Option valuation

Binomial method

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Option valuation

- Context:
  - Many standard and nonstandard options do not have valuation formulas (e.g., American-style options).
  - To value option and measure risk, use approximation methods.
    - Binomial method
    - Trinomial method
    - Quadratic approximation method
    - Finite difference method

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Option valuation

- Purpose:
  - Develop and apply binomial method.
    - Simplest to understand.
    - Broad-ranging applications.

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Binomial method

- Illustration parameters:
  - Consider FX option.
    - Current exchange rate is 50.
    - Domestic interest rate is 5%.
    - Foreign interest rate is 2%.
    - Volatility rate is 20%.
  - Value American-style put option on foreign currency with 55 exercise price and 2-year time to expiration.

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Binomial method

- Step 1: Create asset price lattice.
  - Assume underlying asset price has discrete proportional jumps through option's life.
  - u (>1) is up-step coefficient and d (<1) is down-step coefficient, with $ud = 1$.

$$\begin{array}{lll} & & uuS \\ & uS & \\ S & & udS \\ & dS & \\ & & ddS \end{array}$$

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Binomial method

- Decide on length of time increment.

$$\begin{array}{lll} & & uuS \\ & uS & \\ S & & udS \\ & dS & \\ & & ddS \end{array}$$

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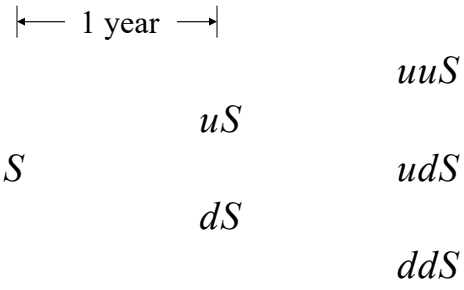
Binomial method

- Set time increment to one year.



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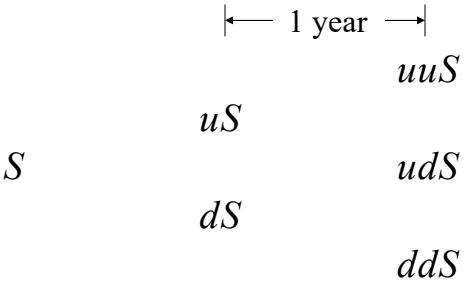
Binomial method

- Set time increment to one year.



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Binomial method

- Compute up-step and down-step coefficients. Up-step coefficient is defined as:

$$u = e^{\sigma\sqrt{\Delta t}}$$

User-defined time increment.

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Binomial method

- Compute up-step and down-step coefficients. Up-step coefficient is defined as:

$$u = e^{\sigma\sqrt{\Delta t}}$$

Standard deviation of asset return.

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Binomial method

- Compute up-step and down-step coefficients. Up-step coefficient is defined as:

$$u = e^{\sigma\sqrt{\Delta t}} \qquad d = \frac{1}{u}$$

Down-step coefficient is reciprocal of up-step coefficient, i.e., tree recombines.

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Binomial method

- Compute up-step and down-step coefficients.

$$u = e^{.20\sqrt{1}} = 1.2214$$

$$d = \frac{1}{1.2214} = .8187$$

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Binomial method

- Complete asset price lattice.
  - Possible asset price paths during option's life:

$$
\begin{array}{ccc}
 & & uuS \\
 & 1.2214(50) & \\
50 & & udS \\
 & .8187(50) & \\
 & & ddS
\end{array}
$$

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Binomial method

- Complete asset price lattice.
  - Possible asset price paths during option's life:

$$
\begin{array}{ccc}
 & & 74.59 \\
 & 61.07 & \\
50 & & 50 \\
 & 40.94 & \\
 & & 33.52
\end{array}
$$

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Binomial method

- Step 2: Value option at expiration.
  - American-style put with $X = 55$.

$$\max(0, 55 - 74.59) = \quad 0$$

$$\max(0, 55 - 50) = \quad 5$$

$$21.48$$

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Binomial method

- Value option at expiration.
  - American-style put with $X = 55$.

$$0$$

$$5$$

$$\max(0, 55 - 33.52) = \quad 21.48$$

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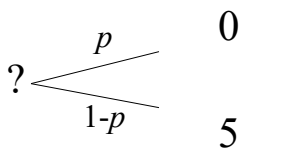
Binomial method

- Step 3: Value option one step back in time by taking present value of expected future value.



21.48

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Binomial method

- Probabilities are identified by equating expressions for expected asset price.

$$Se^{b\Delta t} = puS + (1-p)dS$$

Expected asset price based on expected rate of price appreciation and log-normal price distribution.

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Binomial method

- Probabilities are identified by equating expressions for expected asset price.

$$Se^{b\Delta t} = puS + (1-p)dS$$

Expected asset price based on binomial distribution.

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Binomial method

- Probabilities are identified by equating the expressions for expected asset price.

$$Se^{b\Delta t} = puS + (1-p)dS$$

$$\Rightarrow p = \frac{e^{b\Delta t} - d}{u - d}$$

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Binomial method

- Assume cost of carry rate is 5% - 2% = 3%.

$$e^{.03(1)} = 1.0305$$

$$p = \frac{1.0305 - .8187}{1.2214 - .8187} = .5258$$

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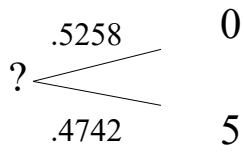
Binomial method

- Value option one step back in time by taking present value of expected future value.



21.48

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Binomial method

- Expected future value (*EFV*) is:

$$EFV = .5258(0) + .4742(5) = 2.37.$$

- Present value (*PV*) of *EFV* is:

$$PV = e^{-r\Delta t}(2.37) = e^{-.05(1)}(2.37) = 2.26.$$

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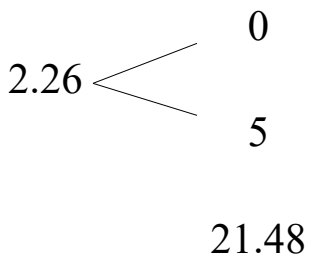
Binomial method

- Substitute value into lattice.



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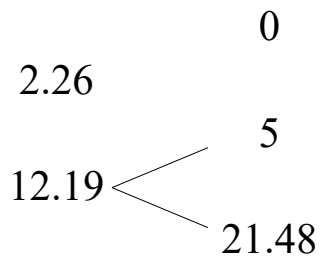
Binomial method

- Compute *PV* at lower node.



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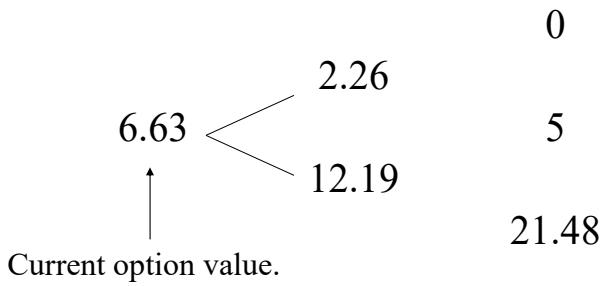
Binomial method

- Compute *PV* at lower node.



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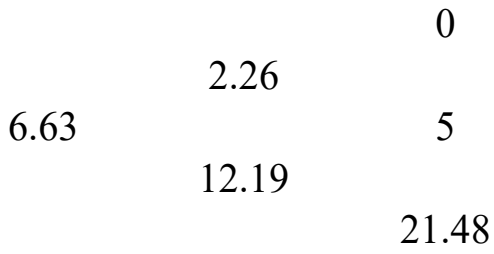
Binomial method

- Current value is for European-style put.
  - Did not account for early exercise.
  - BSM formula (correct) value is 6.40.



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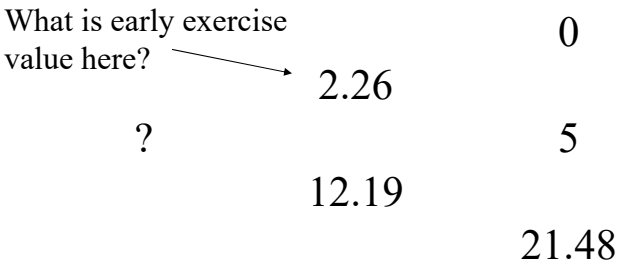
Binomial method

- Step 3a: Check if early exercise is optimal at each node.



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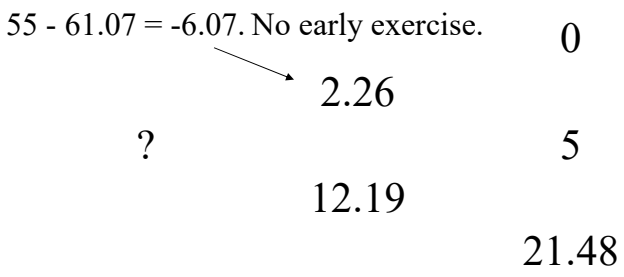
Binomial method

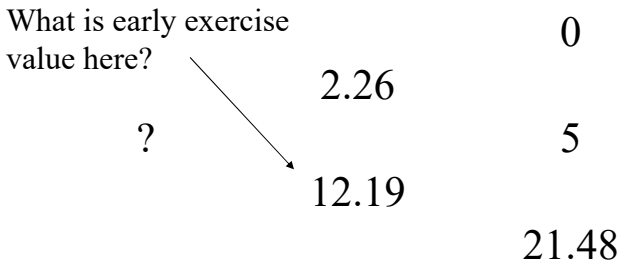
- Check if early exercise is optimal.



29

Binomial method

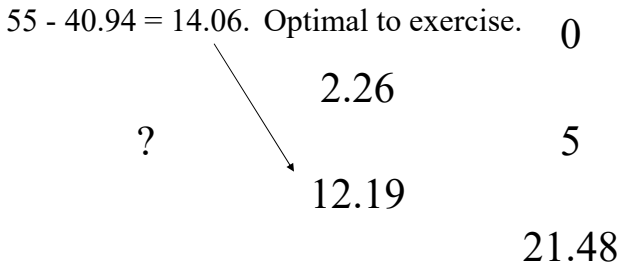
- Check if early exercise is optimal.



30

Binomial method

- Check if early exercise is optimal.



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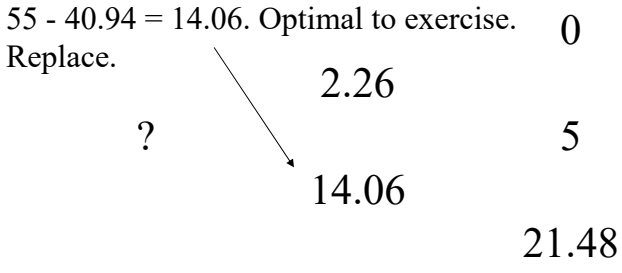
Binomial method

- Check if early exercise is optimal.



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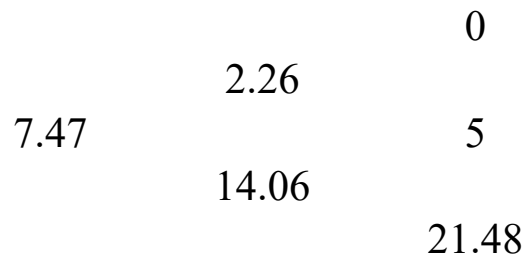
Binomial method

- Continue iterative process.



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Binomial method

- Value of early exercise premium is:

$$7.47 - 6.63 = .84$$

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Binomial method

- Illustration: Nonstandard option valuation.
  - Up-and-out put is put that expires worthless if asset price touches or crosses up-and-out level (i.e., the "knock-out" barrier) any time during option's life.
    - Also called barrier option.
    - Traded in OTC market.
  - Assume put option is American-style and has knock-out barrier of 60.
    - Ceases to exist if exchange rate hits 60.

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Binomial method

- To value up-and-out put simply modify valuation checks at each node.
  - Check for early condition as before.
  - Check "knock-out" condition.

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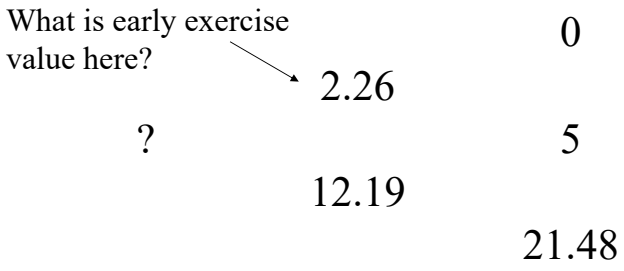
Binomial method

- Check if early exercise is optimal.



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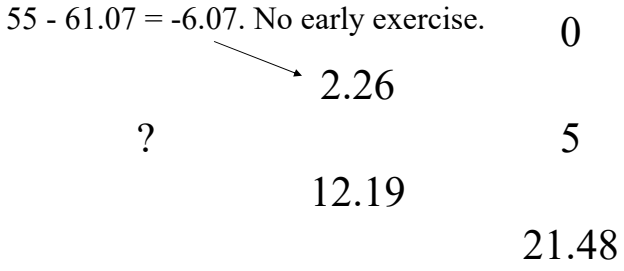
Binomial method

- Check if early exercise is optimal.



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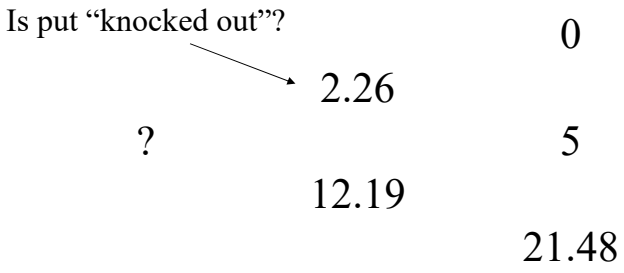
Binomial method

- Check if early exercise is optimal.



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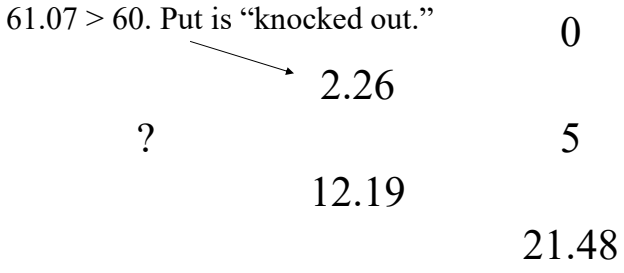
Binomial method

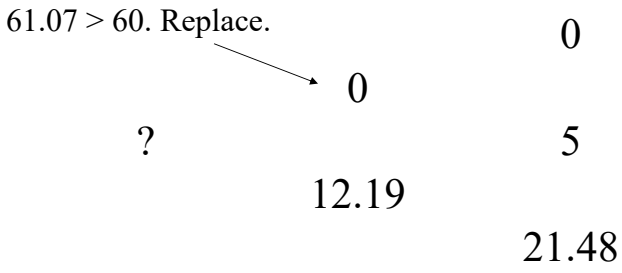
- Check if early exercise is optimal.



40

Binomial method

- Check if early exercise is optimal.



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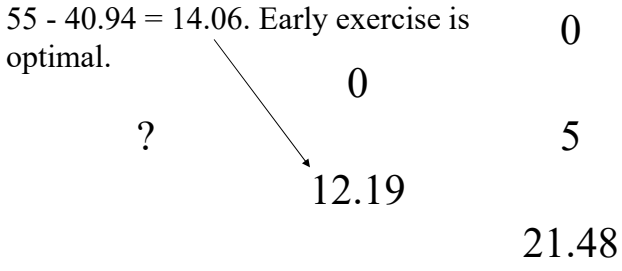
Binomial method

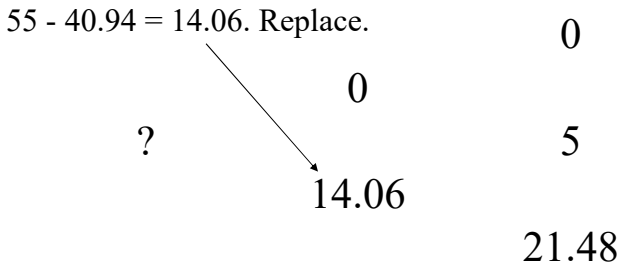
- Check if early exercise is optimal.



42

Binomial method

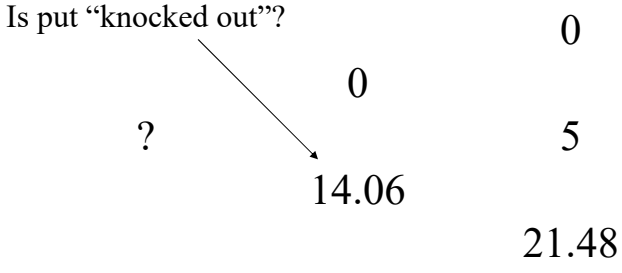
- Check if early exercise is optimal.



43

Binomial method

- Check if early exercise is optimal.



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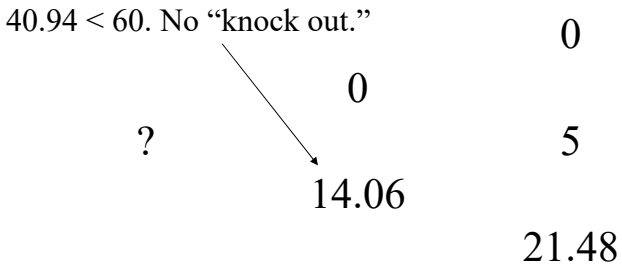
Binomial method

- Check if early exercise is optimal.



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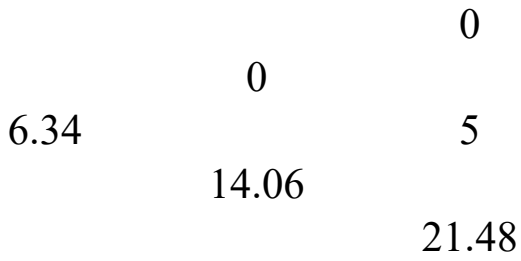
Binomial method

- Value of American-style, up-and-out put is:



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Binomial method

- Value of knock-out feature is:

$$7.47 - 6.34 = 1.13$$

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Binomial method

- Steps of binomial method.
  - 1. Create asset price lattice.
  - 2. Value option at expiration.
  - 3. Step back one time increment and take present value of expected future option value.
    - a. Check boundary conditions.

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Binomial method

- Coefficients and probabilities used in lattice are not unique.
  - Cox, Ross, and Rubinstein (1979) simplification (Method 3)

$$u = e^{\sigma\sqrt{\Delta t}} \qquad d = \frac{1}{u}$$

$$p = \frac{e^{b\Delta t} - d}{u - d}$$

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Binomial method

- Coefficients and probabilities used in lattice are not unique.
  - CRR (1979) correct version (Method 1)

$$u = e^{\sigma\sqrt{\Delta t}} \qquad d = \frac{1}{u}$$

$$p = \frac{1}{2} + \frac{1}{2}\left(\frac{b - .5\sigma^2}{\sigma}\right)\sqrt{\Delta t}$$

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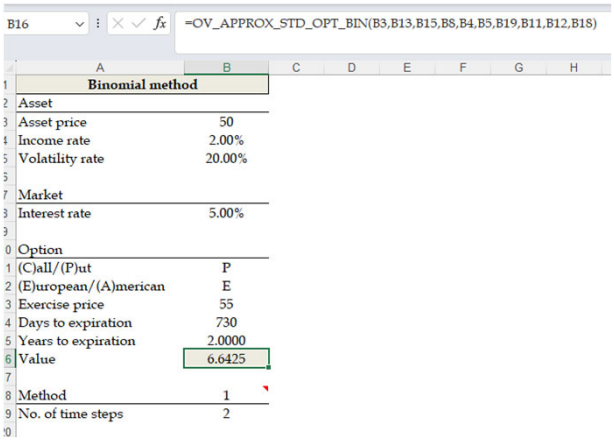
Binomial method

- OPTVAL function



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Binomial method

- Supporting file: Binomial method.xlsx

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Binomial method

- Coefficients and probabilities used in lattice are not unique.
  - JR (1983) (Method 2)

$$u = e^{\left(b-.5\sigma^2\right)\Delta t+\sigma\sqrt{\Delta t}} \qquad d = e^{\left(b-.5\sigma^2\right)\Delta t-\sigma\sqrt{\Delta t}}$$

$$p = \frac{1}{2}$$

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Binomial method

- Qualifications on use.
  - Cruder grid, greater error.
  - More refined grid, greater computational cost.
  - Flexible.
    - Easily adapted to handle other types of contingent claims.

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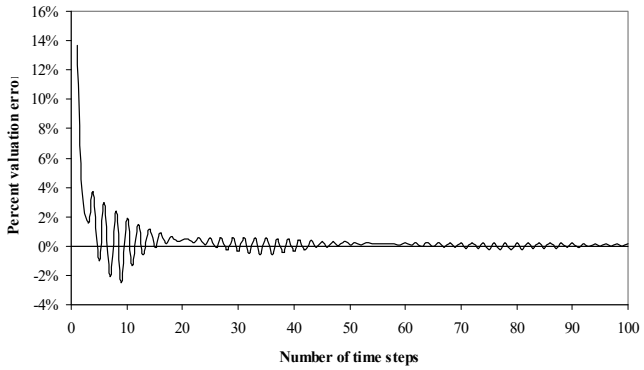
Binomial method

- Percent approximation error.



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Lesson summary

- Many standard and nonstandard options do not have analytical valuations equations (e.g., American-style options).
- Can be valued accurately using approximation methods.
- Develop and apply binomial method.

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