Option valuation

Risk measurement

Risk measurement

- Purpose:
  - Measure option’s risk characteristics day-to-day as asset price moves and as volatility changes.

Risk measurement

- Option value is sensitive to changes in underlying determinants (i.e., S, i, σ, r, T).
  - Sensitivities are:
    - Known by a variety of Greek letters (hence, the "greeks")
    - Key to effective dynamic risk management of portfolios containing options
    - Measured analytically for European-style options and numerically for American-style options.

Risk measurement

- Key greeks are:
  - *delta*: change in option value as asset price changes
  - *gamma*: change in delta as asset price changes
  - *vega*: change in option value as volatility rate changes

Risk measurement

- Analytical measures:
  - For European options, greeks may be computed analytically using BSM option valuation formula.

$$c = Se^{-iT}N\left(d_1\right) - Xe^{-rT}N\left(d_2\right)$$

where

$$d_1 = \frac{\ln\left(Se^{-iT} / Xe^{-rT}\right) + .5\sigma^2 T}{\sigma\sqrt{T}},$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Delta

- Change in option value as asset price changes.
  - Take partial derivative of call option formula with respect to *S*.

$$\Delta_c = \frac{\partial c}{\partial S} = e^{-iT} N(d_1) > 0$$

  - Intuition:
    - Call is right to buy underlying asset at fixed price. Higher is asset price, more valuable is call.

Delta

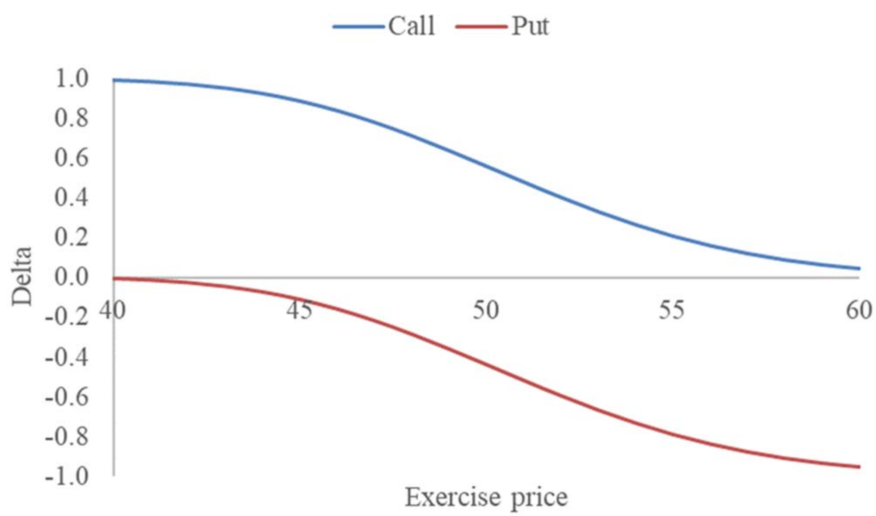
- Change in option value as asset price changes.
  - Take partial derivative of put option formula with respect to *S*.

$$\Delta_p = \frac{\partial p}{\partial S} = -e^{-iT} N(-d_1) < 0$$

  - Intuition:
    - Put is right to sell underlying asset at fixed price. Higher is asset price, less valuable is put.

Delta

- Change in option value as asset price changes.



- <u>Intuition</u>:
  - Call delta becomes smaller as call goes from ITM to OTM.
  - Put delta becomes smaller as put goes from OTM to ITM.

Gamma

- Change in delta as asset price changes.

Call:

$$\gamma_c = \frac{e^{-iT} n(d_1)}{S\sigma\sqrt{T}} > 0$$

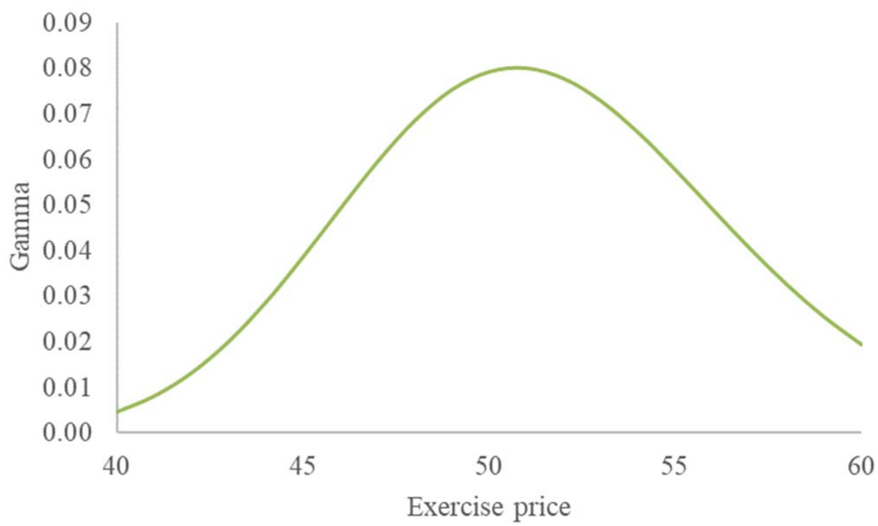
Put:

$$\gamma_p = \frac{e^{-iT} n(d_1)}{S\sigma\sqrt{T}} = \gamma_c > 0$$

- Intuition:
  - Delta increases for both call and put as asset price increases, but at different rates for different levels of option moneyness.

Gamma

- Change in delta as asset price changes.



- Intuition:
  - ATM options have highest gamma.
    - Will be hardest to hedge.

Vega

- Change in option value as volatility rate changes.

Call:

$$Vega_c = Se^{-iT} n(d_1)\sqrt{T} > 0$$

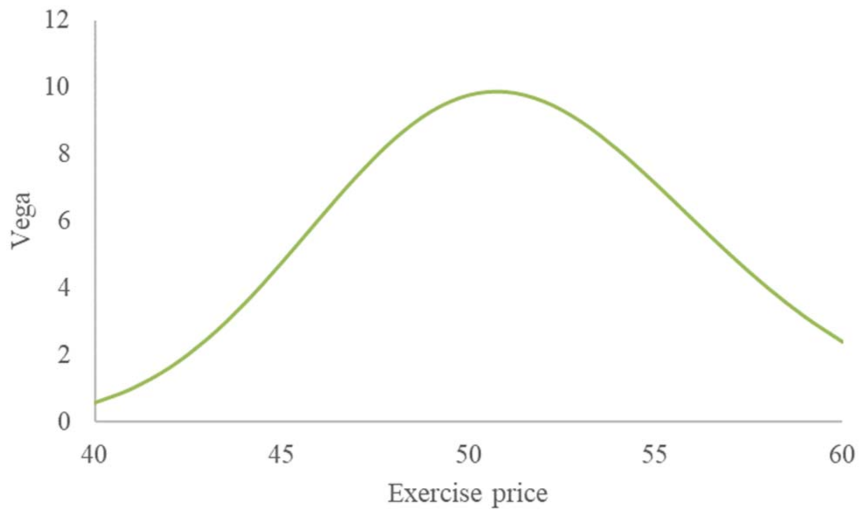
Put:

$$Vega_p = Se^{-iT} n(d_1)\sqrt{T} = Vega_c > 0$$

- Intuition:
  - Vega increases for both call and put as volatility increases, but at different rates for different levels of option moneyness.

Vega

- Change in option value as volatility rate changes.



- Intuition:
  - ATM options have highest vega.
    - Will provide highest exposure on option trading strategies involving directional bets on volatility.

Theta

- Change in option value as option's time to expiration rate changes.
  - Instructive in terms of understanding time erosion.

Call:

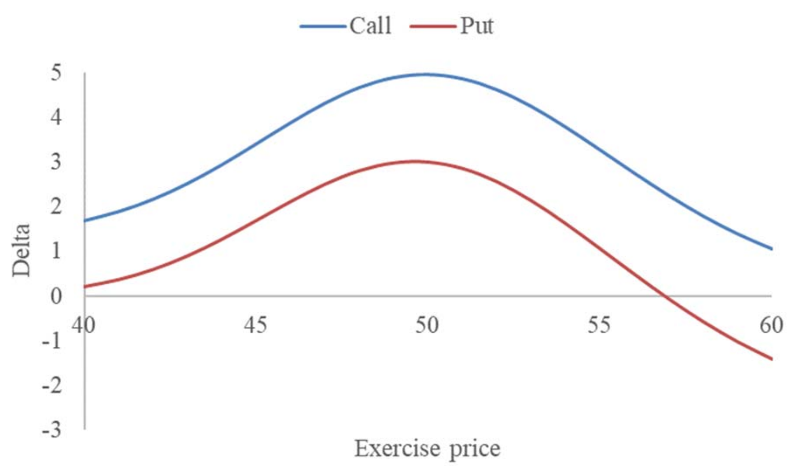
$$\theta_c = Se^{-iT}n(d_1)\frac{\sigma}{2\sqrt{T}} - iSe^{-iT}N(d_1) + rXe^{-rT}N(d_2)$$

Put:

$$\theta_p = Se^{-iT}n(d_1)\frac{\sigma}{2\sqrt{T}} + iSe^{-iT}N(-d_1) - rXe^{-rT}N(-d_2)$$

Theta

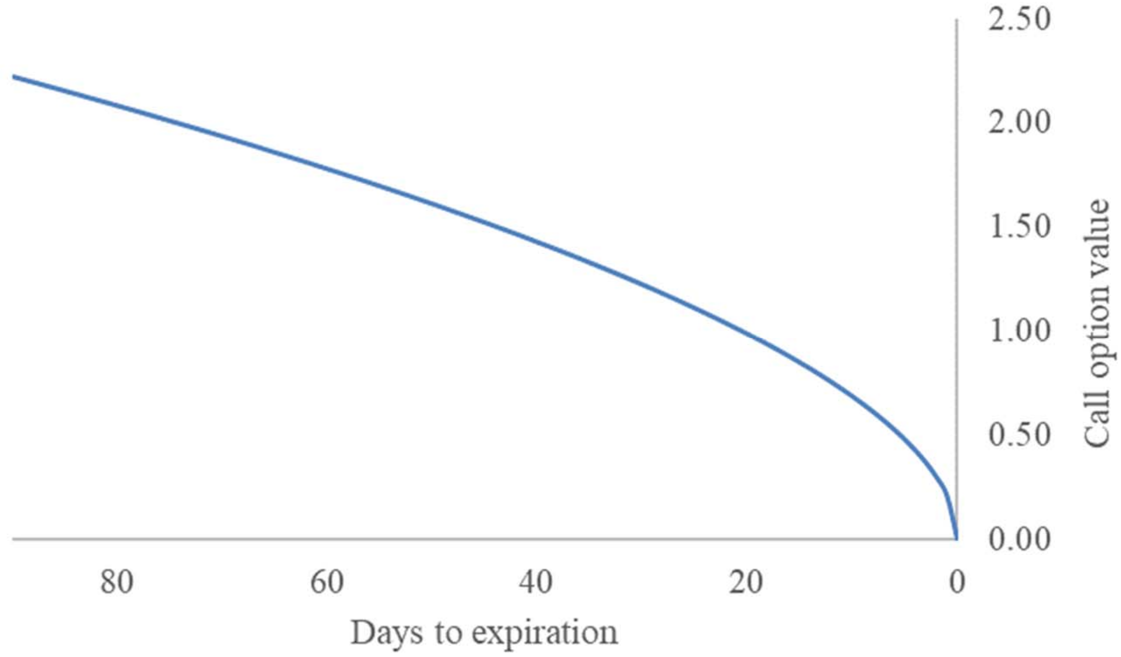
- Change in option value as time elapses.



- <u>Intuition</u>:
  - ATM options have highest theta.
    - Highest time premium.
      - Erodes most quickly.

Time erosion

- Assume ATM call.



Risk measurement

- Numerical measures:
  - For American options, greeks must be computed numerically.

Delta

- Change in option value as asset price changes.
  - Cannot take partial derivative with respect to S.
  - Compute delta as change in option value OV as result of perturbing S by ϕ.

$$\Delta = \frac{OV\left(S+\phi\right)-OV\left(S-\phi\right)}{\left(S+\phi\right)-\left(S-\phi\right)}$$

$$= \frac{OV\left(S+\phi\right)-OV\left(S-\phi\right)}{2\phi}$$

Delta

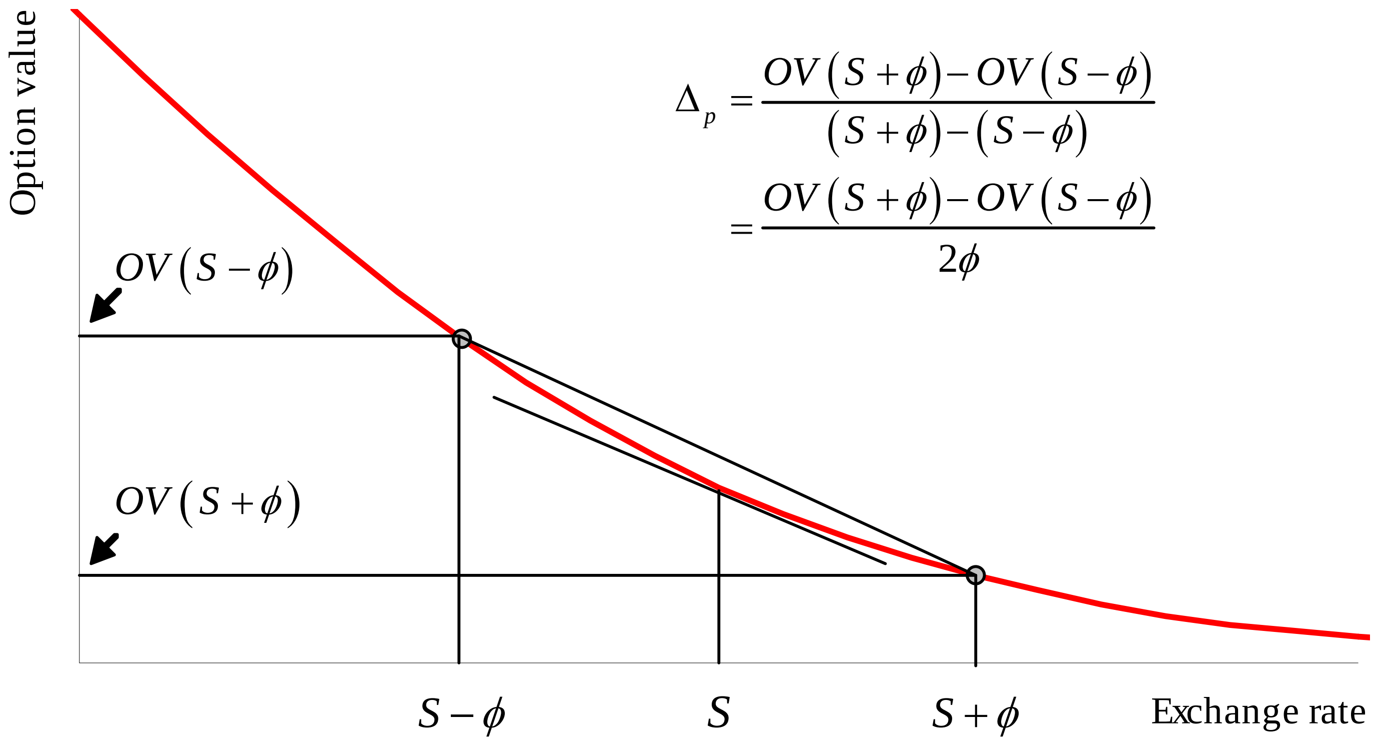
- Illustration:
  - Consider European-style put option.
    - Supporting file: Risk measurement.xlsx

Currency parameters	
Price	50.00
Foreign interest rate	6%
Volatility rate	10%
Domestic interest rate	5%

Option	
Exercise price	50.00
Time to expiration	2.00
(C)all/(P)ut	P

Delta

- Compute *OV* at two asset prices.



$$\Delta_p = \frac{OV(S+\phi) - OV(S-\phi)}{(S+\phi)-(S-\phi)} = \frac{OV(S+\phi) - OV(S-\phi)}{2\phi}$$

Other Greeks

- General numerical procedure:
  - Compute greek as change in option value *OV* as a result of perturbing relevant risk factor *k* by ϕ.

$$\text{Greek}_k = \frac{OV(k+\phi) - OV(k-\phi)}{(k+\phi)-(k-\phi)}$$

$$= \frac{OV(k+\phi) - OV(k-\phi)}{2\phi}$$

where $k = S, \sigma, r, i, T.$

Other greeks

- General numerical procedure:
  - ϕ is arbitrary and can be set at any value you choose and will vary by risk factor.
    - E.g., .50 for delta, .0005 for vega
  - Trade-off:
    - Smaller is perturbation parameter ϕ:
      - Greater should be precision in measurement of Greek.
      - Greater chance that approximation error in option valuation will affect risk measurement in important way.
  - Supporting file: Risk measurement.xlsx

Other greeks

- General numerical procedure:
  - Consider other risk measures.
  - Choices of perturbation parameters are arbitrary.

Risk measures	*Analytical*	*Perturbation parameter*	*Numerical*	*Difference*
Delta	-0.4685	0.5	-0.4685	0.0000
Vega	24.9571	0.05%	24.9571	0.0000
Rho - r	-52.8423	0.05%	-52.8420	-0.0003
Rho - i	46.8459	0.05%	46.8455	0.0004
Theta	0.7082	0.0027397	0.7082	0.0000

  - Conclusion: Can measure risk numerically with great accuracy.

Lesson summary

- Greeks are sensitivities of option value to changes in option determinants.
  - delta and gamma: asset price
  - vega: volatility
  - theta: time to expiration
- Greeks are computed:
  - Analytically for European-style options.
  - Numerically for American-style options.