

INTEREST  
RATE  
FUTURES  
OPTION  
CONTRACTS

Options on Treasury instruments began trading in October 1982. These instruments offer important new ways for managing the interest rate risk of fixed-income portfolios. In particular, interest rate options provide an effective means of managing the convexity risk—a subject first discussed in Chapter 8. In the first section of this chapter, we describe the interest rate option and futures option markets that are currently active in the United States. Of the exchange-traded option contracts, the T-bond futures option is clearly the most popular. Options on T-bonds and T-notes are not as actively traded, although OTC markets for these contracts continue to proliferate. We then proceed, in section 2, with a discussion of short-term interest rate option valuation. Short-term interest rate options provide an interesting new valuation challenge. Using the standard lognormal price distribution assumption from the previous chapters is clearly inappropriate because the underlying asset value cannot exceed a predetermined level.<sup>1</sup> In its place, we substitute the assumption that yield is lognormally distributed and then rederive the European option valuation equations. The third and the fourth sections discuss T-bond and T-bond futures option valuation under the lognormal price and the lognormal yield assumptions, respectively. Section 5 is a detailed discussion of duration/convexity risk management.

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<sup>1</sup>Recall that the assumption of a lognormal price distribution permits the price to rise without limit.

## 15.1 INTEREST RATE OPTION MARKETS

The first interest rate option contracts to trade in the United States were the Chicago Board of Trade's (CBT's) Treasury bond futures option contract and the Chicago Mercantile Exchange's (CME's) Eurodollar futures option contract on October 1, 1982. The Chicago Board Options Exchange (CBOE) introduced options on Treasury bonds, and the American Exchange introduced options on Treasury notes and bills on October 22, 1982. Other interest rate options have been introduced subsequently.

Table 15.1 contains a listing of interest rate instruments from the *Wall Street Journal*. Options on Treasury bonds and notes are traded at the CBOE. These contracts expire on the Saturday after the third Friday of the contract month, are American-style, and require the delivery of a specific Treasury bond or Treasury note. Exchange-traded options on T-bonds and notes are not very active, however. In fact, no active contracts are listed in Table 15.1. The table reports only prices of relatively inactive CBOE options on short-term and long-term bond indexes.

The most active interest rate options are those written on interest rate futures contracts. Of these, the CBT's Treasury bond futures option and the CME's Eurodollar futures option contracts have the greatest trading volume and open interest, as shown in Table 15.1. Upon exercising a T-bond futures option, a long or short position in the nearby T-bond futures contract is assumed. These options are American-style and thus may be exercised at any time up to and including the expiration day. The last day of trading is the Friday preceding, by at least five business days, the first notice day for the corresponding T-bond futures contract. In general, the first notice day of the futures is the first business day of the contract month.

The Eurodollar futures option is also American-style. The expiration day of the Eurodollar futures option contract is the second London business day before the third Wednesday of the contract month, the same as that of the underlying Eurodollar futures. Exercise of the Eurodollar option results in delivery of a position in the Eurodollar futures contract of the same maturity. The Eurodollar futures contract, in turn, fixes the price (or, equivalently, the yield) on a three-month Eurodollar deposit.

Table 15.2 on pages 370–371 shows the large number of Treasury issues outstanding on a given date. Not all of these, nor even the majority of these, have exchange-traded options. If individuals want to buy or sell options on particular bond issues, they usually go to OTC markets where bond option contracts can be tailored in any manner. It is commonplace to see both European- and American-style OTC bond options, including ones with times to expiration of several years.

In Chapter 8, we discussed a number of conventions regarding T-bonds and T-bond price reporting. For example, the decimal part of the reported price represents 32nds. Thus, the reported bid price (in Table 15.2) of 107:07 for the 8½s of May 1997 is actually  $107\frac{7}{32}$ . Two other conventions are that the face value of a Treasury bond is \$100,000 and the bond price is reported as a percentage of par. Thus, the bid price of the 8½s of May 1997 is actually  $107\frac{7}{32} \times \$1000$  or \$107,218.75. Finally, the reported bond price does not include the accrued interest for the current coupon period. For the 8½s of May 1997, this means that from the

TABLE 15.1 Interest rate options and futures.

# INTEREST RATE INSTRUMENTS

Wednesday, November 13, 1991

For Notes and Bonds, decimals in closing prices represent 32nds; 1.01 means 1 1/32. For Bills, decimals in closing prices represent basis points; \$25 per .01.

## OPTIONS

### CHICAGO BOARD

#### OPTIONS ON SHORT-TERM INTEREST RATES

Strike	Nov	Dec	Jan	Nov	Dec	Jan
Price	Nov	Dec	Jan	Nov	Dec	Jan
45		15/16				

Total call volume 45 Total call open int. 1,787

Total put volume 0 Total put open int. 130

IRX levels: High 47.50; Low 46.20; Close 46.20, unch

#### OPTIONS ON LONG-TERM INTEREST RATES

Strike	Nov	Dec	Jan	Nov	Dec	Jan
Price	Nov	Dec	Jan	Nov	Dec	Jan
75		15/16		7/16		
77 1/2		1/2				

Total call volume 17 Total call open int. 2,012

Total put volume 6 Total put open int. 1,470

LTX levels: High 75.40; Low 74.08; Close 74.73, +0.58

## FUTURES

	Open	High	Low	Settle	Chg	Settle	Chg	Yield	Open
<b>TREASURY BONDS (CBT)</b> —\$100,000; pts. 32nds of 100%									
Dec 101-04	101-05	99-15	100-03	-	29	7.991	+.091	280.722	
Mr92	100-08	100-10	98-20	99-07	-	30	8.079	+.095	31,420
June	99-12	99-12	97-30	98-10	-	30	8.173	+.097	10,197
Sept	98-13	98-13	97-06	97-14	-	31	8.264	+.101	2,717
Dec	97-19	97-19	96-10	96-20	-	31	8.350	+.102	4,487
Mr93	96-00	96-05	95-28	95-28	-	32	8.430	+.107	511

Est vol 370,000; vol Tues 245,191; op Int 330,091, +8.975.

	Open	High	Low	Settle	Chg	Settle	Chg	Yield	Open
<b>TREASURY BONDS (MCE)</b> —\$50,000; pts. 32nds of 100%									
Dec 101-02	101-02	99-15	100-04	-	32	7.987	+.100	13,542	

Est vol 6,600; vol Tues 6,396; open Int 13,641, -132.

	Open	High	Low	Settle	Chg	Settle	Chg	Yield	Open
<b>T-BONDS (LIFFE)</b> U.S. \$100,000; pts of 100%									
Dec 101-02	101-03	99-21	100-05	-	0-23	101-03	96-24	5,443	

Est vol 2,273; vol Tues 4,086; open Int 5,480, +721.

	Open	High	Low	Settle	Chg	Settle	Chg	Yield	Open
<b>GERMAN GOV'T. BOND (LIFFE)</b> 250,000 marks; \$ per mark (.81)									
Dec	86.23	86.25	86.02	86.19	+.02	86.44	83.73	75.176	
Mr92	n.a.	n.a.	n.a.	n.a.	n.a.	86.70	85.39	6,879	

Est vol 47,392; vol Tues 51,218; open Int 82,055, -2,487.

	Open	High	Low	Settle	Chg	Settle	Chg	Yield	Open
<b>TREASURY NOTES (CBT)</b> —\$100,000; pts. 32nds of 100%									
Dec 103-23	103-23	102-18	103-06	-	14	7.540	+.061	86,289	
Mr92	102-29	102-29	101-26	102-13	-	14	7.651	+.062	12,194
June				101-18	-	13	7.772	+.058	418

Est vol 30,000; vol Tues 25,215; open Int 98,902, +4,714.

	Open	High	Low	Settle	Chg	Settle	Chg	Yield	Open
<b>5 YR TREAS NOTES (CBT)</b> —\$100,000; pts. 32nds of 100%									
Dec 04-275	104-28	104-12	04-215	-	5.5	6.880	+.040	91,919	
Mr92	04-015	04-015	03-19	03-275	-	6.0	7.071	+.045	10,105

Est vol 19,429; vol Tues 16,648; open Int 102,024, +2,602.

	Open	High	Low	Settle	Chg	Settle	Chg	Yield	Open
<b>2 YR TREAS NOTES (CBT)</b> —\$200,000; pts. 32nds of 100%									
Dec 103-26	103-26	103-17	03-255	-	1/4			13,800	
Mr92	103-11	103-11	03-057	03-105	-	1/2		3,553	

Est vol 1,500; vol Tues 885; open Int 17,353, +153.

	Open	High	Low	Settle	Chg	Settle	Chg	Yield	Open
<b>30-DAY INTEREST RATE (CBT)</b> —\$5 million; pts. of 100%									
Nov	95.14	95.14	95.13	95.14	-.01	4.86	+.01	1,254	
Dec	95.12	95.12	95.08	95.09	-.05	4.91	+.05	1,232	
Ja92	95.16	95.17	95.15	95.17	-.05	4.83	+.05	1,100	
Feb	95.23	95.27	95.23	95.26	-.04	4.74	+.04	962	
Mar	95.18	95.21	95.18	95.21	-.05	4.79	+.05	570	
Apr	95.20	95.20	95.20	95.20	-.05	4.80	+.05	107	
June	95.10	95.11	95.10	95.11	-.04	4.89	+.04	189	

Est vol 725; vol Tues 529; open Int 5,464, +185.

#### TREASURY BILLS (IMM)—\$1 mil.; pts. of 100%

	Open	High	Low	Settle	Chg	Settle	Chg	Yield	Open
Dec	95.38	95.38	95.31	95.34	-.03	4.66	+.03	21,996	
Mr92	95.52	95.52	95.42	95.50	-.02	4.50	+.02	29,371	
June	95.23	95.34	95.23	95.32	-.03	4.68	+.03	3,892	
Sept	95.09	95.09	95.05	95.08	-.02	4.92	+.02	338	
Dec	94.64	94.64	94.64	94.64	.....	5.36	.....	156	

Est vol 6,328; vol Tues 5,830; open Int 55,771, +261.

#### LIBOR-1 MO. (IMM)—\$3,000,000; points of 100%

	Open	High	Low	Settle	Chg	Settle	Chg	Yield	Open
Nov	95.05	95.05	94.98	95.02	-.04	4.98	+.04	6,950	
Dec	94.60	94.60	94.50	94.55	-.10	5.45	+.10	7,336	
Ja92	95.11	95.11	95.02	95.08	-.05	4.92	+.05	9,651	
Feb	95.01	95.08	94.99	95.06	-.05	4.94	+.05	2,515	
Mar	94.96	95.01	94.94	95.00	-.05	5.00	+.05	1,390	
Apr	.....	.....	.....	95.01	-.04	4.99	+.04	163	

Est vol 1,577; vol Tues 2,063; open Int 28,005, +300.

#### MUNI BOND INDEX (CBT)—\$1,000; Times Bond Buyer MBI

	Open	High	Low	Settle	Chg	High	Low	Interest	
Dec	95-23	95-23	95-01	95-08	-	15	95-25	88-16	12,542
Mr92	95-04	95-04	94-07	94-13	-	19	95-04	88-00	827

Est vol 2,500; vol Tues 2,607; open Int 13,370, +850.

The Index: Close 95-09; Yield 6.82.

#### EURODOLLAR (IMM)—\$1 million; pts of 100%

	Open	High	Low	Settle	Chg	Settle	Chg	Yield	Open
Dec	94.85	94.86	94.75	94.80	-.07	5.20	+.07	242,049	
Mr92	94.99	94.99	94.84	94.94	-.05	5.06	+.05	252,314	
June	94.77	94.78	94.62	94.74	-.04	5.26	+.04	145,943	
Sept	94.48	94.50	94.34	94.46	-.03	5.54	+.03	100,739	
Dec	93.94	93.96	93.83	93.93	-.03	6.07	+.03	71,656	
Mr93	93.76	93.77	93.65	93.74	-.03	6.26	+.03	55,423	
June	93.46	93.46	93.36	93.43	-.02	6.57	+.02	44,243	
Sept	93.17	93.19	93.09	93.17	-.01	6.83	+.01	31,658	
Dec	92.79	92.82	92.72	92.82	+.02	7.18	-.02	21,132	
Mr94	92.76	92.81	92.71	92.80	+.03	7.20	-.03	27,888	
June	92.50	92.58	92.46	92.57	+.05	7.43	-.05	17,956	
Sept	92.28	92.37	92.23	92.35	+.06	7.65	-.06	11,828	
Dec	91.97	92.06	91.92	92.05	+.07	7.95	-.07	9,816	
Mr95	91.96	92.05	91.93	92.04	+.07	7.96	-.07	7,178	
June	91.84	91.93	91.82	91.92	+.07	8.08	-.07	6,868	
Sept	91.69	91.78	91.67	91.77	+.07	8.23	-.07	6,419	

Est vol 283,796; vol Tues 130,709; open Int 1,053,277, +4,629.

#### EURODOLLAR (LIFFE)—\$1 million; pts of 100%

	Open	High	Low	Settle	Change	High	Low	Interest
Dec	94.86	94.87	94.75	94.83	-.02	94.94	90.58	17,546
Mr92	94.97	94.99	94.85	94.96	.....	95.06	90.60	10,098
June	94.77	94.78	94.66	94.75	.....	94.83	90.97	5,226
Sept	94.49	94.49	94.47	94.46	.....	94.53	90.97	2,724
Dec	93.97	93.97	93.95	93.93	.....	94.00	91.54	614
Mr93	93.80	93.80	93.78	93.74	+.02	93.80	91.55	545
June	.....	.....	.....	93.43	-.23	93.44	92.60	405
Sept	.....	.....	.....	93.17	+.08	93.09	92.82	137

Est vol 3,771; vol Tues 4,227; open Int 37,295, +327.

#### STERLING (LIFFE)—£500,000; pts of 100%

	Open	High	Low	Settle	Chg	High	Low	Interest
Dec	89.79	89.82	89.78	89.81	+.02	90.35	86.52	52,369
Mr92	90.25	90.29	90.24	90.28	+.04	90.49	86.68	45,849
June	90.34	90.37	90.33	90.36	+.03	90.46	87.45	34,626
Sept	90.31	90.33	90.29	90.32	+.02	90.41	87.46	10,853
Dec	90.22	90.23	90.20	90.23	+.02	90.32	87.55	6,664
Mr93	90.07	90.10	90.07	90.09	+.02	90.16	87.50	4,548
June	89.97	89.97	89.97	89.98	+.02	90.09	87.58	2,095
Sept	89.93	89.95	89.93	89.95	+.02	90.08	88.20	1,746
Dec	89.88	89.90	89.88	89.92	+.04	90.02		

TABLE 15.1

**OTHER INTEREST RATE FUTURES**

Settlement prices of selected contracts. Volume and open interest of all contract months.

**Mortgage-Backed (CBT)**—\$100,000, pts. & 64ths of 100%  
Nov Cpn 8.5 102-04 -6; Est. vol. 0; Open Int. 90  
**5-Yr. Int. Rate Swap (CBT)**—\$25 per ½ b.p.; pts of 100%  
Dec 92.770 -.010; Est. vol. 50; Open Int. 707  
**3-Yr. Int. Rate Swap (CBT)**—\$25 per ½ b.p.; pts of 100%  
Dec 93.490 -.010; Est. vol. 0; Open Int. 456  
**Treas. Auction 5 Yr (FINEX)**—\$250,000, 100 minus yield  
Dec 93.22 -4.0; Est. vol. 100; Open Int. 4

CBT—Chicago Board of Trade. FINEX—Financial Instrument Exchange, a division of the New York Cotton Exchange. IMM—International Monetary Market at Chicago Mercantile Exchange. LIFFE—London International Financial Futures Exchange. MCE—MidAmerica Commodity Exchange.

**FUTURES OPTIONS**

**T-BONDS (CBT)** \$100,000; points and 64ths of 100%  
Strike Calls—Last Puts—Last  
Price Dec-c Mar-c Jun-c Dec-p Mar-p Jun-p  
96 4-07 3-57 3-48 c7 0-44 1-28  
98 2-08 2-31 2-40 0-02 1-19 2-19  
100 0-25 1-31 1-47 0-21 2-17 3-20  
102 0-02 0-51 1-06 1-60 3-32 4-42  
104 0-01 0-26 0-43 3-58 5-05 6-12  
106 c2 0-12 0-25 5-58 6-54 7-54  
Est. vol. 100,000, Tues vol. 57,179 calls, 29,063 puts  
Open Interest Tues 351,393 calls, 279,518 puts  
**T-NOTES (CBT)** \$100,000; points and 64ths of 100%  
Strike Calls—Last Puts—Last  
Price Dec-c Mar-c Jun-c Dec-p Mar-p Jun-p  
101 2-13 2-14 ..... 0-01 0-53 .....  
102 1-14 1-38 ..... 0-02 1-13 .....  
103 0-23 1-08 ..... 0-11 1-45 .....  
104 0-04 0-48 ..... 0-55 .....  
105 0-01 0-32 .....  
106 0-01 .....  
Est. vol. 7,500, Tues vol. 2,665 calls, 1,850 puts  
Open Interest Tues 38,546 calls, 34,901 puts

**MUNICIPAL BOND INDEX (CBT) \$100,000; pts. & 64ths of 100%**

Strike Price	Calls—Settle			Puts—Settle		
	Dec-c	Mar-c	Jun-c	Dec-p	Mar-p	Jun-p
93	2-19	2-00	1-56	0-05	0-40	1-13
94	1-26	1-26	.....	0-11	1-00	.....
95	0-44	.....	.....	0-28	.....	.....
96	0-18	.....	0-44	1-03	.....	2-57
97	.....	.....	.....	.....	.....	.....
98	.....	.....	.....	.....	.....	.....

Est. vol. 10, Tues vol. 64 calls, 2 puts

Open Interest Tues 6,538 calls, 6,435 puts

**5 YR TREAS NOTES (CBT) \$100,000; points and 64ths of 100%**

Strike Price	Calls—Last			Puts—Last		
	Dec-c	Mar-c	Jun-c	Dec-p	Mar-p	Jun-p
10350	1-11	0-62	.....	0-01	0-39	.....
10400	0-44	0-45	.....	0-01	0-54	.....
10450	0-16	0-32	.....	0-05	.....	.....
10500	0-02	.....	.....	0-24	.....	.....
10550	0-01	0-14	.....	.....	.....	.....
10600	.....	.....	.....	.....	.....	.....

Est. vol. 2,500, Tues vol. 315 calls, 960 puts

Open Interest Tues 5,186 calls, 6,365 puts

**5 YR INT. RATE SWAP (CBT) \$12.50 per ¼ b.p.; pts of 100%**

Strike Price	Calls—Last			Puts—Last		
	Dec-c	Mar-c	Jun-c	Dec-p	Mar-p	Jun-p
9260	.....	.....	.....	.....	.....	.....
9270	.....	.....	.....	.....	.....	.....
9280	.....	.....	.....	.....	.....	.....
9290	.....	.....	.....	.....	.....	.....
9300	.....	.....	.....	.....	.....	.....
9310	.....	.....	.....	.....	.....	.....

Est. vol. 0, Tues vol. 0 calls, 0 puts

Open Interest Tues 115 calls, 140 puts

**EURODOLLAR (IMM) \$ million; pts. of 100%**

Strike Price	Calls—Settle			Puts—Settle		
	Dec-c	Mar-c	Jun-c	Dec-p	Mar-p	Jun-p
9425	0.56	0.72	0.56	0.01	0.03	0.09
9450	0.32	0.50	0.39	0.02	0.06	0.16
9475	0.12	0.30	0.25	0.07	0.11	0.26
9500	0.02	0.15	0.14	0.22	0.21	0.39
9525	.0004	0.07	0.08	0.45	0.38	.....
9550	.....	0.03	0.04	0.70	.....	.....

Est. vol. 62,887, Tues vol. 15,575 calls, 7,508 puts

Open Interest Tues 248,142 calls, 360,020 puts

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It is important to review these pricing conventions since similar conventions are used in bond option pricing. For example, prices of T-bond options traded at the CBOE are also reported in 32nds and as a percentage of par. Thus, a reported T-bond option price of 2-24 means that the cost of the option is  $2\frac{24}{32} \times \$1000$ , or \$2,750. To provide a finer demarcation in option price, some OTC T-bond option dealers quote prices in 64ths. The CBOT uses a similar practice for T-bond futures options.<sup>2</sup>

The accrued interest convention also has an impact on T-bond option pricing mechanics. Assume, for example, that a call option with an exercise price of 107 is written on the 8½s of May 1997. If this option were to be exercised on November 13, 1991, the bond holder would pay the exercise price and receive a T-bond that she could immediately sell for \$111,422.55. The exercise price, however, is *not* simply the stated exercise price times \$1,000. If the call is exercised, the option

<sup>2</sup>Recall that the CBOT's T-bond futures contract is quoted in 32nds.

holder must not only pay the stated exercise price but also the accrued interest as of the exercise date. Thus, the total exercise price on November 13, 1991, is  $(107 + 4.20380) \times \$1000$ , or \$111,203.80.

The price of the Eurodollar futures option is expressed in decimal form, representing basis points. Exercising the Eurodollar futures option implies that a futures position is assumed. The December 9475 call option contract implies that the option holder may buy a Eurodollar futures contract at an index level of 94.75.<sup>3</sup> Each basis point of the price of the option is worth \$25, so the price of the December 9475 call is  $12 \times \$25$ , or \$300. (See Table 15.1.) The \$25 value comes from the value of .01 percent of \$1 million for three months (i.e.,  $.0001 \times \$1,000,000 \times 90/360 = \$25$ ).

## 15.2 SHORT-TERM INTEREST RATE OPTION PRICING

The assumption that the underlying commodity has a lognormal price distribution at the option's expiration works well for most commodities, but it is inappropriate for short-term debt instruments such as a T-bill or Eurodollar deposit. The lognormal distribution allows for the possibility of infinitely large prices. T-bills and Eurodollar deposits, which generally mature three months after the option expiration, have a predetermined future value. This fixed future value makes the possibility of an infinitely large price only three months before maturity unreasonable.

To circumvent this problem, we assume that the yield, rather than the price, of the short-term debt instrument is lognormally distributed at the option's expiration. Under this assumption, the yield can fall to zero, in which case the market price of the short-term debt instrument becomes its predetermined maturity value. On the other hand, if the yield rises without limit, the market price of the debt instrument converges to zero.

To make the valuation approach as specific as possible, we focus on the most popular short-term debt option—the CME's Eurodollar futures option. The pricing principles developed here, however, can easily be extended to the other options on short-term debt instruments. Aside from the assumption of lognormal yield, we invoke all of the same assumptions used in Chapter 11. In particular, the assumption of risk-neutral pricing greatly simplifies the development.

Using the risk-neutral valuation approach, the value of the Eurodollar futures option today is the present value of the expected terminal value, that is,

$$c(F, T; X) = e^{-rT} E(\tilde{c}_T). \quad (15.1)$$

The terminal value of the option is, in turn, a function of the Eurodollar futures index level,  $\tilde{F}_T$ , that is,

$$\tilde{c}_T = \begin{cases} \tilde{F}_T - X & \text{if } \tilde{F}_T \geq X \\ 0 & \text{if } \tilde{F}_T < X. \end{cases} \quad (15.2)$$

<sup>3</sup>The translation of the index level to the yield on the \$1,000,000 Eurodollar deposit is provided in Chapter 8.

TABLE 15.2 Treasury instruments.

# TREASURY BONDS, NOTES & BILLS

Wednesday, November 13, 1991

Representative Over-the-Counter quotations based on transactions of \$1 million or more.

Treasury bond, note and bill quotes are as of mid-afternoon. Colons in bid-and-asked quotes represent 32nds; 101:01 means 101 1/32. Net changes in 32nds. n-Treasury note. Treasury bill quotes in hundredths, quoted on terms of a rate of discount. Days to maturity calculated from settlement date. All yields are to maturity and based on the asked quote. Latest 13-week and 26-week bills are boldfaced. For bonds callable prior to maturity, yields are computed to the earliest call date for issues quoted above par and to the maturity date for issues below par. \*When issued.

Source: Federal Reserve Bank of New York.

U.S. Treasury strips as of 3 p.m. Eastern time, also based on transactions of \$1 million or more. Colons in bid-and-asked quotes represent 32nds; 101:01 means 101 1/32. Net changes in 32nds. Yields calculated on the bid quotation. ci-stripped coupon interest. bp-Treasury bond, stripped principal. np-Treasury note, stripped principal. For bonds callable prior to maturity, yields are computed to the earliest call date for issues quoted above par and to the maturity date for issues below par.

Source: Bear, Stearns & Co. via Street Software Technology Inc.

## GOVT. BONDS & NOTES

Maturity					Maturity					Mat.							
Rate	Mo/Yr	Bid	Asked	Chg.	Ask Yld.	Rate	Mo/Yr	Bid	Asked	Chg.	Ask Yld.	Mat.	Type	Bid	Asked	Chg.	Bid Yld.
6 1/2	Nov 91n	100:00	100:02	.....	0.00	8 1/2	Jul 97n	107:07	107:09	- 5	6.92	Nov 99	np	55:03	55:07	- 7	7.59
8 1/2	Nov 91n	100:00	100:02	.....	0.00	8 5/8	Aug 97n	107:26	107:28	- 6	6.94	Feb 00	ci	53:28	54:00	- 8	7.64
14 1/4	Nov 91n	100:01	100:03	.....	0.00	8 3/4	Oct 97n	108:16	108:18	- 4	6.96	Feb 00	np	53:22	53:27	- 9	7.68
7 3/4	Nov 91n	100:04	100:06	.....	3.06	8 7/8	Nov 97n	109:04	109:06	- 7	6.97	May 00	ci	52:28	53:00	- 7	7.64
7 5/8	Dec 91n	100:11	100:13	.....	4.24	7 7/8	Jan 98n	104:06	104:08	- 7	7.01	May 00	np	52:22	52:27	- 7	7.68
8 1/4	Dec 91n	100:14	100:16	.....	4.10	8 1/8	Feb 98n	105:13	105:15	- 6	7.03	Aug 00	ci	51:26	51:30	- 7	7.66
11 5/8	Jan 92n	101:03	101:05	- 1	4.43	7 7/8	Apr 98n	104:06	104:08	- 5	7.04	Aug 00	np	51:23	51:27	- 7	7.68
8 1/8	Jan 92n	100:22	100:24	- 1	4.40	7	May 93-98	100:16	100:24	.....	6.47	Nov 00	ci	50:24	50:28	- 8	7.68
6 5/8	Feb 92n	100:13	100:15	.....	4.65	9	May 98n	109:27	109:29	- 8	7.07	Nov 00	np	50:24	50:28	- 7	7.68
9 1/8	Feb 92n	101:01	101:03	.....	4.59	8 1/4	Jul 98n	105:30	106:00	- 7	7.10	Feb 01	ci	49:14	49:19	- 7	7.76
14 5/8	Feb 92n	102:13	102:15	- 1	4.48	9 1/4	Aug 98n	111:04	111:06	- 8	7.13	Feb 01	np	49:09	49:13	- 8	7.80
8 1/2	Feb 92n	101:01	101:03	.....	4.61	7 1/8	Oct 98n	100:09	100:11	- 5	7.06	May 01	ci	48:14	48:18	- 8	7.78
7 7/8	Mar 92n	101:03	101:05	- 1	4.69	3 1/2	Nov 98	97:24	98:24	+ 24	3.70	May 01	np	48:08	48:12	- 9	7.82
8 1/2	Mar 92n	101:11	101:13	.....	4.63	8 7/8	Nov 98n	109:09	109:11	- 6	7.15	Aug 01	ci	47:16	47:21	- 9	7.78
11 3/4	Apr 92n	102:25	102:27	- 2	4.72	8 7/8	Feb 99n	109:09	109:11	- 6	7.20	Aug 01	np	47:11	47:15	- 8	7.82
8 7/8	Apr 92n	101:26	101:28	.....	4.68	8 1/2	May 94-99	105:00	105:08	- 4	6.20	Nov 01	ci	46:20	46:24	- 7	7.78
6 5/8	May 92n	100:27	100:29	.....	4.77	9 1/8	May 99n	110:23	110:25	- 7	7.24	Feb 02	ci	45:09	45:13	- 12	7.88
9	May 92n	102:00	102:02	- 1	4.78	8	Aug 99n	104:08	104:10	- 6	7.26	May 02	ci	44:11	44:15	- 11	7.90
13 3/4	May 92n	104:11	104:13	- 1	4.73	7 7/8	Nov 99n	103:15	103:17	- 6	7.28	Aug 02	ci	43:12	43:17	- 12	7.92
8 1/2	May 92n	101:28	101:30	- 1	4.83	8 1/2	Feb 95-00	102:15	102:19	- 1	6.97	Nov 02	ci	42:15	42:19	- 11	7.94
8 1/4	Jun 92n	102:01	102:03	- 1	4.82	8 1/2	Feb 00n	107:01	107:03	- 9	7.34	Feb 03	ci	41:16	41:21	- 12	7.97
8 3/8	Jun 92n	102:03	102:05	.....	4.84	8 7/8	May 00n	109:06	109:08	- 12	7.39	May 03	ci	40:21	40:26	- 12	7.98
10 3/8	Jul 92n	103:16	103:18	.....	4.88	8 3/8	Aug 95-00	104:18	104:22	- 3	6.93	Aug 03	ci	39:27	39:31	- 11	7.99
8	Jul 92n	102:02	102:04	- 1	4.92	8 3/4	Aug 00n	108:12	108:14	- 13	7.42	Nov 03	ci	39:00	39:05	- 12	8.00
4 1/4	Aug 87-92	98:13	99:13	- 1	5.07	8 1/2	Nov 00n	106:27	106:29	- 13	7.43	Feb 04	ci	38:04	38:08	- 11	8.03
7 1/4	Aug 92	101:18	101:22	.....	4.94	7 3/4	Feb 01n	101:31	102:01	- 11	7.44	May 04	ci	37:09	37:13	- 12	8.05
7 7/8	Aug 92n	102:00	102:02	- 1	5.04	11 3/4	Feb 01	128:14	128:18	- 10	7.43	Aug 04	ci	36:16	36:21	- 12	8.06
8 1/4	Aug 92n	102:09	102:11	- 1	5.03	8	May 01n	103:22	103:24	- 10	7.44	Nov 04	ci	35:24	35:28	- 12	8.07
8 1/8	Aug 92n	102:11	102:13	- 1	4.99	13 1/8	May 01	138:04	138:08	- 13	7.44	Nov 04	bp	35:23	35:27	- 11	8.08
8 1/8	Sep 92n	102:18	102:20	- 1	5.02	8	Aug 01n	102:29	102:31	- 14	7.44	Feb 05	ci	34:31	35:03	- 11	8.09
8 3/4	Sep 92n	103:03	103:05	- 1	5.02	8	Aug 96-01	103:19	103:23	- 15	7.07	May 05	ci	34:08	34:12	- 11	8.10
9 3/4	Oct 92n	104:04	104:06	- 1	5.02	13 3/8	Aug 01	140:16	140:20	- 8	7.44	May 05	bp	34:09	34:13	- 11	8.09
7 3/4	Oct 92n	102:12	102:14	- 1	5.11	7 1/2	Nov 01n*	100:19	100:20	- 11	7.41	Aug 05	ci	33:18	33:22	- 11	8.10
7 3/4	Nov 92n	102:15	102:17	- 1	5.12	15 3/4	Nov 01	157:21	157:25	- 10	7.45	Aug 05	bp	33:19	33:24	- 12	8.09
8 3/8	Nov 92n	103:02	103:04	- 1	5.13	14 1/4	Feb 02	147:25	147:29	- 14	7.48	Nov 05	ci	32:29	33:01	- 11	8.10
10 1/2	Nov 92n	105:04	105:06	- 1	5.13	11 5/8	Nov 02	130:10	130:14	- 13	7.51	Feb 06	ci	32:05	32:10	- 12	8.12
7 3/8	Nov 92n	102:06	102:08	- 1	5.11	10 3/4	Feb 03	123:31	124:03	- 12	7.54	Feb 06	bp	32:04	32:08	- 11	8.13
7 1/4	Dec 92n	102:06	102:08	.....	5.17	10 3/4	May 03	124:06	124:10	- 12	7.55	May 06	ci	31:17	31:21	- 11	8.12
9 1/8	Dec 92n	104:06	104:08	- 1	5.19	11 1/8	Aug 03	127:04	127:08	- 15	7.58	Aug 06	ci	30:29	31:01	- 11	8.12
8 3/4	Jan 93n	103:28	103:30	- 2	5.23	11 7/8	Nov 03	133:07	133:11	- 23	7.59	Nov 06	ci	30:10	30:14	- 11	8.12
7	Jan 93n	101:30	102:00	- 1	5.27	12 3/8	May 04	137:30	138:02	- 29	7.60	Feb 07	ci	29:16	29:20	- 12	8.17
4	Feb 88-93	96:22	97:22	- 1	5.94	13 3/4	Aug 04	149:07	149:11	- 29	7.63	May 07	ci	28:29	29:01	- 12	8.17
6 3/4	Feb 93	101:22	101:26	- 1	5.24	11 5/8	Nov 04	132:02	132:06	- 23	7.67	Aug 07	ci	28:11	28:15	- 12	8.17
7 7/8	Feb 93	103:00	103:04	- 1	5.26	8 1/4	May 00-05	104:10	104:14	- 8	7.53	Nov 07	ci	27:25	27:29	- 12	8.17
8 1/4	Feb 93n	103:15	103:17	- 1	5.30	12	May 05	135:23	135:27	- 26	7.69	Feb 08	ci	27:03	27:07	- 11	8.20
8 3/8	Feb 93n	103:20	103:22	- 1	5.29	10 3/4	Aug 05	125:12	125:16	- 19	7.71	May 08	ci	26:15	26:19	- 11	8.22
10 7/8	Feb 93n	106:19	106:21	- 2	5.31	9 3/8	Feb 06	114:14	114:18	- 18	7.68	Aug 08	ci	25:30	26:02	- 11	8.22
6 3/4	Feb 93n	101:22	101:24	- 2	5.33	7 5/8	Feb 02-07	99:20	99:24	- 16	7.65	Nov 08	ci	25:14	25:18	- 10	8.22
7 1/8	Mar 93n	102:08	102:12	- 2	5.36	7 7/8	Nov 02-07	101:14	101:18	- 16	7.66	Feb 09	ci	24:25	24:29	- 11	8.25
9 5/8	Mar 93n	105:17	105:19	- 1	5.35	8 3/4	Aug 03-08	104:28	105:00	- 12	7.72	May 09	ci	24:10	24:13	- 10	8.25
7 3/8	Apr 93n	102:20	102:22	- 1	5.38	8 3/4	Nov 03-08	107:19	107:23	- 19	7.75	Aug 09	ci	23:26	23:30	- 10	8.25
7	Apr 93n	102:05	102:07	- 1	5.40	9 1/8	May 04-09	110:21	110:25	- 24	7.76	Nov 09	ci	23:11	23:15	- 10	8.25
7 5/8	May 93n	103:02	103:04	.....	5.43	10 3/8	Nov 04-09	120:28	121:00	- 24	7.78	Nov 09	bp	22:30	23:02	- 9	8.35
8 5/8	May 93n	104:15	104:17	.....	5.44	11 3/4	Feb 05-10	132:10	132:14	- 25	7.78	Feb 10	ci	22:28	23:00	- 10	8.25
												May 10	ci	22:13	22:17	- 10	8.25
												Aug 10	ci	21:30	22:01	- 10	8.26
												Nov 10	ci	21:15	21:19	- 11	8.26
												Feb 11	ci	21:02	21:05	- 10	8.26
												May 11	ci	20:20	20:24	- 10	8.26
												Aug 11	ci	20:07	20:11	- 10	8.26
												Nov 11	ci	19:26	19:30	- 10	8.26
												Feb 12	ci	19:12	19:16	-	

TABLE 15.2 continued

10 1/8	May 93n	106:18	106:20	-	2	5.46	1 10	May 05-10	118:15	118:19	-	23	7.75	Aug 15	ci	14:19	14:22	-	9	8.27
6 3/4	May 93n	101:26	101:28	-	1	5.47	12 3/4	Nov 05-10	141:24	141:28	-	29	7.79	Aug 15	bp	14:20	14:23	-	8	8.26
7	Jun 93n	102:08	102:10	-	1	5.49	13 7/8	May 06-11	152:06	152:10	-	30	7.79	Nov 15	ci	14:10	14:13	-	8	8.27
8 1/8	Jun 93n	103:31	104:01	-	1	5.50	14	Nov 06-11	154:08	154:12	-	31	7.79	Nov 15	bp	14:11	14:14	-	7	8.26
7 1/4	Jul 93n	102:21	102:23	-	1	5.52	10 3/8	Nov 07-12	122:13	122:17	-	25	7.87	Feb 16	ci	14:00	14:04	-	9	8.27
6 7/8	Jul 93n	102:02	102:04	-	1	5.56	12	Aug 08-13	137:25	137:29	-	30	7.88	Feb 16	bp	14:03	14:06	-	8	8.26
7 1/2	Aug 88-93	100:20	100:24	+	2	7.04	13 1/4	May 09-14	150:07	150:11	-	36	7.89	May 16	ci	13:23	13:27	-	9	8.27
8	Aug 93n	103:30	104:00	.....	5.57	11 1/4	Feb 15	135:01	135:05	-	36	7.92	May 16	bp	14:00	14:03	-	8	8.19	
8 5/8	Aug 93	104:30	105:02	.....	5.55	10 5/8	Aug 15	128:19	128:23	-	33	7.92	Nov 16	ci	13:06	13:09	-	8	8.27	
8 3/4	Aug 93n	105:05	105:07	.....	5.58	9 7/8	Nov 15	120:23	120:25	-	30	7.93	Nov 16	bp	13:12	13:15	-	6	8.21	
11 7/8	Aug 93n	110:11	110:13	.....	5.56	9 1/4	Feb 16	113:30	114:00	-	33	7.94	Feb 17	ci	12:31	13:02	-	8	8.26	
6 3/8	Aug 93n	101:09	101:11	.....	5.58	8 3/4	May 17	108:25	108:27	-	31	7.94	May 17	bp	12:26	12:29	-	8	8.26	
6 1/8	Sep 93n	100:27	100:29	.....	5.61	7 1/2	Nov 16	92:21	92:23	-	27	7.93	Aug 17	ci	12:14	12:17	-	8	8.26	
8 1/4	Sep 93n	104:19	104:21	-	1	5.60	8 1/8	Aug 17	110:05	110:07	-	31	7.94	Nov 17	ci	12:05	12:08	-	8	8.27
7 1/8	Oct 93n	102:21	102:23	-	1	5.61	9 1/8	May 18	113:00	113:02	-	32	7.94	Feb 18	ci	11:31	12:02	-	8	8.25
6	Oct 93n	100:21	100:23	.....	5.61	9	Nov 18	111:21	111:23	-	32	7.94	May 18	ci	11:24	11:26	-	7	8.25	
7 3/4	Nov 93n	103:27	103:29	-	1	5.66	8 7/8	Feb 19	110:09	110:11	-	32	7.94	May 18	bp	11:27	11:30	-	8	8.21
8 5/8	Nov 93	105:15	105:19	+	1	5.63	8 1/8	Aug 19	101:31	102:01	-	30	7.94	Aug 18	ci	11:16	11:19	-	8	8.25
9	Nov 93n	106:05	106:07	-	1	5.67	8 1/2	Feb 20	106:06	106:08	-	32	7.94	Nov 18	ci	11:09	11:12	-	7	8.25
11 3/4	Nov 93n	111:10	111:12	-	1	5.65	8 3/4	May 20	109:02	109:04	-	32	7.94	Nov 18	bp	11:12	11:15	-	9	8.21
7 5/8	Dec 93n	103:24	103:26	-	1	5.70	8 1/2	Aug 20	109:02	109:04	-	30	7.94	Feb 19	ci	11:03	11:06	-	8	8.23
7	Jan 94n	102:15	102:17	-	2	5.74	7 7/8	Feb 21	99:07	99:09	-	30	7.94	Feb 19	bp	11:08	11:11	-	8	8.18
6 7/8	Feb 94n	102:09	102:11	.....	5.75	8 1/8	May 21	102:04	102:06	-	31	7.93	May 19	ci	10:29	11:00	-	8	8.22	
8 7/8	Feb 94n	106:10	106:12	-	3	5.81	8 1/8	Aug 21	102:09	102:11	-	30	7.92	Aug 19	ci	10:22	10:25	-	8	8.22
9	Feb 94	106:18	106:22	-	2	5.79	8	Nov 21*	101:12	101:13	-	28	7.88	Aug 19	bp	10:27	10:30	-	7	8.17
8 1/2	Mar 94n	105:24	105:26	-	2	5.84	<b>U.S. TREASURY STRIPS</b>													
7	Apr 94n	102:19	102:21	-	1	5.81	Mat.	Type	Bid	Asked	Chg.	Bid								
4 1/8	May 89-94	96:24	97:24	-	3	5.09	Feb 92	ci	98:26	98:26	+	1	4.87	Feb 20	ci	10:11	10:13	-	7	8.20
7	May 94n	102:17	102:19	-	2	5.87	May 92	ci	97:18	97:19	-	1	4.99	Feb 20	bp	10:14	10:17	-	7	8.16
9 1/2	May 94n	108:06	108:08	-	3	5.90	Aug 92	ci	96:11	96:11	-	1	5.04	May 20	ci	10:05	10:08	-	7	8.19
13 1/8	May 94n	116:17	116:19	-	3	5.89	Nov 92	ci	95:04	95:05	-	2	5.05	Aug 20	bp	10:02	10:05	-	9	8.15
8 1/2	Jun 94n	106:03	106:05	-	1	5.93	Feb 93	ci	93:19	93:20	-	1	5.38	Nov 20	ci	9:26	9:28	-	7	8.17
8	Jul 94n	104:30	105:00	-	2	5.95	May 93	ci	92:08	92:09	-	2	5.46	Feb 21	ci	9:20	9:22	-	7	8.17
6 7/8	Aug 94n	102:08	102:10	-	3	5.95	Aug 93	ci	90:26	90:27	-	3	5.59	Feb 21	bp	9:23	9:26	-	8	8.13
8 5/8	Aug 94n	106:16	106:18	-	2	6.00	Nov 93	ci	89:13	89:15	-	2	5.68	May 21	ci	9:16	9:19	-	7	8.14
8 3/4	Aug 94	106:25	106:29	-	2	5.99	Feb 94	ci	87:24	87:26	-	1	5.89	May 21	bp	9:19	9:21	-	7	8.11
12 5/8	Aug 94n	116:17	116:19	-	2	5.99	May 94	ci	86:13	86:15	-	1	5.93	Aug 21	ci	9:19	9:21	-	7	8.04
8 1/2	Sep 94n	106:12	106:14	-	1	6.03	Aug 94	ci	84:30	85:00	-	3	6.03	Aug 21	bp	9:18	9:20	-	9	8.05
9 1/2	Oct 94n	109:00	109:02	-	2	6.06	Nov 94	ci	83:12	83:15	-	3	6.15	Nov 21	bp	9:19	9:22	-	6	7.97
6	Nov 94n*	100:00	100:01	-	2	5.99	Nov 94	np	83:08	83:11	-	3	6.20	<b>TREASURY BILLS</b>						
8 1/4	Nov 94n	105:27	105:29	-	2	6.07	Feb 95	ci	81:18	81:20	-	4	6.37	Days						
10 1/8	Nov 94	110:28	111:00	-	3	6.06	Feb 95	np	81:24	81:26	-	4	6.30	Maturity	Mat.	Bid	Asked	Chg.	Ask	
11 5/8	Nov 94n	114:31	115:01	-	2	6.07	May 95	ci	80:02	80:05	-	5	6.45	Nov 21 '91	6	4.68	4.58	-0.02	4.65	
7 5/8	Dec 94n	104:10	104:12	-	2	6.07	May 95	np	80:01	80:03	-	4	6.47	Nov 29 '91	14	4.54	4.44	-0.01	4.52	
8 5/8	Jan 95n	106:30	107:00	-	2	6.16	Aug 95	ci	78:20	78:22	-	4	6.52	Dec 05 '91	20	4.41	4.31	-0.03	4.39	
3	Feb 95	97:00	98:00	-	3	6.66	Aug 95	np	78:14	78:17	-	5	6.58	Dec 12 '91	27	4.36	4.26	-0.08	4.34	
7 3/4	Feb 95n	104:17	104:19	-	3	6.17	Nov 95	ci	77:08	77:11	-	5	6.56	Dec 19 '91	34	4.39	4.35	-0.05	4.43	
10 1/2	Feb 95	112:10	112:14	-	3	6.22	Nov 95	np	77:02	77:05	-	5	6.62	Dec 26 '91	41	4.40	4.36	-0.04	4.45	
11 1/4	Feb 95n	114:17	114:19	-	5	6.22	Feb 96	ci	75:14	75:17	-	5	6.74	Jan 02 '92	48	4.43	4.39	-0.02	4.49	
8 3/8	Apr 95n	106:08	106:10	-	4	6.29	May 96	ci	74:04	74:07	-	5	6.77	Jan 09 '92	55	4.50	4.46	-0.02	4.57	
8 1/2	May 95n	106:24	106:26	-	2	6.30	May 96	np	74:01	74:05	-	6	6.79	Jan 16 '92	62	4.55	4.53	-0.01	4.63	
10 3/8	May 95	112:14	112:18	-	3	6.32	Aug 96	ci	72:19	72:22	-	5	6.86	Jan 23 '92	69	4.57	4.55	-0.01	4.67	
11 1/4	May 95n	115:03	115:05	-	4	6.35	Nov 96	ci	71:14	71:17	-	6	6.84	Jan 30 '92	76	4.57	4.55	-0.01	4.67	
12 5/8	May 95	119:14	119:18	-	2	6.31	Nov 96	np	70:10	70:13	-	3	7.17	Feb 06 '92	83	4.62	4.60	.....	4.73	
8 7/8	Jul 95n	107:28	107:30	-	4	6.41	Feb 97	ci	69:20	69:23	-	4	7.02	Feb 13 '92	90	4.63	4.61	.....	4.73	
8 1/2	Aug 95n	106:24	106:26	-	4	6.43	May 97	ci	68:07	68:09	-	4	7.08	Feb 20 '92	97	4.62	4.60	.....	4.74	
10 1/2	Aug 95n	113:07	113:09	-	4	6.46	Aug 97	ci	68:05	68:09	-	4	7.09	Feb 27 '92	104	4.62	4.60	.....	4.74	
8 5/8	Oct 95n	107:08	107:10	-	4	6.48	Nov 97	ci	66:27	66:31	-	4	7.13	Mar 05 '92	111	4.64	4.62	.....	4.76	
8 1/2	Nov 95n	106:28	106:30	-	4	6.50	Aug 97	np	66:26	66:29	-	3	7.14	Mar 12 '92	118	4.66	4.64	+0.01	4.79	
9 1/2	Nov 95n	110:10	110:12	-	6	6.51	Nov 97	ci	65:24	65:28	-	1	7.11	Mar 19 '92	125	4.66	4.64	+0.01	4.79	
11 1/2	Nov 95	117:12	117:16	-	4	6.47	Feb 98	ci	65:24	65:28	-	4	7.16	Mar 26 '92	132	4.66	4.64	.....	4.78	
9 1/4	Jan 96n	109:18	109:20	-	5	6.57	May 98	ci	63:31	64:02	-	8	7.28	Apr 02 '92	139	4.66	4.64	.....	4.80	
7 1/2	Jan 96n	103:09	10																	

If we assume the terminal futures price is lognormally distributed, we would evaluate  $E(\tilde{c}_T)$  in the same manner we did in Chapter 11, and substitute this into (15.1). The valuation equation would be (11.25) with the cost-of-carry rate,  $b$ , set to zero because the underlying instrument is a futures contract.

The assumption of lognormally distributed yield requires a modification of the terminal value function, (15.2), for the call. In Chapter 8, we discussed the fact that the Eurodollar futures price is an index level computed by subtracting the yield on the Eurodollar deposit from 100. In other words, the futures price is  $F = 100 - y$ . If we substitute this definition into (15.2) and rearrange, we find that the terminal call price can be expressed as

$$\tilde{c}_T = \begin{cases} (100 - X) - \tilde{y}_T & \text{if } y_T \leq 100 - X \\ 0 & \text{if } y_T > 100 - X. \end{cases} \quad (15.3)$$

But equation (15.3) looks surprisingly familiar. It is the terminal value function of a European put option, where  $y_T$  has replaced  $S_T$  and where  $(100 - X)$  has replaced  $X$ . Since  $y_T$  is lognormally distributed, the European put formula (11.28) of Chapter 11 can be applied directly. Using the fact that  $y = (100 - F)$ , the expected terminal call price is

$$E(\tilde{c}_T) = (100 - X)N(-d_2) - (100 - F)N(-d_1), \quad (15.4)$$

where

$$d_1 = \frac{\ln[(100 - F)/(100 - X)] + .5\sigma_y^2 T}{\sigma_y \sqrt{T}} \quad \text{and} \quad d_2 = d_1 - \sigma_y \sqrt{T},$$

and  $\sigma_y$  is the standard deviation of the logarithm of the yield ratios,  $\ln(y_t/y_{t-1})$ . Substituting (15.4) into (15.1), the *price of a European call option on a Eurodollar futures contract*<sup>4</sup> is

$$c(F, T; X) = e^{-rT} [(100 - X)N(-d_2) - (100 - F)N(-d_1)]. \quad (15.5)$$

By put-call parity for European futures options, the *price of a European put option on a Eurodollar futures contract* is

$$p(F, T; X) = e^{-rT} [(100 - F)N(d_1) - (100 - X)N(d_2)]. \quad (15.6)$$

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<sup>4</sup>This approach to Eurodollar futures option valuation is described in detail in Emanuel (1985).



**EXAMPLE 15.1**

Using the values reported in Tables 15.1 and 15.2, compute the implied yield volatility of the March 9500 call option on the Eurodollar futures contract. According to the tables, the call price is .15, the underlying futures price is 94.94, and the riskless rate of interest is about 4.6 percent. As of November 13, 1991, the option has 124 days remaining to expiration.

The implied yield volatility for this call is computed by solving

$$.15 = e^{-.046(124/365)}[(100 - 95)N(-d_2) - (100 - 94.94)N(-d_1)],$$

where

$$d_1 = \frac{\ln[(100 - 94.94)/(100 - 95.00)] + .5\sigma_y^2(124/365)}{\sigma_y\sqrt{124/365}}$$

and

$$d_2 = d_1 - \sigma_y\sqrt{124/365}.$$

Without showing the steps of the iterative search that is used to find the implied volatility, the solution is

$$\sigma_y = 10.26\%.$$

Note that this volatility is upward biased since Eurodollar futures options are American style.

**Relation Between Price Volatility and Yield Volatility**

Prior to this chapter, volatility has been defined as the standard deviation of the logarithm of commodity price ratios,  $\ln(S_t/S_{t-1})$ , or the standard deviation of the logarithm of futures price ratios,  $\ln(F_t/F_{t-1})$ . The volatility parameter used in (15.5) and (15.6), however, is the standard deviation of the logarithm of yield ratios,  $\ln(y_t/y_{t-1})$ . Since the scale of these two volatilities appears so different, it is important to understand how these two measures are linked. The yield of the Eurodollar deposit is

$$y = 100 - F, \tag{15.7}$$

so the relation between a yield change and a price change is

$$dy = -dF. \quad (15.8)$$

Multiplying the left-hand side by  $y/y$  and the right-hand side by  $F/F$ , we have

$$y \frac{dy}{y} = F \frac{dF}{F},$$

which can be rearranged as

$$\frac{dy}{y} = \frac{dF}{F} \left( \frac{F}{100 - F} \right). \quad (15.9)$$

In other words, the rate of change in yield equals the rate of change in the index level scaled by the factor  $F/(100 - F)$ . The yield volatility,  $\sigma_y$ , therefore, equals the return volatility,  $\sigma_F$ , times the factor  $F/(100 - F)$ , that is,

$$\sigma_y = \sigma_F \left( \frac{F}{100 - F} \right). \quad (15.10)$$

### EXAMPLE 15.2

In Example 15.1, the implied yield volatility rate from the March 9500 Eurodollar call is shown to be 10.26 percent. Compute the implied return volatility based on this estimate.

The implied futures price volatility is the solution to

$$.1026 = \sigma_F \left( \frac{94.94}{100 - 94.94} \right),$$

which implies that

$$\sigma_F = 0.55\%.$$

### 15.3 TREASURY BOND OPTION PRICING—PRICE-BASED VALUATION

In Table 15.1, only two options are bond options—the CBOE's short-term and long-term interest rate index contracts. These options are American-style and are written on specific Treasury issues (see Table 15.2 for the price of the underlying T-bond or T-note). If we are willing to accept the assumption that long-term bond prices are lognormally distributed at the option's expiration,<sup>5</sup> these options can be priced using the continuous cost-of-carry commodity option framework developed in Chapters 10 and 11.

In doing so, we must account for the treatment of accrued interest. We noted earlier that the reported bond price excludes accrued interest for the current coupon period and, to find the cost of the bond, the accrued interest must be added to the reported bond price. We also noted that the exercise price of a bond option is increased by the accrued interest. Since accrued interest is added to the reported bond price and to the exercise price, one can simply ignore it and use the reported bond price and the stated exercise price in the bond option pricing formula.

The option pricing formulas require the cost of carry for a bond, which is the short-term riskless rate of interest less the coupon yield, that is,  $b = r - y$ . To compute the annualized coupon yield of a bond with price  $B$  for use in the option pricing equations, recognize that a coupon payment,  $C$ , is received each half year, so

$$e^{y(.5)} = \frac{B + C}{B}$$

or

$$y = 2 \times \ln \left( 1 + \frac{C}{B} \right). \quad (15.11)$$

It is worth noting that bonds with high coupon yields tend to depreciate in price (since they initially sell above par) and that bonds with low coupon yields tend to appreciate in price (since they initially sell below par). The value of an option, in turn, depends on the price appreciation or depreciation. The higher (lower) the rate of price appreciation on the bond, the higher (lower) the call price and the lower (higher) the put price. We now apply the commodity option pricing results of Chapters 10 and 11 using this cost-of-carry parameter.

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<sup>5</sup>For short-term options of a year or less, this assumption is plausible, particularly if the underlying bond has a long time to maturity. For long-term options and/or for short-term T-bonds and T-notes, this assumption is less tenable.

**EXAMPLE 15.3**

Assume there exists a European-style call option on the 8½s of May 1997 that we discussed earlier in the chapter. The call has an exercise price of 107, a time to expiration of 100 days, and a current market price of \$1<sup>21</sup>/<sub>32</sub>. Assume that the riskless rate of interest is 4.6 percent. Compute the implied volatility of this option based upon the average of the bid and ask bond prices.

The average of the bid and ask prices for the 8½s of May 1997 is 107<sup>8</sup>/<sub>32</sub> or 107.25. The coupon yield on this bond is

$$y = 2 \times \ln \left( 1 + \frac{4.25}{107.25} \right) = 7.77\%.$$

Substituting into valuation equation (11.25), we get

$$1.656 = 107.25e^{(.046 - .0777 - .046)(100/365)} N(d_1) - 107e^{-.046(100/365)} N(d_2),$$

where

$$d_1 = \frac{\ln(107.25/107) + (.046 - .0777 + .5\sigma^2)100/365}{\sigma\sqrt{100/365}}$$

and

$$d_2 = d_1 - \sigma\sqrt{100/365}.$$

Without showing the steps of the iterative search that is used to find the implied volatility, the solution is

$$\sigma = 8.97\%.$$

---

By far the most active long-term interest rate option market is the CBT's Treasury bond futures options. These options are American-style, and expire on the first Friday preceding, by at least five business days, the first notice day for the corresponding T-bond futures contract. Also, the T-bond futures option has the decimal part of its price reported in 64ths. Under the assumption that the futures price at the option's expiration is lognormally distributed, the valuation of these options is possible using the quadratic approximation described in Chapter 14.

**EXAMPLE 15.4**

Compare the theoretical price of a March 1992 T-bond futures put option, with a strike price of 100 to the quoted price in Table 15.1,  $2^{17/64}$ . The price of the March 1992 futures is  $99^{7/32}$ , and, given that the option expires on February 21, 1992, the time to expiration is 100 days. Assume the riskless rate of interest is 4.6 percent and the volatility rate is 9.00 percent.

The T-bond futures option contract is an American-style option, so we use the quadratic approximation. We begin by computing the value of the corresponding European-style futures put contract using equation (14.20).

$$p = e^{-.046(100/365)}[100N(-d_2) - 99.21875N(-d_1)],$$

where

$$d_1 = \frac{\ln(99.21875/100) + .5(.09)^2(100/365)}{.09\sqrt{100/365}} = -.143,$$

$$d_2 = d_1 - .09\sqrt{100/365} = -.190.$$

The European put option value is 2.32.

Applying the quadratic approximation (14.22), we find that the critical index level  $F^{**}$  below which the put should be exercised immediately is 90.490, considerably below the current futures price of 99.21875. Hence, the value of the early exercise premium should be small. The value of  $A_1$  is 0.092, the value of  $q_1$  is  $-29.619$ . The approximate value of the March 100 put option on the T-bond futures is, therefore,

$$\begin{aligned} P(F, T; X) &= p(F, T; X) + A_1(F/F^{**})^{q_1} \\ &= 2.32 + 0.092(99.21875/90.490)^{-29.619} \\ &= 2.33. \end{aligned}$$

To compare this theoretical value to the observed price, we need to transform the decimal price 2.33 to 64ths, that is,  $2 + (.33 \times 64)/64 = 2^{21/64}$ . In other words, the put option appears  $4/64$  underpriced.

**15.4 TREASURY BOND OPTION PRICING—YIELD-BASED VALUATION**

Yield-based option pricing of T-bond and T-bond futures is facilitated by using the binomial approximation method described in Chapter 13. Instead of modeling movement of the underlying commodity price over the next interval of time, we

model the up and down movement of the yield. The next period up and down state yields are a proportion of the current yield. If the current yield is  $y_0$ , the yield at the end of the first interval is either  $uy_0$  or  $dy_0$ . If the total number of time steps is defined as  $n$ , where  $\Delta t = T/n$  and  $T$  is the time to expiration of the option, there are  $n + 1$  yield nodes at the option's expiration, with an odd number if  $n$  is even and an even number of nodes if  $n$  is odd.

This binomial lattice is illustrated in Figure 15.1. The length of each interval or time step in the figure is  $\Delta t$ . The factors  $u$  and  $d$  are defined as

$$u = e^{\sigma\sqrt{\Delta t}} \quad (15.12a)$$

and

$$d = \frac{1}{u}. \quad (15.12b)$$

The risk-neutral probabilities of up and down movements are

$$p = \frac{1 - d}{u - d} \quad (15.12c)$$

and  $1 - p$ , respectively.

Once the yield lattice is computed, it is necessary to compute the bond price at each node. Bond valuation equations were presented in Chapter 8. Keep in mind

**FIGURE 15.1** Possible Paths that Yield May Follow under the Binomial Model

Yield at end of time interval:								
0	1	2	3	4	...	$n$ (even)	or	$n$ (odd)
						$u^n y_0$		$u^n y_0$
				$u^4 y_0$	...	:		:
		$u^2 y_0$	$u^3 y_0$	$u^2 y_0$	...			:
	$u y_0$		$u y_0$	$y_0$	...	$y_0$		$u y_0$
$y_0$	$d y_0$	$y_0$	$d y_0$	$y_0$	...			$d y_0$
		$d^2 y_0$		$d^2 y_0$	...	:		:
			$d^3 y_0$	$d^4 y_0$	...			:
				$d^4 y_0$	...	$d^n y_0$		$d^n y_0$



and  $1 - p = .506542$ . Note that, in Figure 15.2, possible yields range from  $y_0d^n = 8.00\%(.974168)^{90} = .759\%$  to  $y_0u^n = 8.00\%(1.026517)^{90} = 84.339\%$  at the option's expiration and bond prices range from 237.385 to 9.627.

With the yield/bond price lattice computed, the approximation method starts at the end of the option's life and works back to the present. At the end of the option's life (column  $n$  in the figure), the option value at each yield/bond price node is given by the intrinsic value of the option. In the case of a put option,

$$P_{n,j}(B_{n,j}) = \begin{cases} 0 & \text{if } B_{n,j} > X \\ X - B_{n,j} & \text{if } B_{n,j} \leq X. \end{cases} \quad (15.13)$$

The option values one step,  $\Delta t$ , back in time (in column  $n - 1$ ) are computed by taking the present value of the expected future value of the option. At any point  $j$  in column  $n - 1$ , the yield can move up with probability  $p$  or down with probability  $1 - p$ . The value of the option at time  $n$  if the yield moves up is  $P_{n,j+1}$  and if the yield moves down is  $P_{n,j}$ . The present value of the expected future value of the option is, therefore,

$$P_{n-1,j} = \frac{pP_{n,j+1} + (1-p)P_{n,j}}{r^*}, \quad (15.14)$$

where  $r^* = e^{r\Delta t}$ . Using this present value formulation, all of the option values in column  $n - 1$  may be identified.

Before proceeding back another time increment,  $\Delta t$ , in the valuation, it is necessary to see if any of the computed option values are below their early exercise proceeds at the respective nodes,  $X - B_{n-1,j}$ . If the exercise proceeds are greater than the computed option value, the computed value is replaced with the early exercise proceeds. If they are not, the value is left unchanged. Note that if this step is not performed, the procedure will produce the value of a European put option.

Once the checks are performed, we go to column  $n - 2$  and repeat the steps, and so on back through time. Eventually, we will work our way back to time 0, and the current value of the American put option (in column 0) will be identified.

To complete the binomial method illustration, suppose that the bond price lattice shown in Figure 15.2 underlies a 90-day American put option with an exercise price of 100. Applying the yield-based binomial method, the value of the American put is \$6.157. The value of the corresponding European-style put option using the yield-based method is \$5.713. The early exercise premium of the American put is, therefore, worth about 44.4¢.

## 15.5 MANAGING DURATION AND CONVEXITY

In Chapter 8, we discussed the duration and convexity of a fixed-income portfolio. Modified duration measures the percentage change in the value of a bond for a given change in yield,



$$D_m = -\frac{dB/B}{dy}, \quad (15.15)$$

and convexity indicates how duration changes for a given change in yield,

$$\text{Convexity} = \frac{1}{2} \frac{d^2B}{dy^2} \frac{1}{B}. \quad (15.16)$$

The keys to these expressions are the first and second derivatives of bond price with respect to a change in yield, that is,  $dB/dy$  and  $d^2B/dy^2$ , respectively. In this sense, duration and convexity are like the delta and gamma of an option. In fact, we will now show that bond option deltas and gammas enable a fixed-income portfolio manager to control the duration and convexity of the portfolio.

To understand how to tailor the duration and convexity exposure of a fixed-income portfolio, we need to develop expressions for bond (or bond futures) option price changes as a function of yield changes. The first derivative of option price with respect to a change in yield is

$$\frac{\partial O}{\partial y} = \frac{\partial O}{\partial B} \frac{dB}{dy} = \Delta \frac{dB}{dy}, \quad (15.17)$$

where  $\Delta$  is the delta value of the option. Thus, to change the dollar value exposure of a bond portfolio, we simply combine the exposure in bonds,  $dB/dy$ , with the exposure in  $n$  bond options,  $\Delta dB/dy$ , that is,

$$\text{Desired dollar risk exposure} = \frac{dB}{dy} (1 + n\Delta). \quad (15.18)$$

To reduce the portfolio's risk exposure to zero (for small changes in yields), the optimal number of options is

$$n = -\frac{1}{\Delta}.$$

In the general case, one may use  $N$  different options to hedge a bond portfolio, that is,

$$\text{Desired dollar risk exposure} = \frac{dB}{dy} \left( 1 + \sum_{i=1}^N n_i \Delta_i \right). \quad (15.19)$$

Note that the dollar risk exposure is exactly zero only at the point where the derivative is taken.

**EXAMPLE 15.5**

Assume that a fixed-income portfolio manager holds a 9-percent, 20-year bond whose current yield to maturity is 8 percent. Its current market value is 109.82, its modified duration is 9.606, and its convexity is 70.450. Given the uncertainty about economic events, the manager decides to hedge the interest rate risk of his portfolio by writing call options on this bond. The call has an exercise price of 110 and a time to expiration of 3 months. The current price of this option is 2.900, its delta is .472, and its gamma is .0474. Compute the number of call options to sell against this bond to immunize it from movements in the bond's yield.

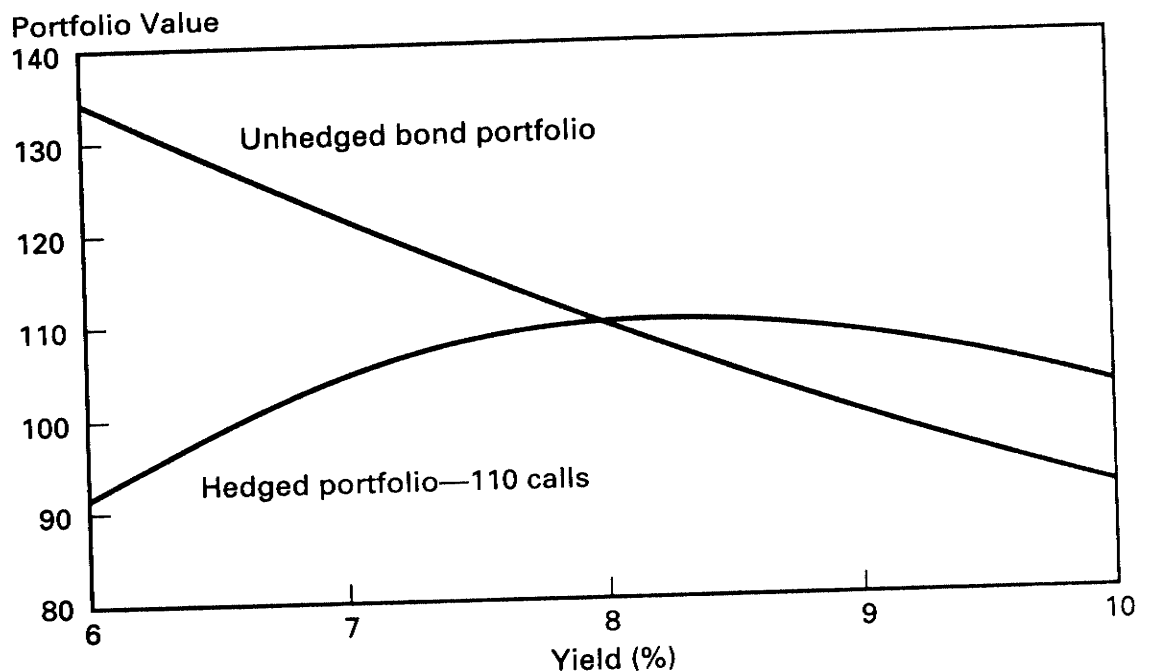
The optimal number of calls to negate the duration exposure is

$$n = -\frac{1}{.472} = -2.119.$$

If we sell this number of options against the underlying bond, the bond portfolio value for given changes in the yield is shown in Figure 15.3.

Figure 15.3 demonstrates that the delta-neutral hedge reduces the range of possible portfolio values. The unhedged portfolio ranges in value from 90 to 135 over the range of yields shown, while the hedged portfolio ranges in value from 90 to 110. But, even with the delta-neutral hedge, the range of possible portfolio values is large over this somewhat limited yield range. In addition, the hedged port-

**FIGURE 15.3** Hedging Duration Exposure Using Bond Options



folio has reduced value if the yield rises *or* falls. To improve upon this hedge, it is possible to use more than one option to hedge both duration and convexity risk.

The zero-risk portfolio given by (15.19) is analogous to the delta-neutral portfolio discussed in Chapter 12. In that chapter, we also noted that a change in delta brought about by a commodity price change introduces gamma risk. A similar situation arises in the case of the hedged bond portfolio. Not only does the bond option delta change as the yield changes, but so does the duration of the bond portfolio. To compensate for these effects, we must consider the second partial derivative of the bond option price with respect to yield, that is

$$\begin{aligned}
 \frac{\partial^2 O}{\partial y^2} &= \frac{\partial(\frac{\partial O}{\partial y})}{\partial y} = \frac{\partial(\frac{\partial O}{\partial B} \frac{dB}{dy})}{\partial y} = \frac{\partial O}{\partial B} \frac{d^2 B}{dy^2} + \frac{dB}{dy} \frac{\partial^2 O}{\partial B \partial y} \\
 &= \Delta \frac{d^2 B}{dy^2} + \frac{dB}{dy} \frac{\partial(\frac{\partial O}{\partial B})}{\partial y} = \Delta \frac{d^2 B}{dy^2} + \frac{dB}{dy} \frac{\partial \Delta}{\partial y} \\
 &= \Delta \frac{d^2 B}{dy^2} + \frac{dB}{dy} \frac{\partial \Delta}{\partial B} \frac{\partial B}{\partial y} \\
 &= \Delta \left( \frac{d^2 B}{dy^2} \right) + \gamma \left( \frac{dB}{dy} \right)^2, \tag{15.20}
 \end{aligned}$$

where  $\gamma = \partial \Delta / \partial B$  is the gamma value of the option. Combining the convexity exposure of the bond with a portfolio that consists of  $N$  bond options, we get dollar convexity exposure

$$\begin{aligned}
 \text{Dollar convexity} &= \frac{d^2 B}{dy^2} + \sum_{i=1}^N n_i \Delta_i \left( \frac{d^2 B}{dy^2} \right) + \sum_{i=1}^N n_i \gamma_i \left( \frac{dB}{dy} \right)^2 \\
 &= \frac{d^2 B}{dy^2} \left( 1 + \sum_{i=1}^N n_i \Delta_i + f \sum_{i=1}^N n_i \gamma_i \right), \tag{15.21}
 \end{aligned}$$

where  $f = \left( \frac{dB}{dy} \right)^2 / \left( \frac{d^2 B}{dy^2} \right)$ .

### EXAMPLE 15.6

Unsatisfied with the effectiveness of the hedge portfolio indicated in Example 15.5, the portfolio manager decides to evaluate the effectiveness of a hedge of both the duration and the convexity risk of his position. Aside from the 110 call described in the last exercise, a 105 call with three months to expiration is available. The current price of the 105 call is 5.720, its delta is .703, and its gamma is .0403. Compute the number of calls to buy/sell against this bond to immunize it from movements in the bond's yield.

To neutralize the duration and the convexity risk of the bond portfolio, we need to solve simultaneously the following equations:

$$n_1\Delta_1 + n_2\Delta_2 = -1$$

and

$$n_1(\Delta_1 + f\gamma_1) + n_2(\Delta_2 + f\gamma_2) = -1.$$

To compute the coefficient  $f$ , we need to know the values of  $dB/dy$  and  $d^2B/dy^2$ . These can be obtained from the modified duration and the convexity figures reported for the bond. That is, from equations (15.15) and (15.16), we know

$$\frac{dB}{dy} = BD_m = 109.82 \times 9.606 = 1,054.93,$$

and

$$\frac{d^2B}{dy^2} = 2B\text{Convexity} = 2 \times 109.82 \times 70.450 = 15,473.64.$$

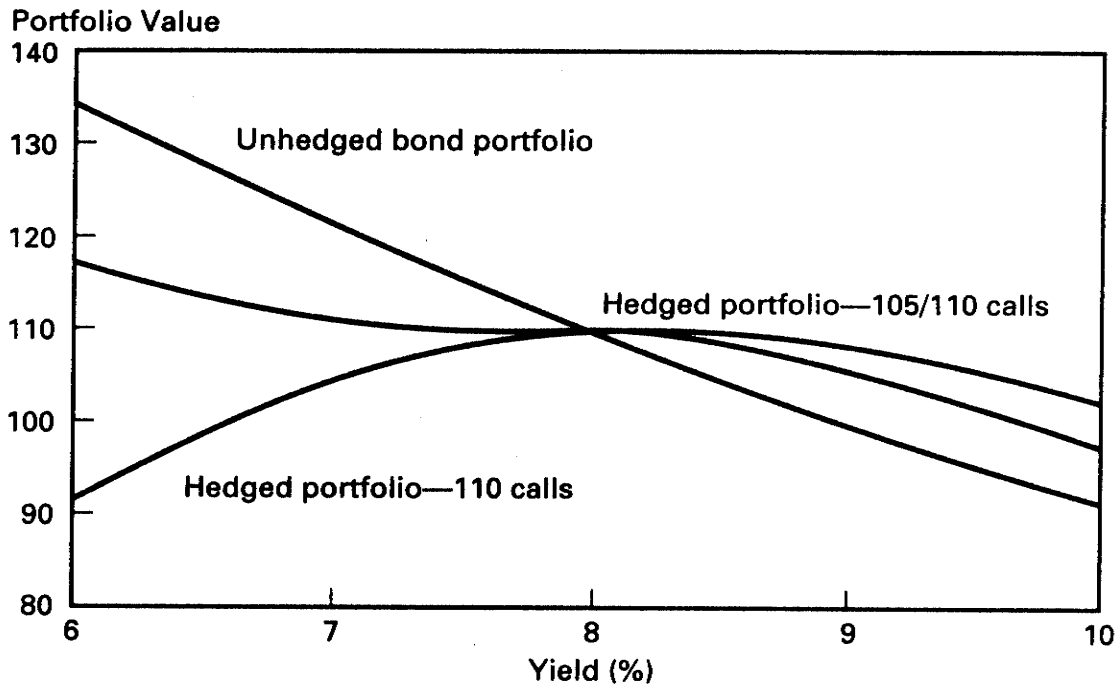
The value of  $f$  is, therefore, 71.921. The deltas and gammas of the individual options are known, and the remaining task is only computational. The optimal composition of the duration/convexity hedge is to sell 3.315 105 calls and to buy 2.819 110 calls.

The effectiveness of this hedge relative to the unhedged portfolio and the duration-hedged portfolio from Example 15.5 can be seen in Figure 15.4. The range of outcomes has been further diminished. Using both calls generates a curve that is much more horizontal at 110, ranging from 103 to 116. In addition, the hedged portfolio rises if the yield falls and falls more slowly if the yield rises. Clearly, this second hedge is more effective than the hedge discussed in Example 15.5.

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## 15.6 SUMMARY

This chapter focuses on the valuation of interest rate options. After reviewing the designs of the U.S. exchange-traded interest option contracts, we discuss short-term interest rate option valuation. Using the standard lognormal price distribution assumption is inappropriate for these options. In its place, we use the assumption that yield is lognormally distributed and rederive the European option valuation equations. In the third section, T-bond and T-bond futures option valuation under

**FIGURE 15.4** Hedging Duration/Convexity Exposure Using Bond Options Portfolio Value

the lognormal price distribution assumption is presented. The fourth section describes the same valuation but under the lognormal yield assumption. The binomial method is also used. Finally, we show how T-bond option contracts can be used to control the duration and the convexity of a fixed-income portfolio.