AIM 02

Return-risk measurement

Return-risk measurement

- Context:
  - Measuring return and risk properly is critical to investment problems.
    - Performance evaluation
    - Portfolio decision-making
- Purpose:
  - Provide framework for measuring return and risk from historical data.
  - Discuss potential historical data sources and their limitations and idiosyncrasies.

Return measurement

- To measure return, consider stock price history.
  - Assume prices are equally-spaced (e.g., daily) through time.

$$S_0, S_1, S_2, \ldots, S_T$$

Holding period returns (HPRs)

- In practice, history of stock prices is used to compute *holding period* or *buy-and-hold* returns.
  - Defined as return earned over defined holding period (e.g., day, month, or year).

Holding period returns (HPRs)

- Daily holding period return
  - With no dividends:

$$R_t = \frac{S_t - S_{t-1}}{S_{t-1}}$$

  - With dividends:

$$R_t = \frac{S_t + D_t - S_{t-1}}{S_{t-1}} = \frac{S_t - S_{t-1}}{S_{t-1}} + \frac{D_t}{S_{t-1}}$$

Price appreciation

Dividend yield

Holding period returns (HPRs)

- Using HPRs can be misleading because they do not aggregate properly through time.
  - E.g., common practice to sum monthly HPR over year to determine annual return or to sum daily returns over month to determine monthly return.
  - This is wrong.

Holding period returns (HPRs)

- Illustration:
  - Assume price moves from 50 to 100 during first day, and then from 100 to 50 during second.
    - Two-day HPR is reported as

$$100\% + (-50\%) = 50\%$$

    - Actual return is 0%.

Continuous returns

- Correct approach is to use continuous returns.
  - Continuous return (or *ln* return) is defined as:

$$R_t = \ln\left(\frac{S_t}{S_{t-1}}\right)$$

    - Can be summed through time (i.e., time aggregation property).
      - *T*-day return equals sum of daily returns.

$$R_T = \ln\left(S_T / S_0\right) = \sum_{t=1}^{T} \ln\left(S_t / S_{t-1}\right) = \sum_{t=1}^{T} R_t$$

Continuous returns

- Illustration:
  - Assume price moves from 50 to 100 during first day, and then from 100 to 50 during second. What is return over two days?

$$\ln(100/50) + \ln(50/100) = \ln(100/100) = 0\%$$

Continuous returns

- Important properties:
  - Mean daily continuous return over T days is:

$$\mu \equiv \frac{\sum_{t=1}^{T} R_t}{T} = \frac{R_T}{T}$$

  - Variance of return is:

$$Var\left(R_T\right) = \sum_{t=1}^{T} Var\left(R_t\right) = \sigma^2 T$$

  - Standard deviation of return (i.e., "volatility") is:

$$\sqrt{Var\left(R_T\right)} = \sigma\sqrt{T}$$

Measuring realized returns

- Many sources of data are available with which to measure *realized* or *historical* returns.
  - At Owen, have access to Bloomberg, Refinitiv (Datastream), Capital IQ, Morningstar, and FactSet.
    - http://researchguides.library.vanderbilt.edu/finance/home
  - Factset: Everyone can get account.
  - Bloomberg: Everyone can get account but must be used in Walker Library.
  - Refinitiv, Capital IQ, and Morningstar: Enquire in library.

Measuring realized returns

- All data sources report total return index (TRI) series.
  - Report for convenience.
    - Do not have to download prices and dividends.
  - Assume reinvestment of dividends.
- Level of *TRI* does not matter.

$$TRI_t = TRI_{t-1}\left(1+R_t\right), \text{ where } R_t = \frac{S_t + D_t - S_{t-1}}{S_{t-1}}$$

Generate daily total HPR returns as $R_t = TRI_t / TRI_{t-1} - 1$

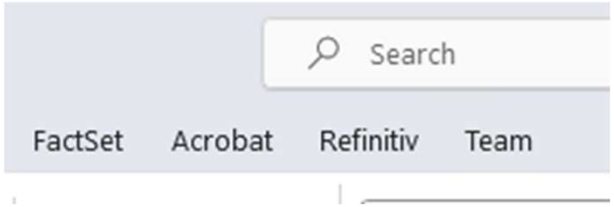
or daily ln returns as $R_t = \ln\left(TRI_t / TRI_{t-1}\right)$

Measuring realized returns

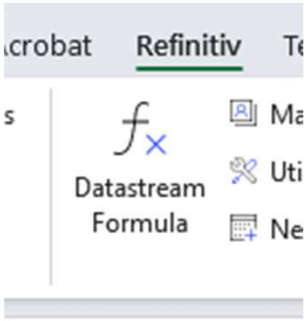
- Illustration:
  - Support file: S&P 500 products.xlsx
    - Downloaded daily data from Refinitiv Datastream.
    - Chose 10-year period 20121231 through 20221230.
    - Chose S&P 500-related series:
      - SPX: S&P 500 price index
      - SPTR: S&P 500 total return index (TRI) (includes dividends)
      - IVV: iShares Core S&P 500 ETF TRI
      - SPXL: Direxion Daily S&P 500 Bull 3*x* ETF TRI
    - Support files:
      - IVV fact sheet.pdf
      - SPXL fact sheet.pdf

Measuring realized returns

- Refinitiv (Datastream): Excel add-in
  - Click on Refinitiv.

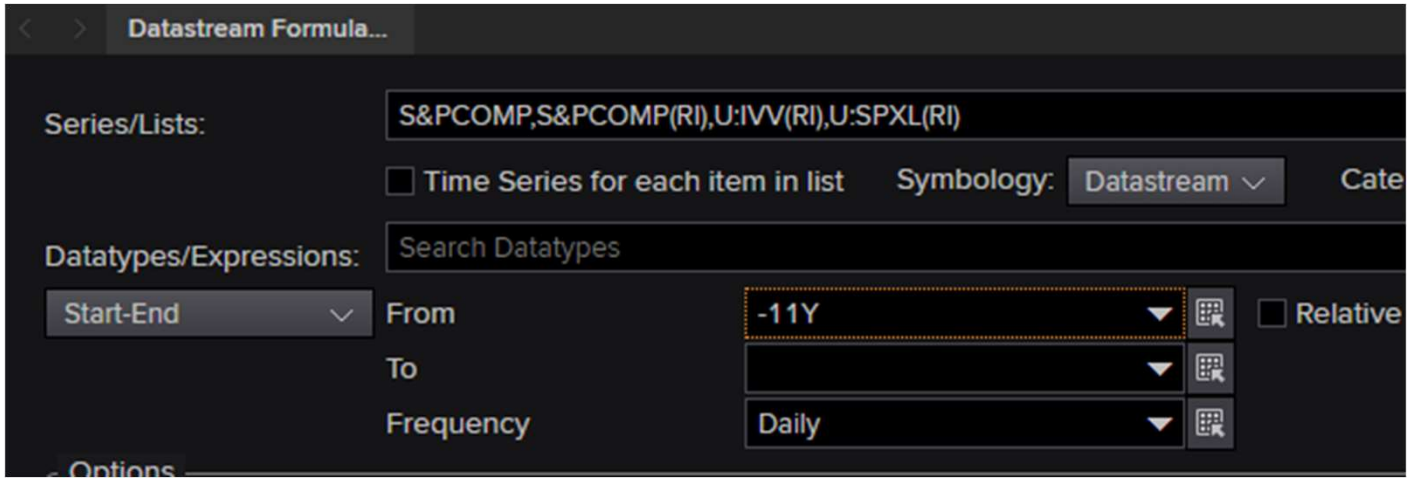


  - Click on Datastream Formula.



Measuring realized returns

- Refinitiv (Datastream): Excel add-in
  - Price data for last 10 years (20121231 – 20221230).



S&PCOMP – S&P 500 price index – SPX
S&PCOMP(RI) – S&P 500 total return index – SPTR
U:IVV(RI) – IVV ETF total return index – IVV
U:SPXL(RI) – SPXL 3x ETF total return index – SPXL

Measuring realized returns

- Problem with Refinitiv (Datastream) and Bloomberg daily data.
  - Both are international data sources and have dates that are not US trading dates.
  - Support file: US trading days 19271231-20231229.xlsx
    - Retain this file for future use.

Measuring realized returns

- Problem with Refinitiv (Datastream) and Bloomberg daily data.
  - Support file: S&P 500 products.xlsx
    - Sheet: Trading days
      - Note days that are not US trading days.

HP returns vs. ln returns

SUMMARY STATISTICS								
	Holding period returns				Ln returns			
Description	SPX	SPTR	IVV	SPXL	SPX	SPTR	IVV	SPXL
n	2,518	2,518	2,518	2,518	2,518	2,518	2,518	2,518
Mean	0.00046	0.00053	0.00053	0.00143	0.00039	0.00047	0.00047	0.00088
StDev	0.01108	0.01109	0.01108	0.03282	0.01112	0.01113	0.01112	0.03338
Skewness	-0.54666	-0.54492	-0.54895	-0.56100	-0.83951	-0.83783	-0.83391	-1.45539
Autocorrelation	-0.14780	-0.14799	-0.14355	-0.13009	-0.14573	-0.14591	-0.14139	-0.12425
Minimum	-0.11984	-0.11980	-0.11571	-0.33872	-0.12765	-0.12760	-0.12298	-0.41358
Median	0.00064	0.00072	0.00065	0.00191	0.00064	0.00072	0.00065	0.00190
Maximum	0.09383	0.09394	0.09450	0.27805	0.08968	0.08978	0.09030	0.24533
Mean (annual)	11.47%	13.40%	13.36%	35.93%	9.91%	11.84%	11.80%	22.07%
StDev (annual)	17.60%	17.60%	17.59%	52.10%	17.66%	17.66%	17.65%	53.00%
CAGR					10.42%	12.57%	12.52%	24.69%
HPR					169.21%	226.54%	225.14%	806.91%

Daily HPR

$$\mu_{daily} T = 0.00046(252) = 11.47\%$$

$$\sigma_{daily} \sqrt{T} = 0.01108\sqrt{252} = 17.60\%$$

Daily ln returns

$$\mu_{daily} T = 0.00039(252) = 9.91\%$$

$$\sigma_{daily} \sqrt{T} = 0.01112\sqrt{252} = 17.66\%$$

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Mean annual HPRs are wrong.

What HPR says you earned.

Amount actually earned.

$$CAGR = e^{0.00039*252} - 1$$

$$= e^{0.00991*1} - 1 = 10.42\%$$

10-year HPR

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Description	SPX	SPTR	IVV	SPXL	SPX	SPTR	IVV	SPXL
n	2,518	2,518	2,518	2,518	2,518	2,518	2,518	2,518
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10-year holding period return

$$\frac{3,839.50}{1,426.19} - 1 = e^{0.00039*2,518} - 1 = 169.21\%$$

Dividend yield

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Dividend yield on S&P 500 = 11.84% - 9.91% = 1.93%

Expense ratio

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IVV charges management fee = 11.84% - 11.80% = 0.04%

SPXL – 3*x* Levered ETF

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$$226.54\% \times 3 = 679.62\% \neq 806.91\%$$

Controversial issue that has received much attention.

- Reinvestment assumptions are different.

Skewness

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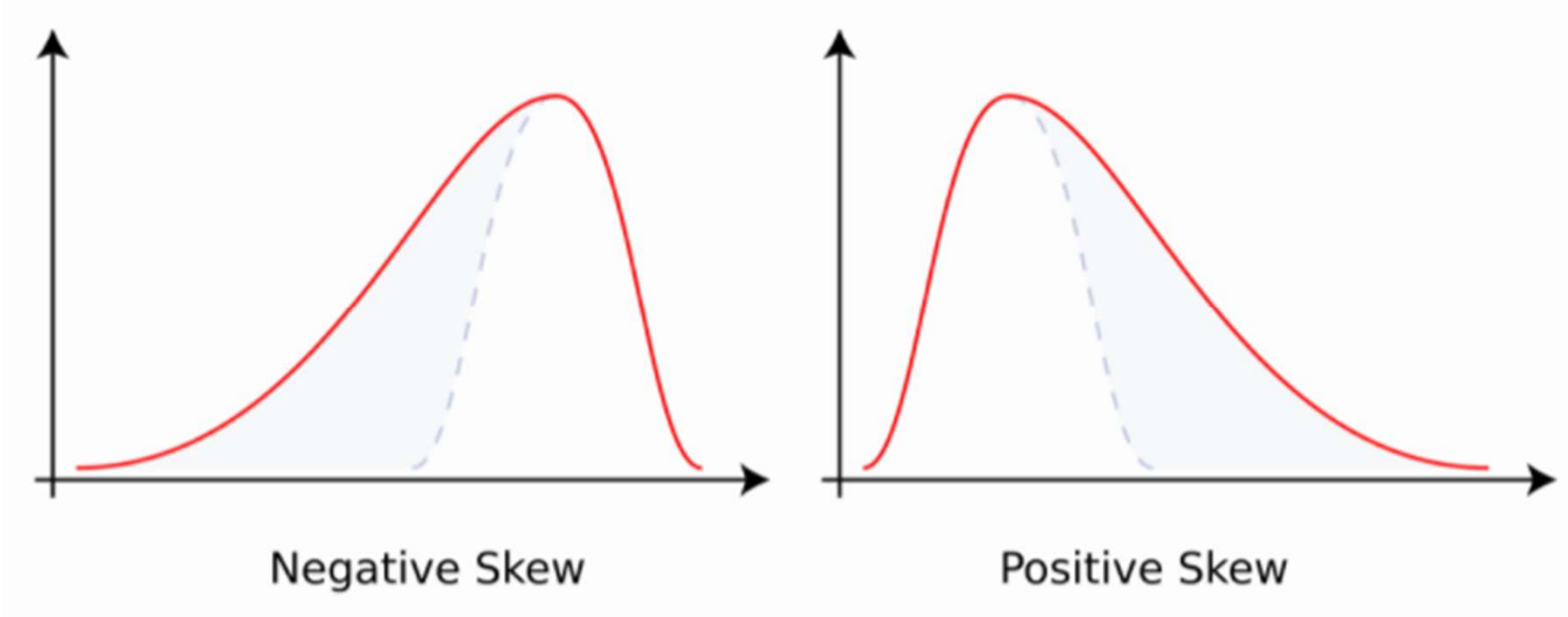

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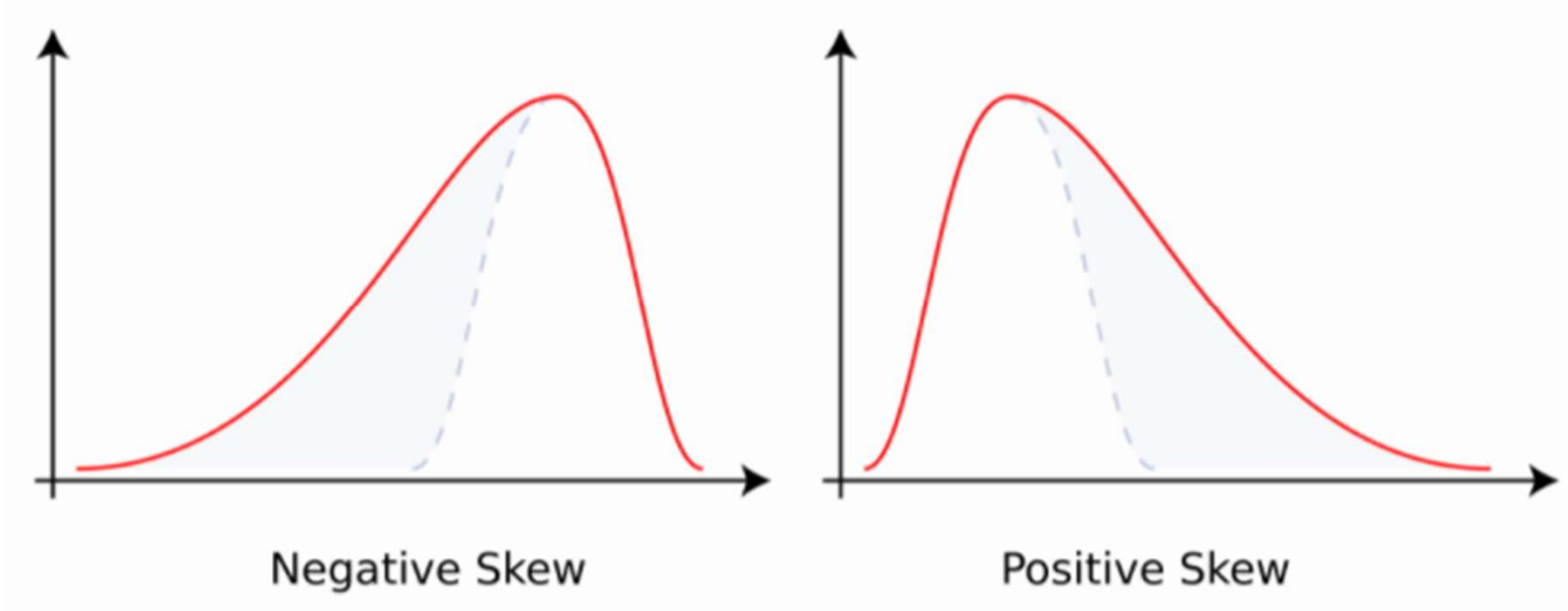

Autocorrelation

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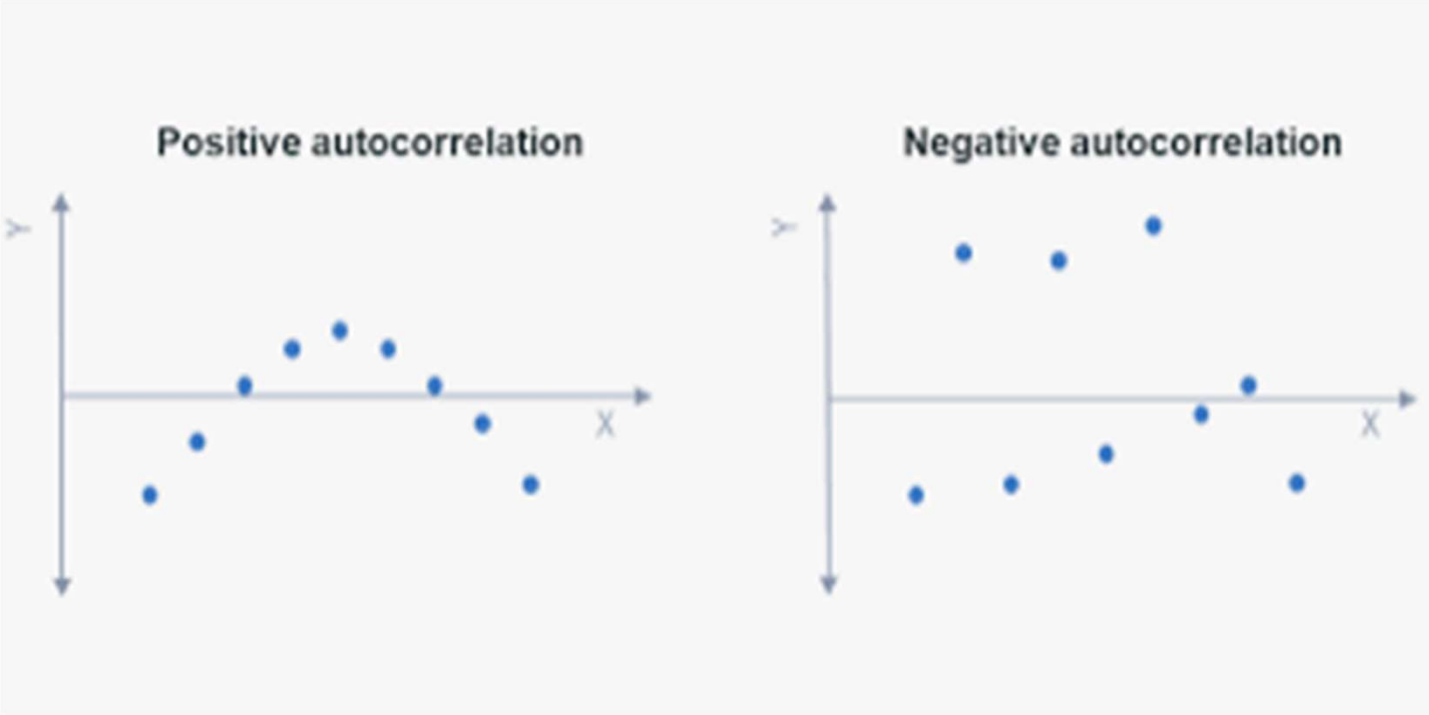

Positive autocorrelation: If yesterday's return is positive, today's return is likely to be positive, and vice versa.

Negative autocorrelation: If yesterday's return is positive, today's return is likely to be negative, and vice versa.

Digression: VBA functions

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Entered separate Excel function for each row. Is both inconvenient and tedious.

Digression: VBA functions

- Best way to handle repetitive computations in Excel is to create library of VBA functions.
  - Support file: VBAs.zip (VBAs.xlsm zipped)
    - Press ALT+F11. Insert module.
    - Program function to compute mean, standard deviation, and skewness.

```
n = ret.count
av = WorksheetFunction.Average(ret)
sd = WorksheetFunction.StDev(ret)
skew = WorksheetFunction.Skew(ret)
```

    - Return outputs in array.

```
aim_stat = Array(n,av,sd,skew)
```

Digression: VBA functions

- Developed series of statistical/financial functions that perform common and repetitive tasks in applied investment management.
  - Support files:
    - AIM installation instructions.pdf
    - AIM.xlam

Return on cash equivalents

- Return of cash-equivalents is said to be “risk-free rate.”
  - No security is risk-free.
    - Depends on:
      - Holding period
      - Credit risk
  - In practice, three overnight benchmarks are used to proxy for return on cash equivalents.
    - Effective fed funds rate (EFFR)
    - London inter-bank offer rate (LIBOR)
    - Secured overnight financing rate (SOFR)

Return on cash equivalents

- Support file: Overnight rates.xlsx
  - Downloaded EFFR and SOFR from FactSet.
  - Downloaded LIBOR from Datastream.
  - Chose 10-year period 20121231 through 20221230.
  - SOFR is new. Series begins 20180402.

Return on cash equivalents

- Created total return index (TRI) for each series.
- Started series with base of 100.
- Updated using daily holding period returns.

$$1 + HPR_t = \left[1 + \left(\frac{N_{t-1}}{100} \right) \times \left(\frac{\#\ \text{of calendar days}}{360} \right) \right]$$

where N_{t-1} is the nominal rate at close yesterday.

Return on cash equivalents

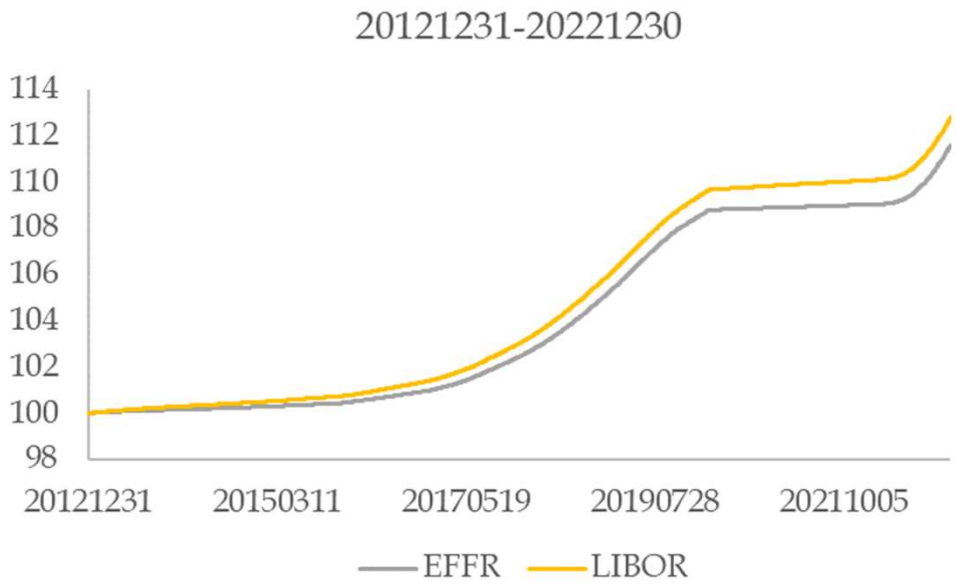
Summary statistics 20121231-20221230		
Description	EFFR	LIBOR
n	2,518	2,518
Mean (daily)	0.0000	0.0000
StDev (daily)	0.0001	0.0001
Skewness	1.2574	1.2185
Autocorrelation	0.9958	0.9940
Minimum	0.0000	0.0000
Median	0.0000	0.0000
Maximum	0.0002	0.0002
Mean (annual)	1.10%	1.20%
StDev (annual)	0.08%	0.08%
CAGR	1.10%	1.21%
HPR	11.58%	12.75%

Correlation matrix		
	EFFR	LIBOR
EFFR	1	0.997
LIBOR	0.997	1

Returns are computed over full 10-year period.

Summary statistics show EFFR and LIBOR are virtually perfect substitutes.

LIBOR return is slightly higher due to credit risk.



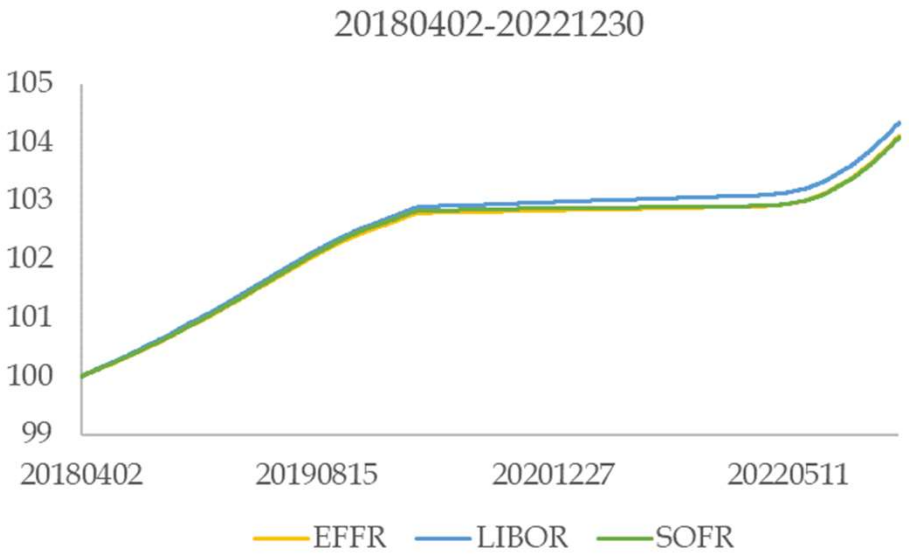
Return on cash equivalents

Summary statistics 20180402-20221231			
Description	EFFR	LIBOR	SOFR
n	1,197	1,197	1,197
Mean (daily)	0.0000	0.0000	0.0000
StDev (daily)	0.0000	0.0000	0.0000
Skewness	0.4201	0.4022	0.4139
Autocorrelation	0.9953	0.9943	0.9898
Minimum	0.0000	0.0000	0.0000
Median	0.0000	0.0000	0.0000
Maximum	0.0001	0.0001	0.0001
Mean (annual)	0.85%	0.89%	0.84%
StDev (annual)	0.05%	0.05%	0.05%
CAGR	0.85%	0.90%	0.85%
HPR	4.11%	4.34%	4.08%

Correlation matrix			
	EFFR	LIBOR	SOFR
EFFR	1	1	0.996
LIBOR	1	1	0.993
SOFR	0.996	0.993	1

Returns are computed over 4.75-year period.

Summary statistics show SOFR and is also close substitute.



Return on cash equivalents

Summary statistics 20121231-20221230		
Description	EFFR	SPTR
n	2,518	2,518
Mean (daily)	0.0000	0.0005
StDev (daily)	0.0001	0.0111
Skewness	1.2574	-0.8378
Autocorrelation	0.9958	-0.1458
Minimum	0.0000	-0.1276
Median	0.0000	0.0007
Maximum	0.0002	0.0898
Mean (annual)	1.10%	11.84%
StDev (annual)	0.08%	17.66%
CAGR	1.10%	12.57%
HPR	11.58%	226.54%

Correlation matrix		
	EFFR	SPTR
EFFR	1	-0.020
SPTR	-0.020	1

Compared EFFR to SPTR.

"Risk-free" security is risk-free only in relative sense.

- Low volatility relative to stocks (0.08% vs 17.66%)
- Little return correlation (-0.02)

Return on cash equivalents

- Use daily effective fed funds rates in moving forward.
  - Support file: EFFR 1954-present.xlsx
    - Retain this file for future use.

Measurement interval

- What is ideal time between price observations (e.g., daily, weekly, monthly)?
  - Re-examine S&P 500 data.

Measurement interval

SUMMARY STATISTICS				
	Ln returns			
Description	SPX	SPTR	IVV	SPXL
n	2,518	2,518	2,518	2,518
Mean (daily)	0.00039	0.00047	0.00047	0.00088
StDev (daily)	0.01112	0.01113	0.01112	0.03338
Skewness	-0.83951	-0.83783	-0.83391	-1.45539
Autocorrelation	-0.14573	-0.14591	-0.14139	-0.12425
Minimum	-0.12765	-0.12760	-0.12298	-0.41358
Median	0.00064	0.00072	0.00065	0.00190
Maximum	0.08968	0.08978	0.09030	0.24533
Mean (annual)	9.91%	11.84%	11.80%	22.07%
StDev (annual)	17.66%	17.66%	17.65%	53.00%
CAGR	10.42%	12.57%	12.52%	24.69%
HPR	169.21%	226.54%	225.14%	806.91%

SUMMARY STATISTICS				
	Ln returns			
Description	SPX	SPTR	IVV	SPXL
n	120	120	120	120
Mean (monthly)	0.00825	0.00986	0.00983	0.01837
StDev (monthly)	0.04273	0.04272	0.04272	0.13858
Skewness	-0.62066	-0.62775	-0.62775	-1.31386
Autocorrelation	-0.17047	-0.17031	-0.16790	-0.13412
Minimum	-0.13367	-0.13183	-0.12940	-0.65719
Median	0.01737	0.01866	0.01808	0.04402
Maximum	0.11942	0.12062	0.11937	0.31653
Mean (annual)	9.90%	11.83%	11.79%	22.05%
StDev (annual)	14.80%	14.80%	14.80%	48.00%
CAGR	10.41%	12.56%	12.51%	24.67%
HPR	169.21%	226.54%	225.14%	806.91%

Observations:

1) Daily mean and volatility are lower than monthly mean and volatility because holding period is less.
2) Volatility using ln monthly returns is lower than volatility using ln daily returns. Why?
3) 10-year HPR is exactly same. Why?

Measurement interval

- Ideal time between price observations (e.g., daily, weekly, monthly) is matter of judgment.
  - From statistics point of view, more information is better.
    - Assumes prices are precise.
  - From financial markets point of view, less information may be more.
    - Prices are noisy.

      $$\sigma_{obs} = \sigma_{true} + \sigma_{noise} \quad \text{where } \sigma_{noise} \text{ is constant.}$$

    - Longer horizon means more market information (i.e. higher true volatility) relative to noise.
      - Referred to as "signal-to-noise" ratio.

Return relations

- In AIM, two primary contexts in which relation between returns are modeled.
  - Pairwise return correlations for portfolio decision-making.
  - Return regressions for characterizing risk relative to market benchmark.

Correlation

- Consider traditional asset categories.
  - Stocks: VTI - CRSP TMI (about 4,000 US stocks)
  - Bonds: BND – Bloomberg ABI (about 10,000 US bonds)
  - Cash-equivalents: EFFR
- Support file: Traditional asset categories.xlsx

Correlation

Summary statistics 20121231-20221231			
Description	EFFR	BND	VTI
n	2,518	2,518	2,518
Mean (daily)	0.0000	-0.0001	0.0004
StDev (daily)	0.0001	0.0031	0.0112
Skewness	1.2574	-1.6906	-0.8569
Autocorrelation	0.9958	0.0028	-0.1191
Minimum	0.0000	-0.0559	-0.1208
Median	0.0000	0.0001	0.0007
Maximum	0.0002	0.0413	0.0907
Mean (annual)	1.10%	-1.57%	9.60%
StDev (annual)	0.08%	4.91%	17.83%
CAGR	1.10%	-1.56%	10.07%
HPR	11.58%	-14.51%	160.90%

Correlation matrix			
	EFFR	BND	VTI
EFFR	1	0.016	-0.021
BND	0.016	1	0.087
VTI	-0.021	0.087	1

Volatilities make sense.

- Stocks are riskier than bonds.
- Bonds are riskier than cash equivalents.

Correlations make sense.

- No correlation between stocks and bonds.
- No correlation between stocks and cash.
- No correlation between bonds and cash.

Realized returns make sense but are to be used cautiously.

- CAGRs of stocks and cash are about right from historical perspective.
- Bonds earned negative return.
  - Would be unreasonable to expect bonds to have negative return moving forward.

Return regression

- Two models:
  - Market model: Return regression

$$R_{i,t} = \alpha + \beta R_{M,t} + \varepsilon_{i,t}$$

Descriptive model

Return regression

- Two models:
  - Market model

$$R_{i,t} = \alpha + \beta R_{M,t} + \varepsilon_{i,t}$$

  - CAPM: Excess return regression

$$XR_{i,t} = \alpha + \beta XR_{M,t} + \varepsilon_{i,t}$$

Economic model

where $XR_{i,t} \equiv R_{i,t} - R_{F,t}$ and $XR_{M,t} \equiv R_{M,t} - R_{F,t}$.

Called *excess returns* or *risk premiums*.

Return regression

- Use excess return regression.
  - Most appropriate from theoretical perspective.

$$R_{i,t} - R_{F,t} = \alpha + \beta\left(R_{M,t} - R_{F,t}\right) + \varepsilon_{i,t}$$

  - CAPM says expected value of "alpha" is 0.

$$E_i = R_F + \left(E_M - R_F\right)\beta_i$$
$$\Rightarrow E_i - R_{F,t} = \left(E_M - R_F\right)\beta_i$$

Market risk vs idiosyncratic risk

- Use excess return regression.
  - Expresses return relation between security and market.

$$XR_{i,t} = \alpha + \beta XR_{M,t} + \varepsilon_{i,t}$$

Estimate parameters.

Market risk vs idiosyncratic risk

- Market return does not explain security return completely.

$$XR_{i,t} = \alpha + \beta XR_{M,t} + \varepsilon_{i,t}$$

Random disturbance unrelated to market and specific to security.

Market risk vs idiosyncratic risk

- Write *total risk* of security as sum of *market risk* and *idiosyncratic risk.*

$$
\begin{aligned}
Var\left(XR_{i,t}\right) &= Var\left(\alpha + \beta XR_{M,t} + \varepsilon_{i,t}\right) \\
&= Var\left(\beta XR_{M,t} + \varepsilon_{i,t}\right) \\
&= \beta^2 Var\left(XR_{M,t}\right) + Var\left(\varepsilon_{i,t}\right)
\end{aligned}
$$

Total risk = Market risk + Idiosyncratic risk

Market risk vs idiosyncratic risk

- Adjusted R-squared from regression is

$$\bar{R}^2 = 1 - \frac{\text{Idiosyncratic risk}}{\text{Total risk}} = 1 - \frac{Var\left(\varepsilon_{i,t}\right)}{Var\left(XR_{i,t}\right)}$$

  - Measures proportion of total risk accounted for by market risk.
  - Measurement of fit (falls between 0 and 1)
    - If 1, all of security return is explained by market return.
    - If 0, none of security return is explained by market return.

Market risk vs idiosyncratic risk

- Market may be any asset category, but category should be all-inclusive.
  - Stocks
    - Common to use S&P 500, but this is wrong.
      - Does not include all US stocks.
    - Use CRSP US Total Market Index
      - Series begins 20110331.
      - Earlier version belonged to University of Chicago business school and dated back to 19251231.
      - Has become industry standard (Bloomberg symbol: CRSPTMT)
  - Bonds
    - Use Bloomberg Aggregate Bond Index.

Excess return regressions

- Support file: Excess return regressions.xlsx
  - Downloaded total return indexes for:
    - CRSPTMT: CRSP total US stock market index
    - SPX: S&P 500
    - AAPL: Shares of Apple
    - LBUSTRUU: Bloomberg aggregate US bond index
    - BND: Vanguard's total bond market ETF
      - Support file: BND fact sheet.pdf
    - TLT: BlackRock's 20+ year US Treasury bond ETF
      - Support file: TLT fact sheet.pdf

Excess return regressions

- Excel has regression tool.

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.9969
R Square	0.9938
Adjusted R Square	0.9938
Standard Error	0.0009
Observations	2518

Excel regression output is tedious and cumbersome.

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.30966	0.30966	401436.9	0
Residual	2516	0.00194	0.00000		
Total	2517	0.31160			

	Coeff	*StErr*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.00002	0.00002	1.36885	0.17117	-0.00001	0.00006	-0.00001	0.00006
X Variable 1	0.97995	0.00155	633.59052	0.00000	0.97692	0.98299	0.97692	0.98299

Excess return regressions

- Created AIM_Regression_SXR.

Excess return regression results		
	SPX	AAPL
n	2,518	2,518
α	0.0000	0.0003
s(α)	0.0000	0.0003
β	0.9800	1.1319
s(β)	0.0015	0.0231
R-squared	0.9938	0.4878
Adj. R-squared	0.9938	0.4876
Std error of estimate	0.0009	0.0131

AIM results are the same.

- Easier to use.

Excess return regressions

Excess return regression results		
	SPX	AAPL
n	2,518	2,518
α	0.0000	0.0003
s(α)	0.0000	0.0003
β	0.9800	1.1319
s(β)	0.0015	0.0231
R-squared	0.9938	0.4878
Adj. R-squared	0.9938	0.4876
Std error of estimate	0.0009	0.0131

Questions:

- Is SPX beta different from one?
- Does SPX outperform?
- Is AAPL aggressive stock?
- What percentage of AAPL's risk is unrelated to market movements?
- Does AAPL outperform?

Excess return regressions

Excess return regression results		
	BND	TLT
n	2,518	2,518
α	0.0000	0.0000
s(α)	0.0000	0.0001
β	1.0080	3.1208
s(β)	0.0129	0.0349
R-squared	0.7094	0.7606
Adj. R-squared	0.7093	0.7605
Std error of estimate	0.0016	0.0045

Questions:

- Is BND beta different from one?
- Does BND outperform?
- Is TLT aggressive bond?
- What percentage of BND's risk is unrelated to market movements?
- Does TLT outperform?

Excess return regressions

Excess return regression results		
	BND	TLT
n	2,518	2,518
α	0.0000	0.0000
s(α)	0.0000	0.0001
β	1.0080	3.1208
s(β)	0.0129	0.0349
R-squared	0.7094	0.7606
Adj. R-squared	0.7093	0.7605
Std error of estimate	0.0016	0.0045

Questions:

- How risky is TLT relative to BND for given change in interest rates?

Excess return regressions

Excess return regression results		
	BND	TLT
n	2,518	2,518
α	0.0000	0.0000
s(α)	0.0000	0.0001
β	1.0080	3.1208
s(β)	0.0129	0.0349
R-squared	0.7094	0.7606
Adj. R-squared	0.7093	0.7605
Std error of estimate	0.0016	0.0045

Questions:

- How risky is TLT relative to BND for given change in interest rates?

  3.1208/1.0080 = 3.1 times

  Why?

Excess return regressions

Vanguard Total Bond Market ETF | BND

ETF attributes

	Total Bond Market ETF	Bloomberg U.S. Aggregate Float Adjusted Index
Number of bonds	10,110	13,133
Average duration	6.5 years	6.2 years
Average effective maturity	8.9 years	8.5 years
Turnover rate[5]	69.3	—
Short-term reserves	0.0	—

TLT

iShares 20+ Year Treasury Bond ETF

Fact Sheet as of 12/31/2022

FUND CHARACTERISTICS

Weighted Average Maturity	25.64 yrs
Standard Deviation (3yrs)	15.13%
Effective Duration	17.40

Questions:

- How risky is TLT relative to BND for given change in interest rates?

  3.1208/1.0080 = 3.1 times

  Why?

  25.64/8.9 = 2.9 times

Estimation objective function

- What is OLS (ordinary least squares) regression?
  - Find values of α and β that minimize sum of squared errors.

$$Min \sum_{t=1}^{T} \varepsilon_{i,t}^{\ 2} = Min \sum_{t=1}^{T} \left(XR_{i,t} - \alpha - \beta XR_{M,t} \right)^2$$

  - Why is OLS appropriate objective function for estimating α and β?
    - May not be.
    - Chosen because mathematically tractable.

Estimation objective function

- Show OLS using brute force.
  - Support file: Excess return regressions.xlsx

Minimize sum of squared errors.		
Alpha	0.0003	
Beta	1.1319	
Sum of squared errors	0.4338	Minimize.

  - Same estimates as regression formulas but computed from brute force.

Estimation objective function

- What is LAD (least absolute deviations) regression?
  - Find values of α and β that minimize absolute deviations.

$$Min\sum_{t=1}^{T}\left|\varepsilon_{i,t}\right| = Min\sum_{t=1}^{T}\left|R_{i,t} - \alpha - \beta R_{M,t}\right|$$

  - Mitigates influence of outliers.
    - Finance data often misreported.

Estimation objective function

- Compare estimates for AAPL.

Minimize sum of squared errors.		
Alpha	0.0003	
Beta	1.1319	
Sum of squared errors	0.4338	Minimize.

Minimize sum of absolute errors.		
Alpha	0.0000	
Beta	1.1340	
Sum of squared errors	22.7658	Minimize.

Difference is small here but can be quite large, particularly for small stocks and growth stocks.

Estimation objective function

- Advantages of LAD
  - Resistant to outliers
  - Robust to departures from normality
- Disadvantages of LAD
  - Computationally expensive
  - Possibility of more than one solution
    - Solved for iteratively

Lesson summary

- AIM involves return/risk tradeoffs.
  - Must measure return and risk properly.
    - Performance measurement
    - Portfolio decision-making
    - Otherwise, GIGO.
  - Must distinguish between holding period and continuous returns.
    - Continuous returns are crucial. HPRs can be misleading.

Lesson summary

- AIM involves return/risk tradeoffs.
  - Must understand properties of return distributions.
    - E.g., mean, standard deviation, skewness, correlation.
  - Must be cognizant of data and estimation issues.
    - E.g., idiosyncrasies of data sources.
  - Must understand return on cash equivalents.
    - Not risk-free in absolute sense.
  - Use excess return regression measure performance.
  - Understand assumptions of measurement technique.
    - E.g., OLS vs LAD regression.