AIM 11

Levered and inverse products

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Applied investment management

- Context:
  - Both physical replication and fully collateralized futures ETPs promise 1*x* daily benchmark return.
  - Levered and inverse (LI) ETPs are *Lx* products created using collateralized futures or total return swaps.
    - Benchmark return is geared (i.e., *L* is not equal to 1).
      - $L > 1$ levered funds (long benchmark).
      - $L < 0$ inverse and levered inverse funds (short benchmark).
    - Usually less than fully collateralized.
    - Dynamic (not passive) futures replication strategy is required.

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Applied investment management

- Context:
  - What was original purpose in creating these products?
    - With FCF products, objective was to create new asset class.
    - With LI products, objective was to create leverage or short sell.
      - Who has leverage and short sales constraints?
    - https://www.sec.gov/newsroom/speeches-statements/schock-statement-single-stock-levered-or-inverse-etfs-071122 ***

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Levered and inverse products

- Purpose:
  - Describe levered and inverse ETPs.
  - Discuss popularity and product offerings.
  - Show how they are created.
  - Identify key design problems.
    - Path dependence and return volatility.
    - Market destabilization.
  - Examine actual price behaviors through time.
  - Simulate expected behavior through time using Monte Carlo simulation.
  - Discuss “Volmageddon” on February 5, 2018.

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Levered and inverse products

- Definition:
  - Levered and inverse ETPs are geared to daily holding period return of benchmark index.
    - *Gear* or *leverage ratio* is *L*.
      - Long exposures: 2*x*, 3*x*, and 4*x*
      - Short exposures:–1*x*, –2*x*, and –3*x*.
  - Benchmark may be any published index.
  - Issuances and redemptions of levered and inverse ETPs are in cash rather than "in-kind" basket transfers.
    - AP buys/sells more units from issuer at end of day.

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Levered and inverse ETPs

- Reasons for popularity:
  - Trade as securities, not derivatives.
    - Anyone can get securities account on Robinhood.
    - Many traders/investors/institutions cannot trade futures.
  - Extreme leverage possible.
    - Buying 3*x* ETP on margin can provide 6*x* exposure.
  - Limited liability.
    - Loss on futures may exceed cash collateral.

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Levered and inverse ETPs

- Product offerings:
  - First products launched in 2006.
    - 20060621: ProShares launched inverse ($-1x$) products on popular stocks indexes:
      - S&P 500, Dow 30, MidCap 400, QQQ
    - 20060711: ProShares launched levered ($2x$) products on same stock indexes.
  - Support file: Levered and inverse ETPs 20231229.xlsx

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Levered and inverse ETPs

- Funds by asset class on 20231229:

Asset class	No. of ETPs	$AUM (M)	Market share
Equity	187	86,765	89.6%
Bond	14	6,125	6.3%
Commodity	15	2,571	2.7%
Volatility	5	838	0.9%
Real Estate	6	323	0.3%
Currency	5	158	0.2%
Multi-Asset	1	19	0.0%
Preferred Stock	1	10	0.0%
	234	96,809	100.0%
Expense ratio		1.00%	
Annual revenue		968	

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Levered and inverse ETPs

- Funds by issuer on 20231229:

Issuer	No. of ETPs	$AUM (M)	Market share
ProShares	97	50,491	52.2%
Rafferty Asset Management	71	37,155	38.4%
BMO Financial Group	21	6,781	7.0%
UBS	17	619	0.6%
First Trust	2	519	0.5%
AXS Investments	4	333	0.3%
GraniteShares	6	301	0.3%
Innovator	8	226	0.2%
Volatility Shares LLC	2	202	0.2%
Deutsche Bank AG	3	93	0.1%
Ameriprise Financial	1	53	0.1%
AdvisorShares	1	28	0.0%
Dynamic Shares LLC	1	7	0.0%
	234	96,809	

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Levered and inverse ETPs

- Funds with over $1B in AUM on 20231229:

Symbol	Name	*L*	*σ*	Assets	Inception	ER	Days
TQQQ	ProShares UltraPro QQQ	3x	45%	20,575	20100209	0.88%	4.0
SOXL	Direxion Daily Semiconductor Bull 3x Shares	3x	75%	8,649	20100311	0.94%	4.0
QLD	ProShares Ultra QQQ	2x	30%	5,673	20060619	0.95%	16.3
TMF	Direxion Daily 20+ Year Treasury Bull 3X Shares	3x	58%	5,065	20090416	1.06%	13.3
SSO	ProShares Ultra S&P 500	2x	24%	4,416	20060619	0.91%	17.8
SPXL	Direxion Daily S&P 500 Bull 3X Shares	3x	35%	4,023	20081105	1.00%	3.7
UPRO	ProShares UltraPro S&P500	3x	35%	3,419	20090625	0.92%	8.3
SQQQ	ProShares UltraPro Short QQQ	-3x	45%	3,305	20100209	0.95%	1.9
TECL	Direxion Daily Technology Bull 3X Shares	3x	44%	3,147	20081217	0.97%	11.6
FNGU	MicroSectors FANG+â„¢ Index 3X Leveraged ETN	3x	57%	3,118	20180122	0.95%	7.1
TNA	Direxion Daily Small Cap Bull 3X Shares	3x	69%	2,525	20081105	1.09%	3.1
FAS	Direxion Daily Financial Bull 3X Shares	3x	41%	2,060	20081106	0.96%	33.9
NRGU	MicroSectors U.S. Big Oil Index 3X Leveraged ETN	3x	57%	1,642	20190409	0.95%	65.0
LABU	Direxion Daily S&P Biotech Bull 3x Shares	3x	103%	1,340	20150528	1.01%	4.9
SH	ProShares Short S&P500	-1x	12%	1,278	20060619	0.88%	4.4
TSLL	Direxion Daily TSLA Bull 1.5X Shares ETF	1.5x	70%	1,093	20220809	1.08%	4.9

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Levered and inverse ETPs

- Funds with over $1B in AUM on 20231229:

Levered and inverse single stock ETPs as of 20241120					
Symbol	Name	Assets	Inception	ER	Days
NVDL	GraniteShares 2x Long NVDA Daily ETF	5,493	20221213	0.0106	3.6
TSLL	Direxion Daily TSLA Bull 2X Shares	2,137	20220809	0.0096	1.6
NVDU	Direxion Daily NVDA Bull 2X Shares	641	20230913	0.0104	5.5
CONL	GraniteShares 2x Long COIN Daily ETF	518	20220809	0.0110	1.0
FBL	GraniteShares 2x Long META Daily ETF	172	20221213	0.0115	7.8
AMZU	Direxion Daily AMZN Bull 2X Shares	138	20220907	0.0106	5.3
GGLL	Direxion Daily GOOGL Bull 2X Shares	128	20220907	0.0105	4.8
MSFU	Direxion Daily MSFT Bull 2X Shares	103	20220907	0.0104	6.0
TSLQ	Tradr 2X Short TSLA Daily ETF	100	20220713	0.0115	1.5
AAPU	Direxion Daily AAPL Bull 2X Shares	94	20220809	0.0104	4.3
NVDS	Tradr 1.5X Short NVDA Daily ETF	53	20220714	0.0115	0.9
TSLS	Direxion Daily TSLA Bear 1X Shares	47	20220809	0.0107	1.4
MSOX	AdvisorShares MSOS Daily Leveraged ETF	36	20220823	0.0113	11.7
BABX	GraniteShares 2x Long BABA Daily ETF	32	20221213	0.0115	7.8
AAPD	Direxion Daily AAPL Bear 1X Shares ETF	28	20220809	0.0106	5.5
NVDD	Direxion Daily NVDA Bear 1X Shares	24	20230913	0.0101	1.0
AAPB	GraniteShares 2x Long AAPL Daily ETF	22	20220809	0.0115	7.4
TSL	GraniteShares 1.25x Long Tesla Daily ETF	10	20220809	0.0115	1.5
MSFD	Direxion Daily MSFT Bear 1X Shares ETF	7	20220907	0.0106	6.1
AMZD	Direxion Daily AMZN Bear 1X Shares ETF	5	20220907	0.0109	7.5
GGLS	Direxion Daily GOOGL Bear 1X Shares ETF	4	20220907	0.0109	9.3

Earliest ones launched 20220713.

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Levered and inverse ETPs

- Rely on Pessina and Whaley (*FAJ* 2021).
  - Use their tables and figures.
- Digression:
  - Finance practitioner publications:
    - *Financial Analysts Journal* or *JPM*
      - Sponsored by CFA Institute
      - Circulation: 250,000 (individuals and libraries)
    - *Journal of Portfolio Management* or *JPM*
      - Circulation: 5,000 (libraries)

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Levered and inverse ETPs

- Who are issuers?

Big guys deliberately stayed away.

Others chose to make them specialties.

Table 1. ETPs by Issuer

Fund Position	Issuer	All Funds: No. of Issues	All Funds: AUM ($)	All Funds: Percent of Total Issues	All Funds: Percent of Total $ AUM	Levered and Inverse Funds: No. of Issues	Levered and Inverse Funds: AUM ($)	Levered and Inverse Funds: Percent of Total Issues	Levered and Inverse Funds: Percent of Total $ AUM
1	iShares	368	1,640,157,916,000	15.8%	38.8%				
2	Vanguard	80	1,121,965,492,600	3.4	26.5				
3	State Street SPDR	140	661,112,371,900	6.0	15.6				
4	Invesco	219	214,569,456,300	9.4	5.1	1	37,027,100	0.4%	0.1%
5	Charles Schwab	25	155,923,124,100	1.1	3.7				
6	First Trust	150	83,892,827,500	6.4	2.0				
7	VanEck	55	41,983,292,900	2.4	1.0				
8	WisdomTree	77	35,742,404,800	3.3	0.8				
9	ProShares	140	33,611,387,700	6.0	0.8	108	24,107,533,200	39.0	47.6
10	J.P. Morgan	34	32,138,565,600	1.5	0.8				
11	PIMCO	16	23,961,158,400	0.7	0.6				
12	Goldman Sachs	22	17,362,803,700	0.9	0.4	1	1,291,261,200	0.4	2.6
13	Fidelity	29	16,466,728,900	1.2	0.4				
14	DWS	37	15,156,061,000	1.6	0.4	4	167,794,600	1.4	0.3
15	FlexShares	29	14,311,807,600	1.2	0.3				
16	Direxion	97	13,010,648,400	4.2	0.3	80	12,467,279,500	28.9	24.6
17	Global X	69	10,042,180,900	3.0	0.2				
18	ALPS	16	8,425,033,600	0.7	0.2				
19	Credit Suisse	20	5,765,620,500	0.9	0.1	14	5,635,376,000	5.1	11.1
20	Pacer Financial	22	5,755,511,400	0.9	0.1				
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
140	Armor Index	1	1,802,700	0.0	0.0				
Total		2,330	4,231,618,595,600	100.0%	100.0%	277	50,630,294,000	100.0%	100.0%

Source: ETFdb.com.

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Levered and inverse ETPs

- Who are issuers?

Why should they bother considering AUM ($) is so low?

51B / 4,232B = 1.2%

Management fees.

Typical ER is 100 bps.

Annual revenue is .01 x 50.6B = 506M.

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Source: ETFdb.com.

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Levered and inverse ETPs

- What benchmarks are most popular?

Table 2. Levered and Inverse ETPs by Asset Class

Asset Class	No. of ETPs	Total AUM	Percent of AUM	Average Dollar Volume	Turnover
Equity	191	42,115,083,000	83.2%	7,133,238,098	16.9%
Commodity	26	3,106,084,200	6.1	1,591,937,806	51.3
Volatility	5	2,513,256,300	5.0	2,866,578,888	114.1
Bond	22	1,524,422,600	3.0	153,728,289	10.1
Real estate	9	866,891,600	1.7	13,355,573	1.5
Multiasset	4	285,840,600	0.6	3,679,366	1.3
Currency	19	214,679,300	0.4	4,419,779	2.1
Alternatives	1	4,036,400	0.0	48,395	1.2
Total	277	50,630,294,000	100.0%		

Stock indexes: 191 funds with 83.2% of $AUM.

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Commodities and volatility are next largest.

- Appear to be used for trading vs investing.

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Levered and inverse ETPs

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Real estate, multi-asset, currency, and alternatives appear as buy-and-hold.

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Levered and inverse ETPs: Flaws

- Levered and inverse ETPs are flawed in <u>two</u> ways:
  - Expected multi-day return is not equal to actual return.
    - Compounding effects resulting from different reinvestment rate assumptions.
      - Performance is benchmark price path dependent.
      - Underperformance increases with benchmark return volatility.
  - End-of-day rebalance effect
    - Can destabilize underlying market.

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Compounding effect

- Compounding effect arises because

$$\prod_{t=1}^{T}(1+LR_t)-1 \neq L\left[\prod_{t=1}^{T}(1+R_t)-1\right]$$

where

L is return multiplier,

R_t is index return on day t, and

T is number of days in holding period.

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Compounding effect

- Compounding effect arises because

$$\prod_{t=1}^{T}(1+LR_t)-1 \neq L\left[\prod_{t=1}^{T}(1+R_t)-1\right]$$

Actual return investor gets if he buys and holds ETP.

Return investor expects to get if he buys and holds ETP.

- Two sides are equal only when T=1 or L=1.

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LI silliness calculator

- Created calculator to show outcomes after 2 days.
  - User sets annualized mean and standard deviation (volatility) of benchmark returns.
  - Support file: LI silliness calculator.xlsx
    - Sheets: No volatility
      No return

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Reinvestment effect: 1*x* ETP

- Sheet: No volatility

Compounding effect (no volatility) over 2-day period								
	Benchmark				Leverage factor			1
Day	Daily Return	Index level	Cum. return	Investor expects	Daily return	ETP price	Cum. return	Implied leverage
0		100.00		100.00		100.00		
1	20%	120.00	20.00%	120.00	20%	120.00	20.00%	1.00
2	20%	144.00	44.00%	144.00	20%	144.00	44.00%	1.00
Volatility	0.00%		Expected	44.00%	Actual	44.00%		

With no leverage ($L = 1$), no difference in performance.

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Reinvestment effect: 2*x* ETP

- Sheet: No volatility

Compounding effect (no volatility) over 2-day period								
	Benchmark				Leverage factor			2
Day	Daily Return	Index level	Cum. return	Investor expects	Daily return	ETP price	Cum. return	Implied leverage
0		100.00		100.00		100.00		
1	20%	120.00	20.00%	140.00	40%	140.00	40.00%	2.00
2	20%	144.00	44.00%	188.00	40%	196.00	96.00%	2.18
Volatility	0.00%		Expected	88.00%	Actual	96.00%		

For L > 1, leverage increases through time (e.g., increased from 2 to 2.18 (=96.0% / 44.0%) due to greater reinvestment.

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Reinvestment effect: –1*x* ETP

- Sheet: No volatility

Compounding effect (no volatility) over 2-day period								
	Benchmark				Leverage factor			-1
Day	Daily Return	Index level	Cum. return	Investor expects	Daily return	ETP price	Cum. return	Implied leverage
0		100.00		100.00		100.00		
1	20%	120.00	20.00%	80.00	-20%	80.00	-20.00%	-1.00
2	20%	144.00	44.00%	56.00	-20%	64.00	-36.00%	-0.82
Volatility	0.00%		Expected	-44.00%	Actual	-36.00%		

For L < 0, leverage diminishes in absolute value through time (e.g., increased from –1 to –0.82) due to lower reinvestment.

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Volatility effect: 1*x* ETP

- Sheet: No return

Compounding effect (no return) over 2-day period							
	Benchmark				Leverage factor		1
Day	Daily Return	Index level	Cum. return	Investor expects	Daily return	ETP price	Cum. return
0		100.00		100.00		100.00	
1	22.14%	122.14	22.14%	122.14	22.14%	122.14	22.14%
2	-18.13%	100.00	0.00%	100.00	-18.13%	100.00	0.00%
Volatility	28.47%	Expected		0.00%	Actual		0.00%
Ln return	20.0%	$((1+R_1)*(1+R_2) - 1) * L$		0.00%	$(1+LR_1)*(1+LR_2) - 1$		0.00%

With no leverage (L = 1), no difference in performance.

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Volatility effect: 2*x* ETP

- Sheet: No return

Compounding effect (no return) over 2-day period							
	Benchmark				Leverage factor		2
Day	Daily Return	Index level	Cum. return	Investor expects	Daily return	ETP price	Cum. return
0		100.00		100.00		100.00	
1	22.14%	122.14	22.14%	144.28	44.28%	144.28	44.28%
2	-18.13%	100.00	0.00%	100.00	-36.25%	91.97	-8.03%
Volatility	28.47%	Expected		0.00%	Actual		-8.03%
Ln return	20.0%	$((1+R_1)*(1+R_2) - 1) * L$		0.00%	$(1+LR_1)*(1+LR_2) - 1$		-8.03%

With leverage (L = 2), investor performed worse than expected.

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Volatility effect: –1*x* ETP

- Sheet: No return

Compounding effect (no return) over 2-day period							
	Benchmark				Leverage factor		-1
Day	Daily Return	Index level	Cum. return	Investor expects	Daily return	ETP price	Cum. return
0		100.00		100.00		100.00	
1	22.14%	122.14	22.14%	77.86	-22.14%	77.86	-22.14%
2	-18.13%	100.00	0.00%	100.00	18.13%	91.97	-8.03%
Volatility	28.47%	Expected		0.00%	Actual		-8.03%
Ln return	20.0%	$((1+R_1)*(1+R_2) - 1) * L$		0.00%	$(1+LR_1)*(1+LR_2) - 1$		-8.03%

With inverse (L = –1), investor performed worse than expected.

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LI silliness calculator

- Created calculator to show outcomes after one year.
  - User sets annualized mean and standard deviation (volatility) of benchmark returns.
  - Support file: LI silliness calculator.xlsx
    - Sheet: One-year horizon examples

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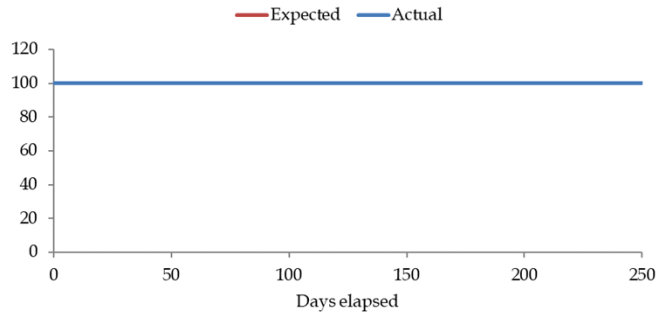
LI silliness calculator

- Example 1: No effect
  - Expected return = 0%, volatility = 0%, $L = 1$



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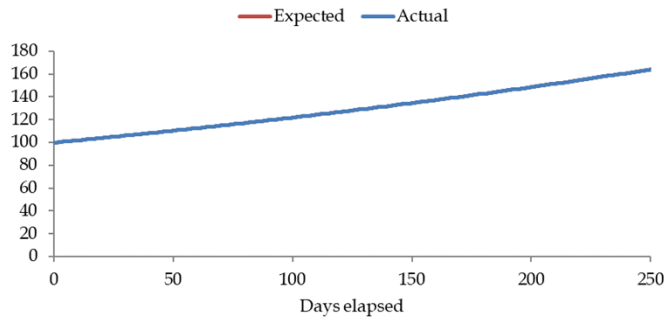
LI silliness calculator

- Example 2: No effect
  - Expected return = 50%, volatility = 0%, $L = 1$



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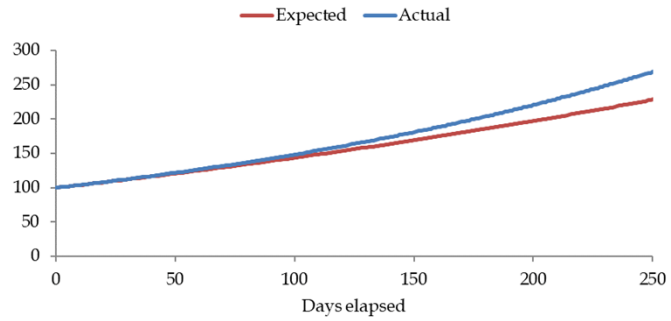
LI silliness calculator

- Example 3: Actual > Expected
  - Expected return = 50%, volatility = 0%, $L = 2$



With certain positive return, re-investment with LI ETF is higher.

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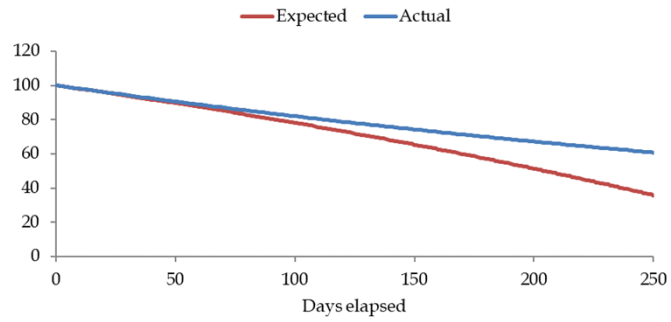
LI silliness calculator

- Example 4: Actual > Expected
  - Expected return = 50%, volatility = 0%, $L = -1$

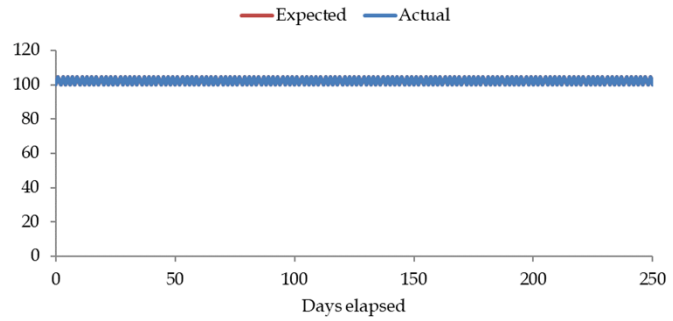


With certain positive return, re-investment with LI ETF is less.

32

LI silliness calculator

- Example 5: No effect
  - Expected return = 0%, volatility = 100%, $L = 1$



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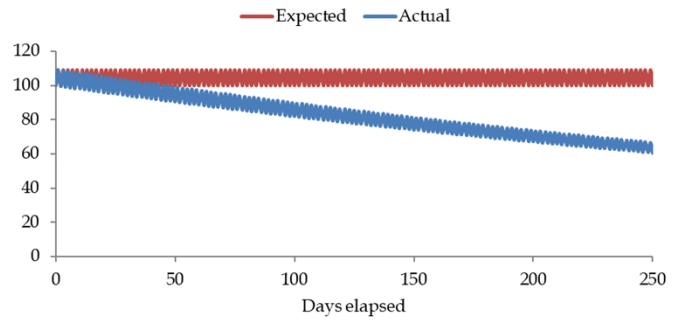
LI silliness calculator

- Example 6: Expected > Actual
  - Expected return = 0%, volatility = 100%, $L = 2$



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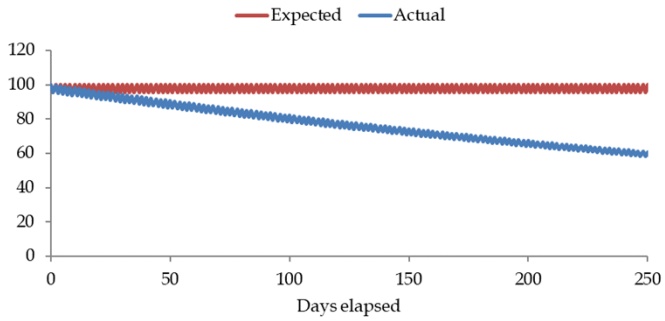
LI silliness calculator

- Example 7: Expected > Actual
  - Expected return = 0%, volatility = 100%, $L = -1$



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Actual price paths

- Addressed issue of compounding mechanics using hypothetical examples to develop intuition.
  - Controlled price path through time.
- LI products are path dependent.
  - Outcome will depend on actual price through time.
  - Can observe past price series.
    - NOTE: Be careful because these are single price paths.

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Price histories: VIX ETFs

- Illustration: ProShares VIX ETFs
  - VIXY: 1*x*
  - UVXY: 2*x* until 20180228; 1.5*x* thereafter.
  - SVXY: –1*x* until 20180228; –0.5*x* thereafter.
  - Support file: VIXY-UVXY-SVXY.xlsx
    - Daily price series begins with launch of UVXY and SVXY on 20111004 and ends 20180131 before de-levering.
    - SPVXSP is underlying (futures) benchmark index.

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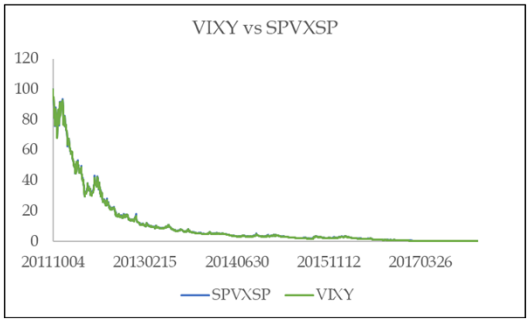
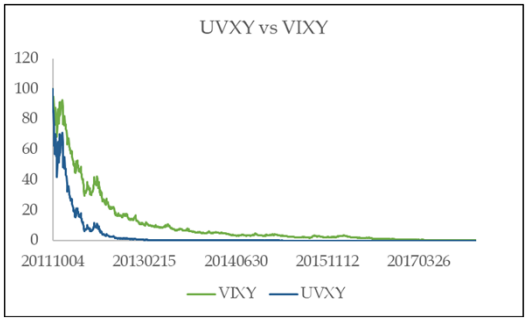
Price histories: VIX ETFs



VIXY behaves like benchmark, SPVXSP.
- Plotted on top of one another.

38

Price histories: VIX ETFs



UVXY (2*x*) does <u>not</u> behave like twice VIXY (1*x*).

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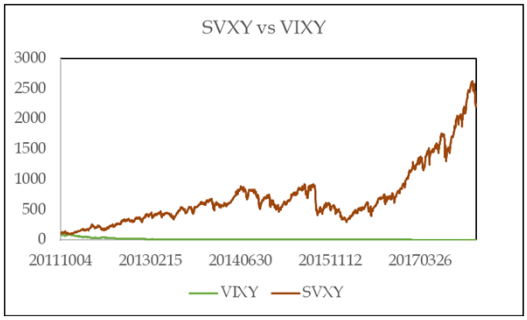
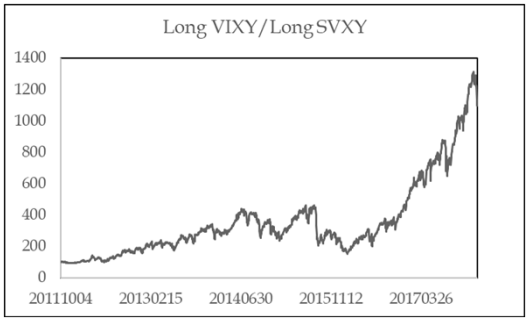
Price histories: VIX ETFs



SVXY (–1*x*) does <u>not</u> behave like inverse of VIXY (1*x*).

40

Price histories: VIX ETFs



Long VIXY and long SVXY appears to have done well.

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Price histories: DWT vs UWT ETNs

- Illustration:
  - Chose two 3*x* products on crude oil.
    - DWT: Citigroup, –3*x*, 1.50%, 20161208
    - UWT: Citigroup, 3*x*, 1.50%, 20161208
  - Downloaded all daily price data from inception through 20190116.
  - Support file: DWT vs UWT.xlsx
    - Computed summary statistics.
    - Regressed DWT on UWT.

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Price histories: DWT vs UWT ETNs

- Illustration:

Summary statistics		
Description	DWT	UWT
n	524	524
Mean (daily)	-0.0016	-0.0013
StDev (daily)	0.0512	0.0518
Skewness	0.1297	-0.7989
Kurtosis	2.2051	2.2265
Autocorrelation	-0.0610	-0.0401
Minimum	-0.2649	-0.2329
Median	-0.0048	0.0042
Maximum	0.1936	0.1562
Mean (annual)	-40.93%	-33.19%
StDev (annual)	81.20%	82.27%
CAGR	-33.59%	-28.24%
HPR	-57.30%	-49.85%

Correlation matrix		
	DWT	UWT
DWT	1	-0.989
UWT	-0.989	1

Volatilities are virtually identical and correlation is near –1.

Both results are expected.

43

Price histories: DWT vs UWT ETNs

- Illustration:

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HPR	-57.30%	-49.85%

Correlation matrix		
	DWT	UWT
DWT	1	-0.989
UWT	-0.989	1

Return results are ludicrous.

DWT (–3x) and UWT (3x) both have negative realized returns (CAGR and HPR).

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Price histories: TSLL vs TSLA

- Illustration:
  - Chose 2*x* ETF on TSLA.
  - TSLL: Direxion, 2*x*, 0.96%, 20220809
  - Downloaded all daily price data from inception through 20251121.
  - Support file: TSLL vs TSLA.xlsx
    - Computed summary statistics.
    - Regressed returns of TSLL on TSLA.

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Price histories: TSLL vs TSLA

- Computed summary statistics.

Summary		
Description	TSLL	TSLA
No. of obs.	575	575
Mean (daily)	-0.0097%	0.0326%
StDev (daily)	6.2887%	3.7195%
Skewness	0.2603602	0.1890515
Kurtosis	3.8450817	2.6092705
Autocorrelation	-0.005438	0.000668
Minimum	-28.6224%	-13.1643%
Median	0.1779%	0.0964%
Maximum	36.2758%	19.8187%
Mean (annual)	-2.46%	8.22%
StDev (annual)	99.83%	59.05%
CAGR	-2.43%	8.57%
HPR	-5.45%	20.62%

Correlations		
	TSLL	TSLA
TSLL	1	0.991
TSLA	0.991	1

Volatility should be 2*x*. Is 99.93/59.05 = 1.69.

TSLL HPR should be 41.24%. Is -5.45%.

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Price histories: TSLL vs TSLA

- Regressed returns of TSLL on TSLA.

Return regression	
Dependent	TSLL
Independent	TSLA
n	575
α	-0.00064
s(α)	0.00036
β	1.67494
s(β)	0.00963
R-squared	0.98140
Adj. R-squared	0.98136
Std error of estimate	0.00858

Intercept should be 0 but is negative and marginally significant.

Slope should be 2 but is significantly less.

Adjusted R-squared is high, indicating strong co-movement between prices.

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Issues

- Levered and inverse ETPs will have payoffs that depend on:
  - Path benchmark index and its return volatility.
- Even if your prediction regarding index return is correct over multi-day holding period, ETP return will not.

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How to develop understanding?

- Monte Carlo simulation is ideal for analyzing levered and inverse ETPs.
  - Support file: LI ETPs (one simulation run).xlsm
    - Sheet: Expected vs actual

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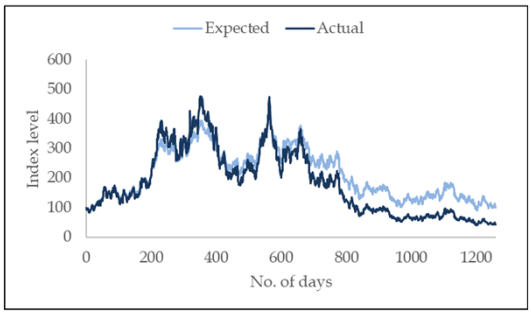
General simulation framework

Simulation of expected vs actual levered ETP return performance		
Simulation parameters		Monte Carlo simulation compares what investor expects from what they actually receives.
Index		
Level	100	
Expected ln return	-20.00%	NOTES:
Expected return volatility	40.00%	1) Expected is realized levered holding period return.
Exchange-traded product		2) Actual is compounded daily levered return.
Leverage ratio	2	3) Hit <F9> to refresh screen.



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General simulation framework

- Monte Carlo simulation is ideal for analyzing levered and inverse ETPs.
  - Support file: LI ETPs (full simulation).xlsx
    - Uses @Risk (must open file through @Risk).
    - Set maximum life to 5 years (1260 days).

Benckmark index	
Level	100
Expected ln return (μ)	-20.00%
Expected volatility (σ)	40.00%
Daily time increment in years	0.003968

Levered product analysis	
Leverage ratio	-3
Stopping criterion	5.0%
Holding period	1,261

Holding period distribution	In days	In years
Mean	978	3.88
Minimum	128	0.51
Median	1,261	5.00
Maximum	1,261	5.00
Prob < 5 years	49.45%	

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Crude oil ETP simulations

Table 4. Simulation of Life in Days for Levered and Inverse ETPs on Three Popular Futures Indexes

		Leverage Ratio					
Measure		-3	-2	-1	1	2	3
A. SPGSCLP (crude oil): CAGR = -15.51%, volatility = 36.58%							
Life in days	Min.	168	459	2,848	1,038	280	123
	Med.	2,698	5,040	5,040	4,411	1,635	859
	Prob. < 20 years (%)	87.7	49.7	2.9	66.8	98.7	100.0
CLR (%)	Min.	-67.9	-52.4	-30.6	-30.8	-52.9	-68.5
	Med.	-1.7	-0.1	0.4	-1.6	-4.1	-7.4
	Max.	178.8	100.6	42.6	42.2	99.9	177.8
LCR – *CLR* (%)	Min.	-86.3	-38.9	-11.7	0.0	-15.4	-51.1
	Med.	3.0	1.5	0.5	0.0	0.5	1.5
	Max.	12.7	6.2	2.0	0.0	2.1	6.4
Prob(*CLR* × *LCR* < 0) (%)		8.4	6.1	4.1	0.0	1.8	3.4

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Crude oil ETP simulations

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Crude oil futures index underlying ETPs.

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Crude oil ETP simulations

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Expected return and volatility.

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Crude oil ETP simulations

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Distribution of life of ETP.

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Crude oil ETP simulations

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Prob(*CLR* × *LCR* < 0) (%)		8.4	6.1	4.1	0.0	1.8	3.4

For -3*x* fund, can be as little as 168 days.
Chance of collapsing within 20 years is 87.7%.

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Crude oil ETP performance

Table 8. Analysis of Tracking Errors and Daily Returns of ETPs Benchmarked to Futures Indexes

		Tracking-Error Summary			Return Summary			
Fund	Expense Ratio	Tracking Difference	Standard Deviation	RATD Ratio	β	H_0: β = *L* (*t*-ratio)	Adj. R^2	Holding-Period Return
A. Crude oil (sample period 28 March 2017–13 March 2020; number of obs. = 745; benchmark HPR = –36.15%)								
DWT, Citigroup, –3x ETN	1.50%	–0.026%	1.674%	–0.431	–2.859	5.19	0.937	–60.37%
OILD, ProShares, –3x ETF	0.49	–0.040	1.676	–0.649	–2.830	6.28	0.936	–63.26
SCO, ProShares, –2x ETF	0.95	–0.022	1.110	–0.549	–1.878	6.80	0.937	–26.21
USO, USCF, 1x ETF	0.73	0.010	0.557	0.485	0.944	–6.24	0.936	–30.68
UCO, ProShares, 2x ETF	0.95	0.009	1.110	0.230	1.849	–8.60	0.937	–69.29
UWT, Citigroup, 3x ETN	1.50	–0.012	1.767	–0.181	2.732	–9.70	0.929	–92.81
OILU, ProShares, 3x ETF	0.49	–0.006	1.759	–0.091	2.719	–10.29	0.930	–92.20

Properties of levered and inverse funds are so strange that all funds can lose.

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Consequences

- Commodity and volatility ETP benchmarks have negative expected returns.
  - Are expected to die quickly.
  - Issuer has right to liquidate.
    - Many have liquidated.
    - Others issue reverse stock splits.
      - Why?

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Record of reverse stock splits

Table 5. Reverse Split Histories of 1x and 2x VIX ETPs since Each Fund's Inception

Date	Reverse Ratio	Since Last Change: Days	Return	Stop
A. VXX (1x): Inception 2009/01/30				
2010/11/09	1:04	648	-89.1%	10.9%
2012/10/05	1:04	696	-81.2	18.8
2013/11/08	1:04	399	-64.4	35.6
2016/08/09	1:04	1,005	-81.2	18.8
2017/08/23	1:04	379	-67.4	32.6
Mean		625.4	-76.7	23.3
Median		648	-81.2%	18.8%
C. TVIX (2x): Inception 2010/11/30				
2012/12/21	1:10	752	-99.1%	0.9%
2013/08/30	1:10	252	-79.7	20.3
2015/06/23	1:10	662	-96.6	3.4
2016/08/09	1:25	413	-87.3	12.7
2017/03/16	1:10	219	-83.7	16.3
2018/06/08	1:10	449	-88.3	11.7
2019/12/02	1:10	542	-83.8	16.2
Mean		469.9	-88.4	11.6
Median		449	-87.3%	12.7%

Date	Reverse Ratio	Since Last Change: Days	Return	Stop
B. VIXY (1x): Inception 2011/01/04				
2013/06/10	1:05	888	-89.1%	10.9%
2016/07/25	1:05	1,141	-81.2	18.8
2017/07/17	1:04	357	-64.4	35.6
Mean		795.3	-78.2	21.8
Median		888	-81.2%	18.8%
D. UVXY (2x): Inception 2011/10/04				
2012/03/08	1:06	156	-85.2%	14.8%
2012/09/07	1:10	183	-88.6	11.4
2013/06/10	1:10	276	-82.4	17.6
2014/01/24	1:04	228	-70.2	29.8
2015/05/20	1:05	481	-88.7	11.3
2016/07/25	1:05	432	-85.4	14.6
2017/01/12	1:05	171	-79.8	20.2
2017/07/17	1:04	186	-73.4	26.6
Mean		264.1	-81.7	18.3
Median		207	-83.8%	16.2%

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Levered and inverse ETPs: Flaws

- Recall earlier slide.
- Levered and inverse ETPs are flawed in <u>two</u> ways:
  - Expected multi-day return is not equal to actual return.
    - Compounding effects resulting from different reinvestment rate assumptions.
      - Performance is benchmark price path dependent.
      - Underperformance increases with benchmark return volatility.
  - End-of-day rebalance effect
    - Can destabilize underlying market.

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End-of-day rebalancing effect

- Levered and inverse products have dynamic futures replication strategies.
  - May require extraordinary volume of trading at close.
- Rebalancing that must be performed at end-of-day (EOD) benchmark index prices to provide promised levered return.
  - Concentration of trading at one instant destabilizes underlying market.

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End-of-day rebalancing effect

- To understand EOD rebalancing, one must understand futures replication strategy.
  - Step-by-step mechanics are provided O'Neill and Whaley (2021).
  - Examine implications here.

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End-of-day rebalancing effect

- Implications of mechanics are *gobsmacking*.
  - Number of futures to buy at close is:

$$\left(n_{F,t} - n_{F,t-1}\right) = \left(\frac{AUM_{t-1}}{F_t}\right)\left(L^2 - L\right)R_{F,t}$$

  - General case:
    - Buy (sell) futures when futures return is positive (negative).
      - Incremental demand may be computed straightforwardly.

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End-of-day rebalancing effect

- Implications of mechanics are *gobsmacking*.
  - Number of futures to buy at close is:

$$\left(n_{F,t} - n_{F,t-1}\right) = \left(\frac{AUM_{t-1}}{F_t}\right)\left(L^2 - L\right)R_{F,t}$$

  - Special cases where demand is 0:
    - If leverage ratio is 1 (ETP is unlevered), no incremental futures hedging is required.
      - Replication strategy is passive.
    - If futures return is 0, no incremental futures hedging is required.

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End-of-day rebalancing effect

- Implications of mechanics are *gobsmacking*.
  - Number of futures to buy at close is:

$$\left(n_{F,t} - n_{F,t-1}\right) = \left(\frac{AUM_{t-1}}{F_t}\right)\left(L^2 - L\right)R_{F,t}$$

  - General case:
    - Incremental futures hedging demand by levered and inverse ETPs is in same direction.
      - If $L = 2$, $L^2 - L = 2$, and, if $L = -1$, $L^2 - L = 2$.
      - Recipe for disaster.
    - Incremental futures demand is in same direction as futures index return.

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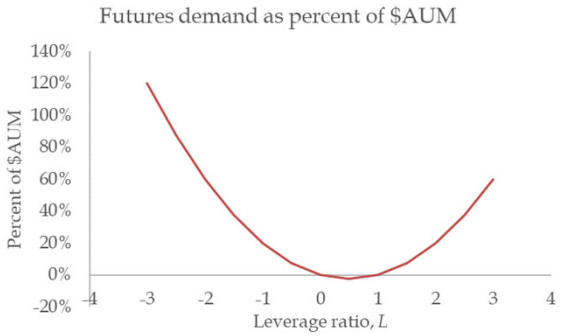
End-of-day rebalancing effect

- Assume benchmark index return is 50%.



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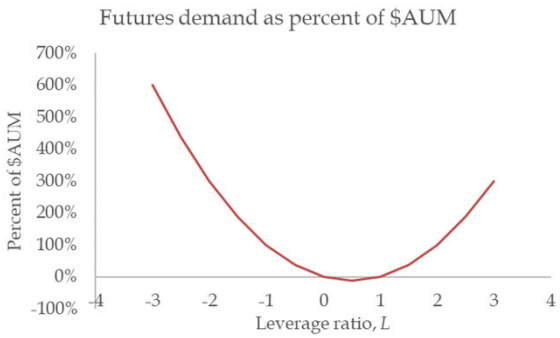
End-of-day rebalancing effect

- Assume benchmark index return is 10%.



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Credit Suisse fiasco #2

- On February 5, 2018, XIV (–1*x* ETN on SPVXSTR) exploded and was quickly delisted.
  - Investors lost virtually all their investment.
- What happened?
- Total end-of-day dollar futures demand is

$$\left(n_{F,t} - n_{F,t-1}\right) F_t = AUM_{t-1} \left(L^2 - L\right) R_{F,t}$$

Know this.

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Credit Suisse fiasco #2

- At close on Friday, Feb. 2, 2018,

Analysis of VIX futures hedging on Feb. 5, 2018		
Symbol	Leverage factor	NAV on 2/2/2018
VXX	1	1,061,669,693
VIXY	1	168,720,939
TVIX	2	349,729,625
UVXY	2	434,306,460
XIV	-1	1,624,858,612
SVXY	-1	1,441,830,320
Total levered and inverse		3,850,725,017

$$AUM_{t-1} = 3{,}850.7 \text{ million}$$

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Credit Suisse fiasco #2

- Recall from levered and inverse products that to maintain leverage ratio L for day t, have buy/sell VIX futures.
- Total end-of-day dollar demand is

$$\left(n_{F,t} - n_{F,t-1}\right) F_t = AUM_{t-1} \left(L^2 - L\right) R_{F,t}$$

Know this.

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Credit Suisse fiasco #2

- What was VIX futures index return on Feb. 5, 2018?

Analysis of VIX futures hedging on Feb. 5, 2018				
	Futures settlement prices			
Futures contract	20180202	20180205	Return	Weights
20180214	15.625	33.225	112.64%	0.35
20180321	14.975	27.975	86.81%	0.65
Index	15.2025	29.8125	96.10%	1.0000

VIX futures index spiked upward by 96.10%.

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Credit Suisse fiasco #2

- What was total dollar VIX futures demand at close on Feb. 5, 2018?

$$AUM_{t-1}\left(L^2 - L\right)R_{F,t} = 3,850,725,017 \times 2 \times 0.9610 = 7,401,294,853$$

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Credit Suisse fiasco #2

- What was outstanding supply of VIX futures at close on Feb. 2, 2018?

Analysis of VIX futures hedging on Feb. 5, 2018

	Futures dollar open interest		
Futures contract	20180202	20180205	Increase
20180214	3,489,093,750	7,402,662,900	3,913,569,150
20180321	3,589,537,450	8,051,988,300	4,462,450,850
	7,078,631,200	15,454,651,200	8,376,020,000

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Credit Suisse fiasco #2

- What was percent increase from open interest on Feb. 2, 2018?

Analysis of VIX futures hedging on Feb. 5, 2018

Futures contract	Futures hedging demand at close on 20180205	Percent of open interest at close on 20180202
20180214	2,590,453,198	74.24%
20180321	4,810,841,654	134.02%
Total	7,401,294,853	

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Credit Suisse fiasco #2

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Lesson summary

- Levered and inverse ETPs:
  - Are dynamically rebalanced derivatives (futures) trading strategies.
  - Produce actual multi-day holding period returns that deviate randomly from what investors expect.
  - Deviations are:
    - Path dependent due to different reinvestment rate assumptions.
    - Underperform directly with return volatility.
  - Expected long-term performance is –100%.
    - Only question is when.

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