays are implicitly excluded. Table A1 illustrates the computation of the contract weights. In the first column, trade dates beginning 20051220 are listed. Recall that 20051220 is the date on which S&P's first began reporting SPVXSP and SPVXSTR. Alongside the trade dates columns are two columns with the VIX futures expiration dates for the two nearby contracts. On 20051220, the two nearby contract expiration dates are 200501221 and 20060118.

The fourth column is an indicator variable that is coded “Yes” if the index futures position is fully rolled from one contract to the next on that day and is left blank otherwise. The roll date is the Tuesday before the contract expiration day. By convention, the expiration date of the monthly VIX futures contract is on Wednesday that is 30 days before the third Friday of the calendar month following the month in which the futures contract expires. The December 2005 VIX

TABLE A1 S&P 500 VIX Short-Term Futures Index weights

	Contract month		Roll	Number of trade days		Weights before rebalance	
Date	1	2	contract	Total	Remaining	1	2
20051220	20051221	20060118	Yes	24	1	0.041667	0.958333
20051221	20060118	20060215		17	17	1	0
20051222	20060118	20060215		17	16	0.941176	0.058824
20051223	20060118	20060215		17	15	0.882353	0.117647
20051227	20060118	20060215		17	14	0.823529	0.176471
20051228	20060118	20060215		17	13	0.764706	0.235294
20051229	20060118	20060215		17	12	0.705882	0.294118
20051230	20060118	20060215		17	11	0.647059	0.352941
20060103	20060118	20060215		17	10	0.588235	0.411765
20060104	20060118	20060215		17	9	0.529412	0.470588
20060105	20060118	20060215		17	8	0.470588	0.529412
20060106	20060118	20060215		17	7	0.411765	0.588235
20060109	20060118	20060215		17	6	0.352941	0.647059
20060110	20060118	20060215		17	5	0.294118	0.705882
20060111	20060118	20060215		17	4	0.235294	0.764706
20060112	20060118	20060215		17	3	0.176471	0.823529
20060113	20060118	20060215		17	2	0.117647	0.882353
20060117	20060118	20060215	Yes	17	1	0.058824	0.941176
20060118	20060215	20060322		20	20	1	0
20060119	20060215	20060322		20	19	0.95	0.05

Note: This table shows how the weights applied to the two nearby monthly VIX futures contracts are determined.

Abbreviations: S&P, Standard and Poor; VIX, volatility index.

futures expire on 20051221. The roll date is, therefore, 20051220, and the indicator variable says "Yes." The next two columns under the general heading "Number of trade days" are trade day counter variables. The "Total" column is the number of trading days between the current nearby contract's roll date, 20051220, and the previous roll date, 20051115,[33] in this case, 24. The "Remaining" column is the number of days remaining until the nearby contract rolls, in this case, 1. Thus, as of the beginning of the day on 20051220, the VIX futures index portfolio has a weight of $1/24 = 0.041667$ invested in the nearby 20051221 contract and $23/24 = 0.958333$ in the second nearby 20060118 contract. These are the weights used to compute the VIX futures index return as of the close of the day. At the instant of the close on 20051220, after the daily return has been computed, the position is rebalanced in preparation for the next day's trading. The 20051221 contract is dropped, and the new nearby futures become the 20060118 contract. Hence, on 20051221, the VIX futures index is fully invested in the nearby 20060118 contract, with nothing in the second nearby contract.

A.2 | Calculation of index return

With the daily index weights in hand, we now turn to computing the index return. Table A2 shows the computations each day. The trade dates are again reported in the first column, followed by the nearby and second nearby contract weights before rebalancing. The fourth and fifth columns are the closing settlement prices of the nearby and second nearby VIX futures, respectively, and the sixth and seventh columns are the weighted-average futures prices as of the close of day $t-1$ and day t based on the weights in that row. The weighted-average price as of the close on day $t-1$ is

$$1.0 \times 12.76 + 0.0 \times 14.53 = 12.76,$$

and the weighted-average price as of the close of day t is

$$1.0 \times 12.59 + 0.0 \times 14.33 = 12.59.$$

On the basis of these EOD levels, the marked-to-market index return for 20051221 is

$$12.59/12.76 - 1 \quad \text{or} \quad -1.3323\%.$$

Note that this return is *exactly* consistent with the reported S&P 500 VIX Short-Term Futures Index levels reported by Bloomberg. At the close on day t, SPVXSP had a level of 98,667.71, falling from a level of 100,000 on day $t-1$,

$$98,667.71/100,000 - 1 \quad \text{or} \quad -1.3323\%.$$

The futures contracts are rebalanced costlessly (since they have been marked-to-market) at the settlement prices[34] to maintain the constant 1-month maturity, with the weights changing from 1(0) to 0.941176(0.058824) for the first (second) nearby VIX futures contracts.

A.3 | Decomposition of index return

Lost in the marked-to-market computation of the index return is the ability to understand its components. Specifically, the daily return of SPVXSP is the sum of two components—calendar rebalances return and VIX futures market return. The easiest way to understand the calendar rebalance effect is to remember that the VIX ETP benchmark is fully collateralized. This means that buying a futures contract at price F costs F, the amount placed in T-bills. Suppose we begin on 20051220 in Table A2, the "roll date" of the 20051221 futures contract. Assume \$100,000 is invested in a 1x VIX ETP benchmarked to SPVXSP. The weighted-average settlement price or, equivalently, the value of each unit of investment is

$$0.041667 \times 11.15 + 0.958333 \times 12.76 = 12.692916,$$

[33]The November 2005 VIX futures contract expiration date was 20051116.

[34]The settlement price is computed as the average of the last bid and ask price quotes to appear during RTHs.

TABLE A2 Computation of S&P 500 VIX short-term index return

Date	Weights before rebalance		Futures settlement prices		Weighted-average futures prices at close on		Computed index return	Reported SPVXSP	Investment portfolio value		Rebalance return	Futures market return
	1	2	1	2	Day $t-1$	Day t			Before rebalance	After rebalance		
20051219			11.11	12.88								
20051220	0.041667	0.958333	11.15	12.76	12.806250	12.692917		100,000.00	100,000.00	99,474.27		
20051221	1	0	12.59	14.33	12.760000	12.590000	−0.013323	98,667.71	98,667.71	97,872.03	−0.005257	−0.008108
20051222	0.941176	0.058824	12.48	14.18	12.692354	12.580001	−0.008852	97,794.30	97,794.30	97,023.06	−0.008064	−0.000794
20051223	0.882353	0.117647	12.21	14.11	12.680000	12.433529	−0.019438	95,893.40	95,893.40	95,039.09	−0.007886	−0.011643
20051227	0.823529	0.176471	12.45	14.20	12.545295	12.758824	0.017021	97,525.57	97,525.57	96,745.02	−0.008909	0.026163
20051228	0.764706	0.235294	12.32	14.11	12.861765	12.741176	−0.009376	96,611.20	96,611.20	95,819.34	−0.008004	−0.001383
20051229	0.705882	0.294118	12.32	14.10	12.846471	12.843530	−0.000229	96,589.08	96,589.08	95,808.02	−0.008196	0.008033
20051230	0.647059	0.352941	12.45	14.09	12.948235	13.028823	0.006224	97,190.24	97,190.24	96,475.89	−0.008086	0.014427
20060103	0.588235	0.411765	12.02	13.88	13.125295	12.785883	−0.025859	94,676.96	94,676.96	93,873.67	−0.007350	−0.018646
20060104	0.529412	0.470588	11.84	13.84	12.895294	12.781176	−0.008850	93,839.11	93,839.11	92,983.22	−0.008485	−0.000368
20060105	0.470588	0.529412	11.83	13.73	12.898824	12.835883	−0.004880	93,381.22	93,381.22	92,575.15	−0.009121	0.004280
20060106	0.411765	0.588235	11.56	13.53	12.947647	12.718823	−0.017673	91,730.89	91,730.89	90,902.66	−0.008632	−0.009120
20060109	0.352941	0.647059	11.33	13.29	12.834706	12.598236	−0.018424	90,040.81	90,040.81	89,224.27	−0.009029	−0.009481
20060110	0.294118	0.705882	11.22	13.04	12.713529	12.504705	−0.016425	88,561.87	88,561.87	87,810.08	−0.009069	−0.007424
20060111	0.235294	0.764706	11.19	12.84	12.611765	12.451765	−0.012687	87,438.32	87,438.32	86,762.03	−0.008489	−0.004234
20060112	0.176471	0.823529	11.30	12.99	12.548823	12.691764	0.011391	88,434.31	88,434.31	87,747.00	−0.007734	0.019274
20060113	0.117647	0.882353	11.32	12.95	12.791177	12.758235	−0.002575	88,206.57				

Note: This table shows how the VIX futures index returns are computed using the index weights and two nearby VIX futures contract prices. SPVXSP is S&P's VIX short-term index level reported by Bloomberg. S&P set the index level equal to 100,000 on December 20, 2005. The final four columns represent the implicit investment portfolio return dynamics taking place within the construction of the SPVXSP.

Abbreviations: S&P, Standard and Poor; VIX, volatility index.

as is shown in the table. Before the calendar rebalance, 100,000/12.692916 units of the VIX ETP are held. But, the EOD rebalancing must occur by index design. The portfolio weights are adjusted to 0 in the 20051221 contract and 1 in the 20060118 contract, and a new weighted-average closing price is computed, that is,

$$1.0 \times 12.76 + 0.0 \times 14.53 = 12.760000.$$

This instantaneous increase in the weighted-average unit price is *not* attributable to market movement. All prices are EOD settlement prices on day t. No time has passed. Instead, it represents an instantaneous rebalancing gain or loss because the amount of collateral must be changed. The amount of the incremental collateral depends on whether the nearby futures price is above or below the second nearby futures price. In this case, it is below as is typical in the VIX futures markets where prices are persistently in contango. The nearby collateralized futures position is sold at 11.15 and the second nearby collateralized futures are purchased at 12.76. Since the purchase price exceeds the sales price, additional collateral is required. Assuming no new investment of capital, we must liquidate a portion of the holdings to pay for the rebalance costs. With \$100,000 invested at the close on 20051220,[35] the portfolio value instantaneously drops in value to

$$100{,}000 \times \left(\frac{12.692916}{12.76}\right) = 99{,}474.26$$

or by –0.5257% purely from the rebalancing activity. All these results are summarized in Table A2. The original \$100,000 investment value is shown in the "Before rebalance" column. After accounting for rebalancing costs, the investment is 99,474.26, as seen in the "After rebalance" column. The "Rebalance return" is –0.5257%, which is the first component of the reported index return on day $t + 1$. Another appropriate label might be "Term structure return."

On the basis of the after rebalance portfolio value, we move forward a day in time. From the close on day $t - 1$ to the close on day t, the weighted-average futures price (i.e., the price per share of the "units" we hold in the investment portfolio) falls from 12.692916 to 12.590000 or by –0.8108%. This would have been the index return had the portfolio weights not changed. In other words, it is the futures market return and is labeled as such in the table. Since we have 99,474.26 invested after rebalancing at the close on day $t - 1$, we have

$$99{,}474.26 \times (1 - 0.008108) = 98{,}667.71$$

invested on day t before rebalancing. The overall rate of return is

$$(1 - 0.005257) \times (1 - 0.008108) - 1 = -1.3323\%,$$

exactly the amount of the return we computed in a more parsimonious, albeit equivalent, fashion earlier.

The remaining rows of Table A2 are computed in a recursive manner. Note that, on 20051229, the futures market return is positive at 0.8033% and the amount of the rebalance return is negative at –0.8196%. The overall investment portfolio return is –0.0229%. Even though the futures market experienced a gain on 20051229, the short-term futures index experienced a loss because the negative rebalance return was greater than the market return. By way of contrast, on the following day, the futures market return is positive, the rebalance return is negative, and the overall return is positive. In the short time-series used to generate the mechanics in Table A2, the daily rebalance returns are all negative.

A.4 | Summary of SPVXSP history

To gain a better sense of the VIX short-term futures index returns, we gather (compute) historical daily data for (a) the total return of the S&P 500 stock index, SPTR, (b) the total return of the VIX cash index, VIXR, (c) the excess return, SPVXSP, and (d) the total return, SPVXSTR, of the S&P 500 VIX Short-Term Futures Index, (e) the difference between the total and excess returns, and (f) the 91-day T-bill return. The sample period is from the first date that Standard & Poor's began reporting VIX futures index levels, December 20, 2005–December 29, 2017. The total return index levels

[35]The choice of 100,000 is meant to mimic S&P arbitrarily setting the base level of the index to be 100,000 on December 20, 2005.

for the S&P 500 index and the VIX as well as the excess and total returns for the futures indexes and the 91-day T-bill rate were downloaded from Bloomberg.

Before examining the summary statistics for the different return series, a discussion about the specific nature of the excess and total return indexes and their relation to the 91-day T-bill rate is warranted. S&P's compute both the excess return and total return VIX Short-term Futures Indexes, and they are reported on Bloomberg as SPVXSP and SPVXSTR, respectively. The excess return of SPVXSP is computed as $ER_t = SPVXSP_t - SPVXSP_{t-1} - 1$. Likewise, the total return of SPVXSTR is computed as $TR_t = SPVXSTR_t / SPVXSTR_{t-1} - 1$. The difference between the returns, $TR_t - ER_t$, is the interest accrual or return on the notional value of the index based on the 91-day US Treasury bill. Approached differently, the daily 91-day T-bill return is defined as

$$TBR_t = \left(\frac{1}{1 - \frac{91}{360} \times TBAR_{t-1}} \right)^{\frac{\Delta_t}{91}} - 1,$$

where Δ_t is the number of days between the current and previous trading days, and $TBAR_{t-1}$ is the most recent weekly high discount rate for 91-day Treasury bills effective on the preceding business day.[36] Generally, the rates are announced by the US Treasury on each Monday. On Mondays that are bank holidays, Friday's rates apply. The 91-day Treasury bill discount rate, $TBAR_t$, is reported as the Bloomberg series USB3TMA. Thus, it should be the case that the two approaches should be equivalent, that is,

$$\frac{SPVXSTR_t}{SPVXSTR_{t-1}} = \frac{SPVXSP_t}{SPVXSP_{t-1}} + TBR_t. \tag{A1}$$

Table A3 contains summary statistics for the different return series over the 3027-day sample period December 20, 2005–December 29, 2017. The results in Panel A are interesting in a number of respects. First, the total holding period return of the S&P 500 is 173.81%. This translates to a compound annual growth rate of 8.75%. Over the same period, the VIX cash index produced a total holding period of −1.34%. This, too, is expected. Volatility follows a mean-reverting process, so changes in volatility should average to be about 0% over long periods of time. The total holding period returns for SPVXSP and SPVXSTR are −99.95% and −99.95%, respectively. As dramatic as these returns are, they are also expected. The VIX futures price curve is typically in contango. Across the days in the sample period, the slope of the term structure at 30 days to maturity is positive in 82.9% of the time (not reported in the table). The futures indexes fall as calendar rebalancing realizes its contango losses.

Second, the implied T-bill return, 13.69%, differs from the actual T-bill return, 13.65%. This arises from a subtle, but increasingly important, issue. At the beginning of the futures index series on 20051220, the excess return index SPVXSP is reported on Bloomberg to four decimal places while the total return index is reported to two decimal places. Since both indexes are at a level of 100,000 at the beginning of the series, the error in the computation of returns is not noticeable. But, with the precipitous declines in both indexes over the period, the index levels are now at levels well below 100, and the implications for the computation of benchmark returns are becoming more and more important. Consider, for example, the returns of the last day of the sample period. The excess return index SPVXSP was at 40.8995 on 20171228 and 41.7190 on 20171229, implying that the daily excess return for the VIX futures index was 2.0037%. The total return index SPVXSTR levels on the same days were 46.50 and 47.43, for a daily total return of 2.0000%. Note that the implied T-bill return is actually negative at −0.0037%. On the basis of the 91-day T-bill discount rate USB3TMA, the daily 91-day T-bill return was actually 0.0040%.

Third, the estimated volatility rates show the riskiness of the market volatility products. Where the S&P 500 index has an annualized total return volatility of 19.42% (about normal since the inception of the S&P 500 index), the volatility rate of the VIX cash index level is 115.82%, about six times higher than the stock index level. Clearly, the volatility of volatility is high. Oftentimes, buyers of 1*x* VIX ETPs consider them to be a proxy for investment in the VIX cash index level. This is not the case. We have already discussed the difference in return performance. The CAGR of VIX is −0.11%, while the CAGR of SPVXSP is −47.68%. A dramatic difference in volatility is also apparent. The return

[36]See S&P VIX Futures Indices: Methodology (2018, p. 8).

TABLE A3 Summary statistics for daily returns of selected stock market and market volatility indexes during the period December 20, 2005 and December 29, 2017

	SPTR	VIXR	SPVXSP	SPVXSTR	Difference	T-bill
Panel A: Summary statistics based on daily returns						
n	3027	3027	3027	3027	3027	3027
Mean (%)	0.033	0.000	−0.257	−0.253	0.004	0.004
Standard deviation (%)	1.224	7.296	3.988	3.988	0.009	0.008
Skewness	−0.35142	0.71611	0.62437	0.62323	3.06345	3.24719
Median (%)	0.071	−0.519	−0.583	−0.583	0.001	0.001
Minimum (%)	−9.460	−35.059	−20.885	−20.885	−0.012	0.000
Maximum (%)	10.958	49.601	28.299	28.298	0.068	0.069
Holding period return (%)	173.81	−1.34	−99.96	−99.95	13.69	13.65
CAGR (%)	8.75	−0.11	−47.68	−47.12	1.07	1.07
Volatility (%)	19.42	115.82	63.31	63.31	0.14	0.13
Panel B: Correlation matrix based on daily returns						
SPTR	1.000	−0.738	−0.747	−0.747	−0.005	−0.008
VIXR	−0.738	1.000	0.884	0.884	0.036	−0.003
SPVXSP	−0.747	0.884	1.000	1.000	0.001	0.018
SPVXSTR	−0.747	0.884	1.000	1.000	0.003	0.019
Difference	−0.005	0.036	0.001	0.003	1.000	0.563
T-bill	−0.008	−0.003	0.018	0.019	0.563	1.000

Note: Column headings are: (a) SPTR, the total return on S&P 500 index, (b) VIXR, the relative change of the VIX cash level, (c) SPVXSP and (d) SPVXSTR, the excess and total returns on the S&P 500 VIX Short-Term Futures Indexes, (e) the difference between the total and excess returns, and (f) the 91-day US Treasury bill return. Returns are the natural logarithm of the daily index level ratios. Holding period return is the realized rate of return over the entire period, and the compound annual growth rate (CAGR) is the holding period return expressed on an annualized basis. Volatility is defined as the daily return standard deviation times the square root of the number of trading days in a year.

volatility of VIX is 115.82% on an annualized basis, while the return volatility of the SPVXSP is a muted 63.31%, about half. Finally, the correlation between VIXR and the futures indexes is direct and strong, 0.884, but by no means perfect.

Fourth, the S&P 500 return distribution is negatively skewed, induced by infrequent but sizable negative price shocks. Conversely, VIX levels occasionally experience upwards jumps, and, hence, the returns of VIX, SPVXSP and SPVXSTR are positively skewed. The T-bill returns are strongly positively skewed reflecting the fact that interest rates were generally low during the sample period and could go below 0%.

Finally, the results in Panel B of Table A3 show the stock index returns and volatility returns are highly negatively correlated. The correlation of SPTR with VIXR, for example, is –0.738, and the correlation with SPVXSP is –0.747. Holding other factors constant, this suggests that a buy-and-hold investment in a direct VIX ETPs such as VXX or TVIX is a good way to diversify the risk of a well-diversified stock portfolio. This potential benefit, however, is strongly outweighed by the negative returns earned by holding direct VIX ETPs over long periods of time.