


# AIM 08 Performance measurement

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## AIM 08.1 Tracking performance

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# Tracking performance

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- Context:
  - ETP performance is often measured by its ability to track its benchmark precisely.
- Purpose:
  - Develop framework for evaluating how well ETP tracks its benchmark.
    - No need for risk-adjustment.
      - Factors affecting ETP affect benchmark.
    - Difference between ETP return and benchmark return is called “tracking error.”
      - Term is applied inconsistently and often incorrectly.

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## Promised return

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- ETPs promise daily holding period return of benchmark.
  - Tracking error analysis should use holding period returns.
  - Ln returns are unnecessary because no time aggregation is required.
- Some ETPs (i.e., levered and inverse funds) are geared by factor  $L$ .

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## Tracking error definition

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- *Tracking error (TE)* is

$$TE_t = R_{F,t} - LR_{B,t}$$

where  $R_{F,t} \equiv$  fund return on day  $t$ , and  
 $R_{B,t} \equiv$  benchmark index return on day  $t$ .

- In ideal world, daily tracking error ( $TE$ ) is 0.

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## Reasons why $TE$ is not 0

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- Expense ratio ( $ER$ )
  - Each day issuers levies fee.
    - $ER$  times  $n/365$  times \$AUM, where  $n$  is 1 for trading days and 3 for weekends.
  - Contributes to  $TE < 0$ .

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## Reasons why $TE$ is not 0

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- Transaction and rebalancing costs
  - Most ETPs are passive.
    - Trading is nondiscretionary. E.g.,
      - Securities in benchmark index experience corporate event like cash dividend.
      - Index composition changes
  - Contributes to  $TE < 0$ .

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## Reasons why $TE$ is not 0

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- Sampling and optimization
  - Sometimes ETP manager does not some securities in index because they are illiquid and expensive to trade.
    - E.g., bond indexes
  - Choose subset of index securities whose return matches benchmark return.
    - Maximize return correlation with benchmark.
  - No predictable effect on  $TE$ .

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## Reasons why $TE$ is not 0

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- Cash drag
  - Oftentimes benchmark pays daily dividends, while ETP pays quarterly (e.g., S&P 500 index vs SPY).
    - Daily dividends are too small to warrant cost-efficient reinvestment in the benchmark securities.
    - Held in cash causing *cash drag*.
  - Usually  $TE < 0$ .
    - Depends on difference between benchmark return and rate of interest on cash equivalents.

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## Reasons why $TE$ is not 0

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- Timing
  - Index rebalancing occurs instantaneously (e.g., Russell 2000).
  - ETP cannot transact in instantaneously.
  - Usually  $TE < 0$ .
    - Price impact from chasing securities with other funds with same benchmark.

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## Reasons why $TE$ is not 0

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- Securities lending
  - ETFs may lend securities in fund.
  - Securities lending income subsidizes fund operating costs, thereby permitting lower expense ratios.
  - Contributes to  $TE > 0$ .

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## Tracking difference definition

- *Tracking difference* (technically, *mean or average tracking error*) is

$$TD = \sum_{t=1}^T (R_{F,t} - LR_{B,t}) / T$$

where  $T$  is number of daily returns in sample time-series.

- Measures benefit/cost of holding ETP on daily basis.
- Variation in  $TE$  is not considered.

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## Standard deviation of $TE$

- *Standard deviation of tracking error* is

$$\sigma_{TE} = \sqrt{\frac{\sum_{t=1}^T (TE_t - TD)^2}{T - 1}}$$

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## Standard error of $TD$

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- *Standard error of tracking difference is*

$$\sigma_{TD} = \frac{\sigma_{TE}}{\sqrt{T}}$$

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## Risk-adjusted tracking difference

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- *Risk-adjusted tracking difference (RATD) is*

$$RATD = \frac{TD}{\sigma_{TD}}$$

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## Risk-adjusted tracking difference

- Risk-adjusted tracking difference (RATD) is

$$RATD = \frac{TD}{\sigma_{TD}}$$

- Assuming multiple ETPs on same benchmark index, choose highest.
- Also *t*-test of null hypothesis that *TD* is 0.

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## Industry mislabeling

- Industry defines term “tracking error” as

$$\text{Tracking error} = \sqrt{\sum_{t=1}^T (R_{F,t} - LR_{B,t})^2 / T}$$

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## Industry mislabeling

- Industry defines term “tracking error” as

$$\text{Tracking error} = \sqrt{\sum_{t=1}^T (R_{F,t} - LR_{B,t})^2 / T}$$

- Called *root mean squared error (RMSE)* in statistics.

$$RMSE = \sqrt{\sum_{t=1}^T (R_{F,t} - LR_{B,t})^2 / T}$$

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## Industry mislabeling

- Not sensible measure.
  - Does not measure performance.
  - Does not properly measure variability.
    - Can show that RMSE includes not only standard deviation of tracking error but also squared mean tracking error or squared tracking difference.

$$RMSE = \sqrt{\sigma_{TE}^2 + TD^2}$$

- Manager is penalized for both positive (outperformance) and negative (underperformance) *TD*.

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## S&P 500 products

- Support file: S&P 500 ETF tracking performance.xlsx.
  - Downloaded from Bloomberg.
    - Collected daily total return data for S&P 500 total return index and SPY, IVV, and VOO ETFs.

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## Results

Sample period	
Begins	20121231
Ends	20221230
No. of days	3,651
No. of years	10.00

Ten-year sample period

Tracking difference is about Equal to expense ratio.

Tracking performance			
Description	SPY	IVV	VOO
No. of daily returns ( <i>n</i> )	2,518	2,518	2,518
Tracking difference ( <i>TD</i> )	-0.000005	-0.000001	-0.000001
Standard deviation of <i>TE</i>	0.000562	0.000511	0.000511
Standard error of <i>TD</i>	0.000011	0.000010	0.000010
Risk-adjusted tracking difference ( <i>RATD</i> )	-0.4094	-0.1216	-0.1197
Prob <i>TD</i> = 0	0.6823	0.9032	0.9048
Autocorrelation	-0.5767	-0.4642	-0.4190
Minimum	-0.0077	-0.0041	-0.0053
Median	0.0000	0.0000	0.0000
Maximum	0.0104	0.0042	0.0055
Root mean square tracking error ( <i>RMSE</i> )	0.0006	0.0005	0.0005
Annualized <i>TD</i> (252 days)	-0.0012	-0.0003	-0.0003
Correlation between ETP and benchmark	0.9988	0.9989	0.9989

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*RATD* is different from 0.

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Game of "catch-up."

- Which direction?

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- Which direction?

Perfect substitutes from  
correlation perspective.

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# Lesson summary

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- ❑ Developed comprehensive framework for analyzing tracking error.
  - Key elements are:
    - ❑ Tracking difference (TD)
    - ❑ Risk-adjusted tracking difference (RATD)
- ❑ Showed industry definition of "tracking error" is root mean squared error.
  - Cannot distinguish between under- and over-performance.
- ❑ Provided intuition for TE measurements.

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