AIM 04 Performance measurement

AIM 04.3 Multiple risk factor performance

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Multiple risk factors

- Context:
  - Performance evaluation notes have discussed how to:
    - Analyze tracking error.
    - Assess single-factor risk-adjusted performance.
    - Isolate managerial skill using portable alpha strategies.
  - Now turn to assessing performance of funds with multiple market price risk factor exposures.
    - Some risks may be known (e.g., convertible bond funds).
    - Some risks may be unknown (e.g., hedge funds).

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Multiple risk factors

- Purpose:
  - Develop framework for:
    - Assessing performance when measurement for multiple price risk factors.
    - Hedging unwanted market risk exposures.
    - Transporting investor skill to different asset categories.

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Steps in risk assessment process

- Try to identify all tradable market risk factors that may affect fund returns.
- Regress excess fund returns on excess returns of price risk factors.
  - Excess returns can be based on:
    - ETP return
    - Futures return
- Eliminate insignificant factors and re-estimate.
- Intercept is risk-adjusted performance.

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Return to BOND

- Recall BOND analysis (BOND analysis II.xlsx).
  - Bought PIMCO's BOND ETF.
  - Short sold Vanguard's BND to:
    - Hedge long-term interest rate risk, and
    - Isolate manager's alpha.
  - Took cash from short sale and bought VTI.
    - Overlaid managerial skill from PIMCO's BOND fund onto stock market (where alpha is hard to find).
  - Problem arose.
    - Alpha changed.
    - Beta was not equal to one.

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Return to BOND

- Bought VTI with investment funds (cash generated from short sale) while holding BOND alpha.

Regression on VTI	
	Alpha+VTI
n	2,701
α	$\boxed{0.000045}$
s(α)	0.000028
β	$\boxed{1.010690}$
s(β)	0.002496
R-squared	0.983801
Adj. R-squared	0.983795
Std error of estimate	0.001453

Why isn't alpha equal to 0.000049 and beta equal to one?

Answer: Because BOND also has stock market risk exposure.

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Return to BOND

- Regress excess return of BOND on excess returns of BND and VTI.
  - BOND has two-factor risk exposures.

$$XR_{BOND,t} = \alpha_{BOND} + \beta_{BND,t} XR_{BND,t} + \beta_{VTI} XR_{VTI,t} + \varepsilon_{BOND,t}$$

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Return to BOND

- BOND has exposures to long-term interest rate risk and US stock market risk.

Excess return regression of BOND on BND and VTI

Multiple R	0.8644
R Square	0.7471
Adjusted R Square	0.7469
Standard Error	0.0015
Observations	2,701

	Coeff	*StErr*	*t Stat*	*P-value*
Intercept	0.000045	0.000028	1.59	0.1110
BND	0.806364	0.009099	88.62	0.0000
VTI	0.010748	0.002504	4.29	0.0000

BOND has significant exposures to bonds and stocks.

Abnormal performance is 0.000045.

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Return to BOND

- Can hedge exposures by short selling 0.806364 BND and 0.010748 VTI.

Regression of alpha porfolio on excess returns of BND and VTI

Multiple R	0.0000
R Square	0.0000
Adjusted R Square	-0.0007
Standard Error	0.0015
Observations	2,701

	Coeff	*StErr*	*t Stat*	*P-value*
Intercept	$\boxed{0.000045}$	0.000028	1.59	0.1110
BND	0.000000	0.009099	0.00	1.0000
VTI	0.000000	0.002504	0.00	1.0000

Alpha portfolio has no market risk, no investment, and outperformance.

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Return to BOND

- Want stock market beta of one.
  - Short sell 0.806364 BND and buy (1 - 0.010748) VTI.

Regression of alpha porfolio + VTI on VTI excess return

Multiple R	0.9917
R Square	0.9835
Adjusted R Square	0.9835
Standard Error	0.0015
Observations	2,701

	Coeff	*StErr*	*t Stat*	*P-value*
Intercept	$\boxed{0.000045}$	0.000028	1.59	0.1109
VTI	1.000000	0.002496	400.59	0.0000

Maintained alpha from BOND, and overlaid it on stock market risk.

Viola, portable alpha!

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Review steps

- Try to identify all tradable market risk factors that may affect fund returns.
- Regress excess fund returns on excess returns of price risk factors.
  - Excess returns can be based on:
    - ETP return
    - Futures return
- Eliminate insignificant factors and re-estimate.
- Intercept is risk-adjusted performance.

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Two more applications

- Apply technique in two additional applications.
  - Alpha Beta Fund
    - Uses convertible bond trading strategy.
    - Hedge stock market exposure using futures.
  - Gamma Fund
    - Also uses convertible bond trading strategy.
    - Create bond market risk using futures.
    - Overlay stock market alpha on bond performance.

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Alpha Beta Fund

- Hedge fund manager claims to be able to identify under- and over-priced convertible bonds.
  - Provides historical record showing abnormal risk-adjusted performance when benchmarked against the S&P 500.
    - Historical data for hedge funds in typically monthly.
- Support file: Alpha Beta Fund.xlsx

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Alpha Beta Fund

- Take record and perform due diligence.

ALPHA BETA CONVERTIBLE BOND FUND					
Month	Fund value	Money market	Excess fund return	Index futures price	Index futures return
0	122,413,208			801.67	
1	117,241,194	0.001054	-0.044223	755.82	-0.058893
2	110,732,395	0.002881	-0.059998	675.54	-0.112294
3	101,269,649	0.003038	-0.092368	676.54	0.001482
4	101,950,545	0.002871	0.003830	632.49	-0.067335
5	96,205,445	0.001979	-0.059981	585.56	-0.077088
6	99,833,854	0.002115	0.034906	610.15	0.041133
7	97,448,951	0.002049	-0.026227	617.96	0.012723
59	158,983,305	0.002891	-0.018803	744.87	-0.040201
60	169,143,914	0.002823	0.059128	787.50	0.055649

60 months of audited fund values provided by manager.

Money market is return on cash equivalents.

Futures price (returns) on stock index (e.g., S&P 500).

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Regress 60 months of excess returns of fund *P* on excess returns of stock index price risk factor.

Using futures: $R_{P,t} - r_t = \alpha + \beta_{SI} R_{SIF,t} + \varepsilon_t$ or

Using ETP: $R_{P,t} - r_t = \alpha + \beta_{SI} \left(R_{SI,t} - r_t \right) + \varepsilon_t$

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Alpha Beta Fund

- Excess return regression results show:

Alpha Beta regression results			
Multiple R	0.6442		
R Square	0.4150		
Adjusted R Square	0.4049		
Standard Error	0.0503		
Observations	60		
	Coeff.	*StErr.*	*t Stat*
Intercept	0.0031	0.0065	0.48
Index futures	0.6558	0.1022	6.41

Manager's claim is true. On a risk-adjusted basis, fund outperformed S&P 500 by 31 basis points per month or (0.31% x 12 =) 3.72% annually.

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Alpha Beta Fund

- Before concluding manager has great investment skills, **think**.
  - Convertible bonds have two market risk exposures.
    - Equity risk exposure
    - Long-term interest rate risk exposure
  - His performance regression specification is wrong.
    - Has missing variables problem.

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Alpha Beta Fund

- Correct regression specification also requires long-term interest rate risk factor:

$$R_{P,t} - r_t = \alpha + \beta_{SI} R_{SIF,t} + \beta_{TB} R_{TBF,t} + \varepsilon_t$$

Use T-bond futures returns to proxy for long-term interest rate exposure.

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Alpha Beta Fund

- Collect historical price data for bond futures.

Alpha Beta regression results

Multiple R	0.9938		
R Square	0.9876		
Adjusted R Square	0.9872		
Standard Error	0.0074		
Observations	60		
	Coeff.	*StErr.*	*t Stat*
Intercept	-0.00301	0.00096	-3.14
Index futures	0.37332	0.01598	23.36
Bond futures	0.90874	0.01770	51.35
Correlation between SIF and TBF:			0.344

Regression has great explanatory power.

Fund returns clearly depend on both stock and bond price risk factors.

Stock index return coefficient fell because it previously also proxied for bond price movements.
– Called *missing variable bias*.

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Alpha Beta Fund

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Bond futures	0.90874	0.01770	51.35
Correlation between SIF and TBF:			0.344

Adjusting for market price risks properly, manager underperforms by 30 basis points a month or 3.6% annually.

Unsavory character tried to dupe us!

Avoid mistake by:
– Using common sense.
– Identifying complete list of market factors.
– Applying knowledge of statistics.

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Setting futures hedge

- To demonstrate use of futures to manage risk, hedge stock market and interest rate risk exposures of Alpha Beta using futures.
  - Will have negative abnormal performance.

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Setting futures hedge

- Sell S&P 500 e-mini futures.
  - Index futures denomination is 50.
  - Number of contracts to sell is

$$n_{SIF} = \beta_{SI}\left(\frac{\text{Fund value}}{\text{Contract denomination}_{SI} \times \text{Futures price}_{SI}}\right)$$

- Sell T-bond futures.
  - Bond futures denomination is 1,000.
  - Number of contracts to sell is

$$n_{TBF} = \beta_{TB}\left(\frac{\text{Fund value}}{\text{Contract denomination}_{TB} \times \text{Futures price}_{TB}}\right)$$

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Setting futures hedge

- Solving.

$$n_{SIF} = 0.37332\left(\frac{169{,}143{,}914}{50 \times 787.50}\right) = 1{,}503.68$$

$$n_{TBF} = 0.90874\left(\frac{169{,}143{,}914}{1{,}000 \times 146.39}\right) = 1{,}050.00$$

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Setting futures hedge

- To isolate *portable alpha* for this fund, hedge stock market and interest rate risk exposures.
  - Portable alpha return is

$$\begin{aligned} PA_t &= R_{P,t} - r_t - \hat{\beta}_{SI} R_{SIF,t} - \hat{\beta}_{TB} R_{TBF,t} \\ &= R_{P,t} - r_t - 0.37332 R_{SIF,t} - 0.90874 R_{TBF,t} \end{aligned}$$

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Setting futures hedge

- Re-assess.

Systematic market price risks
Have been eliminated while
Isolating alpha.

Guaranteed loser.

Alpha Beta regression results before hedge

Multiple R	0.9938		
R Square	0.9876		
Adjusted R Square	0.9872		
Standard Error	0.0074		
Observations	60		
	Coeff.	*StErr.*	*t Stat*
Intercept	-0.00301	0.00096	-3.14
Index futures	0.37332	0.01598	23.36
Bond futures	0.90874	0.01770	51.35

Alpha Beta regression results after hedge

Multiple R	0.0000		
R Square	0.0000		
Adjusted R Square	-0.0351		
Standard Error	0.0074		
Observations	60		
	Coeff.	*StErr.*	*t Stat*
Intercept	-0.00301	0.00096	-3.14
Index futures	0.00000	0.01598	0.00
Bond futures	0.00000	0.01770	0.00

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Gamma Fund

- Competing manager also claims to be able to identify under- and over-priced convertible bonds.
  - Provides historical record showing abnormal risk-adjusted performance when benchmarked against the S&P 500.
- Support file: Gamma Fund.xlsx

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Gamma Fund

- Will not be duped this time.
  - Immediately assess performance using S&P 500 futures and T-bond futures returns.

$$R_{P,t} - r_t = \alpha + \beta_{SI} R_{SIF,t} + \beta_{TB} R_{TBF,t} + \varepsilon_t$$

Gamma Fund regression results

Multiple R	0.9954
R Square	0.9909
Adjusted R Square	0.9906
Standard Error	0.0078
Observations	60

	Coeff.	*StErr.*	*t Stat*
Intercept	0.00189	0.00101	1.88
Index futures	1.24456	0.01679	74.12
Bond futures	-0.00490	0.01860	-0.26

Strong explanatory power.

Only index futures has significant coefficient. Bond futures is irrelevant.

Remove bond futures return variable, and re-run regression.

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Gamma Fund

Gamma Fund regression results using SIF and TBF

Multiple R	0.9954
R Square	0.9909
Adjusted R Square	0.9906
Standard Error	0.0078
Observations	60

	Coeff.	*StErr.*	*t Stat*
Intercept	0.00189	0.00101	1.88
Index futures	1.24456	0.01679	74.12
Bond futures	-0.00490	0.01860	-0.26

Gamma Fund regression results using SIF only

Multiple R	0.9954
R Square	0.9909
Adjusted R Square	0.9907
Standard Error	0.0077
Observations	60

	Coeff.	*StErr.*	*t Stat*
Intercept	0.00186	0.00099	1.87
Index futures	1.24304	0.01564	79.49

Adjusted R-squared improves. Good sign.

Coefficient and significance of index futures variable virtually unchanged.

Inclusion of irrelevant explanatory variable has no great cost.

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Gamma Fund

- Know Gamma manager has managerial skill.
  - Alpha is positive.
- Problem is you believe:
  - Stock market investors are nervous and want to eliminate their market exposure while retaining Gamma Fund's alpha.
  - Replace with long-term bond exposure 1.5 times higher than T-bonds.

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Gamma Fund

- You want to make *tactical decision*.
  - Retain alpha of Gamma Fund.
  - Eliminate stock market exposure.
  - Take on long-term interest rate exposure 1.5 times higher than bond market.
- This tactical decision is also called *market timing decision*.

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Gamma Fund

- Since you want long-term interest exposure, return to two price risk-factor regression.
- Excess return of stock index hedged portfolio is

$$\begin{aligned} XR_{TB,t} &= R_{P,t} - r_t - \left(\hat{\beta}_{SI} R_{SIF,t} + \hat{\beta}_{TBF} R_{TBF,t}\right) + 1.5 R_{TBF,t} \\ &= R_{P,t} - r_t - \left(1.24304 R_{SIF,t} + \left(-0.00490 - 1.5\right) R_{TBF,t}\right) \end{aligned}$$

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Gamma Fund

- Re-assess.

Regression results hedging SIF and levering TBF

Multiple R	0.9962
R Square	0.9923
Adjusted R Square	0.9921
Standard Error	0.0078
Observations	60

	Coeff.	*StErr.*	*t Stat*
Intercept	0.00189	0.00101	1.88
Index futures	0.00000	0.01679	0.00
Bond futures	1.50000	0.01860	80.66

Hedged stock market exposure and levered T-bond exposure, all while maintaining manager’s alpha.

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Lesson summary

- Performance measurement must account for all systematic market price risk exposures, e.g., equity, interest rate, volatility, commodity, etc.
  - Failure to do so will misstate performance due to missing variable bias.
  - Including irrelevant risk factors is non-issue.
  - Price risk factors must be tradable and low cost.
  - Generate excess returns for price risk factors using:
    - ETPs
    - Futures

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