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Trading Relative Performance with Alpha Indexes

Jacob S. Sagi and Robert E. Whaley

Relative performance is central to investment management, and yet relative performance securities do not trade directly. Complex trading strategies must be devised to capture relative gains. The authors introduce a suite of relative performance indexes and index derivatives that offer new and attractive payoff structures. They demonstrate a variety of ways in which these products can provide a more efficient and cost-effective means of realizing investment objectives than can traditional futures and option markets.

Relative performance is at the heart of investment management. Many stock portfolio managers focus on identifying under- and overpriced stocks in hopes of "beating the market." Commonly referred to as "stock pickers," these managers take long and short positions in stocks on the basis of their company-specific analyses and price predictions. Other stock portfolio managers operate globally and focus on identifying under- and overpriced stock markets; these managers are also stock pickers, but of country-specific rather than company-specific performance. Large institutional investors, such as pension fund managers and university endowments, spread fund wealth across many asset categories, including stocks, bonds, and real estate. They constantly monitor the relative performance of each asset category in deciding how to allocate fund wealth.

As these examples illustrate, investment managers pit the performances of individual securities and security portfolios, both domestic and international, against one another. Although relative performance remains the central focus, few vehicles exist for trading relative performance directly. If a stock picker believes that a particular stock will outperform the market, she can buy the stock and sell the market by using such index products as exchange-traded funds (ETFs) or index futures. But long stock/short market is only one payoff structure, and such a position can entail unlimited downside. Suppose that the stock picker prefers a call-like payoff structure for the relative performance or otherwise wishes to limit the downside. In that case, although the investor could buy a call on the stock and a put on the market, doing so would entail paying unnecessarily for the market volatility embedded in the call and put option premiums. To avoid this scenario, the investor would have to take dynamically shifting positions in the stock and a market ETF (or in their respective derivative products). For the typical institutional or retail investor, constantly migrating funds from one security to another in response to a change in expected performance is cumbersome and costly. Exchange-traded products for relative performance promise to provide a simple and cost-effective means for implementing existing return–risk management strategies and for creating new return–risk management strategies to add to investors' arsenals.

Jacob S. Sagi is the FMRC Associate Professor of Finance and Robert E. Whaley is the Valere Blair Potter Professor of Management and co-director of the Financial Markets Research Center at the Owen Graduate School of Management, Vanderbilt University, Nashville, Tennessee.

In October 2010, NASDAQ OMX laid the groundwork for introducing relative performance index products by computing and disseminating in real time several indexes that measure the relative total return of a single stock ("target component") against the Standard & Poor's Depositary Receipt (SPDR) ETF ("benchmark component").[1] Among the names currently available are AAPL, F, GE, IBM, and RIMM. Although these indexes themselves do not trade, the U.S. SEC approved NASDAQ OMX PHLX's application to list exchange-traded option contracts on 7 February 2011, and the first relative performance index option market was launched on 18 April 2011. **Table 1** reports the company names for which options are or will be listed. The purpose of our study was to conduct an academic analysis of a suite of relative performance indexes and associated derivatives (futures and options), which includes the new NASDAQ OMX Alpha Indexes.[2]

Table 1. List of NASDAQ OMX Alpha Indexes with SEC-Approved Option Contract Markets

				Stock Return	
Stock Name	Stock Ticker	Alpha Index Ticker	Launch Date	Volatility	Correlation with SPY Return
Amazon.com	AMZN	AMZSY		32.60%	60.40%
Apple	AAPL	AVSPY	18 Apr 2011	26.70	71.10
Cisco Systems	CSCO	CSCSY		31.90	61.70
Ford Motor Co.	F	FRDSY		38.10	70.60
General Electric Co.	GE	GESPY	2 May 2011	27.40	82.30
Google	GOOG	GOOSY	20 Apr 2011	27.80	65.10
Hewlett-Packard Co.	HPQ	HPQSY		24.90	68.00
International Business Machines	IBM	IBMSY	6 May 2011	17.80	78.30
Intel Corp.	INTC	INTSY	4 May 2011	25.30	77.30
Coca-Cola Co.	KO	KOSPY		15.50	63.70
Merck & Co.	MRK	MRKSY	5 May 2011	20.60	65.70
Microsoft Corp.	MSFT	MSFSY		22.00	73.00
Oracle Corp.	ORCL	ORCSY		24.40	67.50
Pfizer	PFE	PFESY		21.30	66.20
Research in Motion Ltd.	RIMM	RIMSY		38.50	44.90
AT&T	T	ATTSY		15.10	70.70
Target Corp.	TGT	TGTSY		20.20	66.00
Verizon Communications	VZ	VZSPY		16.00	60.30
Wal-Mart Stores	WMT	WMTSY	3 May 2011	14.00	50.30
Mean				24.22%	66.48%

Notes: All the indexes listed are outperformance indexes with the benchmark being the SPDR ETF (ticker symbol SPY). The launch dates are when options on the different alpha indexes began trading. Missing dates are to be announced. The annualized volatility of each stock and the correlation with SPY were calculated by using daily return data for calendar year 2010. We obtained the data from Thomson Reuters Datastream. Over the sample period, SPY daily returns had a realized annual volatility rate of 17.9 percent.

Relative Performance Indexes

A relative performance index measures the total return performance of a target security relative to the adjusted total return performance of a benchmark, such as the S&P 500 Index. The total daily security return includes both price appreciation and dividends: $R_{S,t+1} \equiv (S_{t+1} - S_t + D_{S,t+1})/S_t$, where S_t is the target security price at the end of day t and $D_{S,t}$ is the dividend (or other distribution) paid by the security during day t. The total daily benchmark return is defined similarly: $R_{M,t+1} \equiv (M_{t+1} - M_t + D_{M,t+1})/M_t$, where M_t is the benchmark price level at the end of day t and $D_{M,t}$ is the dividend paid by the benchmark during day t. Structurally, a relative performance index is defined by the updating rule

$$I_{b,t+1} = I_{b,t}\frac{\left(1+R_{S,t+1}\right)}{\left(1+R_{M,t+1}\right)^b}, \quad (1)$$

where b is a relative risk adjustment coefficient and can be set equal to any value to reflect the security's systematic variation with the benchmark.[3] Where $b = 0$, the relative performance index is a total return index. Where $b = 1$, the right-hand side of Equation 1 corresponds to the ratio of the target and benchmark returns and the relative performance index is an outperformance index.[4]

As defined by Equation 1, the relative performance index has several noteworthy features. First, the performance measure is based on dividends as well as price appreciation in order to put all stocks on an equal footing. For example, because AAPL does not pay dividends, comparing its price appreciation to that of IBM (which often distributes a dividend yield of 2 percent or more) unfairly handicaps IBM when comparing the performances of the two stocks. Second, the index, like the value of an actual portfolio, is always positive and can be replicated by using a continuously rebalanced dividend-paying portfolio consisting of the target, the benchmark, and risk-free bonds. Mimicking such a portfolio could be prohibitively expensive from an investor's standpoint. Futures and option contracts on relative performance indexes, however, would create buy-and-hold opportunities and generate significant trading cost savings. Third, the ratio of the levels of the index at two different points in time is easily interpreted, particularly in the case where $b = 1$. If the current level of the index is 150 and its level three months ago was 120, the target security outperformed the benchmark by 150/120 – 1 = 25 percent. Finally, relative performance indexes can be readily extended to multi-asset benchmarks. For example, an asset-pricing purist may want to pit target security performance against a benchmark that includes a number of asset classes, such as stocks, bonds, real estate, and commodities. In that case, the returns of asset categories would be included in the denominator of Equation 1, effectively assigning each category its own relative risk adjustment coefficient. Appendix A provides the multifactor version of Equation 1.

Futures and Options on Relative Performance Indexes

Valuation equations for futures and option contracts on relative performance indexes can be derived analytically under the valuation assumptions of Black and Scholes (1973) and Merton (1973). Specifically, markets are assumed to be frictionless (e.g., no trading costs or different tax rates on different forms of income), and market participants can borrow or lend risklessly at a constant annualized interest rate r. The total returns on the target and benchmark securities evolve as multivariate geometric Brownian motion with constant drifts (μ_S and μ_M), volatilities (σ_S and σ_M), and instantaneous return correlation (ρ_{SM}). Under these Black–Scholes/Merton (BSM) assumptions, a relative performance index with constant relative risk adjustment coefficient b will evolve as

$$I_{b,t} = I_{b,0} \exp\left[\left(\mu_S - b\mu_M + \frac{b\sigma_m^2 - \sigma_s^2}{2}\right)t + \sigma_S B_{S,t} - b\sigma_M B_{M,t}\right], \tag{2}$$

where $B_{S,t}$ and $B_{M,t}$ are the Brownian motion variables associated with securities S and M, respectively. Under the BSM assumptions, we can see that a risk-free hedge can be formed between the index and its derivative products. The practical implication of this finding is that a risk-averse investor will value an index derivative product the same as a risk-neutral investor.[5] By now, this approach is classic. In a risk-neutral world, every security is expected to grow at the risk-free rate and is discounted to the present at the risk-free rate. So, the first step in valuing any derivative security is to calculate the distribution of future derivative payoffs by replacing μ_S and μ_M in Equation 1 with the risk-free interest rate. The risk-neutral evolution of the relative performance index is

$$I_{b,t}^{RN} = I_{b,0} \exp\left\{\left[(1-b)r + \frac{b\sigma_m^2 - \sigma_s^2}{2}\right]t + \sigma_S B_{S,t} - b\sigma_M B_{M,t}\right\}. \tag{3}$$

We can then discount the payoffs from this "risk-neutral" distribution of derivative payoffs to the present at the risk-free rate. For example, the price of a futures contract that expires at time T is calculated as $E\left(I_{b,T}^{RN}\right)$, where E( ) denotes the expectation of the quantity inside the parentheses, and the value of a European-style call option with exercise price X and time to expiration T is $e^{-rT}E\left[\max\left(I_{b,T}^{RN} - X, 0\right)\right]$. In developing analytical equations for valuing futures and option contracts on relative performance indexes, we assume that the index options are European-style and that both the futures and the options expire at the same time.[6]

The Relative Performance Index as a Dynamically Rebalanced Portfolio. As mentioned earlier, the relative performance index represents the value of a dynamically rebalanced replication portfolio. Showing the nature of the rebalancing activity is critical in developing an understanding of how relative performance indexes help complete the market. To begin, we use a simple discrete-time illustration.

The mimicking portfolio has three constituent securities: the target, the benchmark, and risk-free bonds. For every dollar long in target S, the portfolio is short b dollars in benchmark M. For simplicity, our illustration, whose time-series dynamics are shown in **Table 2**, assumes $b = 1$. On Day 0, the portfolio is long \$100 in the target, short \$100 in the benchmark, and long \$100 in risk-free bonds. Because the sales proceeds from the benchmark offset the purchase price of the target exactly, the value of the mimicking portfolio equals the value of the risk-free bonds, or \$100.

In our illustration, the subsequent levels of the total return indexes of the target and the benchmark are set arbitrarily. Over the first day, the target advances by 4.17 percent and the benchmark advances by 7.16 percent. Because the benchmark return is larger, the relative performance index falls

Table 2. Simulation of Replicating Portfolio for the $b = 1$ Relative Performance Index

	Total Return Indexes		Relative Performance Index		Hedge Portfolio					Mimicking Portfolio Value
						Dollar Income				
Day	Target	Benchmark	Level	Gain	Interest Income	Security	Benchmark	Net Gain	Payout	
0	$100.00	$100.00	100.00							$100.00
1	104.17	107.16	97.21	–2.79	$0.070	$4.17	$7.160	–$2.92	–$0.130	97.21
2	108.61	111.53	97.38	0.17	0.068	4.143	3.964	0.247	0.075	97.38
3	110.68	110.36	100.29	2.91	0.068	1.856	–1.022	2.946	0.038	100.29
4	111.01	113.83	97.52	–2.77	0.070	0.299	3.153	–2.784	–0.017	97.52
5	112.62	110.70	101.73	4.21	0.068	1.414	–2.682	4.164	–0.048	101.73

Notes: At the beginning of each day, the portfolio is rebalanced so that the position in cash and the position in the target security are set equal to the level of the index and the position in the benchmark is *short* an amount equal to the level of the index. The table documents the daily income from these positions, as well as the payout that results from rebalancing.

to 104.17/107.16 = 97.21, or by 2.79. The objective of the mimicking portfolio is to match the appreciation of the relative performance index.

Over the first day, the bond position produces $0.07 in interest income,[7] the target security position produces a gain of $4.17, and the benchmark security position produces a loss of $7.16. The net gain across the three positions is –$2.92. To bring the mimicking portfolio value to the level of the relative performance index, an additional investment of $0.13 is made in risk-free bonds (i.e., the mimicking portfolio has a negative payout or dividend on Day 1). The mimicking portfolio is then rebalanced. The long position in the target security is reduced from $104.17 to $97.21, producing a gain of $6.96. The short position in the benchmark security is likewise reduced to the same dollar value, producing a loss of $9.95. The difference, $2.99, is subtracted from the available risk-free bonds, $100.20, and the value of the risk-free bonds in the mimicking portfolio becomes $97.21, exactly the level of the relative performance index.

On Day 2, the interest income from the investment of $97.21 in risk-free bonds is $0.068, bringing the balance to $97.278. The long position in the target security rises in value from $97.21 to $97.21($108.61/$104.17) = $101.353, for a gain of $4.143, and the short position in the benchmark rises in value from $97.21 to $97.21($111.53/$107.16) = $101.174, for a loss of $3.964. To align the value of the mimicking portfolio with the relative performance index level, $0.075 is paid out, which brings the risk-free bond balance to $97.203. The mimicking portfolio is then rebalanced. The long position in the target goes from $101.353 to the new index level of $97.38, and the short position in the benchmark goes from $101.174 to $97.38. Adding the difference, $0.179, to the value of the risk-free bonds, $97.203, the new risk-free bond balance settles, not coincidentally, at the level of the relative performance index, $97.38.

Under the BSM assumptions and continuous (instead of daily) rebalancing, we can show that the portfolio payout is a constant proportion of the index level: $\delta = r - \left(\sigma_M^2 - \rho_{SM}\sigma_S\sigma_M\right)$. In other words, the relative performance index is the value of a portfolio that has a constant proportional payout rate, or *dividend yield*.[8] Note that although the index itself does not pay dividends, its value at any time is the same as the value of a portfolio that does pay dividends.[9] Note also that the dividend yield may be negative in some cases—for instance, when the risk-free interest rate is low or the correlation is low. In most cases, however, it will be positive. Suppose, for example, that the total return volatilities of AAPL and SPY (the ticker symbol for the SPDR ETF) are 26.7 percent and 17.9 percent, respectively, and that the correlation between the two is 71.1 percent.[10] If the risk-free interest rate is 2 percent, the portfolio that mimics the $b = 1$ AAPL versus SPY index would pay a constant continuous dividend yield of 2.19 percent.

Valuation of Relative Performance Index Futures. The value of a futures contract written on a relative performance index can be derived by calculating the "risk-neutral" expected value of the index at the expiration of the futures contract. That is, from Equation 3,

$$F_b = \mathrm{E}\left(I_{b,T}^{RN}\right) = I_b e^{(r-\delta)T}, \tag{4}$$

where T is the time remaining to expiration of the futures contract and $\delta = br - (b\sigma_M)/2[(1 + b)\sigma_M - 2\rho_{SM}\sigma_S]$. The term δ can be viewed as the generalization of the previously mentioned payout rate to the case where b can be different from 1. Note that Equation 4 is the usual cost-of-carry relationship for a stock with a payout yield of δ.[11]

Valuation of Relative Performance Index Options. Under the BSM assumptions, the simplest way to value European-style options on relative performance indexes is to first apply Black's futures option formula (1976). Because the futures price and the relative performance index level are identical at the futures/option expiration, the value of a European-style option on the relative performance index equals the value of a European-style option on the index futures.[12] The value of a European-style call option on a relative performance index futures contract is

$$C_b = e^{-rT}\left[F_b N(d_1) - XN(d_2)\right], \tag{5}$$

where X is the exercise price of the option, $N(d)$ is the cumulative normal density function with upper integral limit d, and the upper integral limits are

$$d_1 = \frac{\ln(F_b/X) + 0.5\sigma^2 T}{\sigma\sqrt{T}}$$

and

$$d_2 = d_1 - \sigma\sqrt{T}.$$

Because the underlying source of uncertainty is the ratio of two lognormally distributed prices, the volatility rate in the expressions for d_1 and d_2 is

$$\sigma = \sqrt{\sigma_S^2 + b^2\sigma_M^2 - 2b\rho_{SM}\sigma_S\sigma_M}. \tag{6}$$

Then, to value a European-style call option on a relative performance index, we can substitute Equation 4 into both Equation 5 and the expressions for the upper integral limits d_1 and d_2 that accompany Equation 5 to obtain

$$C_b = I_b e^{-\delta T} N(d_1) - Xe^{-rT} N(d_2), \tag{7}$$

where

$$d_1 = \frac{\ln(I_b/X) + (r - \delta + 0.5\sigma^2)T}{\sigma\sqrt{T}}$$

and

$$d_2 = d_1 - \sigma\sqrt{T}.$$

The value of a European-style put option on a relative performance index follows straightforwardly from put–call parity.[13] More specifically, the payoff that results from purchasing a call option and selling a put option (with the same exercise price and time remaining to expiration) equals the value of the index at expiration less the exercise price. The present value of these payoffs yields the put–call parity relationship for European-style options:

$$C_b - P_b = I_b e^{-\delta T} - Xe^{-rT}. \tag{8}$$

Substituting Equation 7 into Equation 8 and isolating the put value, we obtain

$$P_b = e^{-rT} XN(-d_2) - I_b e^{-\delta T} N(-d_1). \tag{9}$$

Hedging Relative Performance Index Futures and Options. The valuation equations for the futures on relative performance indexes (Equation 4) and for the options on relative performance indexes (Equation 7 and Equation 9) allow us to develop analytical expressions for the metrics used in dynamic risk management (i.e., delta, gamma, and vega), fully discussed in Appendix B. Several results are noteworthy. First, because the underlying payoffs depend on changes in two distinct securities, delta risk management will necessarily require a simultaneous position in both security S and its corresponding benchmark M. As one might suspect from the links between portfolio formation and relative performance indexes, for every dollar of security S used to hedge a relative performance index derivative, one must take a position of $-b$ dollars in the benchmark M. Thus, although at first blush a relative performance index derivative might seem twice as complicated to hedge as a derivative product on a single underlying asset, in practice the hedging position in M is completely determined by the hedging position in S.

A second noteworthy result is that the futures vega is not zero because the futures price depends on the volatilities of S and M. Indeed, we must consider a new type of vega, one that corresponds to price sensitivity to changes in the correlation between S and M. Finally, the gamma with respect to the benchmark and the cross-gamma (i.e., the sensitivity of the benchmark delta to both the benchmark and the stock) of the futures price are not zero. Intuitively, the current value of the index is inversely proportional to the cumulative performance of the benchmark, and such inverse dependence is necessarily a convex function. This intuition implies that the gamma of the index with respect to the benchmark will not be zero and that extra care should be taken when delta hedging index derivatives against large benchmark movements.

Using Relative Performance Index Products

With the valuation mechanics for the relative performance index products in hand, we can now turn to a series of illustrations of the potential benefits of these products. To keep matters simple and realistic, we will focus on the NASDAQ OMX Alpha Indexes ("alpha indexes"), in which the risk adjustment coefficient equals 1—that is, $b = 1$.[14] Options on the alpha indexes began trading on the NASDAQ OMX PHLX on 18 April 2011, with the launch of contracts on AVSPY (i.e., the alpha index that pits the total return performance of Apple against the total return performance of the SPDR ETF). Option contracts on Google's alpha index were launched on 20 April and those on General Electric's alpha index on 2 May (see Table 1 for a list of all the SEC-approved option contract markets, together with the ticker symbols of both the stock and the alpha index). More option contract markets are expected to follow. Markets for option contracts on relative performance indexes where $b \neq 1$ and for futures contracts on alpha indexes are in the planning stage.

Estimating and Trading Correlation. One of the most intriguing benefits of relative performance index derivatives is that they provide a means of estimating and trading the expected future correlation between the returns of the target and the benchmark. When $b = 1$, the dividend yield term becomes $\delta = r - \sigma_M(\sigma_M - \rho_{SM}\sigma_S)$ and the futures price is

$$F = Ie^{\sigma_M(\sigma_M - \rho_{SM}\sigma_S)T}. \quad (10)$$

Note that, depending on whether σ_M is greater or less than $\rho_{SM}\sigma_S$, the futures contract may trade at a premium or at a discount relative to the underlying index. More important, if the futures price, F, for a contract with a maturity of T is observable, one can use Equation 10 to solve for ρ_{SM}:

$$\rho_{SM} = \frac{\sigma_M - \dfrac{\ln(F/I)}{T\sigma_M}}{\sigma_S}. \quad (11)$$

The alpha index level, I, is readily observable, and the expected return volatilities for the target and the benchmark, σ_S and σ_M, can be accurately estimated by using the volatilities implied by the prices of exchange-traded options on the target and the benchmark. Note that, by definition, the correlation estimate in Equation 11, like implied volatilities from stock option prices, is forward looking.

A by-product of being able to estimate the implied correlation from alpha index derivatives prices is that the beta of the target security can be inferred. The definition of beta is

$$\beta_{SM} \equiv \rho_{SM}\frac{\sigma_S}{\sigma_M}. \quad (12)$$

The implied correlation from alpha index futures (or option) prices, together with the implied volatilities from exchange-traded stock and index option prices, can be used to compute expected future beta. Such an estimation approach may be particularly useful given that current approaches to estimating beta involve using a time series of past return data and assuming that beta is constant over the entire time-series history. Hence, typical beta estimates are inherently backward looking and behave like a moving average over time. Finance theory, however, has long recognized that the beta of a stock is forward looking and changes with the nature of a company's business, its financial and operating leverage, the macroeconomy, and other factors—an obvious conundrum.

Implied correlations can also be computed by using the prices of alpha index options and the call and put valuations (Equations 7 and 9). To illustrate the computation of implied correlation, we can use the bid–ask quote midpoints of at-the-money call options on the AAPL, SPY, and AVSPY at the close on Friday, 29 April 2011. AAPL's share price closed at \$349.70, and the \$350 call expiring on 21 May 2011 closed at \$7.625. With a 1.475 percent interest rate, AAPL's call-option-implied volatility rate over the next 20 days was 23.38 percent. At the same time, SPY closed at \$136.78, and the \$137 May call closed at \$1.45, producing a call-option-implied volatility of 11.77 percent. Finally, the AVSPY index closed at 131.71, and the 132 May call closed at 2.45. Using the AAPL and SPY implied volatilities in the call option formula (Equation 7) and the call price of \$2.45, the implied correlation between AAPL and SPY returns over the next 20 days is 0.4417. To put these values in context, we used the daily prices (and dividends, if applicable) of AAPL, SPY, and AVSPY over the 82-day period of 3 January 2011–29 April 2011 to compute the historical return volatilities and return correlation. The annualized historical volatilities of AAPL and SPY were 22.03 percent (23.38 percent implied) and 12.09 percent (11.47 percent implied), and the historical correlation between the returns of AAPL and SPY was 0.5883 (0.4417 implied).

The difference in the historical and implied parameter estimates has two important implications. Let us first consider historical versus implied beta. Using Equation 12 and four months of daily historical return data for the period ended 30 April 2011, we computed AAPL's beta: 1.0718. As mentioned earlier, before the advent of alpha index options, the backward-looking, historical estimator of beta was all that was available to investors and other analysts. Using market information at the close on 30 April 2011, we found that the forward-looking, call-option-implied estimate of beta is a much more defensive-looking 0.8773.[15] This finding is sensible: The historical estimate uses data that overlap an earnings announcement, and the option is due to mature before the next earnings announcement. Recent studies have found that systematic risk increases in the period around an earnings announcement, which suggests that AAPL's correlation with the market should be lower between earnings announcements (see, e.g., Patton and Verardo 2010; Savor and Wilson 2011). The rich array of research projects that will be possible as the historical data for alpha index option prices accumulate is an exciting prospect. Among the questions that will undoubtedly be explored are the following: What is the time-series behavior of the difference between implied and historical correlation/beta? Is implied correlation a better predictor of future realized correlation than historical correlation? Is implied beta a better predictor of future realized beta than historical beta? Are implied betas systematically related to news events at a macro or company-specific level?

Another exciting prospect is the ability to trade correlations. For the typical stock, a higher correlation implies lower prices for index futures and call options. A trader who believes correlations will increase (decrease) can sell (buy) index futures or call options to benefit from his insights. Using our AVSPY example, let us suppose that a trader believes that the implied correlation of the at-the-money May call, 0.4417, is an aberration and that the price of the call will quickly adjust to the historical correlation level of 0.5883. If his view is correct, the call price would fall from $2.45 to $2.16, or 12 percent. Selling the call would generate a profit of $0.29. Naturally, if the correlation movement is not immediate, the alpha index may move higher, which would cancel the decline in the option value arising from the correlation increase. To isolate the expected correlation increase from the risk of index movement, the short call option position can be hedged by using the deltas calculated in Appendix B.[16] If volatility risk is also a concern, the vegas in Appendix B can also be used in setting the appropriate hedge.

Efficiency Gains to Trading Relative Performance by Using Index Derivatives. Absent a specific view on individual stock performance, an equity investor should hold a well-diversified stock portfolio. With a strong view that a particular stock will outperform the market, however, an investor may want to devote a large portion of portfolio wealth to the individual stock rather than to the market. But buying the stock directly is not a "clean" way to implement the view that the stock will outperform the market. To illustrate, let us consider the first day of trading of options on AVSPY: Monday, 18 April 2011. On 15 April 2011, AAPL closed at $327.46, SPY closed at $132.04, and AVSPY closed at 128.00. On the morning of 18 April, Standard & Poor's cut its long-term outlook on U.S. debt to negative. The market dropped quickly at the open and then partly recovered during the day, with SPY closing at $130.56, down 1.12 percent. AAPL's share price increased to $331.56 over the day, posting a gain of 1.34 percent. But AAPL was not immune to the market's drop. Had an investor "hedged the market risk" of AAPL by buying AVSPY (instead of AAPL shares), the gain would have been 2.58 percent (i.e., AVSPY closed at 131.30).[17] By construction, AVSPY is less exposed than AAPL to events that move the entire market.

Alpha Index Derivatives and Buy-and-Hold Strategies. We now turn to comparing alpha index futures and option positions with alternative buy-and-hold strategies that use existing exchange-traded products. In highlighting the differences between the alpha indexes and traditional products, we will show how these new instruments help "complete the market" for investors interested in relative performance.

Long Stock/Short Benchmark. Consider an investor who has $100 to invest and wants to speculate that the price of a particular stock will rise relative to a benchmark. One possible trading strategy that uses currently traded securities is to buy $100 of the target company's shares, sell $100 of the benchmark ETF, and buy $100 of risk-free bonds.[18] The overall value of this three-security, passive position is $100. Assuming that alpha index futures (alpha futures) are also traded, the investor could also pursue a similarly purposed, passive strategy by buying $100 of risk-free bonds and an equal amount of alpha futures.[19] Over a very short horizon, the benefits of both strategies are the same. Over longer periods, however, the passive long target/short benchmark position can become unbalanced, exposing the investor to more downside risk. Suppose, for example, that the target and the benchmark are priced at $100 and the alpha index is at a level of 100 at the beginning of the investment horizon. The long target/short benchmark position has a value of $100, as does the fully collateralized alpha futures position. Now suppose that over the investment horizon, the stock falls to $50 and the benchmark rises to $200. The gain on the long target/short benchmark position is –$150, whereas the gain on the alpha futures is –$75. The total value of the long–short strategy becomes negative because the loss exceeds the value of the risk-free bonds, illustrating the unlimited liability of the short benchmark position. The value of the fully collateralized alpha futures position, however, is $25, well above its minimum level of $0.

Figure 1 and **Figure 2** provide a general comparison of the position values at the end of the investment horizon. Figure 1 shows the fully collateralized alpha index futures position value as a function of the end-of-horizon target price and the benchmark price. Note the limited liability of this position. As the target price falls, the position value approaches but does not go below zero (holding the benchmark price constant). As the target price rises while the benchmark price is held constant, however, the position value increases with the target price. If the target price and the benchmark price rise together at the same rate, the position value holds constant at a level of $100. If the target price rises while the benchmark falls, the position value increases at an increasing rate. In comparison, Figure 2 shows the value of the long target/

Figure 1. End-of-Horizon Value of a Long Fully Collateralized Alpha Index Futures Position with an Initial Investment of $100

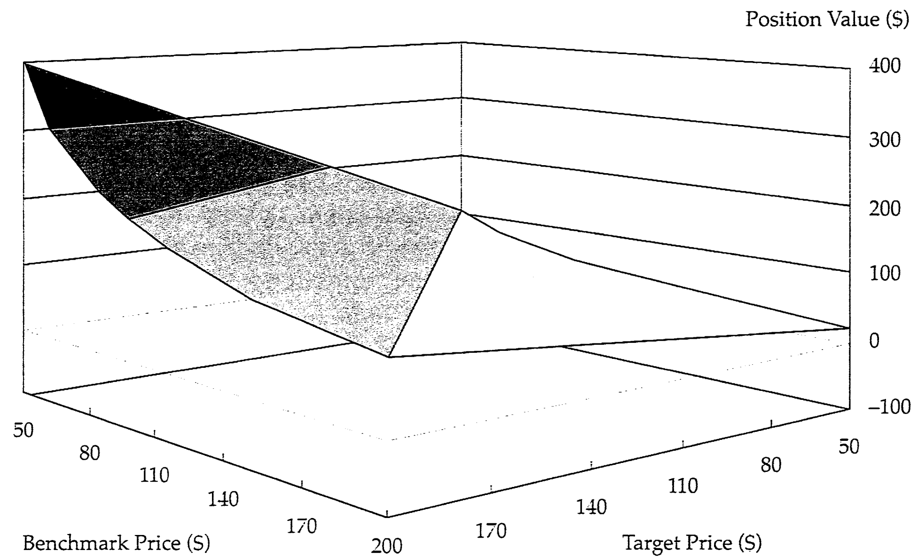
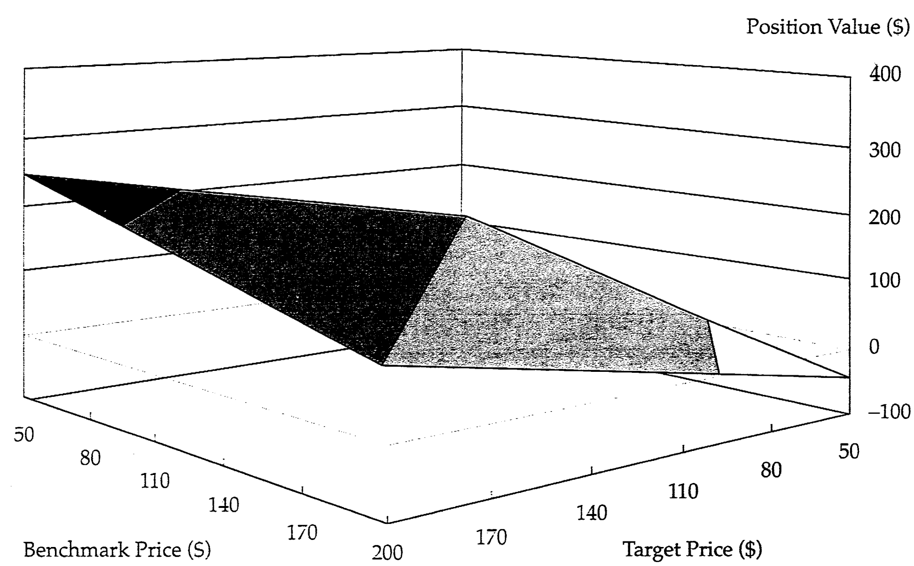


Figure 2. End-of-Horizon Value of a Long Target/Short Benchmark/Long Risk-Free Bond Position with an Initial Investment of $100



short benchmark/long risk-free bond position. Note that the surface is flat and does not have limited liability; the unlimited liability is not driven by the target—as the target price falls, the position value falls to zero—but, rather, by the benchmark. As the benchmark price rises, the position value can eventually become negative. At a target price of $50 and a benchmark price of $200, the position value is –$50, as noted in the previous example. As in the fully collateralized alpha index futures position, the position value remains at a level of $100 if the target and the benchmark rise together at the same rate. If the target price rises while the benchmark price falls, however, the position value increases at a constant rate.

In general, demonstrating that one strategy (e.g., the fully collateralized alpha futures position) dominates another (e.g., the long target/short benchmark/long risk-free bond position) is impossible. The reason is that the choice of strategy depends on the nature of investor preferences, which, in turn, are a function of the properties of the strategy return distributions, among other things. Nonetheless, we can glean some information about the difference in return performance of the two strategies at hand by using Monte Carlo simulation, which requires certain parameter assumptions. We can assume that the expected returns of the target and the benchmark, as well as the risk-free rate, are equal to zero to negate the effects of different rates of price appreciation. The volatility rates of the target (AAPL) and the benchmark (SPY) are set equal to 26.7 percent and 17.9 percent, respectively. These values are drawn from Table 1, as is the assumed correlation between the returns, 0.711. We focus on simulating the realized *return difference* between the two strategies. With a three-month investment horizon and 20,000 simulation runs, the standard deviation of this difference is 0.86 percent and the span between the 10th and 90th percentiles of the return differential distribution (the 10–90 return differential) is 1.75 percentage points (pps). In this case, the dispersion in the return difference between the two strategies is modest because the investment horizon is short. With a two-year horizon, the standard deviation of the return differential is 7.69 percent and the 10–90 return differential is 13.9 pps.

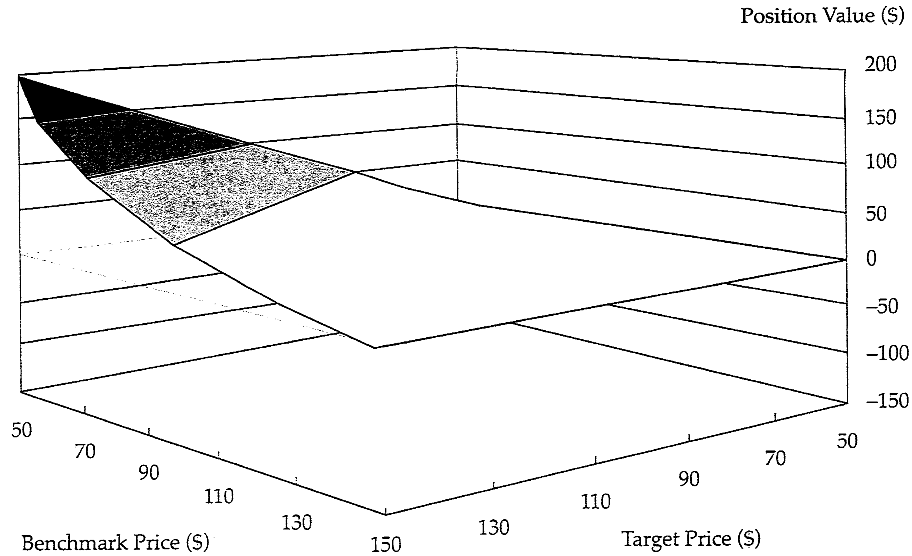
Long Target Call/Short Benchmark Call. Investors often prefer to use optionlike structures in their trading strategies. To capture relative performance, an investor could buy an at-the-money call option on the stock and sell an at-the-money call option on the benchmark. Again, our illustration assumes initial target, benchmark, and index levels of $100. Instead of buying the target and selling the benchmark directly, however, the investor buys a target call option with an exercise price of $100 and sells a benchmark call option with an exercise price of $100. If the target price falls to $50 and the benchmark price rises to $200, as in the previous example, the long target call would expire worthless at the end of the investment horizon because the target price, $50, is below the exercise price, $100. The benchmark call, however, is $100 in the money at expiration. Because the investor is short the call, he loses $100. As an alternative strategy, the investor could choose to buy an at-the-money call on the alpha index (alpha call). At the end of the call's life, the alpha index is at 25, and the call expires worthless because the alpha call strategy has limited liability.

To examine the potential upside of the alpha call, we can reverse the price movements by letting the target price go from $100 to $200 and the benchmark price go from $100 to $33.33. The long target call/short benchmark call strategy would have a payoff of $100 at expiration because the target call is $100 in the money and the benchmark call is worthless. The alpha index, however, rises to a level of 600, leaving the alpha call $500 in the money. The alpha call strategy has greater upside potential.

Figure 3 and **Figure 4** provide a general comparison of the end-of-horizon position values. Figure 3 shows the value of the alpha call as a function of the target and benchmark prices. The call option has an exercise price of 100. Note that as the target price falls and the benchmark price rises, the alpha call goes to its lowest possible value, $0. As the target price rises, the call value rises. As the benchmark price falls, however, the call value rises at an increasing rate because the benchmark price is in the denominator of the alpha index. Figure 4 shows the value surface of the long target call/short benchmark call portfolio. The value of this position is scaled to match the value of the at-the-money alpha call option in Figure 3 to ensure that we are comparing strategies with equal dollar investments. In general, the scale factor is greater than 1 because the proceeds from the sale of the benchmark call are used to offset the purchase price of the target call. As in the alpha call, the value of the long target call/short benchmark call position falls as the target price falls and/or the benchmark price rises. Unlike the alpha call value, however, the position value may become negative. Moreover, as the target price rises and the benchmark price falls, the position value rises, albeit not to the same levels as the alpha call value.

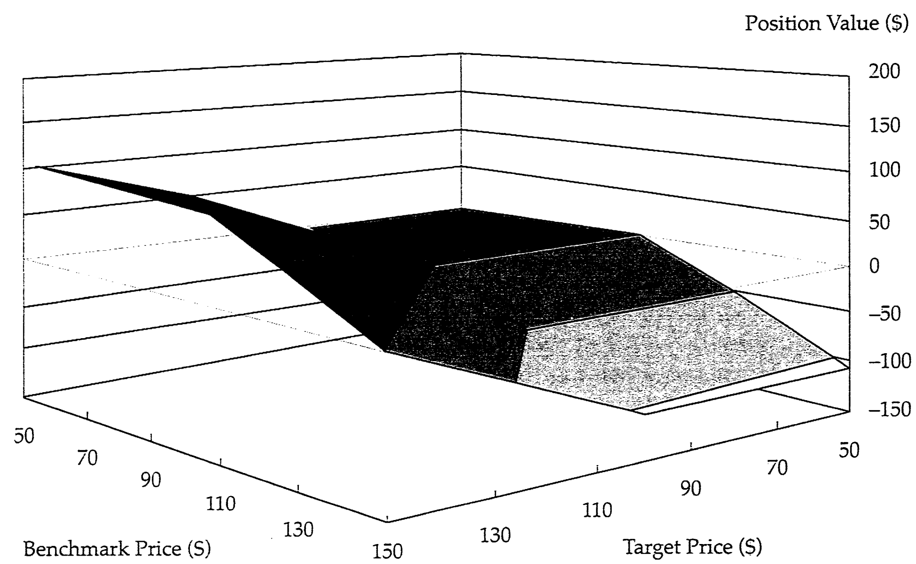
The dramatic difference in the performances of these two strategies can also be demonstrated with Monte Carlo simulation. Using the parameters

Figure 3. End-of-Horizon Value of an At-the-Money Call Option on an AVSPY Alpha Index



Note: The alpha call option has an exercise price of 100 and is written on an alpha index whose level is 100 at the beginning of the horizon.

Figure 4. End-of-Horizon Value of a Long Target/Short Benchmark Call Position



Note: Both call options have exercise prices of 100, and the position is normalized so that the beginning-of-horizon value equals that of the AVSPY call option in Figure 3.

from our AVSPY illustration, with a three-month investment horizon and 20,000 simulation runs, the standard deviation of the return differential is a whopping 261 percent and the 10–90 return differential is 611 pps.

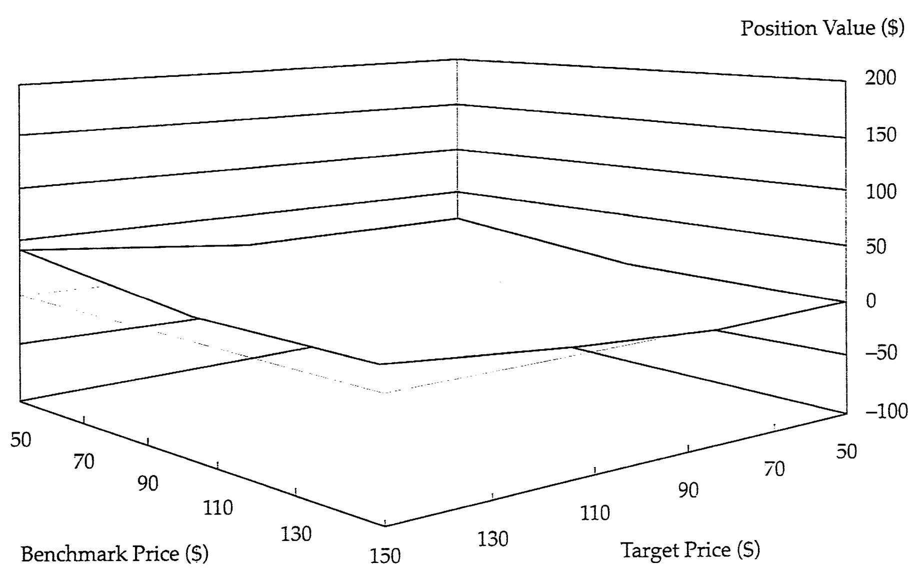
Long Target Call/Long Benchmark Put. Another option strategy designed to capture relative performance is to buy an at-the-money target call and an at-the-money benchmark put. Let us assume that the target price falls from $100 to $50 and the benchmark price rises from $100 to $200. The long target call expires out of the money, as does the long benchmark put. Hence, the "straddle" expires worthless, similar to the alpha call. If the reverse happens—the target price rises to $200 and the benchmark falls to $50—the target call is $100 in the money, the put is $50 in the money, and the straddle value is $150. The alpha call, however, has a value of $300.

Figure 3 and **Figure 5** provide a general comparison of the end-of-horizon values of the two strategies. Figure 3 shows the value surface of the alpha call as a function of the target and benchmark prices. Figure 5 shows the value surface of the long target call/long benchmark put position at expiration. Here, too, the call–put position is scaled so that its initial value matches that of the alpha call. In general, the scale factor is less than 1 because two options are being purchased and the target call alone has a higher premium than the alpha call. Like the alpha call value, the value of the long target call/long benchmark put position falls as the target price falls and/or the benchmark price rises. And like the alpha call, the call–put position never falls below zero. As the target price rises and the benchmark price falls, however, the call–put position value does not rise to the same levels as the alpha call. Although not quite as dramatic as the previous return differential comparison, the standard deviation of the three-month holding period return differential between the long alpha call and the long target call/long benchmark put is quite substantial at 113 percent; the 10–90 return differential is 281 pps.

Another important difference between the long target call/long benchmark put strategy and the alpha call strategy is the exposure to unexpected changes in market volatility. In general, the value of the former strategy is highly affected by changes in market volatility because both the call and the put are exposed to market risk and move in the same direction in reaction to news about market volatility. In contrast, the value of the alpha call strategy has much less volatility risk because the alpha index is designed to reduce or eliminate the market risk embedded in stock returns.[20]

Costs of Dynamic Replication. The three main strategy comparisons illustrate the distinctive features of alpha index derivatives. These products can simultaneously provide downside protection and reduce exposure to market volatility. Although existing exchange-traded products can be combined to create a passive, directional bet on relative performance, they cannot be combined to create a passive, directional bet on the alpha index or its derivative products. To do so would require dynamic trading strategies, which are prohibitively expensive, at least for retail investors.

Figure 5. End-of-Horizon Value of a Long Target Call/Long Benchmark Put Position



Note: All options have exercise prices of 100, and the position is normalized so that the beginning-of-horizon value equals that of the AVSPY call option in Figure 3.

To get a rough sense of the trading costs in replicating alpha products, we performed a set of simulations in which we assumed that the investor was to pay a proportional cost of $0.01 per share; a fixed cost of $1.00 for trading any quantity of the target, benchmark, or bond; and an additional 0.20 percent net fee for a short sale.[21] The trading strategy aimed to replicate a $1,000 investment in a three-month at-the-money alpha call option on the AVSPY alpha index by using standard delta-hedging techniques.[22] The volatility and correlation parameters are drawn from Table 1.

The simulation results are quite striking. To achieve a standard tracking error no larger than about 5 percent, the average trading costs are around 110 percent of the value of the position! The reason is that hedging an at-the-money call requires frequent rebalancing (about 300 times over the three months to achieve the 5 percent standard tracking error). Each time the position is rebalanced, the investor must pay $1 per trade—or $3 in total to trade in risk-free bonds, AAPL, and SPY—amounting to about $900 over the life of the option. The remaining $200 in average costs is generated by the proportional transaction costs and is still staggeringly large. In other words, even a $1 million position in at-the-money calls would require an expenditure of roughly 20 percent of the position value in trading costs if the standard tracking error is to be kept below 5 percent.

Table 3 documents simulation results of tracking errors and trading costs for various three-month AVSPY alpha call options. The initial investment is assumed to be $10,000, and the replicating portfolio is assumed to be rebalanced daily (90 trading periods after the initial position setup) over the life of the option. Note that the at-the-money alpha call now has a tracking error of 9 percent as opposed to the 5 percent in the earlier example because the replicating portfolio is rebalanced only 90 times as opposed to 300. Out-of-the-money calls are the most difficult and costly to replicate because the high leverage implicit in such options requires large positions in the target and benchmark relative to the option value. Even small fluctuations in the underlying prices can greatly unbalance the portfolio. In addition, the large trades required for rebalancing entail higher costs. The 10 percent out-of-the-money alpha call has an average tracking error of 34 percent and an average total trading cost of 36 percent of the option's value. Overall, the lesson from this exercise is that although forward/futures positions on an alpha index might be relatively inexpensive to implement through dynamic trading, replicating alpha options is not.

Alpha Index Options Are Relatively Inexpensive. The fact that alpha index products are less exposed to market risk than the typical underlying target means that alpha index options are less expensive than standard stock options. The reason is simple: Stock options are based on the total risk of the stock (i.e., the sum of market risk and idiosyncratic risk), whereas alpha index options are essentially based on the difference in risk between the target and the benchmark. Because option value varies directly with volatility, alpha index options are cheaper.

To illustrate, let us return once more to the AVSPY example. Suppose that an investor decides to buy an at-the-money call option with an exercise price of 100 and three months remaining to expiration. The investor estimates that the expected future return volatility of AAPL is 26.7 percent and

Table 3. Transaction Costs and Tracking Error for Delta-Hedging Portfolios of Positions in Three-Month Futures and Options Written on the AVSPY Alpha Index

Derivative Product	Standard Tracking Error	Average Fixed Costs	Average Proportional Costs	Average Total Costs
Futures	0.10%	$274	$ 30	3%
Call option				
$X = 80$	0.50%	$274	$ 160	4%
$X = 90$	2	274	400	7
$X = 100$	9	272	1,300	16
$X = 110$	34	268	3,300	36

Notes: This table reports the standard deviation of the tracking error, defined as the difference between the option value and the value of the tracking portfolio at expiration *in the absence of transaction costs*. The table also reports a breakdown of the accrued trading costs at the option expiration. The fixed costs can accrue to more than $273 because costs are capitalized. They can also be below that value for deep-out-of-the-money options because certain price paths lead to essentially zero option values well before the option expiration.

that of SPY is 17.9 percent and that the expected correlation between AAPL and SPY returns is 0.711. The risk-free interest rate is 2 percent. Under the BSM assumptions, the value of an at-the-money call option on AAPL is $11.53. At the same time, the value of an at-the-money call option on AVSPY is $7.24, corresponding to a discount of 37 percent. The volatility used in the valuation of the alpha index option, given by Equation 6, is 18.8 percent. So long as $\rho_{SM} > \sigma_M/2\sigma_S$ (i.e., so long as the beta of the stock is greater than 1/2), the volatility of the index (i.e., σ) is guaranteed to be smaller than the volatility of the target (i.e., σ_S) and the value of an alpha call is likewise guaranteed to be lower than the value of a target call. Note that because the average stock has a beta of 1, alpha calls should be cheaper than target calls for most stocks.

Alpha Index Products Are Based on Total Return. Earlier, we argued that to make a fair comparison between the performances of two securities, one should consider their *total return*, which includes income distribution as well as price appreciation. There are other advantages to using total return in measuring performance. Standard exchange-traded stock futures and option contracts are based on only the price appreciation of the underlying stock. Because a relative performance index incorporates dividends into the performance calculation, its payoffs implicitly include the automatic reinvestment of security income without incurring transaction costs. In addition, relative performance index options are not susceptible to the trading games played in the stock option market when deep-in-the-money options remain unexercised.[23]

Conclusion

In this article, we introduced certain relative performance index products: option and futures contracts written on an index that tracks the total return performance of a target security relative to that of a benchmark security. In particular, we illustrated how relative performance derivatives can provide investors with an efficient and cost-effective means of tailoring their risk–reward opportunity set. We also derived valuation formulas for option and futures contracts on relative performance indexes as well as their associated "Greeks" for those interested in the dynamic risk management of relative performance index products.

In April 2011, the NASDAQ OMX launched trading in options on its alpha indexes—relative performance indexes in which the relative risk adjustment is set equal to 1 (i.e., $b = 1$). These new indexes track the value of a continuously rebalanced portfolio that is long the target security, short the benchmark security in an equal dollar amount, and long risk-free bonds. If the target security outperforms the benchmark over the next instant of time, the equivalent long target/short benchmark position is increased, as is the long position in risk-free bonds. Likewise, if the target security underperforms the benchmark, the equivalent long target/short benchmark position is reduced, as is the long position in risk-free bonds. By dynamically rebalancing the portfolio in such a manner, the investor is ensured downside protection in the sense that the portfolio value can never become negative. But therein lies the dilemma. Such dynamic risk management strategies are beyond the reach of most investors because of excessive trading costs and tracking error. At the same time, investors cannot replicate such a strategy's payoff structure by using buy-and-hold strategies with currently traded securities. Consequently, alpha index products help complete the market. Because these products are claims on the index itself, they provide investors with a means of creating buy-and-hold strategies with the desired payoff structure in a more efficient and cost-effective manner.

Options on alpha indexes provide several additional benefits. Generally speaking, they are cheaper than options on the target security because their prices depend largely on a security's idiosyncratic risk, not its total risk. They also offer a different payoff structure than related positions that use standard calls and puts on the underlying target and benchmark securities. In particular, the new index call options simultaneously have downside protection and an accelerated upside relative to static positions in existing instruments. Finally, an exciting aspect of the new index options is that they present investors with the ability to trade the correlation between the target and benchmark securities. Indeed, we have shown that option-implied correlations (which could be used to calculate option-implied target capital asset pricing model betas if the benchmark is a market index) are both forward looking and potentially more informative about future correlations than are historical estimates.

We are grateful for financial support from NASDAQ OMX and for comments by Dan Carrigan, Paul Jiganti, Eric Noll, Mark Rubinstein, and Walt Smith.

This article qualifies for 1 CE credit.

Appendix A. Multi-Asset Relative Performance Indexes

The benchmark used in the definition of the complex of relative performance indexes in Equation 1 can be extended to include multi-asset benchmarks. Suppose, for example, that the desired benchmark has n different asset classes (i.e., stocks, bonds, real estate, and commodities) and that each asset class constitutes w percent of the overall benchmark portfolio, where $\sum_{i=1}^{n} w_i = 1$. Under these assumptions, the updating rule for the multi-asset relative performance index is

$$I_{b,w;t+1} = I_{b,w;t} \frac{\left(1+R_{S,t+1}\right)}{\prod_{i=1}^{n}\left(1+R_{i,t+1}\right)^{w_i b}},$$

where b is as before and w is a vector of benchmark asset allocation weights (i.e., w_i, $i = 1, \ldots, n$). With a relative performance so defined, a change in the log index corresponds to the difference between the instantaneous performance of target security S and the weighted and scaled instantaneous performances of the benchmark securities—that is,

$$\ln I_{b,w;t+1} - \ln I_{b,w;t} = \ln\left(1+R_{S,t+1}\right) - b\sum_{i=1}^{n} w_i \ln\left(1+R_{i,t+1}\right).$$

The futures and option valuation equations for these multifactor benchmarks are also analytically tractable and are available from the authors upon request.

Appendix B. Risk Metrics for Derivatives on Relative Performance Indexes

Under the BSM option valuation assumptions, we showed that the value of a futures contract written on a relative performance index with a constant relative risk adjustment coefficient of b is

$$F_b = I_b e^{(r-\delta)T}, \tag{B1}$$

where

F_b = the futures price

I_b = the relative performance index level

r = the annualized risk-free interest rate

T = the futures time remaining to expiration in years

δ = $br - (b\sigma_M)/2[(1+b)\,\sigma_M - 2\rho_{S,M}\sigma_S]$

σ_S and σ_M = the volatility rates of the security and the benchmark

$\rho_{S,M}$ = the correlation between the returns of the security and the benchmark

We also showed that the values of the European-style call and put options on the relative performance index are

$$C_b = I_b e^{-\delta T} N(d_1) - Xe^{-rT} N(d_2) \tag{B2}$$

and

$$P_b = e^{-rT} XN(-d_2) - I_b e^{-\delta T} N(-d_1), \tag{B3}$$

where

X = the exercise price of the option

T = the time remaining to option expiration (i.e., same time as futures)

$N(d)$ = the cumulative normal density function with upper integral limit d

σ = $\sqrt{\sigma_S^2 + b^2\sigma_M^2 - 2b\rho_{S,M}\sigma_S\sigma_M}$

The upper integral limits are

$$d_1 = \frac{\ln\left(I_b/X\right) + \left(r-\delta+0.5\sigma^2\right)T}{\sigma\sqrt{T}}$$

and

$$d_2 = d_1 - \sigma\sqrt{T}.$$

The put–call parity relationship for European-style options is

$$C_b - P_b = I_b e^{-\delta T} - Xe^{-rT}. \tag{B4}$$

We can compute the risk metrics (i.e., delta, gamma, and vega) of the futures and options on the basis of these equations.

Futures Deltas

Under Equation B1, the delta of the futures with respect to the underlying relative performance index is

$$\Delta_{F,I} = e^{(r-\delta)T}. \tag{B5}$$

The deltas with respect to the prices of the security and the benchmark are

$$\Delta_{F,S} = \frac{I_b}{S} e^{(r-\delta)T} \tag{B6}$$

and

$$\Delta_{F,M} = -b\frac{I_b}{M} e^{(r-\delta)T}. \tag{B7}$$

Option Deltas

Under Equation B2 and Equation B3, the deltas of the call and put with respect to a change in the underlying index are

$$\Delta_{c,I} = e^{-\delta T} N(d_1) \tag{B8}$$

and

$$\Delta_{p,I} = -e^{-\delta T} N(-d_1). \tag{B9}$$

Because the alpha options may be hedged by using shares of the target or the benchmark, the deltas with respect to the per share target price, S, are

$$\Delta_{c,S} = \frac{I_b}{S}\Delta_{c,I} \tag{B10}$$

and

$$\Delta_{p,S} = \frac{I_b}{S}\Delta_{p,I}, \tag{B11}$$

and the deltas with respect to the per share benchmark price, M, are

$$\Delta_{c,M} = -b\frac{I_b}{M}\Delta_{c,I} \tag{B12}$$

and

$$\Delta_{p,M} = -b\frac{I_b}{M}\Delta_{p,I}. \tag{B13}$$

Futures Gammas

Equation B5 shows that the delta is not a function of the relative performance index level; so, the futures gamma with respect to the index is zero, which is also true of the gamma with respect to the target, S. The gamma with respect to the benchmark price is not zero, however, and is given by

$$\Gamma_{F,MM} = b(b+1)\frac{I_b}{M^2}e^{(r-\delta)T}. \tag{B14}$$

The cross-gammas are

$$\Gamma_{F,SM} = \Gamma_{F,MS} = -b\frac{I_b}{SM}\Delta_{F,I}. \tag{B15}$$

Option Gammas

The gamma of a call option with respect to the underlying index is the same as that for the put option:

$$\Gamma_I = \Gamma_{c,I} = \Gamma_{p,I} = e^{-\delta T}\frac{n(d_1)}{I_b\sigma\sqrt{T}}, \tag{B16}$$

where $n(d_1) = \frac{1}{\sqrt{2\pi}}e^{-d_1^2/2}$ is the normal density function evaluated at d_1. The gammas of the call option with respect to the underlying target and benchmark prices are given by

$$\Gamma_{c,S} = \frac{I_b^2}{S^2}\Gamma_I, \tag{B17}$$

$$\Gamma_{c,M} = b\frac{I_b\left[bI_b\Gamma_I + (b+1)\Delta_I\right]}{M^2}, \tag{B18}$$

and

$$\Gamma_{c,SM} = \Gamma_{c,MS} = -b\frac{I_b\left(I_b\Gamma_I + \Delta_I\right)}{SM}. \tag{B19}$$

Under put–call parity (Equation B4), the put gammas are related through the futures gamma:

$$\Gamma_{p,S} = \Gamma_{c,S}, \tag{B20}$$

$$\Gamma_{p,M} = \Gamma_{c,M} - e^{-rT}\Gamma_{F,M}, \tag{B21}$$

and

$$\Gamma_{p,SM} = \Gamma_{p,MS} = \Gamma_{c,MS} - e^{-rT}\Gamma_{F,MS}. \tag{B22}$$

Futures Vegas

Unlike the usual cost-of-carry model, the pricing relationship for a futures contract on a relative performance index is a function of volatility. The vegas with respect to σ_S, σ_M, and ρ_{SM} are

$$Vega_{F,\sigma_S} = -b\rho_{SM}\sigma_M TF_b, \tag{B23}$$

$$Vega_{F,\sigma_M} = b\left[(1+b)\sigma_M - \rho_{SM}\sigma_S\right]TF_b, \tag{B24}$$

and

$$Vega_{F,\rho_{SM}} = -b\sigma_S\sigma_M TF_b. \tag{B25}$$

Option Vegas

The call option vegas with respect to σ_S, σ_M, and ρ_{SM} are

$$Vega_{c,\sigma_S} = I_b e^{-\delta T} n(d_1) T\left(\frac{\sigma_S - b\rho_{SM}\sigma_M}{\sigma\sqrt{T}}\right) + e^{-rT}N(d_1)Vega_{F,\sigma_S}, \tag{B26}$$

$$Vega_{c,\sigma_M} = I_b e^{-\delta T} n(d_1) T\left[\frac{b(b\sigma_M - \rho_{SM}\sigma_S)}{\sigma\sqrt{T}}\right] + e^{-rT}N(d_1)Vega_{F,\sigma_M}, \tag{B27}$$

and

$$Vega_{c,\rho_{SM}} = -I_b e^{-\delta T} n(d_1) T \left(\frac{b\sigma_S \sigma_M}{\sigma \sqrt{T}} \right) + e^{-rT} N(d_1) Vega_{F,\rho_{SM}}. \quad \text{(B28)}$$

Under put–call parity (Equation B4), the vegas of the call, put, and futures are related through $Vega_{c,i} - Vega_{p,i} = e^{-rT} Vega_{F,i}$, where $i = \sigma_S$, σ_M, and ρ_{SM}. Hence, the put vegas are

$$Vega_{p,\sigma_S} = Vega_{c,\sigma_S} - e^{-rT} Vega_{F,\sigma_S}, \quad \text{(B29)}$$

$$Vega_{p,\sigma_M} = Vega_{c,\sigma_M} - e^{-rT} Vega_{F,\sigma_M}, \quad \text{(B30)}$$

and

$$Vega_{p,\rho_{SM}} = Vega_{c,\rho_{SM}} - e^{-rT} Vega_{F,\rho_{SM}}. \quad \text{(B31)}$$

Notes

1. See www.nasdaqtrader.com/micro.aspx?id=alpha.
2. NASDAQ OMX Alpha Indexes is a trademark of the NASDAQ OMX Group.
3. The value of b can be set to the security's price elasticity or beta with respect to the benchmark. The daily updating rule is used for illustrative purposes only. In practice, the index is updated continually throughout the trading day.
4. Note that this measure of outperformance is *proportional relative performance*. The more usual definition of outperformance is the degree to which the price of the target exceeds the price of the benchmark, or *arithmetic relative performance* (see, e.g., Margrabe 1978; Fischer 1978). Rubinstein (1991) also focused on the valuation of absolute performance options. More closely related to our study is the work by Reiner (1992), who valued foreign equity options struck in a domestic currency. Reiner analyzed the special case of setting the risk adjustment coefficient equal to 1 (i.e., the exchange rate is acting like 1/benchmark) and considered only price appreciation.
5. For a simple numerical example that shows the equivalence of risk-averse and risk-neutral option valuation under a binomial model, see Whaley (2006, pp. 202–206).
6. The NASDAQ OMX PHLX options are European-style and are cash-settled on the morning of the third Friday of the contract month.
7. For illustrative purposes only, the interest income is based on a simple rate of 7 bps a day.
8. Merton (1973) was the first to value securities by using the constant proportional dividend yield assumption.
9. Similarly, although the S&P 500 does not pay dividends, the portfolio it mimics does.
10. These figures correspond to the daily returns for calendar year 2010 that we used in generating Table 1.
11. For a development of the cost-of-carry relationship, see Whaley (2006, pp. 125–127).
12. For a proof, see Whaley (2006, p. 198).
13. Stoll (1969) was the first to derive put–call parity for European-style options.
14. Most of our discussion also applies qualitatively to other cases in which $b > 0$.
15. By definition, implied correlations and betas apply to the life of the alpha index option. Thus, one can deduce a term structure of implied correlations/betas from alpha index options of varying maturities, corresponding to the market's forecast of how a company's systematic risk is expected to change over time.
16. This action assumes that volatilities are constant. If that is not the case, the trader would have to account for changing volatilities in constructing the hedging portfolio.
17. Although buying the AVSPY alpha index directly is not possible, a synthetic long position in the index may be formed by buying a call and selling a put with the same exercise price in the option market and investing the present value of the exercise price in risk-free bonds. Similarly, when alpha index futures contracts begin trading, the value of a direct investment in the AVSPY alpha index can be mimicked by a fully collateralized long futures position.
18. This strategy assumes that the investor can short the benchmark ETF costlessly and maintain full use of the cash proceeds. An equivalent and perhaps more viable strategy is to buy futures on the target, sell futures on the benchmark ETF, and buy risk-free bonds. Note that all the target companies in the 19 currently available NASDAQ OMX Alpha Indexes, as well as the benchmark SPDR ETF, have single stock futures traded on the OneChicago exchange.
19. The position can also be constructed by using options as detailed in Note 17.
20. In particular, if a stock's beta is fixed at 1, the alpha call value depends on only the stock's idiosyncratic volatility.
21. We also assumed that the proportional costs for trading the short-term bond were negligible. Our cost assumptions were conservative and understated the true trading costs currently faced by retail investors.
22. Details regarding the delta hedge simulation are available from the authors upon request.
23. For example, Pool, Stoll, and Whaley (2008) showed that many long call option holders unwittingly fail to exercise outstanding call option positions on stocks when doing so is optimal (just before the ex-dividend day) and thus forfeit the ex-dividend call option price drop to market makers and proprietary companies.

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