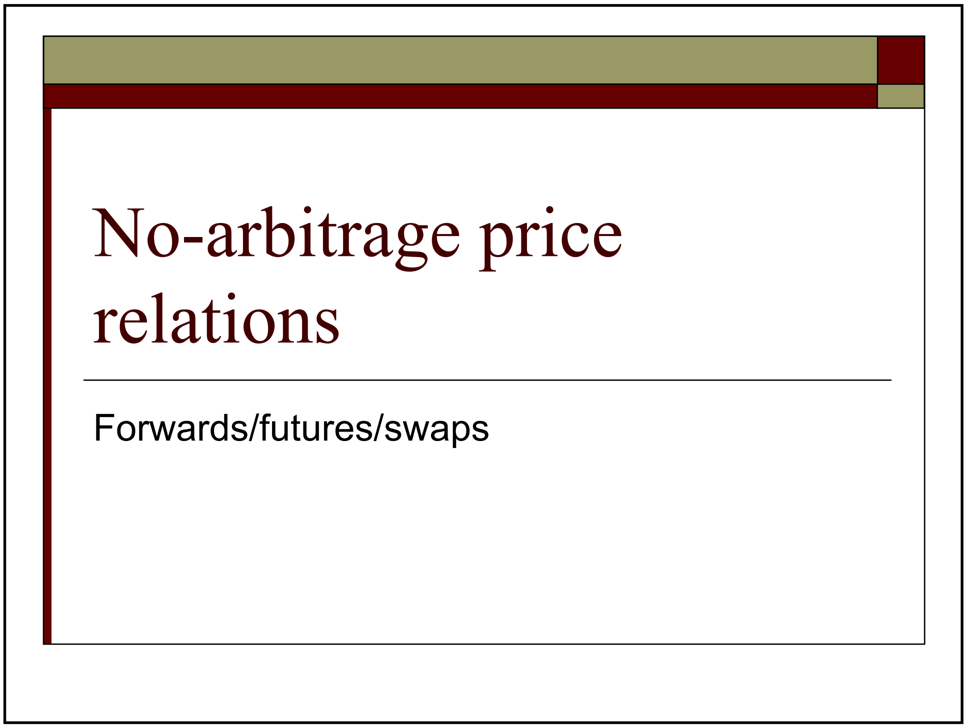
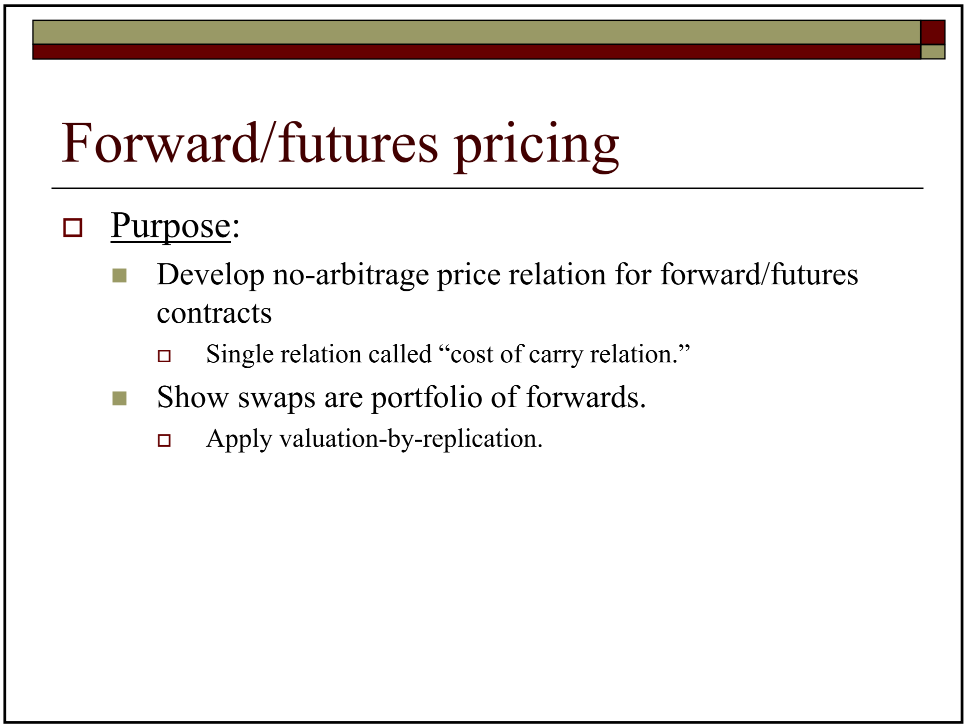


No-arbitrage price relations

Forwards/futures/swaps



Forward/futures pricing

- Purpose:
  - Develop no-arbitrage price relation for forward/futures contracts
    - Single relation called “cost of carry relation.”
  - Show swaps are portfolio of forwards.
    - Apply valuation-by-replication.

Forward/futures pricing

- Assumptions:
  - Individuals prefer more wealth to less.
    - Implies
      - Perfect substitutes have same price
      - No costless arbitrage opportunities
  - Markets are frictionless.
    - No trading costs
    - No taxes
    - Freedom to short, with full use of proceeds
    - Can trade any quantity at any time

Forward/futures pricing

- Assumptions:
  - Individuals can borrow or lend without limit at known continuously compounded risk-free interest rate, r.

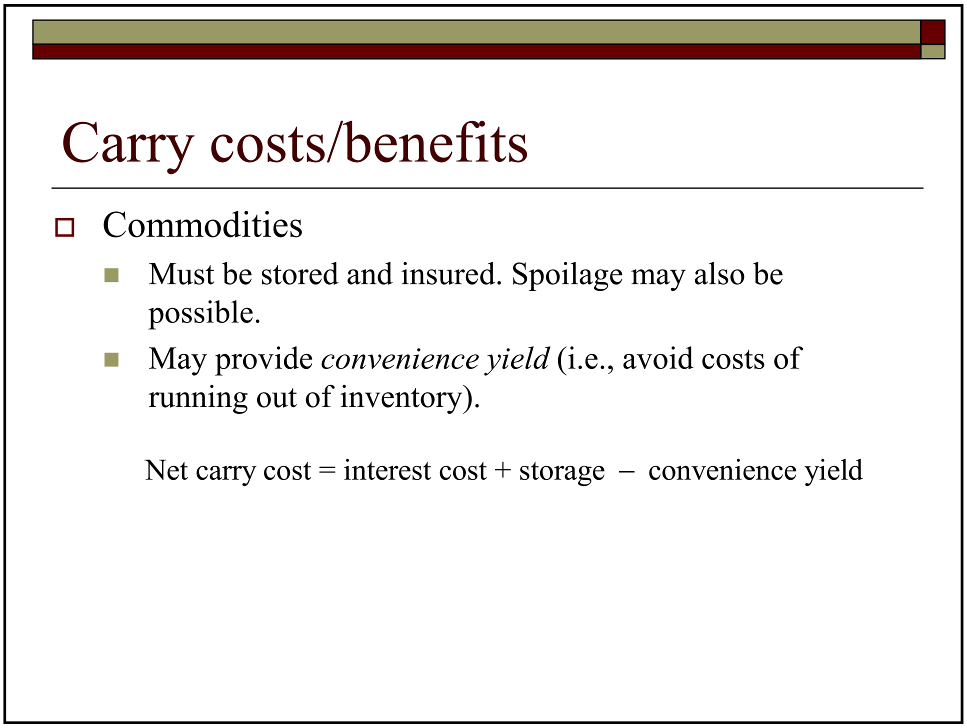
Forward/futures pricing

- Notation:

$f_0(\tilde{f}_T) =$ current (random terminal) forward price
$F_0(\tilde{F}_T) =$ current (random terminal) futures price
$S_0(\tilde{S}_T) =$ current (random terminal) asset price
$T =$ time to expiration of futures

Carry costs/benefits

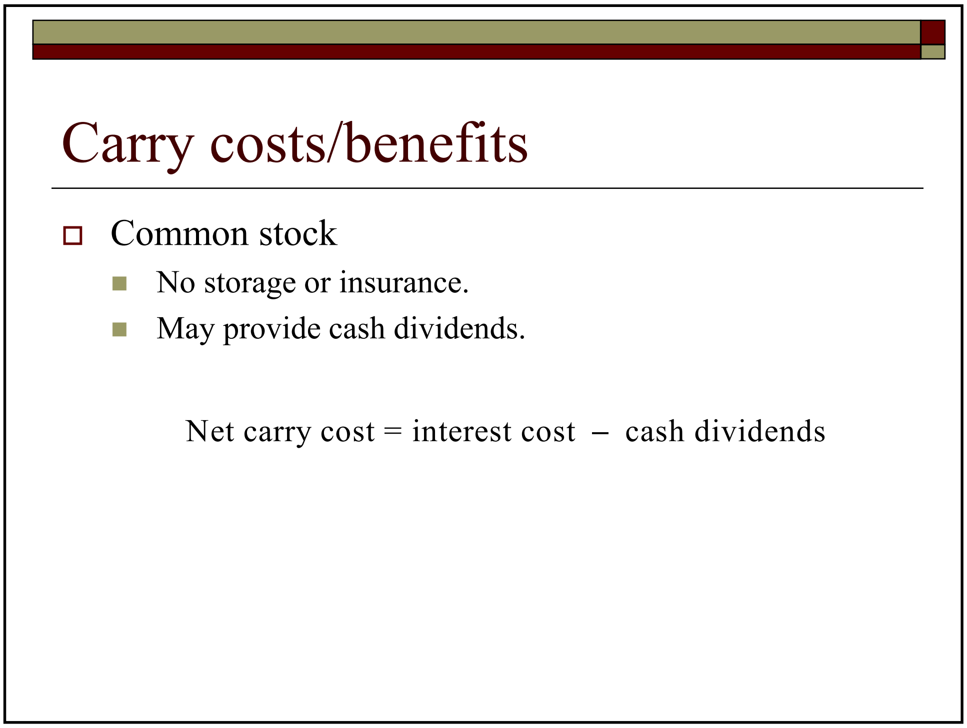
- *Carry* refers to any costs/benefits from holding asset.
  - Always includes opportunity cost of funds required to buy asset.
    - Interest rate, *r*
  - May include other costs/benefits depending on nature of asset.



Carry costs/benefits

- Commodities
  - Must be stored and insured. Spoilage may also be possible.
  - May provide *convenience yield* (i.e., avoid costs of running out of inventory).

Net carry cost = interest cost + storage − convenience yield



Carry costs/benefits

- Common stock
  - No storage or insurance.
  - May provide cash dividends.

Net carry cost = interest cost − cash dividends

Carry costs/benefits

- Coupon-bearing bond
  - No storage or insurance.
  - May provide coupon payment.

Net carry cost = interest cost − coupon receipt

Carry costs/benefits

- Currency
  - No storage or insurance.
  - Provides foreign interest income.

Net carry cost = domestic interest payment
− foreign interest income

Carry costs/benefits

- ☐ Futures contract
  - No storage or insurance.
  - No interest cost.
  - No interest receipt or other benefit.

Net carry cost = 0

Forward/futures pricing

- ☐ No-arbitrage rule 1: Forward/futures price equals asset price at expiration.

$$\tilde{f}_T = \tilde{F}_T = \tilde{S}_T$$

  - If forward price is greater,
    - ☐ Sell forward, buy asset, and deliver.
  - If forward price is less,
    - ☐ Buy forward, take delivery, and then sell asset in spot market.

Forward/futures pricing

- No-arbitrage rule 2: Current forward price equals current futures price.

$$f_0 = F_0$$

  - If forward price is greater,
    - Sell forward and buy "telescoping" futures position.
  - If forward price is less,
    - Buy forward and sell "telescoping" futures position.

Forward/futures pricing

- Illustration:
  - Consider buying 2-day forward on wheat at its current price of $5.50 per bushel.
    - What are cash flows if held to expiration?

Telescoping futures position

- Illustration:
  - Cash flows:

Day	0	1	2
Long forward	0		$\tilde{f}_2 - 5.50$

No investment today.

Telescoping futures position

- Illustration:
  - Cash flows:

Day	0	1	2
Long forward	0		$\tilde{f}_2 - 5.50$

Amount you will pay on day 2.

Telescoping futures position

- Illustration:
  - Cash flows:

Day	0	1	2
Long forward	0		$\tilde{f}_2 - 5.50$ $= \tilde{S}_2 - 5.50$

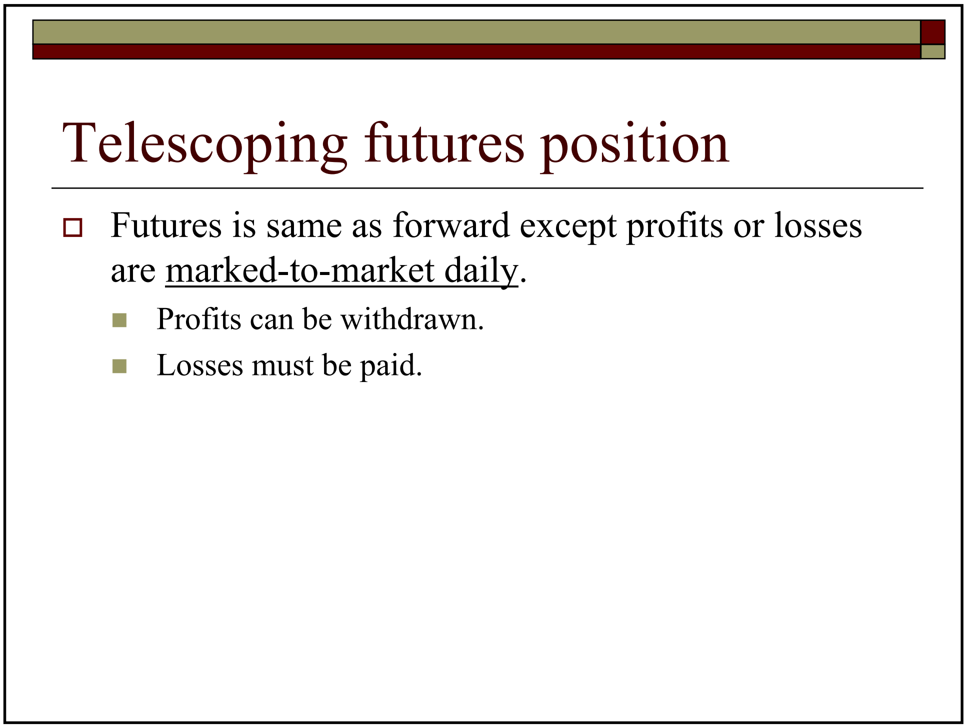
At expiration, forward price equals asset price by no-arbitrage rule 1.

Telescoping futures position

- Illustration:
  - Cash flows:

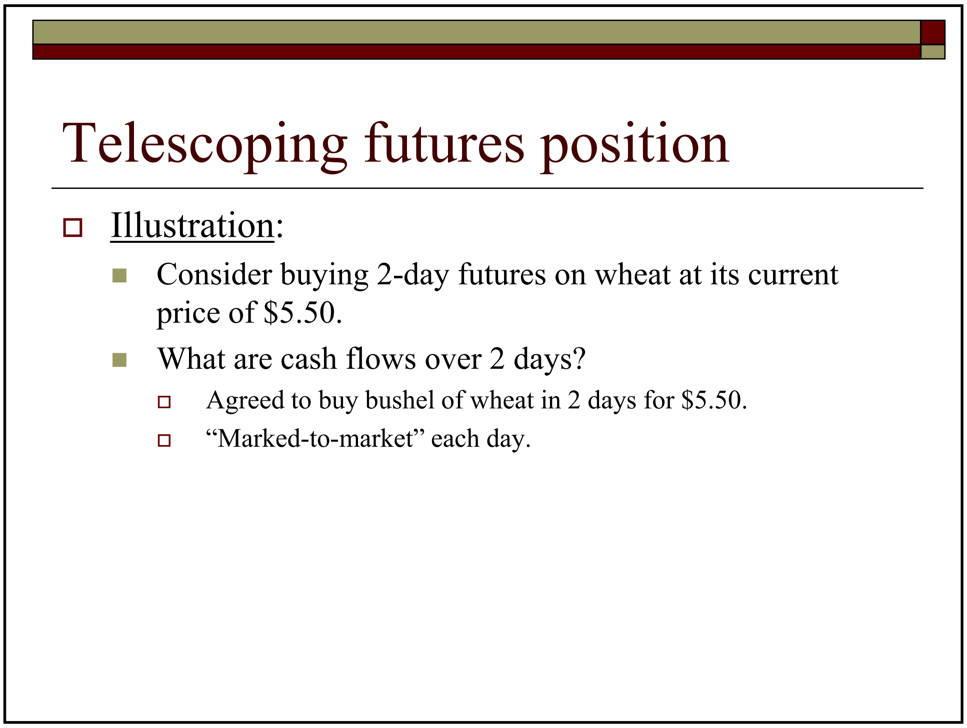
Day	0	1	2
Long forward	0		$\tilde{f}_2 - 5.50$

Price of forward contract at expiration.



Telescoping futures position

- Futures is same as forward except profits or losses are marked-to-market daily.
  - Profits can be withdrawn.
  - Losses must be paid.



Telescoping futures position

- Illustration:
  - Consider buying 2-day futures on wheat at its current price of $5.50.
  - What are cash flows over 2 days?
    - Agreed to buy bushel of wheat in 2 days for $5.50.
    - “Marked-to-market” each day.

Telescoping futures position

- ☐ Illustration:
  - Cash flows:

Day	0	1	2
Long futures	0	$\tilde{F}_1 - 5.50$	$\tilde{F}_2 - \tilde{F}_1$ $= \tilde{S}_2 - \tilde{F}_1$

No investment today.

Telescoping futures position

- ☐ Illustration:
  - Cash flows:

Day	0	1	2
Long futures	0	$\tilde{F}_1 - 5.50$	$\tilde{F}_2 - \tilde{F}_1$ $= \tilde{S}_2 - \tilde{F}_1$

Marked-to-market at end of day 1, at which time profit/loss is received/paid.

Telescoping futures position

- Illustration:
  - Cash flows:

Day	0	1	2
Long futures	0	$\tilde{F}_1 - 5.50$	$\tilde{F}_2 - \tilde{F}_1$
			$= \tilde{S}_2 - \tilde{F}_1$

Marked-to-market at end of day 2, at which time profit/loss is received/paid.

Telescoping futures position

- Illustration:
  - Cash flows:

Day	0	1	2
Long futures	0	$\tilde{F}_1 - 5.50$	$\tilde{F}_2 - \tilde{F}_1$
			$= \tilde{S}_2 - \tilde{F}_1$

$$\text{total profit}_2 = \left(\tilde{F}_1 - 5.50\right) + \left(\tilde{S}_2 - \tilde{F}_1\right) = \underline{\tilde{S}_2 - 5.50}$$

Same as long forward position.

Telescoping futures position

- Forward/futures comparison:
  - Ignored interest on first day’s marking-to-market.
  - Assume daily interest is r_d .

$$r_d = r / 365$$

  - Now, re-consider illustration.

Telescoping futures position

- Illustration:
  - Cash flows:

Day	0	1	2
Long futures	0	$\tilde{F}_1 - 5.50$	$\tilde{F}_2 - \tilde{F}_1$ $= \tilde{S}_2 - \tilde{F}_1$

$$\text{total profit}_2 = \left(\tilde{F}_1 - 5.50\right) e^{r_d} + \left(\tilde{S}_2 - \tilde{F}_1\right) \neq \tilde{S}_2 - 5.50$$

Telescoping futures position

- Forward/futures comparison:
  - If short-term daily interest rate, r_d, is known, can duplicate forward position by telescoping futures position.
    - Each day number of futures held is

      $$e^{-r_d t}$$

      where t is number of days to expiration.
    - Position “telescopes” upward in size (i.e., # of contracts) until exactly one contract is held on day T.

Telescoping futures position

- <u>Illustration</u>:
  - <u>Cash flows</u>:

Day	0	1	2
Units	$e^{-r_d(2-0)}$	$e^{-r_d(2-1)}$	1
Long futures	0	$\left(\tilde{F}_1 - 5.50\right)e^{-r_d}$	$\tilde{F}_2 - \tilde{F}_1$ $= \tilde{S}_2 - \tilde{F}_1$

$$\text{total profit}_2 = e^{-r_d}\left(\tilde{F}_1 - 5.50\right)e^{r_d} + \left(\tilde{S}_2 - \tilde{F}_1\right) = \tilde{S}_2 - 5.50$$

Telescoping futures position

- Forward/futures comparison:
  - If interest rates are known, telescoping futures position is equivalent to forward position.
  - Under no-arbitrage principle, futures price equals forward price.
- Henceforth, we treat forwards and futures as indistinguishable and use only futures price notation, F.

Cost of carry relation

- No-arbitrage rule 3: Futures price equals spot price times net cost of carry factor.

$$F = Se^{(r-i)T} \left(\text{or } F = Se^{bT}\right)$$

  - Called *net cost of carry relation*.
  - Assumes non-interest carry costs/benefits are at continuous rate.
    - r is *interest rate*.
    - i is *income rate*.
    - $b=r-i$ is *net cost of carry rate*.
  - Most important concept in futures pricing.

Cost of carry relation

- Net cost of carry relation is based on cash flows at time *T*.

$$F = Se^{(r-i)T}$$

- Prepaid net cost of carry relation is based on present value of cash flows.

$$Fe^{-rT} = Se^{-iT}$$

- We use both relations depending on application.

Cost of carry relation

- Illustration:
  - Stock index futures contract
    - Price (F) is \$457.
    - Time to expiration (T) is one year
  - Underlying stock index
    - Level (S) is \$450.
    - Dividend yield rate is 3% annually
  - Market
    - Risk-free interest rate (r) is 5% annually

Cost of carry relation

- <u>Illustration</u>:
  - Cost of carry relation is violated.

$$457 < 450e^{(.05-.03)1} = 459.09$$

Futures price is too low relative to index level
plus carry costs. Buy index futures; sell index portfolio;
lend proceeds.

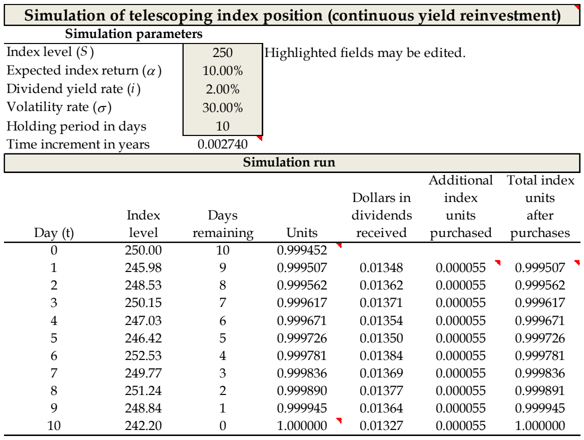
Cost of carry relation

- <u>Digression</u>: Under continuous dividend yield assumption:
  - All dividend income is immediately reinvested in more units of stock portfolio.
  - Reinvestment implies:

	No. of units$_0$	No. of units$_T$
Buy	-1	e^{iT}
Buy	$-e^{-iT}$	1

Cost of carry relation

- ☐ Supporting file: Telescoping index position.xlsx



Simulation of telescoping index position (continuous yield reinvestment)

Simulation parameters

Index level (S)	250	Highlighted fields may be edited.
Expected index return (α)	10.00%	
Dividend yield rate (i)	2.00%	
Volatility rate (σ)	30.00%	
Holding period in days	10	
Time increment in years	0.002740	

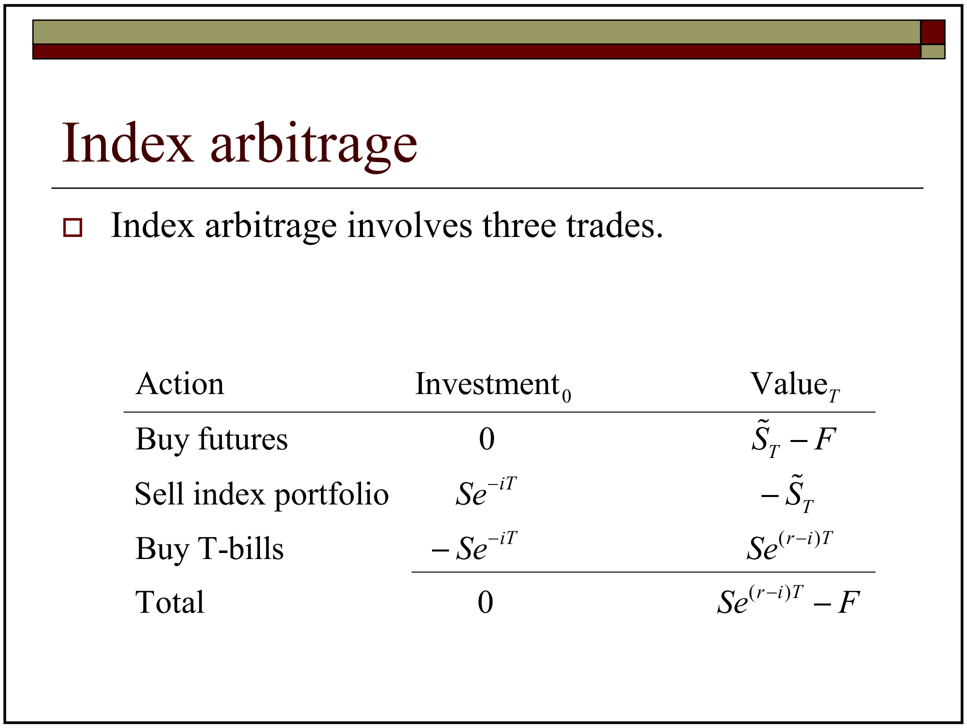
Simulation run

Day (t)	Index level	Days remaining	Units	Dollars in dividends received	Additional index units purchased	Total index units after purchases
0	250.00	10	0.999452			
1	245.98	9	0.999507	0.01348	0.000055	0.999507
2	248.53	8	0.999562	0.01362	0.000055	0.999562
3	250.15	7	0.999617	0.01371	0.000055	0.999617
4	247.03	6	0.999671	0.01354	0.000055	0.999671
5	246.42	5	0.999726	0.01350	0.000055	0.999726
6	252.53	4	0.999781	0.01384	0.000055	0.999781
7	249.77	3	0.999836	0.01369	0.000055	0.999836
8	251.24	2	0.999890	0.01377	0.000055	0.999891
9	248.84	1	0.999945	0.01364	0.000055	0.999945
10	242.20	0	1.000000	0.01327	0.000055	1.000000

Cost of carry relation

- ☐ Digression: Under continuous dividend yield assumption:
  - If index portfolio is sold short, must pay dividends.

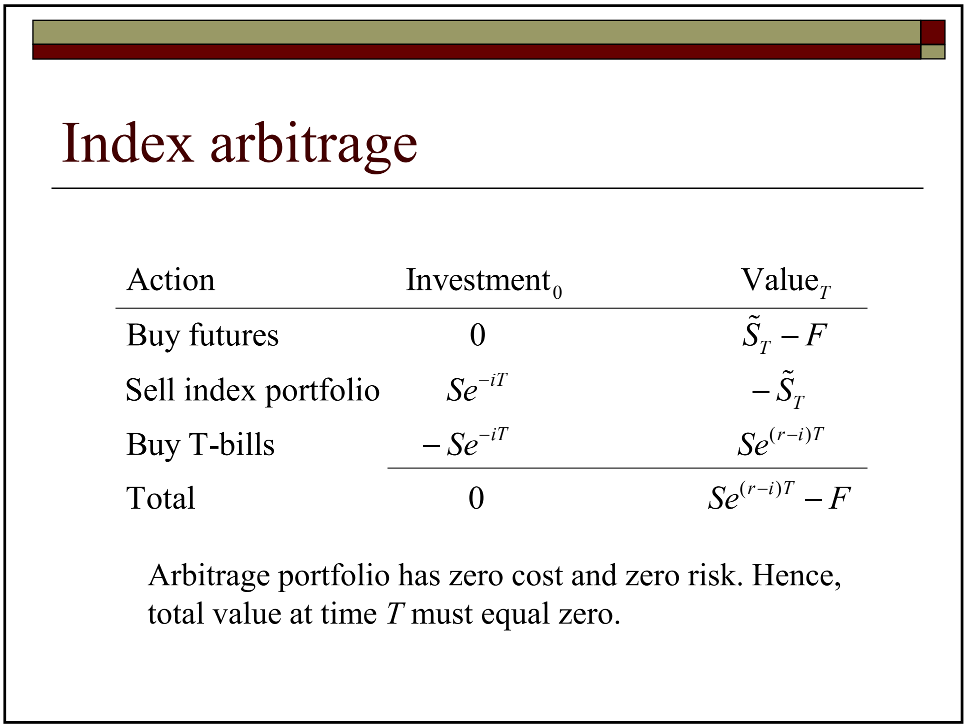
	No. of units$_0$	No. of units$_T$
Sell	1	$-e^{iT}$
Sell	e^{-iT}	-1



Index arbitrage

- Index arbitrage involves three trades.

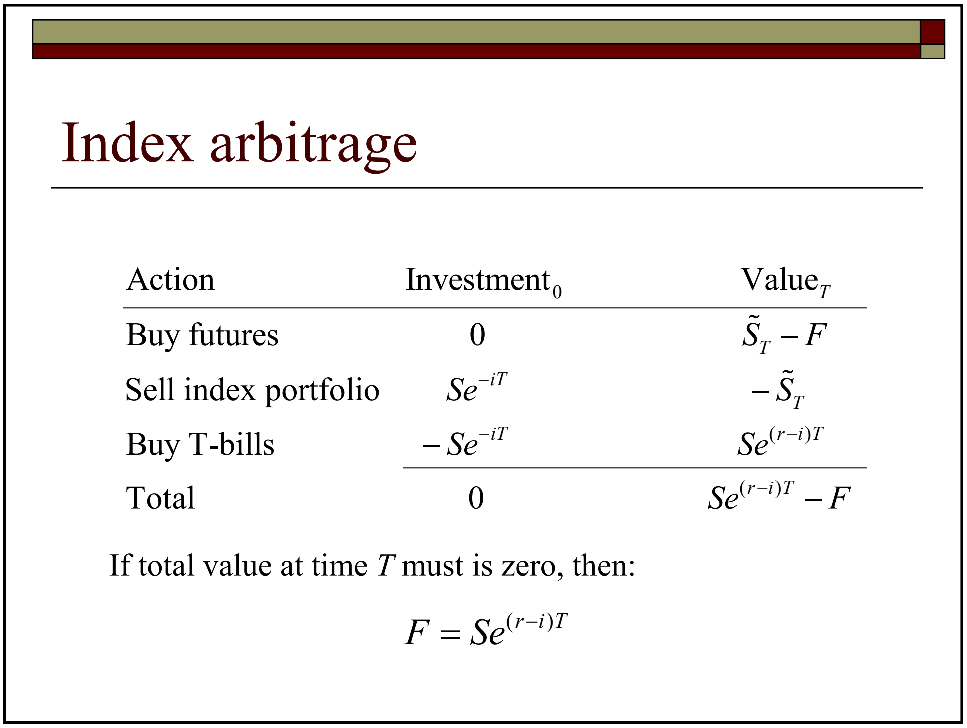
Action	Investment$_0$	Value$_T$
Buy futures	0	$\tilde{S}_T - F$
Sell index portfolio	Se^{-iT}	$-\tilde{S}_T$
Buy T-bills	$-Se^{-iT}$	$Se^{(r-i)T}$
Total	0	$Se^{(r-i)T} - F$



Index arbitrage

Action	Investment$_0$	Value$_T$
Buy futures	0	$\tilde{S}_T - F$
Sell index portfolio	Se^{-iT}	$-\tilde{S}_T$
Buy T-bills	$-Se^{-iT}$	$Se^{(r-i)T}$
Total	0	$Se^{(r-i)T} - F$

Arbitrage portfolio has zero cost and zero risk. Hence, total value at time T must equal zero.

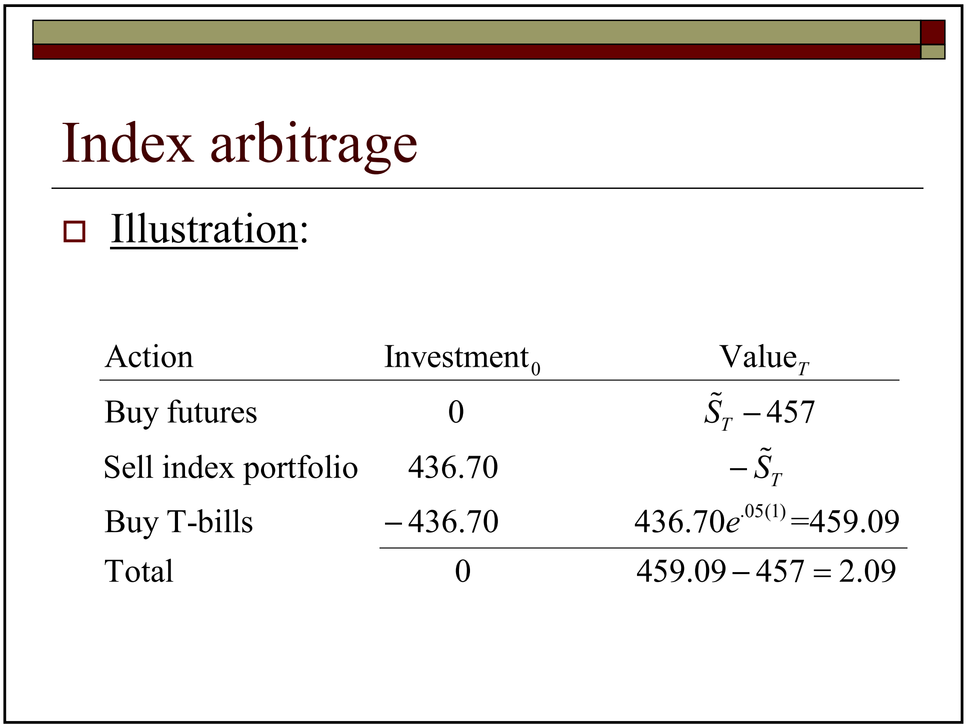


Index arbitrage

Action	Investment_0	Value_T
Buy futures	0	$\tilde{S}_T - F$
Sell index portfolio	Se^{-iT}	$-\tilde{S}_T$
Buy T-bills	$-Se^{-iT}$	$Se^{(r-i)T}$
Total	0	$Se^{(r-i)T} - F$

If total value at time T must is zero, then:

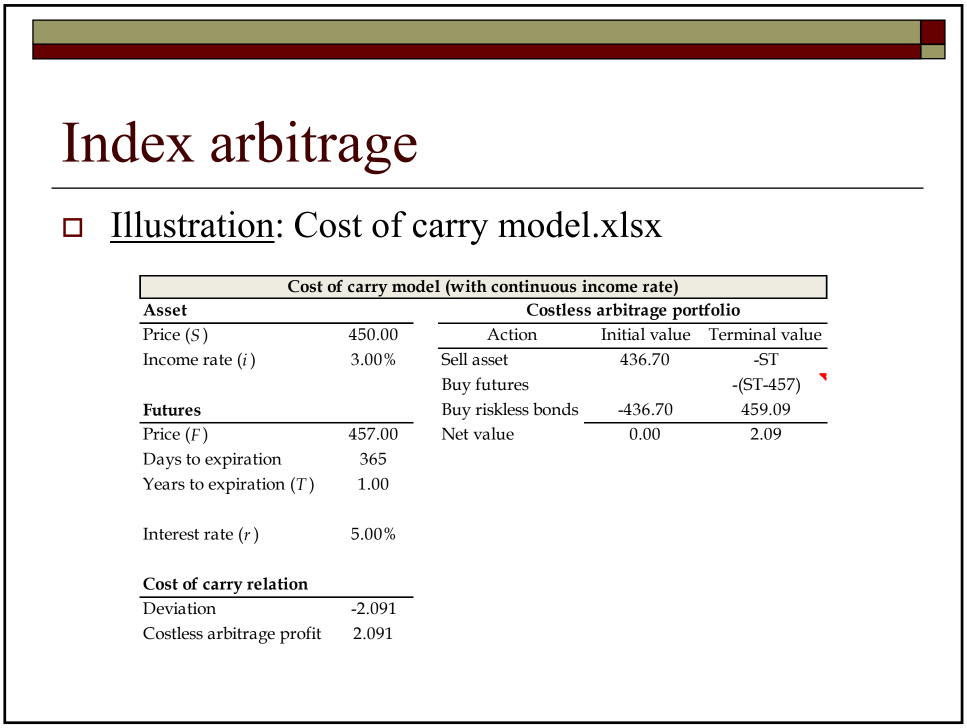
$$F = Se^{(r-i)T}$$



Index arbitrage

- Illustration:

Action	Investment_0	Value_T
Buy futures	0	$\tilde{S}_T - 457$
Sell index portfolio	436.70	$-\tilde{S}_T$
Buy T-bills	-436.70	$436.70e^{.05(1)} = 459.09$
Total	0	$459.09 - 457 = 2.09$



Index arbitrage

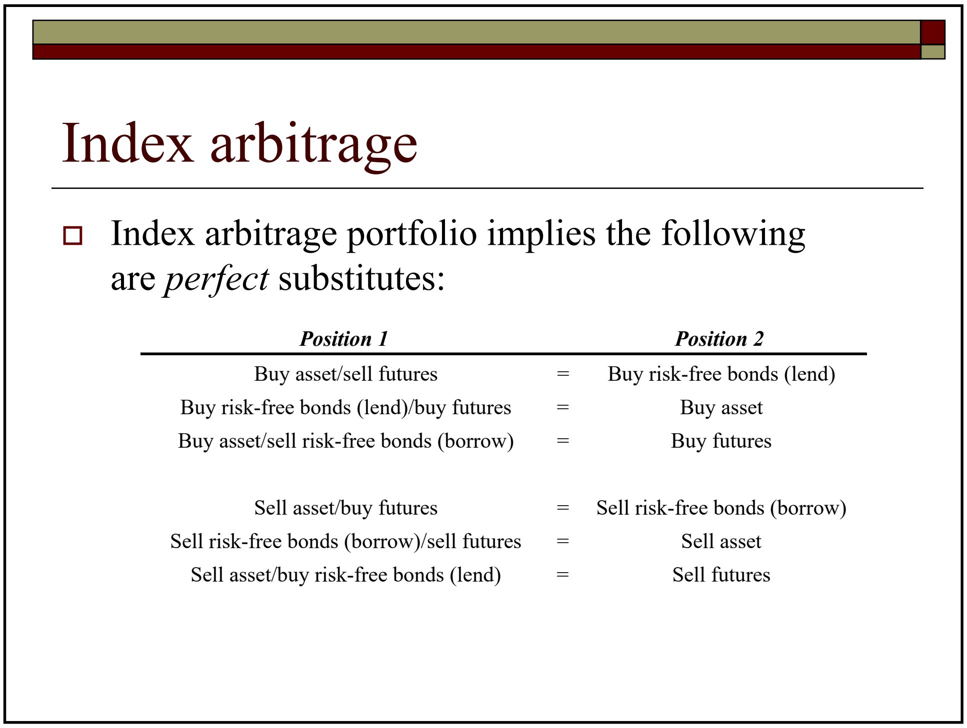
- Illustration: Cost of carry model.xlsx

Cost of carry model (with continuous income rate)

Asset	
Price (*S*)	450.00
Income rate (*i*)	3.00%
Futures	
Price (*F*)	457.00
Days to expiration	365
Years to expiration (*T*)	1.00
Interest rate (*r*)	5.00%
Cost of carry relation	
Deviation	-2.091
Costless arbitrage profit	2.091

Costless arbitrage portfolio

Action	Initial value	Terminal value
Sell asset	436.70	-ST
Buy futures		-(ST-457)
Buy riskless bonds	-436.70	459.09
Net value	0.00	2.09



Index arbitrage

- Index arbitrage portfolio implies the following are *perfect* substitutes:

Position 1		***Position 2***
Buy asset/sell futures	=	Buy risk-free bonds (lend)
Buy risk-free bonds (lend)/buy futures	=	Buy asset
Buy asset/sell risk-free bonds (borrow)	=	Buy futures
Sell asset/buy futures	=	Sell risk-free bonds (borrow)
Sell risk-free bonds (borrow)/sell futures	=	Sell asset
Sell asset/buy risk-free bonds (lend)	=	Sell futures

Index arbitrage

- ☐ Consider long futures/long T-bills.

Action	Investment_0	Value_T
Buy futures	0	$\tilde{S}_T - F$
Buy T-bills	$-Se^{-iT}$	$Se^{(r-i)T}$
Total	$-Se^{-iT}$	$\tilde{S}_T + Se^{(r-i)T} - F$ $= \tilde{S}_T$

Same as long index portfolio.
Called valuation-by-replication.

Futures return-risk relation

- ☐ Under CAPM, expected return/risk relation for assets is:

$$E_S = r + (E_M - r)\beta_S$$

- ☐ Will show expected return/risk relation for futures is:

$$E_F = (E_M - r)\beta_F$$

Futures return-risk relation

- Start with cost of carry relation at time *t*.

$$F_t = S_t e^{(r-i)(T-t)}$$

- Re-write in log form at time *t*.

$$\ln F_t = (r-i)(T-t) + \ln S_t$$

- Write in log form at time *t*+1.

$$\ln F_{t+1} = (r-i)(T-t-1) + \ln S_{t+1}$$

Futures return-risk relation

- Difference to find futures return.

$$\ln(F_{t+1} / F_t) = -(r-i) + \ln(S_{t+1} / S_t)$$

- Futures return is price appreciation only.

$$R_F = RA_F = \ln(F_{t+1} / F_t)$$

- Rate of price appreciation on asset is:

$$RA_S = \ln(S_{t+1} / S_t)$$

- Rate of return on asset is:

$$R_S = RA_S + i$$

Futures return-risk relation

- Relation between (random) futures return and (random) asset return is:

$$\begin{aligned}\tilde{R}_F &= -(r-i)+RA_S \\ &= (i+RA_S)-r \\ &= \tilde{R}_S - r\end{aligned}$$

- Relation between asset return and futures return is:

$$\tilde{R}_S = r + \tilde{R}_F$$

- <u>Supporting file</u>: Futures return-risk relation.xlsx

Futures return-risk relation

- Implies:
  - Expected futures return equals expected asset return less risk-free rate.

$$E_F = E_S - r$$

  - Variance of futures return equals variance of asset return.

$$Var\left(\tilde{R}_F\right) = Var\left(\tilde{R}_S\right)$$

Futures return-risk relation

- Implies:
  - Correlation between futures return and asset return.

$$\rho_{F.S} = \frac{Cov\left(\tilde{R}_F, \tilde{R}_S\right)}{\sqrt{Var\left(\tilde{R}_F\right) Var\left(\tilde{R}_S\right)}} = \frac{Cov\left(\tilde{R}_S - r, \tilde{R}_S\right)}{\sqrt{Var\left(\tilde{R}_S - r\right) Var\left(\tilde{R}_S\right)}} = \frac{Var\left(\tilde{R}_S\right)}{Var\left(\tilde{R}_S\right)} = 1$$

  - Beta of futures equals beta of underlying asset.

$$\beta_F = \frac{Cov\left(\tilde{R}_F, \tilde{R}_M\right)}{Var\left(\tilde{R}_M\right)} = \frac{Cov\left(\tilde{R}_S - r, \tilde{R}_M\right)}{Var\left(\tilde{R}_M\right)} = \beta_S$$

Futures return-risk relation

- CAPM says expected asset return is:

$$E_S = r + (E_M - r)\beta_S$$

- Implies expected return on futures equals *risk premium* of asset.

$$E_F = E_S - r = (E_M - r)\beta_F$$

Exchange-traded products

- Many exchange-traded products (ETPs) are created as fully-collateralized futures positions.
  - Long futures/long T-bills.

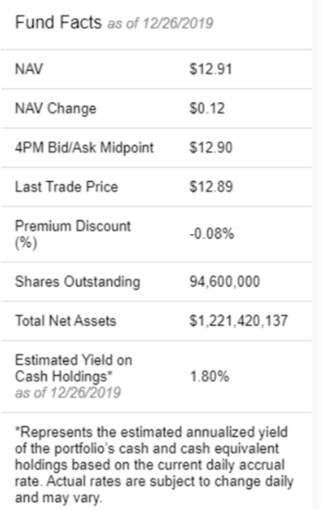
United States Oil Fund (USO)

- Link:
  http://www.uscfinvestments.com/uso

United States Oil Fund (USO)

- Description:
  - The United States Oil Fund® LP (USO) is an exchange-traded security designed to track the daily price movements of West Texas Intermediate ("WTI") light, sweet crude oil. USO issues shares that may be purchased and sold on the NYSE Arca.

United States Oil Fund (USO)



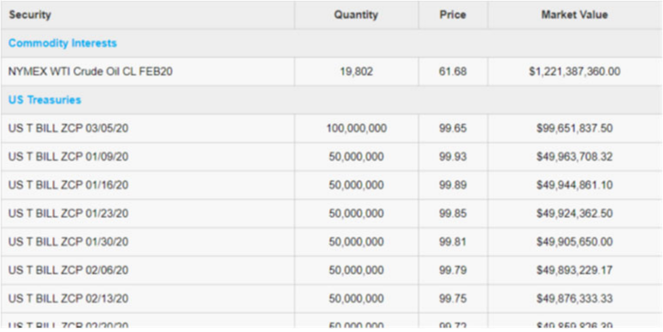
Fund Facts *as of 12/26/2019*

NAV	$12.91
NAV Change	$0.12
4PM Bid/Ask Midpoint	$12.90
Last Trade Price	$12.89
Premium Discount (%)	-0.08%
Shares Outstanding	94,600,000
Total Net Assets	$1,221,420,137
Estimated Yield on Cash Holdings* *as of 12/26/2019*	1.80%

*Represents the estimated annualized yield of the portfolio's cash and cash equivalent holdings based on the current daily accrual rate. Actual rates are subject to change daily and may vary.

United States Oil Fund (USO)

- Daily holdings



Security	Quantity	Price	Market Value
Commodity Interests			
NYMEX WTI Crude Oil CL FEB20	19,802	61.68	$1,221,387,360.00
US Treasuries			
US T BILL ZCP 03/05/20	100,000,000	99.65	$99,651,837.50
US T BILL ZCP 01/09/20	50,000,000	99.93	$49,963,708.32
US T BILL ZCP 01/16/20	50,000,000	99.89	$49,944,861.10
US T BILL ZCP 01/23/20	50,000,000	99.85	$49,924,362.50
US T BILL ZCP 01/30/20	50,000,000	99.81	$49,905,650.00
US T BILL ZCP 02/06/20	50,000,000	99.79	$49,893,229.17
US T BILL ZCP 02/13/20	50,000,000	99.75	$49,876,333.33

Fully collateralized futures position.

United States Oil Fund (USO)

- Monthly roll period (USO Prospectus, p. 45)
  - Starts at end of day on date two weeks prior to expiration of nearby contract.
    - E.g., two weeks prior to Jan. 20, 2012
  - 25% of nearby contracts are sold, and 25% of second nearby are purchased each day over four-day period.
  - At end of four days, USO is long only second nearby contracts.

FX forward/futures pricing

- Consider long asset/short futures.

Action	Investment_0	Value_T
Buy index portfolio	$-Se^{-iT}$	$\tilde{S}_T$
Sell futures	0	$-\left(\tilde{S}_T - F\right)$
Total	$-Se^{-iT}$	$F = Se^{(r-i)T}$

Same as buying T-bills.
Called *valuation by replication*.

FX forward/futures pricing

- Illustration:
  - Currency market is tightly arbitraged.
    - USD/BP spot rate is \$2.00
    - USD/BP 3-month forward rate is \$1.98
    - 3-month US interest rate is 3%
  - What is 3-month interest rate in Britain?

FX forward/futures pricing

$$1.98 = 2.00e^{b(3/12)}$$

$$b = \frac{\ln(1.98/2.00)}{3/12} = -4.02\%$$

Implied net cost of carry rate

FX forward/futures pricing

Implied British 3-month interest rate is:

$$b = -4.02\% = r - i = 3.00\% - i$$

$$i = 7.02\%$$

FX forward/futures pricing

- Supporting file: Implied foreign rate.xlsx

Compute implied foreign interest rate	
Spot rate (*S*)	2.00
Forward rate (*F*)	1.98
Months to expiration (*T*)	3
US interest rate (*r*)	3.00%
Implied carry rate (*b*)	-4.02%
Implied UK interest rate (*i*)	7.02%

Discrete costs of carry

- Sometimes *continuous* cost of carry assumption is inappropriate.
  - Single stock futures trade in many countries.
    - Were launched in U.S. in 2002.
    - Were delisted in 2020.

Discrete costs of carry

- No-arbitrage rule 5: *Net cost of carry relation* is:

$$F = Se^{rT} - I_t e^{r(T-t)}$$

where interest is constant, continuous rate and underlying asset pays discrete amount I_t at time t during futures life.

Discrete costs of carry

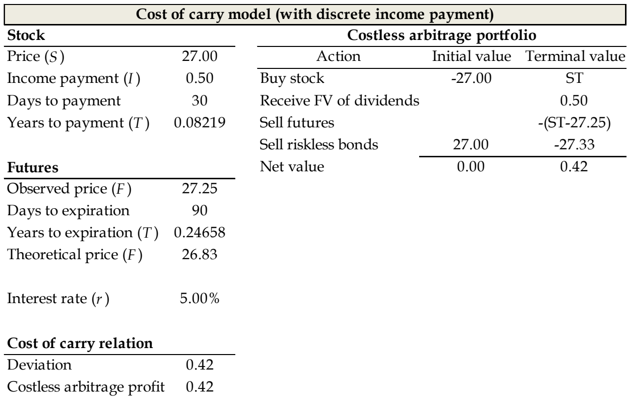
Action	Investment_0	Value_T
Sell stock futures	0	$-(\tilde{S}_T - F)$
Buy stock	$-S$	$\tilde{S}_T + I_t e^{r(T-t)}$
Sell T-bills	S	$-Se^{rT}$
Total	0	$F-(Se^{rT} - I_t e^{r(T-t)})$

Discrete costs of carry

- Illustration:
  - Suppose Foster's Brewing Co. currently has share price of AD 27 and plans on paying AD .50 cash dividend in 30 days. The price of 90-day futures on Foster's is 27.25. Is costless arbitrage profit possible if risk-free rate is 5%.

Discrete costs of carry

- Illustration: Cost of carry model.xlsx



Cost of carry model (with discrete income payment)

Stock	
Price (*S*)	27.00
Income payment (*I*)	0.50
Days to payment	30
Years to payment (*T*)	0.08219

Futures	
Observed price (*F*)	27.25
Days to expiration	90
Years to expiration (*T*)	0.24658
Theoretical price (*F*)	26.83
Interest rate (*r*)	5.00%

Cost of carry relation	
Deviation	0.42
Costless arbitrage profit	0.42

Costless arbitrage portfolio

Action	Initial value	Terminal value
Buy stock	-27.00	ST
Receive FV of dividends		0.50
Sell futures		-(ST-27.25)
Sell riskless bonds	27.00	-27.33
Net value	0.00	0.42

Discrete costs of carry

- Illustration:

$$F = 27e^{.05(90/365)} - .50e^{.05(60/365)} = 26.83$$

Swap contracts

- Illustration:
  - Manage chain of filling stations and want to lock-in price of future deliveries of unleaded gasoline over next 6 months.
    - Total monthly demand is 10 million gallons
    - No ability to store
    - Interest rate is 5%
  - Supporting file: Unleaded gasoline.xlsx

Swap contracts

Alternative 1: Buy strip of NYMEX futures.

Buy strip of unleaded gasoline futures

		Present value of	
Months to maturity	Dollars per gallon	Futures price	Cash payment
1	0.5358	0.53357	5,335,721
2	0.5437	0.53919	5,391,880
3	0.5490	0.54218	5,421,802
4	0.5497	0.54061	5,406,143
5	0.5472	0.53592	5,359,179
6	0.5427	0.52930	5,293,007
		Total	32,207,732

Swap contracts

Alternative 2: Buy OTC swap contract. Promises delivery of 10 million gallons per month for 6 months at fixed price of $.55 per gallon. Like futures, OTC contract demands no up-front payment.

Swap contracts

Alternative 2: Buy OTC contract.

Buy fixed-price contract in OTC market

		Present value of	
Months to maturity	Fixed price	Fixed price	Cash payment
1	0.5500	0.54771	5,477,131
2	0.5500	0.54544	5,454,357
3	0.5500	0.54317	5,431,678
4	0.5500	0.54091	5,409,093
5	0.5500	0.53866	5,386,602
6	0.5500	0.53642	5,364,205
		Total	32,523,066

Swap contracts

- Difference in alternatives:
  - OTC agreement’s cost is $32,523,066
  - Cost of futures strip is $32,207,732
  - Difference is $315,334
    - OTC dealer’s fee (i.e., margin)

Swap contracts

- What drives size of OTC swap dealer's margin?
  - Cost of hedging
  - Premium for counterparty default risk
  - Competition
  - Prospect of repeated business
  - Profit

Setting swap price quotes

- How does OTC dealer set price quotes?
  - Identifies "fair value" using actively-traded hedge instruments.
  - Adds or subtracts margin, depending upon whether customer wants to buy or sell.

Setting swap price quotes

- Suppose oil refiner contacts OTC dealer and requests fixed price per gallon on swap contract.
  - Refiner promises to deliver 10 million gallons per month for 6 months.

Setting swap price quotes

- Identify fair value.
  - Unleaded gasoline futures contracts are actively-traded on NYMEX.
  - Find prices of futures contracts whose expirations match refiner's desired delivery dates.
  - Set PV(OTC) = PV(strip) and solve.
    - Assume risk-free rate is 5%.

Setting swap price quotes

- Identify fair value.
  - Present value of futures strip.

Buy strip of unleaded gasoline futures

		Present value of	
Months to maturity	Dollars per gallon	Futures price	Cash payment
1	0.5358	0.53357	5,335,721
2	0.5437	0.53919	5,391,880
3	0.5490	0.54218	5,421,802
4	0.5497	0.54061	5,406,143
5	0.5472	0.53592	5,359,179
6	0.5427	0.52930	5,293,007
		Total	32,207,732

Setting swap price quotes

- Identify fair value.
  - Find fixed-price where PV = 32,207,732.

Compute fair price

		Present value of	
Months to maturity	Fixed price	Fixed price	Cash payment
1	0.5447	0.54240	5,424,026
2	0.5447	0.54015	5,401,473
3	0.5447	0.53790	5,379,014
4	0.5447	0.53566	5,356,648
5	0.5447	0.53344	5,334,375
6	0.5447	0.53122	5,312,195
		Total	32,207,732

Setting swap price quotes

- Based on fair value, OTC dealer sets price quote.
  - Refiner wants to sell gasoline so OTC dealer sets bid price by subtracting margin from fair value.

Setting swap price quotes

- Suppose dealer wants $1 million margin. What bid price will he quote?
  - Set PV(OTC) = PV(strip) - $1,000,000.
  - Solve for fixed price.

Setting swap price quotes

- ☐ Suppose dealer wants $1 million margin.

		Present value of	
Months to maturity	***Fixed price***	***Fixed price***	***Cash payment***
1	0.5278	0.5256	5,255,618.91
2	0.5278	0.5234	5,233,766.05
3	0.5278	0.5212	5,212,004.07
4	0.5278	0.5190	5,190,332.56
5	0.5278	0.5169	5,168,751.17
6	0.5278	0.5147	5,147,259.51
		Total	31,207,732.27

Setting swap price quotes

- ☐ OTC dealers frequently do not provide “two-sided” quotes.
  - Know which side of the market customer is on.
  - Simply adjust fair value to build in margin.

Controversies considered

Controversy	Year	Amount (millions)
AWA	1987	$50
ABN Amro	1991	$70
Barings Bank	1995	$1,300
Gibson Greetings	1994	$23
Metallgesellschaft	1994	$1,400
Orange County	1994	$1,700
State of Wisconsin	1995	$130
Long Term Capital Mgt.	1998	$4,505
		$9,178

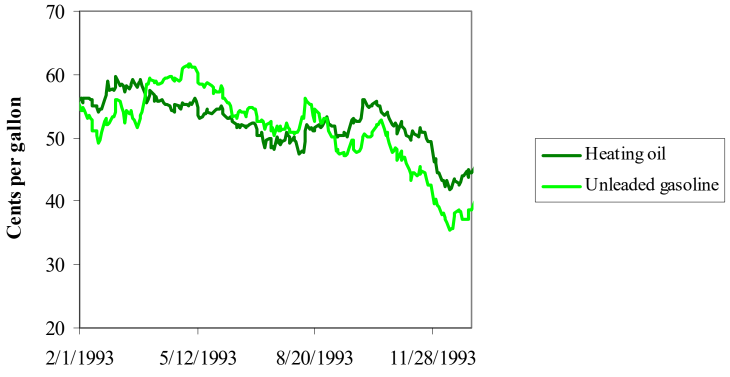
Metallgesellschaft - The Story

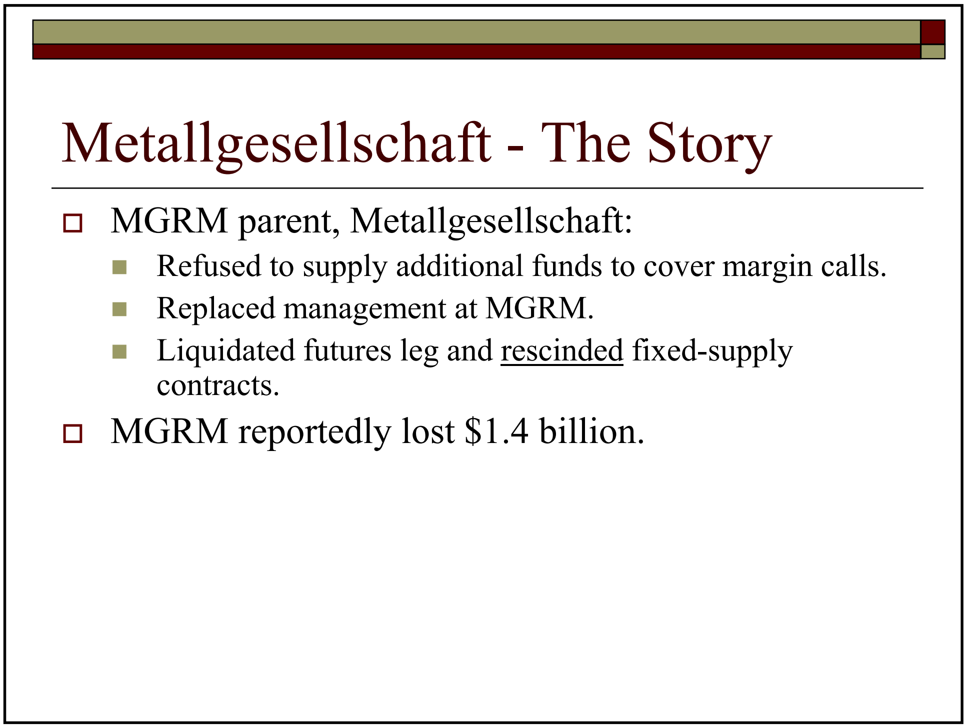
- In December 1991, MG Refining and Marketing (MGRM) began program of selling fixed supply contracts on heating oil and unleaded gasoline.
  - Firm-fixed contracts promised delivery of fixed amount of oil product (at fixed price) each month over contract's life

Metallgesellschaft - The Story

- ☐ MGRM marketing program was very successful.
  - 160 million barrels under contract as of Dec/93.
- ☐ MGRM hedged by buying heating oil and unleaded gasoline futures contracts on the NYMEX.

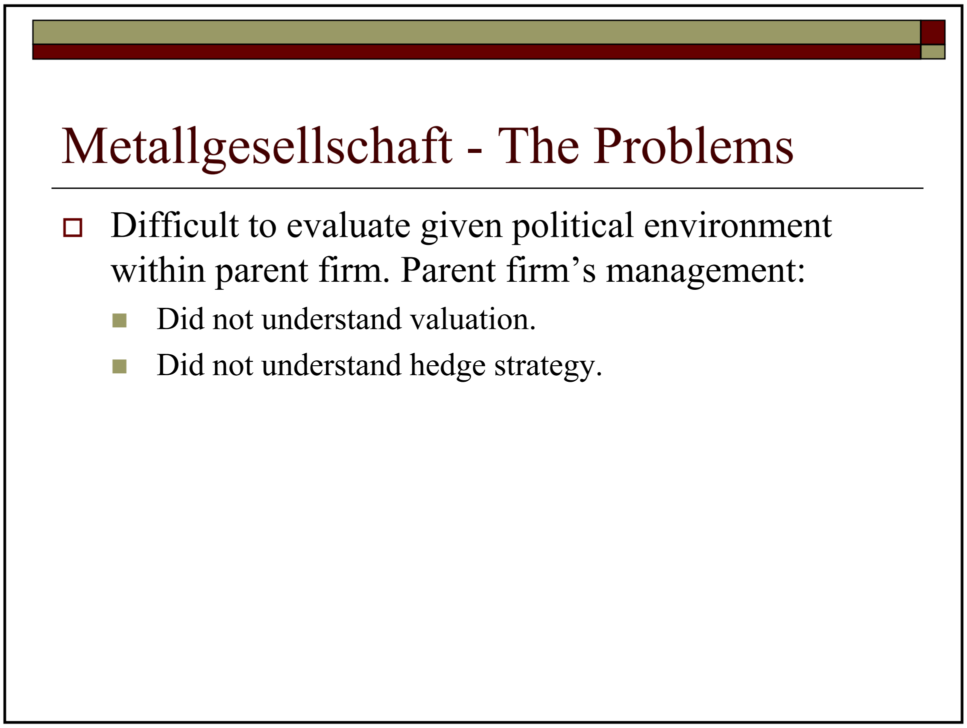
Metallgesellschaft - The Story





Metallgesellschaft - The Story

- MGRM parent, Metallgesellschaft:
  - Refused to supply additional funds to cover margin calls.
  - Replaced management at MGRM.
  - Liquidated futures leg and rescinded fixed-supply contracts.
- MGRM reportedly lost $1.4 billion.



Metallgesellschaft - The Problems

- Difficult to evaluate given political environment within parent firm. Parent firm’s management:
  - Did not understand valuation.
  - Did not understand hedge strategy.

Metallgesellschaft - The Problems

- Upon margin calls in futures market, parent refused to supply additional capital.
- Value of fixed-supply contracts were estimated to be nearly $800 million when they were rescinded.

Lesson summary

- *Cost of carry* relation for pricing forward/futures depends only on no-arbitrage.
  - E.g., firm needs British pounds in three months. To lock-in cost today:
    - Buy BP today and hold them for three months, or
    - Buy three-month BP forward.

$$Se^{-iT} = Fe^{-rT} \Rightarrow F = Se^{(r-i)T}$$

Lesson summary

- Cost of carry relation for particular asset depends on how carry costs accrue.
- Many “commodity swaps” are “fixed-price, fixed-supply” contracts.
  - Contracts are portfolios of forwards.
  - Fairness of prices can be deduced from exchange-traded products.