

Simulating Supply*

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Much has been written about the controversy surrounding the financial crisis at MG Refining & Marketing (MGRM), a US subsidiary of Germany's Metallgesellschaft AG. MGRM's positions in petroleum derivatives generated losses reportedly topping US\$1 billion in late 1993. These losses, and the aftermath within the company, generated enormous public controversy, including several rounds of academic debate regarding MGRM's use of derivatives. Were the positions in derivatives a speculative gamble, or part of a viable programme to sell petroleum products forward at fixed prices and hedge using nearby futures contracts?

In 1991, MGRM developed a series of contracts for long-term customers whereby MGRM committed to deliver petroleum products at fixed prices over a period of up to 10 years. These contracts were highly successful: by December 1993, the company had sold forward approximately 160 million barrels of petroleum. To hedge these commitments, MGRM purchased short-dated futures contracts and over-the-counter (OTC) derivatives with total underlying volume equal to the total commitments, a so-called one-to-one stacked hedge. When the futures approached maturity, they were "rolled" into new positions by selling the maturing futures and purchasing a new set of nearby futures. The unusual steady decline in petroleum prices in autumn 1993 led to severe "roll" losses, and triggered a series of articles that argued both for and against MGRM's use of derivatives and the overall viability of the company's combined marketing and hedging strategy.

Using various analyses, critics of MGRM generally concluded that the company took long futures/OTC derivatives positions far in excess of a variance-minimising hedge (Pirrong, 1997, and Mello and Parsons, 1995). The "hedge", therefore, contained a huge bet that oil prices would rise. Since oil prices fell in 1993, the one-to-one hedge lost more money than a variance-minimising hedge would have, simply because the hedge ratio was larger than it should have been.

Supporters of MGRM counter that the combined marketing and hedging strategy was almost certain to produce a profit if left in place (Culp and Miller, 1995a). The decision to close out the derivative positions was a case of mistaking a funding problem for an all-out bankruptcy. Further, the variance-minimising hedge championed by MGRM's critics is not always optimal. Some companies have a comparative advantage in bearing certain financial risks, and should be willing to capitalise on it (Stulz, 1996). In the case of MGRM, the company was clearly banking on a relatively stable phenomenon in petroleum futures contracts in which the "roll" generates positive profits much of the time (Edwards and Canter, 1995).

This chapter contributes to the debate by analysing the portfolio of supply contracts sold by MGRM as of December 1993. We first describe the nature of the long-term, fixed-supply contracts sold by the company. Next, historical petroleum price data are used to simulate the cashflows resulting from the supply contracts and various hedging strategies. We find that the simulated strategy always produces a positive profit when completed, but there is a 33% probability of hitting a cash position of -US\$1 billion along the way. We then calculate what MGRM's cash position would have been at the end of March 1997 had the programme been fully funded and left in place. We find that the programme would have earned in excess of US\$1.1 billion, covering even the most pessimistic accounts of the losses sustained from MGRM's programme.¹ We close with some concluding remarks.

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SIMULATING SUPPLY Metallgesellschaft's supply contracts

Table 1 contains a summary of MGRM's portfolio of long-term supply contracts as of December 1993.² As the Table shows, MGRM had sold long-term supply contracts on a total volume of around 6.7 billion gallons of petroleum products. Around 57% of the total committed volume was heating oil and the rest was unleaded gasoline. Around 63% of the total volume was committed under "firm-fixed" contracts with fixed delivery schedules and the rest was committed under "45-day" contracts with flexible delivery schedules.

Under the terms of the firm-fixed supply contracts, MGRM agreed to provide unleaded gasoline or heating oil at a fixed monthly rate over a fixed term at a fixed price. Around 88.5% of the firm-fixed contracts had a term of 10 years at contract inception, 11.2% had five years and 0.3% had two years. Table 1 reports the weighted (by contract volume) average fixed price of the contracts by underlying product and by contract maturity. The average fixed price for the two-year firm-fixed contracts on unleaded gasoline, for example, is US\$0.6501 a gallon.

Around 62% of the total volume of the firm-fixed contracts had "exit letters" that provided immediate cash settlement when the spot month futures price reached a pre-defined "exit price". The cash settlement amount equals half the difference between the exit price and the fixed price times the remaining volume of the contract. For the contracts with exit letters, Table 1 includes the weighted (by contract volume) average exit price. The five-year firm-fixed contracts with exit letters on unleaded gasoline, for example, had an average exit price of US\$0.6414.

Under the terms of the 45-day supply contracts, as with the firm-fixed contracts, MGRM agreed to provide unleaded gasoline or heating oil in fixed total volume over a fixed term at a fixed price agreed upon today. The 45-day contracts differ in that the contract holder chooses the amount and timing of contract deliveries. Any undelivered portion of the total volume will be delivered at the end of the contract's term. About 82.6% of the 45-day contracts had a term of 10 years at contract inception, and 17.4% had five years. The weighted (by contract volume) average fixed prices of the four types of 45-day contracts are also provided in Table 1. The average fixed price of the five-year contracts on heating oil, for example, is US\$0.6223.

Hedging supply contracts

MGRM chose to hedge its long-term supply commitments by buying nearby futures contracts (or, alternatively, OTC forward or swap contracts) with underlying volume equal to the contracted delivery volume, a so-called "one-to-one stacked hedge" - "one-to-one" because the underlying futures volume equals the committed volume and "stacked" because only the nearby futures are used. As the futures contracts neared expiry, they were

Table 1. Summary of MGRM's long-term fixed-supply contracts: December 1993

| Term of contract in years | Type of contract | Millions of gallons outstanding | Weighted average fixed price (US\$) | Weighted average exit price (US\$) |
|---------------------------|------------------|---------------------------------|-------------------------------------|------------------------------------|
| Heating oil | | | | |
| 2 | Firm-fixed | 15.18 | 0.6012 | |
| 5 | Firm-fixed | 222.36 | 0.5925 | |
| 5 | Firm-fixed | 54.17 | 0.5815 | 0.6239 |
| 10 | Firm-fixed | 1,054.13 | 0.5890 | |
| 10 | Firm-fixed | 1,592.35 | 0.5728 | 0.6286 |
| 5 | 45-day | 322.73 | 0.6223 | |
| 10 | 45-day | 542.70 | 0.6207 | |
| | Subtotal | 3,803.63 | | |
| Unleaded gasoline | | | | |
| 2 | Firm-fixed | 5.78 | 0.6501 | |
| 5 | Firm-fixed | 31.09 | 0.6339 | |
| 5 | Firm-fixed | 12.69 | 0.6209 | 0.6414 |
| 10 | Firm-fixed | 292.79 | 0.5886 | |
| 10 | Firm-fixed | 942.48 | 0.5924 | 0.6270 |
| 5 | 45-day | 107.19 | 0.6409 | |
| 10 | 45-day | 1,504.91 | 0.6199 | |
| | Subtotal | 2,896.94 | | |
| | Total | 6,700.57 | | |

"rolled", ie, the contracts were sold and new near-month contracts were bought. As deliveries were made on the long-term supply contracts, the futures position was commensurately reduced.

Ideally, MGRM or any other supplier could trade in derivatives with maturity equal to the contracted delivery of the underlying asset. MGRM's commitments stretched out up to 10 years, however, whereas futures contracts on petroleum are only actively traded out a few months. The unusual steady decline of petroleum prices in autumn 1993 led to large losses on the futures contracts that were only partially offset by cash gains on the delivery specified in the supply contracts. Critics maintain that MGRM overhedged, and was in fact speculating on oil prices.

Most of MGRM's critics support a minimum-variance hedging strategy that seeks to take positions in exchange-traded or OTC derivatives to minimise exposure to risk. The standard argument for a firm to minimise cashflow variance is that it reduces the probability of bankruptcy and associated costs of financial distress. Should a firm such as MGRM employ a minimum-variance hedge? There are at least two reasons why not. First, supply contracts expose the provider to the downside risk that prices will rise, but also the upside "risk" that prices will fall. A minimum-variance hedge reduces both sources of "risk". But an option-like payout structure could minimise the downside risk while preserving the upside potential of the strategy. The exit letter, for example, limits the supplier's losses when prices rise by prematurely terminating the contract. Second, a one-to-one stacked hedge can generate relatively stable profits by exploiting the historical tendency for futures prices to rise as they approach maturity. The choice of hedging strategy involves a risk-return trade-off that should be considered when comparing alternative trading strategies. In the rest of this section, we describe two simple hedging strategies and compare their performance using simulation.

FIXED HEDGE RATIOS

A fixed hedge, such as MGRM's, involves buying derivatives with underlying volume equal to a fixed percentage of the committed volume. MGRM's reason for using the one-to-one stacked hedge was presumably to guard against spikes in the price of heating oil and unleaded gasoline. In the event of a price spike, MGRM would have a sufficient number of futures contracts to match contracted deliveries or to cover early liquidation payouts. Since these payouts were calculated as half the remaining volume times the liquidation premium, a hedge ratio of 0.5 might seem appropriate. Recall, though, that early liquidations occur when spot prices rise above the exit price - a hedge of greater than 0.5 would generate extra revenue for MGRM since its long futures position would be larger. In addition, MGRM's implementation of the "stack-and-roll" strategy was intended to capture profits from relatively stable backwardation in the unleaded gasoline and heating oil futures prices.

MATURITY-SPECIFIC HEDGE RATIOS

A "tailed" hedge hedges only the present value (not the full value) of a known future commitment to deliver at a fixed price. Assuming monthly deliveries over a full 10-year contract term and a 5% annualised interest rate, tailing the hedge requires only 79% of the number of futures contracts required under a one-to-one hedge. With monthly deliveries over a full five-year contract term, tailing the hedge requires about 88% of the number of contracts of a one-to-one hedge. And with monthly deliveries over a two-year contract term, tailing the hedge requires about 95% of the number of contracts of a one-to-one hedge. Note that tailing the hedge implies that for a given supply contract, the appropriate hedge ratio grows over time as the contract nears maturity, and is effectively equivalent to the one-to-one hedge for short-dated commitments. Tailing the hedge is widely regarded as a necessary part of variance-minimisation, and is used in this chapter as the main alternative to the fixed hedge ratio strategies.

ASSESSING HEDGE PERFORMANCE BY SIMULATION

With details of MGRM's supply contracts and descriptions of alternative hedging strategies in hand, we are now in position to answer the question: was the one-to-one stacked hedge a reasonable strategy? Our goal is to conduct risk analysis of MGRM's supply programme as the company might have done on November 15, 1991, directly prior to implementation of the strategy. To proceed, we first estimate parameters of models of spot and futures prices. Next, we use the models to generate random price paths and to simulate profits from the

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supply programme with alternative hedging strategies. The simulated profit distributions of the strategies will allow us to evaluate their risk-return trade-offs and to determine whether the one-to-one stacked hedge was reasonable.

MODEL ESTIMATION

Our first task is to estimate parameters of models of spot and futures prices. The data are monthly observations of unleaded gasoline and heating oil futures contracts traded on the New York Mercantile Exchange (Nymex) from December 1985 to November 1991, a total of 72 observations. The upper half of Figure 1 shows a plot of the spot price data. We use the nearby futures as the spot price and the second nearby as the hedging instrument.

The model of spot prices we estimate specifies mean-reversion in expected price changes and correlation between heating oil and gasoline prices:⁵

$$\ln\left(\frac{S_{i,t}}{S_{i,t-1}}\right) = \alpha(\beta_i - S_{i,t-1}) + \varepsilon_{i,t} \quad i = 1,2$$

$$(\varepsilon_{1,t}, \varepsilon_{2,t}) \sim \text{BVN}(0, \sigma_1^2, 0, \sigma_2^2, \rho) \quad (1)$$

where BVN denotes the bivariate normal function. Mean reversion appears to be an important feature in the oil prices shown in Figure 1. For example, oil prices spiked temporarily in autumn 1990 as a result of the Gulf War. The correlation between heating oil and gasoline prices, also apparent in Figure 1, results from the common dependence on crude oil. The specification of model (1) also ensures positive prices.

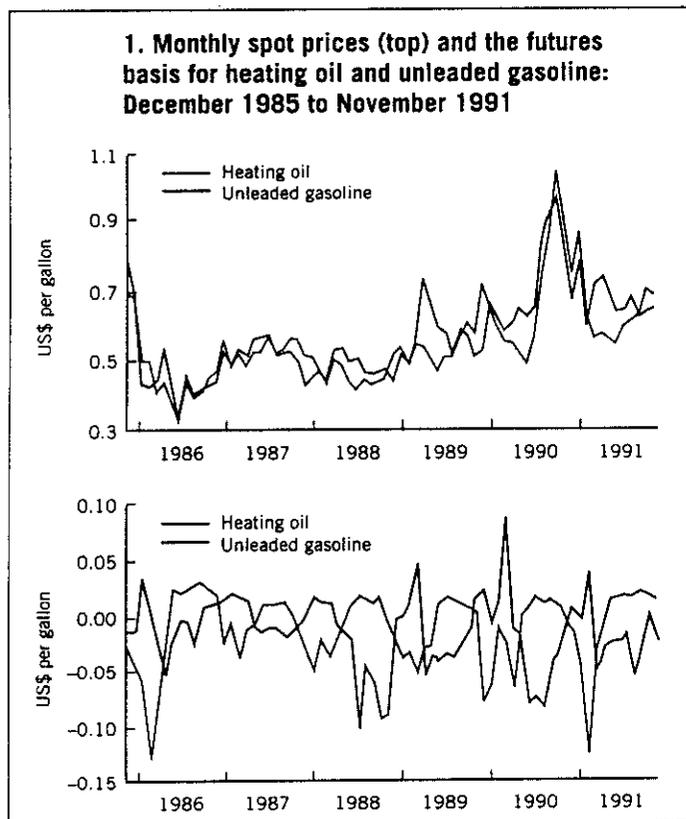
To estimate the historical features of futures prices, we first define the basis as

$$b_{i,t} = \ln\left(\frac{F_{i,t}}{S_{i,t}}\right) \quad (2)$$

The basis implied by the spot and futures prices in our data set are plotted in the lower half of Figure 1. We estimate parameters of the following model for the basis for unleaded gasoline and heating oil:

$$b_{i,t} = \alpha_i b_{i,t-1} + \beta_i S_{i,t-1} + \varepsilon_{i,t} \quad i = 1,2$$

$$(\varepsilon_{1,t}, \varepsilon_{2,t}) \sim \text{BVN}(0, \sigma_1^2, 0, \sigma_2^2, \rho) \quad (3)$$



This model allows for serial correlation in the level of the basis, a feature that stands out in Figure 1, and a dependence on the underlying spot price. The relation of the basis to the spot price may arise from the inverse relation between convenience yield and the level of inventory. When inventories are low, the spot price is high since the commodity is scarce, and the convenience yield from holding the commodity is also high, hence the basis is low. So we should expect a minus sign on b in equation (3). Correlation between shocks to the basis of gasoline and heating oil is motivated by the asynchronous demand cycles of the petroleum products and their impact on inventories and convenience yield.

Parameters of the models, presented in Table 2, are estimated using maximum likelihood. For both models, all parameters are significant at all usual levels using heteroscedasticity-consistent standard errors. Note that for the spot price dynamics, the correlation between gasoline and heating oil is about 0.7. For the basis dynamics, the correlation is about -0.4, consistent with asynchronous convenience yields for gasoline and heating oil.

SIMULATION PROCEDURE

For a single simulation, a series of random spot and futures prices is generated consistent with mean-reverting spot price dynamics and the model of basis dynamics estimated above. For a given randomly generated pair of spot price and basis, the simulated futures price is constructed as

$$F_t = S_t e^{b_t} \quad (4)$$

For each simulation, the initial spot and futures prices are set equal to their values on November 15, 1991, approximately one month prior to the implementation of the MGRM supply contract programme, to replicate the market conditions that existed at the time. The fixed prices of the contracts were set at US\$0.08 above the prevailing spot price on November 15, 1991 to reflect the "margin" built into each supply contract. For those contracts that included an exit provision, the exit price of the contract was set at US\$0.1354 above the spot for heating oil and US\$0.1144 above for unleaded gasoline. The exit price consists of the US\$0.08 margin plus a weighted average of the premiums of the exit prices above the fixed prices shown in Table 1.

The terminal profits from each simulation run are recorded when all contracts have been completely honoured. To illustrate, consider the simulated profits from a hedged, 10-year, firm-fixed commitment. In the first month, the monthly deliverable quantity of either unleaded gasoline or heating oil is delivered at the fixed price for a gain (loss) equal to the difference between the fixed price and the prevailing spot price times the delivered amount. At the same time, the futures position realises a gain (loss) equal to the size of the hedge position times the nearby futures price change. The gain (loss) on the sale of the oil product is then added to the gain (loss) on the futures position, and then the sum is carried forward for 119 months to account for the time value of money. In addition, for those supply contracts with an exit letter, the simulated spot price is used to determine whether the exit provision is triggered. If the spot price exceeds the exit price, the cashflow generated from the exit clause is calculated as half the difference between the fixed price and the exit price times the remaining volume of the contract. The contract is then terminated. The procedure is repeated for months two to 120, with the number of gallons in the futures hedge being reduced each month by the monthly deliverable quantity. In each simulation run, the terminal profits of competing hedging strategies are calculated.

The simulation is performed 5,000 times to cover the full gamut of possible price paths. For each simulation, we analyse the performance of fixed hedge ratios between zero and 1.0, in increments of 0.05, as well as a tailed strategy using a 5% interest rate.

SUMMARY STATISTICS

Table 3 lists some summary statistics for the simulations categorised by the hedging strategy. For each hedging strategy, we calculated the minimum and maximum terminal profit levels over the 5,000 simulations. We also recorded the average terminal profit level, the standard deviation, the percentage of the simulations resulting in a positive terminal profit and the percentage of the simulations that witnessed a cash balance of less than -US\$1 billion at some point during the simulation.

Table 2. Parameter estimates of models of spot price and futures basis dynamics for heating oil and unleaded gasoline

| Panel A: Spot price dynamics | | | |
|------------------------------|----------|-----------|-------------|
| Parameter | Estimate | Std error | t-statistic |
| α_{OH} | 0.342 | 0.111 | 3.065 |
| β_{OH} | 0.539 | 0.038 | 14.368 |
| σ_{OH} | 0.110 | 0.010 | 11.176 |
| α_{Gas} | 0.391 | 0.104 | 3.745 |
| β_{Gas} | 0.560 | 0.035 | 15.989 |
| σ_{Gas} | 0.116 | 0.013 | 9.017 |
| ρ | 0.705 | 0.079 | 8.899 |
| Panel B: Basis dynamics | | | |
| Parameter | Estimate | Std error | t-statistic |
| α_{OH} | 0.663 | 0.089 | 7.473 |
| β_{OH} | -0.009 | 0.005 | -1.803 |
| σ_{OH} | 0.025 | 0.003 | 7.694 |
| α_{Gas} | 0.424 | 0.146 | 2.906 |
| β_{Gas} | -0.015 | 0.008 | -1.846 |
| σ_{Gas} | 0.029 | 0.003 | 9.248 |
| ρ | -0.358 | 0.104 | -3.436 |

Table 3. Summary statistics of simulated payouts from MGRM's supply contracts and a range of hedging strategies (US\$ million)

| Ratio | Mean | Std dev | Min | Max | % > 0 | % ≤ 0 |
|--------|----------|----------|---------|----------|--------|-------|
| 0.00 | 1,348.60 | 365.56 | 382.47 | 2,567.13 | 100.00 | 0.00 |
| 0.05 | 1,481.37 | 348.35 | 570.46 | 2,547.17 | 100.00 | 0.00 |
| 0.10 | 1,614.15 | 342.14 | 722.85 | 2,597.81 | 100.00 | 0.00 |
| 0.15 | 1,746.92 | 347.52 | 756.76 | 2,754.94 | 100.00 | 0.00 |
| 0.20 | 1,879.70 | 363.99 | 790.67 | 2,977.78 | 100.00 | 0.00 |
| 0.25 | 2,012.47 | 390.13 | 824.58 | 3,298.50 | 100.00 | 0.00 |
| 0.30 | 2,145.24 | 424.17 | 858.50 | 3,619.23 | 100.00 | 0.00 |
| 0.35 | 2,278.02 | 464.36 | 870.03 | 3,939.95 | 100.00 | 0.00 |
| 0.40 | 2,410.79 | 509.26 | 759.75 | 4,260.67 | 100.00 | 0.00 |
| 0.45 | 2,543.57 | 557.73 | 649.46 | 4,581.39 | 100.00 | 0.08 |
| 0.50 | 2,676.34 | 608.92 | 539.18 | 4,902.12 | 100.00 | 0.58 |
| 0.55 | 2,809.12 | 662.20 | 428.89 | 5,222.84 | 100.00 | 1.60 |
| 0.60 | 2,941.89 | 717.09 | 318.61 | 5,543.56 | 100.00 | 3.22 |
| 0.65 | 3,074.67 | 773.27 | 208.33 | 5,864.28 | 100.00 | 5.82 |
| 0.70 | 3,207.44 | 830.46 | 98.04 | 6,185.00 | 100.00 | 9.58 |
| 0.75 | 3,340.22 | 888.47 | -12.24 | 6,505.73 | 99.98 | 14.12 |
| 0.80 | 3,472.99 | 947.16 | -122.53 | 6,826.45 | 99.98 | 18.46 |
| 0.85 | 3,605.76 | 1,006.39 | -232.81 | 7,147.17 | 99.98 | 22.48 |
| 0.90 | 3,738.54 | 1,066.09 | -343.10 | 7,467.89 | 99.96 | 25.94 |
| 0.95 | 3,871.31 | 1,126.18 | -453.38 | 7,788.62 | 99.96 | 29.54 |
| 1.00 | 4,004.09 | 1,186.59 | -563.66 | 8,109.34 | 99.92 | 33.36 |
| Tailed | 3,303.34 | 807.29 | 315.73 | 6,253.29 | 100.00 | 4.16 |

The last two columns show the percentage of simulations resulting in a positive terminal profit and the percentage of simulations leading to a cash deficit above US\$1 billion at some point during the simulation.

Table 4. Cash balance probabilities over a range of time horizons and catastrophe levels for three hedging strategies: no hedge, a one-to-one hedge and a 5% tailed hedge

| Month | No hedge | | | One-to-one hedge | | | Tailed hedge | | |
|-------|----------|-----------|----------|------------------|-----------|----------|--------------|-----------|----------|
| | ≤ \$0 | ≤ -\$500m | ≤ -\$1bn | ≤ \$0 | ≤ -\$500m | ≤ -\$1bn | ≤ \$0 | ≤ -\$500m | ≤ -\$1bn |
| 1 | 7.72 | 0.00 | 0.00 | 62.64 | 19.38 | 1.26 | 62.52 | 3.92 | 0.00 |
| 2 | 10.96 | 0.00 | 0.00 | 66.10 | 30.94 | 5.82 | 65.48 | 11.20 | 0.10 |
| 3 | 14.08 | 0.00 | 0.00 | 67.26 | 35.80 | 8.82 | 66.76 | 15.56 | 0.26 |
| 4 | 13.82 | 0.00 | 0.00 | 66.68 | 36.04 | 10.48 | 65.62 | 17.42 | 0.34 |
| 5 | 12.64 | 0.00 | 0.00 | 65.96 | 36.02 | 11.86 | 65.08 | 16.96 | 0.38 |
| 6 | 11.76 | 0.00 | 0.00 | 62.76 | 35.68 | 11.50 | 61.30 | 17.44 | 0.70 |
| 7 | 11.42 | 0.00 | 0.00 | 61.58 | 34.48 | 11.84 | 59.84 | 16.44 | 0.76 |
| 8 | 10.82 | 0.00 | 0.00 | 60.92 | 33.12 | 10.92 | 58.96 | 15.30 | 0.84 |
| 9 | 10.34 | 0.00 | 0.00 | 57.54 | 31.86 | 10.36 | 54.98 | 14.40 | 0.86 |
| 10 | 9.62 | 0.00 | 0.00 | 54.24 | 29.58 | 10.14 | 51.08 | 13.42 | 0.80 |
| 11 | 9.34 | 0.00 | 0.00 | 52.08 | 26.76 | 10.06 | 48.88 | 12.72 | 0.78 |
| 12 | 8.54 | 0.00 | 0.00 | 50.38 | 25.50 | 8.58 | 46.64 | 10.88 | 0.76 |
| 24 | 1.10 | 0.00 | 0.00 | 21.86 | 8.58 | 2.40 | 16.32 | 2.38 | 0.18 |
| 36 | 0.04 | 0.00 | 0.00 | 7.54 | 2.28 | 0.42 | 3.86 | 0.32 | 0.00 |
| 48 | 0.00 | 0.00 | 0.00 | 2.66 | 0.64 | 0.12 | 0.82 | 0.06 | 0.00 |
| 60 | 0.00 | 0.00 | 0.00 | 0.96 | 0.28 | 0.06 | 0.24 | 0.04 | 0.00 |
| 72 | 0.00 | 0.00 | 0.00 | 0.52 | 0.10 | 0.02 | 0.06 | 0.00 | 0.00 |
| 84 | 0.00 | 0.00 | 0.00 | 0.24 | 0.08 | 0.00 | 0.02 | 0.00 | 0.00 |
| 96 | 0.00 | 0.00 | 0.00 | 0.12 | 0.04 | 0.00 | 0.02 | 0.00 | 0.00 |
| 108 | 0.00 | 0.00 | 0.00 | 0.10 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 |
| 120 | 0.00 | 0.00 | 0.00 | 0.08 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 |

The hedge ratio has a dramatic impact on the minimum terminal profit level. For the case of zero hedging, the minimum is about US\$382 million. As the hedge ratio increases to 0.35, the minimum profit level increases, then begins to decline. For hedge ratios between 0.00 and 0.70, none of the 5,000 simulations results in a negative terminal profit. For hedge ratios above 0.70, the maximum loss increases in magnitude with the hedge ratio. The negative outcomes are quite rare, however, as all hedging strategies record a positive terminal profit over 99.9% of the time.⁴ This result indicates that the combined marketing and hedging programme is almost always profitable if funded to maturity.⁵

The hedge ratio also has a dramatic impact on the maximum terminal profit level. The maximum profit without hedging is about US\$2.6 billion. The maximum profit rises with the hedge ratio and reaches a maximum of more than US\$8.1 billion for the one-to-one hedge ratio. The average profit also increases with the hedge ratio, from US\$1.3 billion without hedging to US\$4 billion with a hedge ratio of 1.0. This results from the "roll" profits referred to earlier. The increase in average profits is generally accompanied by an increase in the standard deviation of profits. The minimum variance hedge is 0.10. The variation in profitability for higher hedge ratios, however, may be desirable if the variation is on the upside, a point we will return to shortly.

Perhaps the most interesting result from Table 3 is the last column, the percentage of simulations resulting in a cash position of less than -US\$1 billion at some point during the simulation. Hedge ratios of up to 0.4 result in a "catastrophe probability" of 0%. For higher hedge ratios, the probability increases significantly. Indeed, for a hedge ratio of 1.0, the probability of a catastrophe is about 33%. This result indicates that the cash problems at MGRM could have been anticipated.

How did MGRM's strategy compare with the tailed hedge? The tailed hedge is a lower risk strategy with lower expected returns, as can be seen by comparing the last two lines of Table 3. The tailed hedge also has a higher minimum profit level, a lower maximum profit level and a lower catastrophe probability of about 4%. To assess the risk-return trade-off between the two strategies, it is necessary to consider the shape of their distributions, as discussed below. But first, we explore the catastrophe levels in greater detail.

Table 4 lists the percentage of the 5,000 simulations for which the cash balance was at or below three levels (zero, -US\$500 million and -US\$1 billion) over time for three hedging strategies (no hedge, one-to-one hedge and a 5% tailed hedge). For the case of no hedging, the probability of achieving a negative cash balance reaches a maximum of 14% in month three, declines to about 9% in month 12 and falls to zero shortly thereafter. The probability of achieving a balance of -US\$500 million or worse is zero. For both of the other strategies, the probability of achieving a negative cash balance is much higher early on, reaching a

maximum of about 67% in month three. However, for both of the other strategies, the probability declines rapidly so that, by the end of the programme, there is virtually a 100% probability of breaking even. The probability of achieving a balance of -US\$500 million or worse reaches a maximum of 36% in month four for the one-to-one hedge and 17.4% in month six for the tailed hedge. Further, the probability of achieving a balance of -US\$1 billion or worse reaches a maximum of about 12% in months five to seven for the one-to-one hedge and about 1% for the tailed hedge in month nine. These results indicate that the worst-case outcomes occur in the first year of the programme. In addition, the probability of breaking even approaches 100% rather quickly for all hedging strategies considered.

HISTOGRAMS OF SIMULATED PROFITS

Comparison of the risk-return trade-offs of alternative hedging strategies hinges on the shape of the simulated distributions. If variation of outcomes occurs on the upside, then variation is desirable. Figure 2 is a histogram of the simulated payouts for the supply contracts with a one-to-one hedge and a tailed hedge. With the one-to-one hedge, the probability distribution is "flattened", with a dramatic increase in the probability of extremely good outcomes. There is also a slightly higher probability of an extremely bad outcome but its effect pales by comparison. Figure 2 illustrates clearly why variance-minimisation is not always an appropriate hedging objective.

Projected cash balance

The remaining question surrounding the MGRM hedging controversy is "What would have happened if the original hedging strategy had been left in place?" We answer this question by simulating the monthly net cashflows of MGRM's fixed-supply contract assuming a one-to-one stacked hedge. Using the actual history of petroleum prices from December 16, 1993 to March 1997, we calculate what the balance would have been at the end of March 1997 if the stacked hedge were left in place.

MECHANICS OF THE NET CASHFLOW CALCULATION

The fixed supply contract positions shown in Table 1 form the basis of our analysis. MGRM is assumed to have these positions in place as of December 31, 1993. Along with the fixed-supply contracts, MGRM is assumed to be long 3.8 billion gallons of the nearby heating oil futures contract and 2.9 billion gallons of the nearby unleaded gasoline futures contract.

The deliveries on the fixed-supply contracts as well as the rolls of the futures contract positions are assumed to occur on the fifteenth day of each month. Each month, the net cashflow for each hedged supply contract position is calculated. To illustrate, consider the net cashflow calculations for the two-year firm-fixed heating oil contracts on their first delivery date, January 17, 1994. On January 17, the two-year firm-fixed contracts had 15,183,970 gallons outstanding. Since the contracts had rateable monthly deliveries over two years, the amount assumed to be delivered on January 17 is $15,183,970/24$ or 632,665 gallons. The average fixed price for these two-year contracts was US\$0.6012 a gallon and the price of heating oil on January 17 was US\$0.5267 a gallon. Hence, the two-year fixed supply contracts on heating oil produced a cashflow of $632,665 \times (\text{US}\$0.6012 - \text{US}\$0.5267) = \text{US}\$47,134$. The remaining contract volume is then reduced by 632,665 gallons.

To hedge the futures contracts, a one-to-one stacked hedge is used. To hedge the 15,183,970 gallons of two-year firm-fixed contracts outstanding, 15,183,970 gallons of the nearby heating oil futures were assumed to be bought on December 16, 1993 and sold on January 17, 1994. The nearby futures contract in this case was

2. Simulated payouts from MGRM's supply contracts with a one-to-one stacked hedge and a 5% tailed hedge

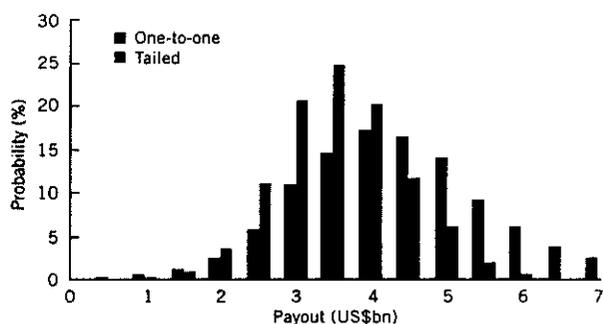


Table 5. Simulated cashflows from MGRM's supply contracts and a one-to-one stacked hedge: January 1994 to March 1997 (US\$ million)

| Delivery | Firm-fixed hedge | 45-day hedge | Exit cashflow | 45-day cashflow | Total | |
|-------------------|------------------|--------------|---------------|-----------------|-------|----------|
| Heating oil | 55.99 | 569.28 | 133.88 | -35.45 | -1.73 | 721.97 |
| Unleaded gasoline | 13.03 | 231.33 | 197.12 | -14.23 | -1.53 | 425.72 |
| Both products | 69.02 | 800.60 | 331.00 | -49.68 | -3.27 | 1,147.68 |

the February 1994 contract. Its price on December 16, 1993 was US\$0.4549 a gallon and its price on January 17, 1994 was US\$0.5267. Consequently, the long futures hedge on the two-year firm-fixed heating oil contracts produced a gain of $15,183,970 \times (\text{US}\$0.5267 - \text{US}\$0.4549)$ or US\$1,090,209. The net cashflow for the two-year firm-fixed heating oil contracts was therefore US\$1,137,343.

SIMULATION RESULTS

A summary of the net cashflows is provided in Table 5. The results indicate that had the long-term fixed supply contracts and their one-to-one stack-and-roll futures hedge programme been left in place, the aggregate net cashflow would have been US\$1.1 billion, covering even the most pessimistic of MGRM's reported losses in 1993. Though this evidence may be discounted by some as resulting from a lucky string of price changes since 1993, it nonetheless answers a question in the minds of those that have followed the Metallgesellschaft controversy.

Conclusions

This chapter contributes to the debate surrounding the financial crisis at MGRM by analysing the portfolio of supply contracts sold by the firm in 1993. Using historical data on spot and futures prices of heating oil and unleaded gasoline, we simulate the terminal future value of cashflows from the contracts and a variety of hedging strategies, assuming that each strategy is funded, as necessary, until all the committed product is delivered. We find that though MGRM's combined supply and hedging programme is almost always profitable after all contracts are honoured, there is a significant probability that the firm's cash balance drops below -US\$1 billion at some point. Further, the worst-case outcomes are most likely to occur in the first year of the programme. These results indicate that the program is indeed a "cash cow", but that, to be milked, a substantial capital commitment may be necessary early on. Further, we show that variance-minimisation is not always an optimal goal. MGRM's hedging strategy has more variance than a tailed hedge, but the variation seems to exist on the upside. In sum, our results suggest that the short-term losses at MGRM were a predictable intermediate outcome of a viable marketing/hedging strategy.

1 An analysis of the actual losses sustained by MGRM is performed in Culp and Miller (1995b).

2 We are grateful to Siegfried Holdapp for providing us with MGRM's contract information.

3 We estimated parameters of several simpler models of spot and futures price dynamics using maximum likelihood. The simpler models were easily rejected in favour of the models described in the chapter using likelihood ratio statistics. Details are available from the authors.

4 For very low hedge ratios, however, a significant number of the simulations are not profitable. With a zero hedge, for example, about 30% of the simulations lose money.

5 For a more detailed explanation of this phenomenon, see Culp and Miller (1995a).

