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VALUATION OF AMERICAN CALL OPTIONS ON DIVIDEND-PAYING STOCKS

Empirical Tests

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This paper examines the pricing performance of the valuation equation for American call options on stocks with known dividends and compares it with two suggested approximation methods. The approximation obtained by substituting the stock price net of the present value of the escrowed dividends into the Black–Scholes model is shown to induce spurious correlation between prediction error and (1) the standard deviation of stock return, (2) the degree to which the option is in-the-money or out-of-the-money, (3) the probability of early exercise, (4) the time to expiration of the option, and (5) the dividend yield of the stock. A new method of examining option market efficiency is developed and tested.

1. Introduction

Perhaps the most significant development in the financial economics literature of the last decade is the option valuation work of Black and Scholes (1973). Under a somewhat stringent set of assumptions they derive the first closed form solution to the call option pricing problem. Their most exacting assumption disallows income distributions on the underlying security. It is somewhat disconcerting when, for example, less than five percent of the options listed on the Chicago Board Options Exchange are written on non-dividend-paying stocks.

In this paper a model for pricing American call options on dividend-paying stocks is presented and compared with prior approximate solutions to the problem. Section 3 reviews prior empirical work on option pricing models. That work is concerned with three issues: (1) estimating the standard deviation of stock return, (2) testing alternative option pricing models, and (3) testing the efficiency of options markets. After a description of the data, these issues are addressed in turn. A new method of estimating stock volatility is presented in section 5. Using the standard deviation estimates, model option value is

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calculated and compared with approximations previously used. In section 7 the profitability of a riskless hedging strategy of buying options undervalued by the model and selling options overvalued by the model is examined. Finally, the paper is summarized and conclusions are drawn.

2. Theory of option valuation

The general equilibrium pricing solution to the call option pricing problem derived by Black and Scholes (1973) incorporates the following assumptions:

(A.1) All individuals can borrow or lend without restriction at the instantaneous riskless rate of interest, r, and that rate is constant through the life of option, T.

(A.2) Stock price movement through time is described by the stochastic differential equation

$$\mathrm{d}P/P = \mu\,\mathrm{d}t + \sigma\,\mathrm{d}z,$$

where μ is the instantaneous expected rate of return on the stock, σ is the instantaneous standard deviation of stock return (assumed to be constant over the life of the option), the $\mathrm{d}z$ is a standard unit normally distributed variable.

(A.3) The capital market is free from transaction costs (e.g., brokerage fees, transfer taxes, short selling and indivisibility constraints) and tax differentials between dividend and capital gain income.

(A.4) The stock pays no dividends during the option's time to expiration.

The value of a European call,[1] denoted $c(P, T, X)$, provided by Black and Scholes is

$$c(P, T, X) = PN_1(d_1) - X\,\mathrm{e}^{-rT}N_1(d_2), \tag{1}$$

where

$$d_1 = \{\ln(P/X) + (r + 0.5\sigma^2)T\}/\sigma\sqrt{T}, \qquad d_2 = d_1 - \sigma\sqrt{T},$$

X is the option's contracted exercise price, and $N_1(d)$ is the univariate cumulative normal density function with upper integral limit d.

The assumed absence of income distributions on the underlying security causes the Black–Scholes formula to overstate the value of an American call option on a

[1]Assuming the common stock underlying the call option has no income distributions to shareholders, the value of the American call is equal to the value of the European call. See Merton (1973, p. 144) or Smith (1976, pp. 8–11).

stock with dividend payments during the option's time to expiration. A dividend paid during the option's life reduces the stock price at the ex-dividend instant, and thereby reduces the probability that the stock price will exceed the exercise price at the option's expiration.

In order to introduce a discrete dividend payment into the option pricing problem it is usually assumed that:

(A.4′) The stock pays a certain dividend, D, at the ex-dividend instant, t ($t<T$), and the stock price simultaneously falls by a known amount, αD.

With the amount and timing of the dividend payment known, a simple approximation for the value of the American call is the value of a European call, $c(S,T,X)$, where S is the stock price net of the present value of the escrowed dividend payment, $S_\tau = P_\tau - \alpha D\,\mathrm{e}^{-r(t-\tau)}$ for $\tau<t$ and $S_\tau = P_\tau$ for $t \leqq \tau \leqq T$. Note that by using the lower stock price the model's price is adjusted downward to allow for the stock price decline at the ex-dividend date.

Unfortunately, this approximation ignores a second dividend-induced effect in that it presumes that the call will not be exercised prior to expiration. Smith (1976, pp. 13–14) demonstrates that the American option holder may benefit from exercising early, just prior to the ex-dividend instant. To compensate for this possibility Black (1975, pp. 41, 61) recommends an approximate value equal to the higher of the values of a European call where the stock price net of the present value of the escrowed dividend is substituted for the stock price and a European call where the time to ex-dividend is substituted for the time to expiration, that is,

$$\max\,[c(S,T,X),c(P,t,X)]. \tag{2}$$

The first option within the maximum value operator assumes the probability of early exercise is zero, while the second option assumes it is one.

The American call option on a stock with a known dividend, however, may be characterized by an early exercise probability between zero and one. For some time, this option pricing problem was thought to be insoluble.[2] If the stock price follows a lognormal diffusion process, there exists some non-zero probability that the dividend cannot be paid. Roll (1977) and Geske (1979) resolve the problem by assuming that the stock price net of the present value of the escrowed

[2]Schwartz (1977) provides a numerical method by which the value of an American call on a stock with known dividends can be approximated.

dividend follows the lognormal process. That is, if Assumption (A.2) is amended such that:

(A.2′) Stock price movement through time is described by the stochastic differential equation

$$\mathrm{d}S/S = \mu\,\mathrm{d}t + \sigma\,\mathrm{d}z,$$

where S represents the stock price net of the present value of the escrowed dividend,

the solution to American call option pricing problem, as provided by Whaley (1981), is

$$\begin{aligned} C(S,T,X) = {} & S[N_1(b_1) + N_2(a_1, -b_1; -\sqrt{t/T})] \\ & - X\,\mathrm{e}^{-rT}[N_1(b_2)\,\mathrm{e}^{r(T-t)} + N_2(a_2, -b_2; -\sqrt{t/T})] \\ & + \alpha D\,\mathrm{e}^{-rt} N_1(b_2), \end{aligned} \tag{3}$$

where

$$a_1 = \{\ln(S/X) + (r + 0.5\sigma^2)T\}/\sigma\sqrt{T}, \qquad a_2 = a_1 - \sigma\sqrt{T},$$

$$b_1 = \{\ln(S/S_t^*) + (r + 0.5\sigma^2)t\}/\sigma\sqrt{t}, \qquad b_2 = b_1 - \sigma\sqrt{t},$$

and $N_2(a,b;\rho)$ is the bivariate cumulative normal density function with upper integral limits a and b, and correlation coefficient ρ. S_t^* is the ex-dividend stock price determined by

$$c(S_t^*, T-t, X) = S_t^* + \alpha D - X, \tag{4}$$

above which the option will be exercised just prior to the ex-dividend instant.

Note that if the American call is neither a dominant nor a dominated security its value is bounded from below by the Black approximation. The first term within the maximum value operator of expression (2) represents the price of the call if there were no chance of early exercise. Since the right to early exercise has a non-negative value, $C(S,T,X) \geqq c(S,T,X)$. The second term represents the price of a call with maturity t. At t, its payoffs are 0, if $S_t + \alpha D < X$, and $S_t + \alpha D - X$, if $S_t + \alpha D \geqq X$. At t, however, the American call is worth $c(S_t, T-t, X)$, if $S_t + \alpha D < X$, and $\max[c(S_t, T-t, X), S_t + \alpha D - X]$, if $S_t + \alpha D \geqq X$. In the absence of dominance, therefore, $C(S,T,X) \geqq c(P,t,X)$, and, if the arguments are combined, $C(S,T,X) \geqq \max[c(S,T,X), c(P,t,X)]$.

3. Review of option pricing tests

Previous empirical analyses of option pricing models have been concerned to varying degrees with three issues — volatility estimation, model specification and market efficiency. While a detailed description of each study is beyond the scope of this paper, a brief overview of the salient features relating to these issues is warranted.

3.1. Volatility estimation

Of the determinants in the call option pricing formulas, all but one are known or can be estimated with little difficulty. The exercise price and the time to expiration are terms written into the option contract; the stock price and the riskless rate are easily accessible market-determined values. The dividend information, if it is required, can be fairly accurately estimated by casual inspection of the stock's historical dividend series. The problem parameter is the expected volatility of the stock return.

An obvious candidate to proxy for the volatility expectation is an historical estimate obtained from the stock's realized return series. Black and Scholes (1972), as well as Galai (1977) and Finnerty (1978), use this estimate in valuing calls using the Black–Scholes option pricing model. Black and Scholes, however, recognize that a substantial amount of the observed deviation of the model's price from the market price may be attributable to an 'errors-in-the-variables' problem. In fact, they note that there is a tendency of the model to overprice options with high standard deviation estimates and to underprice options with low standard deviation estimates.

Latane and Rendleman (1976) investigate the predictive ability of a weighted implied standard deviation vis-à-vis the historical estimate. If there are n options on a stock at a particular point in time, n implied standard deviations, $\hat{\sigma}_j$, $j=1,\ldots,n$, may be obtained by setting the option's market price equal to the model price,

$$C_j = C_j(\sigma_j),$$

and solving for σ_j, where all of the remaining arguments of $C(\cdot)$ are assumed to be known. If these estimates are then weighted and averaged,

$$\hat{\sigma} = \sum_{j=1}^{n} \omega_j \hat{\sigma}_j \Big/ \sum_{j=1}^{n} \omega_j,$$

where ω_j is the weight applied to the jth estimate, a weighted implied standard deviation is realized.

Previous researchers employ various weighting schemes. Schmalensee and

Trippi (1978) and Patell and Wolfson (1979), for example, use an equal weighted average, $\omega_j = 1/n, j = 1,\ldots,n$. Latane and Rendleman, on the other hand, weight according to the partial derivative of the call price with respect to the standard deviation of stock return, that is, $\omega_j = \partial C_j/\partial\hat{\sigma}_j$, $j = 1,\ldots,n$. In doing so, the standard deviation estimates of options which are theoretically more sensitive to the value of σ are weighted more heavily than those which are not. Chiras and Manaster (1978) follow a similar logic in using the elasticity of the call price with respect to standard deviation, $\omega_j = (\partial C_j/\partial\hat{\sigma}_j)(\hat{\sigma}_j/C_j), j = 1,\ldots,n$.

Regardless of the weighting scheme, however, there appears to be strong empirical support in favor of an implied volatility measure. Latane and Rendleman and Chiras and Manaster correlate the historical and the implied measures on the actual standard deviation of return[3] and conclude that the implied estimate is a markedly superior predictor. The market apparently uses more information than merely an historical estimate in assessing the stock's volatility expectation.

3.2. Model specification

The valuation equation most commonly employed in past research has been either the Black–Scholes relation $c(P, T, X)$ or the approximation $c(S, T, X)$. Black and Scholes, for example, examine Over-the-Counter (OTC) option prices during the period May 1966 through July 1969. They apply their model directly with no dividend adjustment since the OTC options are protected. For protected options the exercise price is reduced by the amount of the dividend on the ex-dividend date. For unprotected options this is tantamount to reducing the stock price by the present value of the escrowed dividends. Galai compares the Black–Scholes price with and without the dividend adjustment by using unprotected Chicago Board Options Exchange (CBOE) pricing data and concludes that the latter model provides a more adequate description of the observed structure of call option prices. Studies by Merton, Scholes and Galdstein (1978) and MacBeth and Merville (1979) employ the approximation $c(S, T, X)$.

Chiras and Manaster account for dividends by transforming the payments into a constant, continuous dividend yield and applying the Merton (1973a) model. While this method uses dividend information in valuing the option, the transformation of discretely-timed dividend payments to a continuous dividend yield effectively assumes away the American option holder's early exercise dilemma.

Schmalensee and Trippi apply the Black–Scholes model without dividend adjustment to CBOE options to compute implied standard deviations, but try to minimize the dividend problem by concentrating on options whose underlying

[3]While Latane and Rendleman (1976) and Chiras and Manaster (1978) refer to the standard deviation computed with stock returns generated during the option's life as the 'actual standard deviation of return', it is only an estimate of the realized volatility.

stocks have low dividend yields. Latane and Rendleman and Finnerty use the Black–Scholes formula directly, with no consideration of the effects of the dividend payments.

Of the tested option pricing models, it appears that the Black–Scholes formula with the stock price net of the present value of the escrowed dividends provides the best explanation of the observed structure call option prices to date. Galai demonstrates a substantial increase in trading profits by employing this approximation instead of the Black–Scholes model with the stock price cum dividend. Black and Scholes, Black, and MacBeth and Merville, however, report certain systematic biases in the application of the model that are worth investigating in the present study. For instance, the degree of under- and over-pricing appears to be related to the standard deviation of the stock return — options on high-risk stocks tend to be overpriced and the options on low-risk stocks tend to be underpriced. Further, the degree of under- and over-pricing appears to be related to the difference between the stock price and the exercise price (i.e., how far in-the-money or out-of-the-money the option is), and the time to expiration of the option.

3.3. Market efficiency

The tests of option market efficiency usually involve an option trading strategy that is designed to create a riskless portfolio, which should, in an efficient capital market, yield the riskless rate of interest. The key insight into the Black–Scholes development is, in fact, the premise that risk substitutes have the same equilibrium rate of return. By taking a long (short) position in one call option and a short (long) position of $\partial C/\partial P$ shares of the stock, a riskless hedge is created, which, if continuously rebalanced through time, leads to a partial differential equation whose solution, subject to the terminal date boundary conditions, is the Black–Scholes call option pricing model.

From the standpoint of empirical investigation, continuous rebalancing is not possible, and discrete readjustment of the portfolio position is substituted. Undervalued (overvalued) options are identified, and are bought (sold) and hedged against a short (long) position of $\partial C/\partial P$ shares of the stock. All of the hedge positions are aggregated and the excess dollar returns [i.e., dollar returns on the hedge portfolio less the investment cost times (one plus the riskless rate of interest)] computed. The process is repeated on each trading date so that a time series of excess dollar returns is generated. A regression of the portfolio returns on a stock market index is usually included to verify that the hedge position is riskless, and, if so, the intercept term and its corresponding standard error provide a means of testing whether significantly positive (negative) returns are earned.

Latane and Rendleman use this procedure in testing option market efficiency, however, they aggregate separately the undervalued and the overvalued positions. The problem with this approach is that the returns on each portfolio have downward bias since the call options have a predictable decrease in value over the trading interval Δt, all other things remaining constant. For a detailed discussion on this point, see Boyle and Emanuel (1980).

Black and Scholes, Galai, and Finnerty combine the undervalued and the overvalued positions into one portfolio, long in the former group and short in the latter. If the characteristics of the long options (i.e., the stock prices, exercise prices, times to expiration and standard deviations of stock return) are nearly the same as those of the short options, the portfolio will be approximately riskless over the interval Δt.

Chiras and Manaster choose to hedge each undervalued option against an overvalued option on the same stock, thereby eliminating the need for investment in the stock. Unfortunately the requisite of being able to identify both an undervalued and an overvalued option on the same stock unduly restricts the number of options that may be included in the sample. In fact, the technique is so restrictive that Chiras and Manaster are left with an average of less than 12 options in each of their 23 cross-sections.[4]

4. Data

The data employed in this study consisted of weekly closing price observations for all Chicago Board Options Exchange (CBOE) call options written on 91 dividend-paying stocks during the 160 week period January 17, 1975 through February 3, 1978. The prices of the stocks, options and Treasury Bills were compiled from various issues of the *Wall Street Journal*. Wherever necessary, adjustments were made for stock splits and stock dividends. The riskless rate appropriate to each option was estimated by interpolating the effective yields of the two Treasury Bills whose maturities most closely preceeded and exceeded the option's time to expiration.[5] The dividend information was gleaned from *Standard and Poor's Stock Reports*, and the weekly stock return data were generated from the *Center for Research in Security Prices* (CRSP) daily return file. The market index was the value-weighted portfolio of NYSE and AMEX securities provided on the CRSP return file.

Three exclusion criteria were imposed on the option pricing information. First, the option's underlying stock had to have *exactly* one dividend paid during the option's remaining life. Without a dividend paid, the American call option formula would have reduced to the simple Black–Scholes model, and, since the focus of study is on investigating the effects of the dividend payment on the observed structure of call option prices, options with expiration dates before the

[4]See Chiras and Manaster (1978, p. 230).

[5]The rate of interest for a particular Treasury Bill was assumed to be the annualized, arithmetic average of the rates implied by the bid and ask prices.

stock's next ex-dividend date were eliminated. With more than one dividend paid before expiration, the structure of the valuation equation would have become more complex, requiring, among other things, the evaluation of an approximation for a trivariate or higher-order multivariate cumulative normal density function. Since the computational cost of such an integral approximation is high,[6] options with times to expiration including more than one ex-dividend date were, likewise, excluded.

The second restriction eliminated options whose premia were below fifty cents. CBOE regulation generally prohibits traders from establishing new positions in these options so that the reported prices may not accurately reflect transacting prices.

The remaining constraint was imposed to facilitate the market efficiency test design. For each cross-section t, all options employed were required to have three consecutive weekly prices. The first week's price, C_{t-1}, was required to compute the implied standard deviation of stock return.[7] This estimate was, in turn, employed in the valuation process of C_t at t in order to establish whether the option was under- or over-priced. The price, C_{t+1}, was used to compute the holding period return of the option over the trading interval t to $t+1$.

Descriptive statistics of the remaining sample data are included in table 1. The means, standard deviations, mean absolute deviations and percentile ranges of the 15,582 sets of option pricing information are reported. On average, the sample options are on-the-money, with the mean stock price only slightly exceeding the mean exercise price. The riskless rate standard deviation is low during the sample period, indicating that interest rate uncertainty is not a potential problem.

Certain limitations of the data should be noted. All of the option pricing formulas require that the stock price be known at the exact instant the option is priced. However, the stock price is the 3:00 p.m. EST Friday closing in New York,[8] while the option price is the 3:00 p.m. CST Friday closing in Chicago. Further, the closing price is the price for the last transaction, which may have occurred before the market closing and which may be either a bid or an ask price. To the extent that the closing quotations may not accurately reflect the prices at which the securities may be transacted, random noise in the empirical results should be expected.

In all applications of the American call option formula and the two approximation techniques to follow, the coefficient α preceding the dividend variable was assumed to be equal to one. There are several reasons why the

[6]For further discussion, see Milton (1972).

[7]The motivation for using the previous week's implied standard deviation in the option valuation process is discussed in the next section.

[8]During the sample period, all options, with exception of those written on Houston Oil and Minerals Corporation, were written on stocks listed on the New York Stock Exchange. Houston Oil was listed on the American Stock Exchange.

Table 1

Distributions of the prices and the parameters of the sample's 15,582 CBOE call options (with a single dividend to be paid during the option's life and a price of at least \$0.50 per option) during the period January 17, 1975 through February 3, 1978.

		Call price C	Stock price P	Exercise price X	Years to expiration T	Annualized riskless rate r	Amount of escrowed dividend D	Years to ex-dividend t
Mean		\$4.139	\$48.838	\$48.288	0.3178	0.0569	\$0.3756	0.1645
Standard deviation		5.240	39.547	38.735	0.1246	0.0067	0.3696	0.1133
Mean absolute deviation		2.856	22.521	22.516	0.0930	0.0057	0.2172	0.0788
Deciles	0.10	1.000	20.500	20.000	0.1753	0.0485	0.1100	0.0521
	0.20	1.313	27.125	25.000	0.2192	0.0507	0.1500	0.0740
	0.30	1.750	31.500	30.000	0.2521	0.0525	0.2000	0.0986
	0.40	2.155	35.250	35.000	0.2712	0.0541	0.2500	0.1233
	0.50	2.750	39.250	40.000	0.2959	0.0555	0.2750	0.1452
	0.60	3.375	44.375	45.000	0.3288	0.0576	0.3313	0.1671
	0.70	4.250	49.750	50.000	0.3534	0.0607	0.4000	0.1890
	0.80	5.625	58.875	60.000	0.3863	0.0641	0.5000	0.2110
	0.90	8.250	76.875	80.000	0.4822	0.0666	0.6500	0.2904

coefficient may be less than one, however. Differential income tax rates between dividends and capital gains, transaction costs, and the time interval between the ex-dividend and dividend payment dates may cause the stock price decline at the ex-dividend instant to be an amount less than the dividend. Since there was no manageable way of handling these institutional restrictions,[9] they were ignored.

5. Implied standard deviation estimation

5.1. Procedure

The methodology implemented to impute the estimate of the future volatility of the stock return differs from the previous studies in three important ways. First, rather than explicitly weighting the implied standard deviations of a particular stock where the weights are assigned in an ad hoc fashion, the call prices are allowed to provide an implicit weighting scheme that yields an estimate of standard deviation which has as little prediction error as is possible.[10] At a point in time options written on the same stock may be represented as

$$C_j = C_j(\sigma) + \varepsilon_j, \tag{5}$$

where C_j is the market price of the option, $C_j(\sigma)$ is the model's price (where all argument values are known, with exception of σ), and ε_j is a random disturbance term. The estimate of σ is then determined by minimizing the sum of squared residuals,

$$\min_{\{\hat{\sigma}\}} \sum_{j=1}^{n} e_j^2, \tag{6}$$

where e_j is the observed residual and $\hat{\sigma}$ is the estimated parameter.

The nonlinear estimation procedure applied to minimize the sum of squared residuals is the first order linearization process described by Eisner and Pindyck (1973, pp. 30–34).[11] As adapted to the present problem, the iterative technique begins with an expansion of C_j into a Taylor series around some initialization value σ_0, that is,

$$C_j = C_j(\sigma_0) + \left.\frac{\partial C_j}{\partial \sigma}\right|_{\sigma_0} (\sigma - \sigma_0) + \ldots \text{higher-order terms} \ldots + \varepsilon_j. \tag{7}$$

[9]Actually, the time lapse between the ex-dividend and dividend payment dates may be handled quite easily. During the sample period the average riskless rate was 5.69% per year and the average time between the ex-dividend and dividend payment dates was 26 days or 0.071 years. The value of α is therefore approximately $e^{-0.0569(0.071)} = 0.996$.

[10]Implied standard deviations are accurate only insofar as the call option model is correctly specified. In general, the implied volatility will not be the 'best' estimator of the stock's future standard deviation of return.

[11]For a review of nonlinear estimation techniques, see Spang (1962) or, more recently, Goldfeld and Quandt (1972).

Ignoring the higher-order terms and gathering known values on the left-hand side of the equation,

$$C_j - C_j(\sigma_0) + \sigma_0 \left.\frac{\partial C_j}{\partial \sigma}\right|_{\sigma_0} = \sigma \left.\frac{\partial C_j}{\partial \sigma}\right|_{\sigma_0} + \varepsilon_j. \tag{8}$$

Applying ordinary least squares (OLS) to (8) yields an estimate of σ. If that estimate satisfies an acceptable tolerance,

$$|(\hat{\sigma}_1 - \sigma_0)/\sigma_0| < \kappa, \tag{9}$$

where κ is a small positive constant, $\hat{\sigma}_1$ is the estimate of σ. If, the tolerance test fails, eq. (5) is linearized around the realized parameter estimate $\hat{\sigma}_1$, and OLS is reapplied. The process is repeated for $\hat{\sigma}_i$, $i=2,\ldots$, until the tolerance criterion is satisfied.

The second difference between the implied volatility estimation procedure is with respect to the timing of the estimation. It has apparently become an accepted practice to compute the implied standard deviation at the same instant at which the option is priced.[12] Conceptually, this procedure is difficult to understand in that, at an instant in time, the valuation equation is assumed to price options correctly (when the implied volatility is computed), and, yet, simultaneously, is assumed to price options incorrectly (when the model is used to identify whether the option is under- or over-priced). As an empirical matter, this procedure eliminates from study all single options written on a stock at a particular instant. With only one option, the model will exactly price the option since the volatility estimate at t is that standard deviation which equates the observed call price to the model's price. Moreover, even if two or more options are available, Phillips and Smith (1980, pp. 189–192) point out that contemporaneous estimation of volatility and valuation of options leads to a selection bias which systematically identifies bid prices as undervalued options and ask prices as overvalued options. To circumvent these problems, the implied volatility is computed at $t-1$. The conceptual problem is alleviated, at least in part, a larger sample size is retained, and a potential source of selection bias is eliminated.

Finally, unlike the previous empirical studies which compute a single volatility estimate on the basis of all of the options written on a stock at a particular point in time, the present study uses only those options which share a common maturity. Patell and Wolfson (1979, pp. 119–123) argue and demonstrate empirically that the standard deviation implied by the price of a longer-lived option written on a stock is greater than the standard deviation implied by the price

[12]Latane and Rendleman (1976), Chiras and Manaster (1978) and MacBeth and Merville (1979), among others, use this approach.

of a shorter-lived option if there is an anticipated information event between the expirations of the two options. Given that the difference between option lives is typically three months for CBOE call options, an information release, such as, for example, an earnings announcement, may be expected, and, hence, maturity-specific implied stock volatilities, rather than a single implied volatility common to all maturities, are most appropriate.

5.2. Results

For each stock, implied standard deviations were computed weekly using the previous period's call prices and a tolerance criterion of $\kappa = 0.0001$.[13] The numbers of options included in each computation are summarized in table 2. The 15,582 sets of call price information yielded 9,318 implied standard deviations, each computation including an average of 1.67 options of common maturity. Of the 9,318 computations, 936 were estimates for a second option maturity, or, equivalently, 1.11 estimates of the stock's volatility were obtained each week.

Descriptive statistics for the implied standard deviations computed on the basis of the valuation equation $C(S, T, X)$ are reported in table 3. On average, $\hat{\sigma}$ was 0.3004 on an annualized basis. The percentile ranges indicate that the distribution of volatilities is skewed to the left, with the lowest value being 0.0493 and the highest being 1.0379. Implied volatility estimates were also computed on the basis of the approximation models $c(S, T, X)$ and $\max[c(S, T, X), c(P, t, X)]$, and the results were nearly identical to those of $C(S, T, X)$.

Table 2

The number of options of common maturity included in each implied standard deviation computation.

Number of maturities of options written on a stock	Number of options of common maturity written on a stock						Number of implied standard deviation estimates
	1	2	3	4	5	6	
1	4,056	3,202	880	219	22	3	8,382
2	455	432	43	2	2	2	936
Total number of implied standard deviation estimates							9,318
Total number of call prices used							15,582

[13] With the maximum absolute relative error (κ) set at 1 one-hundredth of 1 percent, an average of slightly more than 4 iterations were required for convergence.

Table 3

Distribution of the 9,318 implied standard deviations computed using the 15,582 sample call option prices and the valuation equation for an American call option on a stock with a known dividend.

		Implied standard deviation of stock return $\hat{\sigma}$
Mean		0.3004
Standard deviation		0.1082
Mean absolute deviation		0.0855
Deciles	0.10	0.1801
	0.20	0.2104
	0.30	0.2340
	0.40	0.2577
	0.50	0.2826
	0.60	0.3092
	0.70	0.3423
	0.80	0.3865
	0.90	0.4529

6. Tests of option valuation

6.1. Procedure

Using the implied standard deviation estimates developed according to the procedure outlined in the preceding section, options were valued according to: (1) the American call option valuation equation, $C(S, T, X)$, (2) the Black approximation, $\max[c(S, T, X), c(P, t, X)]$, and (3) the Black-Scholes formula applied to the stock price net of the present value of the escrowed dividend, $c(S, T, X)$.[14] For each model the following cross-sectional regressions, designed to examine the difference between actual (C) and model ($\hat{C}$) option values, were estimated:

$$\text{Test 1:} \quad C_j = \alpha_0 + \alpha_1 \hat{C}_1 + \mu_j,$$

$$\text{Test 2:} \quad \frac{C_j - \hat{C}_j}{\hat{C}_j} = \alpha_0 + \alpha_1 \hat{\sigma}_j + \mu_j,$$

[14]Using the previous week's implied standard deviation may have introduced an 'errors-in-the-variables' problem in the tests of this section. Patell and Wolfson (1979) document that the implied standard deviation of common stock return is 'high' near earnings announcements dates, and, to the extent that such an information release may have occurred between volatility estimation at week $t-1$ and option valuation at week t, there will be inaccuracy in the test results.

$$\text{Test 3:}\quad \frac{C_j-\hat{C}_j}{\hat{C}_j}=\alpha_0+\alpha_1\left(\frac{S_j-X_j e^{-r_j T_j}}{X_j e^{-r_j T_j}}\right)+\mu_j,$$

$$\text{Test 4:}\quad \frac{C_j-\hat{C}_j}{\hat{C}_j}=\alpha_0+\alpha_1 p_j+\mu_j,$$

$$\text{Test 5:}\quad \frac{C_j-\hat{C}_j}{\hat{C}_j}=\alpha_0+\alpha_1 T_j+\mu_j,$$

$$\text{Test 6:}\quad \frac{C_j-\hat{C}_j}{\hat{C}_j}=\alpha_0+\alpha_1 d_j+\mu_j,$$

where p_j denotes the probability of early exercise, d_j denotes the dividend yield, μ_j is a disturbance term, and all other notation is as it was previously defined.

6.2. Results

To begin, all observations were pooled, and the grand means and standard deviations of the option prices were computed. The results were as follows:

Value	Mean	Standard deviation
Observed	\$4.1388	\$5.2400
$C(S, T, X)$	4.1291	5.1025
$\max[c(S, T, X), c(P, t, X)]$	4.1198	5.1025
$c(S, T, X)$	4.1071	5.1036

While the American call formula provided prices which are closer to the observed prices, all of the formulas yielded prices which are, on average, within three and a half cents of the observed market price.

The simple linear regression of market price on the model value is in the spirit of Theil's (1966) 'line of perfect forecast'. With perfect prediction the values of the coefficients α_0 and α_1 in the regression

$$C_j=\alpha_0+\alpha_1\hat{C}_j+\mu_j, \tag{10}$$

should be indistinguishable from zero and one, respectively. The estimate of α_0 and its corresponding standard error provide a means of testing the degree of bias in the valuation equation; the estimate of α_1 and its standard error provide a means of testing the degree of inefficiency.[15] The coefficient of determination from the regression reflects the degree to which the valuation equation is able to explain the variation in the observed call prices.

Observed market prices were regressed on each model's prices in each of the 160 cross-sections, where the number of observations included in the regressions averaged 97.3875 and ranged from 30 to 157. The average values of the weekly parameter estimates, $\bar{\hat{\alpha}}_0$ and $\bar{\hat{\alpha}}$, the Student t ratios testing the hypotheses that the average value of the intercept term is equal to zero, $t(\bar{\hat{\alpha}}_0)$,[16] and that the average value of the slope term is equal to one, $t(\bar{\hat{\alpha}}_1)$, and the two-tailed probability levels that the t-values will be exceeded in absolute magnitude by a random variable following a Student t distribution $p[t(\bar{\hat{\alpha}}_0)]$ and $p[t(\bar{\hat{\alpha}}_1)]$, are reported as Test 1 of table 4. The first-order serial correlation of the parameter estimates, $\rho(\hat{\alpha}_0)$ and $\rho(\hat{\alpha}_1)$, and the average of the coefficients of determination, $\bar{R}^2$, are also included.

All of the models seem to perform extremely well, with the explained variation being greater than 98 percent in all cases. Although there do not appear to be perceptible differences between the models, it is likely as a result of prediction error being small in relation to the magnitude of call price. In fact, when call price was regressed on the boundary condition $\max[0, S-X\,\mathrm{e}^{-rT}]$ in each of the 160 cross-sections, the average coefficient of determination was 87 percent. In subsequent tests the focus will be on relative prediction error so that the attenuating influence of heteroscedasticity will be reduced. Before leaving the results of Test 1, however, it is interesting to note that all models demonstrate a slight tendency to overprice low-priced options and to underprice high-priced options (i.e., $\bar{\hat{\alpha}}_1 > 1$).

The remaining tests focused on identifying systematic behavior in the relative prediction error [i.e., $(C-\hat{C})/\hat{C}$] of the valuation models. As a preliminary investigation, the relative prediction errors of all cross-sections were, again, pooled, and the grand means and standard deviations computed. The results were as follows:

Model	Mean	Standard deviation
$C(S,T,X)$	0.0108	0.2382
$\max[c(S,T,X), c(P,t,X)]$	0.0148	0.2396
$c(S,T,X)$	0.0215	0.2524

[15]When the forecast model underestimates high (low) values and overestimates low (high) values (i.e., $\hat{\alpha}_1 \neq 1$), it is said to be inefficient.

[16]The Student t ratio for the intercept term, for example, was computed as $t(\bar{\hat{\alpha}}_0) = \sqrt{160}\,\bar{\hat{\alpha}}_0/s(\hat{\alpha}_0)$. This methodology is not unlike that employed by Fama and MacBeth (1973, pp. 619–624).

The American call formula more clearly dominates when these figures are considered: both the mean and the standard deviation of the relative prediction error are lower than they are for either of the suggested approximations.

Test 2 addressed the underpricing of low-risk and overpricing of high-risk options issue. Both Black and Scholes and MacBeth and Merville document this phenomenon, but both authors' models exclude the premium for early exercise. If the probability of early exercise is correlated with the standard deviation of stock return, the American call formula should reduce the systematic pricing discrepancy.

The results reported for Test 2 in table 4 indicate that there exists a significantly positive relationship between prediction error and stock volatility for all models. The American call serves to reduce the magnitude of the slope coefficient and the coefficient of determination, but the hypothesis that there is no relationship between the variables is soundly rejected. All models appear to overprice options on high-risk stocks and to underprice options on low-risk stocks.

Test 3 attempted to uncover systematic underpricing and overpricing of in-the-money and out-of-the-money options — again, a result that has been the cause of consternation. The findings indicate that the relationship is not as strong as one may have been led to believe. The null hypothesis of a zero slope coefficient cannot be rejected at the 5 percent level for any of the valuation models. Again, the American call formula did better than the alternative models, with its slope coefficient being lower and less significant.

The fourth regression examined whether pricing inaccuracy was related to the probability of early exercise. In this test the independent variable p was computed as

$$\int_{S_t^*}^{\infty} L(S_t)\,\mathrm{d}S_t,^{17}$$

where S_t^* represents the ex-dividend stock price above which the American option holder will exercise just prior to the ex-dividend instant and $L(\cdot)$ is a log-normal density function. Since the simple approximation does not account for the prospect of early exercise and since the Black approximation does so only in an ad hoc fashion, the results should range from a strong relationship for the simple model to no relationship for the American call formula.

[17] When a riskless hedge can be formed between the call option and its underlying stock, the value of the American call on a stock with a known dividend is the sum of the discounted conditional expected value of the option's worth if exercised just prior to the ex-dividend instant, $\mathrm{e}^{-rt}[E(S_t \mid S_t > S_t^*) - X + \alpha D]\,\mathrm{prob}(S_t > S_t^*)$, and the discounted conditional expected value of the option's worth if exercised at expiration, $\mathrm{e}^{-rT}[E(S_T \mid S_T > X \text{ and } S_t \leqq S_t^*) - X]\ \mathrm{prob}(S_T > X \text{ and } S_t \leqq S_t^*)$. In the American call option formula (3), $N_1(b_2)$ represents $\mathrm{prob}(S_t > S_t^*)$ and $N_2(a_2, -b_2; -\sqrt{t/T})$ represents $\mathrm{prob}(S_T > X \text{ and } S_t \leqq S_t^*)$. For the present purpose of measuring the probability of early exercise, therefore, $N_1(b_2)$ was used.

Table 4

Average values of parameter estimates of the 160 weekly cross-sectional regressions testing for systematic relation between call option model relative prediction error and call option model determinants over the period January 17, 1975 through February 3, 1978.[a]

Test[b]	Model[c] for $\hat{C}$	$\bar{\hat{\alpha}}_0$	$t(\bar{\hat{\alpha}}_0)$	$p[t(\bar{\hat{\alpha}}_0)]$	$\rho(\hat{\alpha}_0)$	$\bar{\hat{\alpha}}_1$	$t(\bar{\hat{\alpha}}_1)$	$p[t(\bar{\hat{\alpha}}_1)]$	$\rho(\hat{\alpha}_1)$	$\bar{R}^2$
1. $C=\alpha_0+\alpha_1\hat{C}$	$C(S,T,X)$	−0.0508	−3.92[d]	0.00	0.51	1.0088	2.61[d]	0.01	0.28	0.98
	$\max[c(S,T,X),c(P,t,X)]$	−0.0425	−3.30	0.00	0.50	1.0092	2.74	0.01	0.28	0.98
	$c(S,T,X)$	−0.0300	−2.27	0.02	0.53	1.0091	2.70	0.01	0.29	0.98
2: $\dfrac{C-\hat{C}}{\hat{C}}=\alpha_0+\alpha_1\hat{\sigma}$	$C(S,T,X)$	0.1413	13.70	0.00	−0.09	−0.4326	−14.54	0.00	0.02	0.06
	$\max[c(S,T,X),c(P,t,X)]$	0.1541	14.78	0.00	−0.08	−0.4628	−15.32	0.00	0.04	0.07
	$c(S,T,X)$	0.1831	14.94	0.00	−0.03	−0.5467	−14.49	0.00	0.10	0.08
3: $\dfrac{C-\hat{C}}{\hat{C}}=\alpha_0+\alpha_1\left(\dfrac{S-X\mathrm{e}^{-rT}}{X\mathrm{e}^{-rT}}\right)$	$C(S,T,X)$	0.0103	2.24	0.03	0.12	−0.0780	−1.76	0.08	0.14	0.05
	$\max[c(S,T,X),c(P,t,X)]$	0.0144	3.11	0.00	0.12	−0.0802	−1.80	0.07	0.14	0.05
	$c(S,T,X)$	0.0209	4.31	0.00	0.15	−0.0818	−1.83	0.07	0.14	0.05
4: $\dfrac{C-\hat{C}}{\hat{C}}=\alpha_0+\alpha_1 p$	$C(S,T,X)$	0.0095	2.03	0.04	0.07	0.0117	1.19	0.23	0.07	0.01
	$\max[c(S,T,X),c(P,t,X)]$	0.0111	2.36	0.02	0.07	0.0347	3.54	0.00	0.09	0.02
	$c(S,T,X)$	0.0101	2.14	0.03	0.07	0.0963	7.03	0.00	0.13	0.03
5: $\dfrac{C-\hat{C}}{\hat{C}}=\alpha_0+\alpha_1 T$	$C(S,T,X)$	0.0199	1.82	0.07	−0.10	−0.0389	−1.19	0.24	−0.08	0.02
	$\max[c(S,T,X),c(P,t,X)]$	0.0308	2.68	0.01	−0.08	−0.0628	−1.77	0.08	−0.05	0.02
	$c(S,T,X)$	0.0611	3.43	0.00	0.00	−0.1475	−2.42	0.02	0.16	0.02
6: $\dfrac{C-\hat{C}}{\hat{C}}=\alpha_0+\alpha_1 d$	$C(S,T,X)$	0.0060	1.17	0.24	0.06	0.4612	0.96	0.34	0.09	0.02
	$\max[c(S,T,X),c(P,t,X)]$	0.0019	0.37	0.71	0.08	1.4309	2.91	0.00	0.11	0.02
	$c(S,T,X)$	−0.0067	−1.29	0.20	0.09	3.1658	5.73	0.00	0.17	0.02

[a]Test specification notation:

- $\hat{\sigma}$ = implied deviation of stock return.
- S = stock price net of the present value of the escrowed dividend,
- X = exercise price,
- r = annualized riskless rate of return,
- T = years to expiration,
- p = probability of early exercise,
- d = D/P = quarterly dividend divided by stock price.

Column heading notation:

- $\bar{\hat{\alpha}}_i$ = average value of parameter estimate,
- $t(\bar{\hat{\alpha}}_i)$ = Student t value testing the hypothesis that the average value of the parameter estimate, $\bar{\hat{\alpha}}_i$, is equal to zero,
- $\rho[t(\bar{\hat{\alpha}}_i)]$ = two-tailed probability that $t(\bar{\hat{\alpha}}_i)$ will be exceeded in absolute magnitude by a random variable following a Student t distribution,
- $\rho(\hat{\alpha}_i)$ = first-order serial correlation of the weekly parameter estimate $\hat{\alpha}_i$,
- $\bar{R}^2$ = average value of the coefficient of determination from the 160 regressions.

[b]For convenience the subscript j and the disturbance term μ_j are omitted from the test specifications.

[c]The models are $C(S,T,X)$, the value for an American call option on a stock with a known dividend; $\max[c(S,T,X), c(P,t,X)]$, the approximate call option value using the higher of the values of a European call, where the stock price net of the present value of the escrowed dividend is substituted for the stock price variable, and a European call, where the time to ex-dividend is substituted for the time to expiration variable; and $c(S,T,X)$, the approximate call option value using the value of a European call, where the stock price net of the present value of the escrowed dividend is substituted for the stock price variable.

[d]All t-ratios except those of Test 1 are computed on the basis of a null hypothesis that $\bar{\hat{\alpha}}_i = 0$. Those of Test 1 are computed on the basis of $\bar{\hat{\alpha}}_0 = 0$ and $\bar{\hat{\alpha}}_1 = 1$.

The results of Test 4 reflect the expected behavior. The relationship between the prediction error of the approximation $c(S,T,X)$ and the early exercise probability is strongly significant, with the t-value of the slope coefficient exceeding 7. The Black approximation apparently improves matters, but the relationship remains significant. The American call formula almost completely eliminates the association.

The fifth regression was included to test for the previously reported relationship between pricing error and the time to expiration of the option. The results show that while there exists a significantly negative relationship for the commonly applied approximation $c(S,T,X)$ (i.e., it underprices short-lived options and overprices long-lived options) it virtually disappears when the American call formula is used.

The final regression tested for systematic relationship between prediction error and the dividend yield on the stock.[18] Here there should be a significant relationship using the approximation methods, but none with the American call formula. The magnitude of the early exercise premium is importantly influenced by the amount of the dividend payment, and, since neither of the approximations account for the premium adequately, prediction error should be found.

The results of Test 6 are consistent with expectations. With the approximation methods the relationship between prediction error and dividend yield is strongly significant, and with the American call formula it is virtually non-existent.

As a precautionary measure, the prediction errors of the three models were regressed on various combinations of the independent variables in Tests 2 through 6. The aforementioned results seemed to be robust in that whenever standard deviation was included as an explanatory variable, it appeared in a significantly negative fashion, with its coefficient being of the same order of magnitude as that in Test 2. None of the remaining variables had a significant impact on the prediction error of $C(S,T,X)$, independent of what other variables were included in the regression.

Finally, it should be noted that Tests 1 through 6, along with the multiple regressions, were performed on each cross-section as well as on the pooled observations, although only the former results are reported. Chow (1960) tests of the joint hypothesis that $\alpha_{0\tau}=\alpha_0$ and $\alpha_{1\tau}=\alpha_1$, $\tau=1,\ldots,160$, were computed for each of the tests, and at the 0.0001 percent significance level all values of $F(318,15262)$ were in excess of the critical value.

In summary, of the models tested, the American call option pricing formula provides the best description of the observed structure of call option prices. For options whose prices exceeded $0.50 and whose remaining lives included exactly one ex-dividend date, the relative prediction error of $C(S,T,X)$ was markedly lower than those of the alternative models, and was not systematically related to:

[18]The dividend yield variable was computed by dividing the escrowed quarterly dividend by the stock price.

(1) the degree to which the option was in-the-money or out-of-the-money, (2) the probability of early exercise, (3) the option's remaining time to expiration, or (4) the stock's dividend yield. The American call formula did not, however, eliminate, although it did reduce, the association between prediction error and the standard deviation of stock return. Further research into this relationship is clearly warranted.

The simple approximation $c(S, T, X)$ had results which were, for the most part, consistent with previous findings. Its prediction errors were related to the stock's volatility, to the option's time to expiration, and to all but one of the other variables considered. The degree to which the options is in-the-money or out-of-the-money apparently does not significantly affect the model's prediction error, contrary to previous evidence.

The Black approximation yielded results very much similar to those of the simple approximation, although its mean and standard deviation of relative prediction error were lower. The early exercise premium is apparently an integral component in pricing the American call option.

7. Test of market efficiency

7.1. Procedure

The empirical evidence shows that the correspondence between observed and model prices is closest for $C(S, T, X)$. Nevertheless sufficient deviation could exist to permit trading profits. In an efficient options market, no costless arbitrage opportunities can exist. If a portfolio is formed by buying 'undervalued' and selling 'overvalued' options in proportions such that no wealth is used and no risk is assumed, its expected return is equal to zero.[19,20]

Underlying the weighting scheme employed to form the costless arbitrage position is the premise that asset prices are determined by security market relation[21]

$$E(\tilde{r}_i) = r + \beta_i[E(\tilde{r}_m) - r], \tag{11}$$

where $E(\tilde{r}_i)$ and $E(\tilde{r}_m)$ are the instantaneous expected returns on asset i and the market, respectively, r is the riskless rate of interest, and $\beta_i \equiv \operatorname{cov}(\tilde{r}_i, \tilde{r}_m)/\operatorname{var}(\tilde{r}_m)$ is

[19]The null hypothesis of a zero expected return on the costless arbitrage portfolio jointly tests the propositions that: (1) the American call option model is correctly specified, (2) the implied volatility of common stock return is an accurate reflection of the expected volatility, and (3) the options market is efficient.

[20]Costless arbitrage portfolio test methodology has been used in previous tests of capital asset pricing. See, for example, Black and Scholes (1974) and Watts (1978).

[21]The continuous time version of the capital asset pricing model was derived by Merton (1973b).

asset i's instantaneous relative systematic risk. For portfolios the relation is

$$E(\tilde{r}_p) = r + \beta_p[E(\tilde{r}_m) - r], \tag{12}$$

where $E(\tilde{r}_p) = \sum \omega_i E(\tilde{r}_i)$, $\beta_p = \sum \omega_i \beta_i$ and $\sum \omega_i = 1$, and ω_i denotes the proportional investment in security i.

The riskless hedge position consists of two portfolios: the first containing undervalued options (denoted by the subscript u), and the second containing overvalued options (o). Applying eq. (12), the expected returns on these portfolios are

$$E(\tilde{r}_u) = r + \beta_u[E(\tilde{r}_m) - r], \tag{13}$$

and

$$E(\tilde{r}_o) = r + \beta_o[E(\tilde{r}_m) - r], \tag{14}$$

respectively. If the undervalued securities are bought and the overvalued sold, the expected return on the hedge position (h) is

$$E(\tilde{r}_h) = E(\tilde{r}_u) - E(\tilde{r}_o) = (\beta_u - \beta_o)[E(\tilde{r}_m) - r]. \tag{15}$$

If the constraint $\beta_u = \beta_o$ is additionally imposed, the expected return of this zero-investment, zero-risk hedge portfolio becomes

$$E(\tilde{r}_h) = 0 \cdot [E(\tilde{r}_m) - r] = 0. \tag{16}$$

On first appearance the practical matter of creating a riskless arbitrage portfolio may seem as simple as investing equally in the options of the undervalued and the overvalued portfolios, and then hedging one portfolio against the other. Unfortunately this scheme does not ensure a perfect risk hedge, that is, $\beta_u = \beta_o$. To circumvent this problem a modest adjustment is made in the equal-weighted averaging procedure. First, the options in the undervalued (overvalued) portfolio are ranked in descending order of systematic risk. Two portfolios are then formed: the first by equal-weighted investment of one dollar in the high-risk options, and the second by equal-weighted investment of one dollar in the low-risk options. Where the number of options in the undervalued option portfolio is odd, the low-risk portfolio contains the extra option. Wealth is then allocated between the two portfolios such that

$$\omega_u^H \beta_u^H + \omega_u^L \beta_u^L = \beta^*, \tag{17a}$$

$$\omega_u^H + \omega_u^L = 1, \tag{17b}$$

$$\omega_u^H, \omega_u^L > 0, \tag{17c}$$

where ω_u^H is the proportion of wealth invested in the high-risk, undervalued option portfolio and ω_u^L is the proportion in the low-risk, undervalued option portfolio. β^* is an arbitrary value of β, as is to be described subsequently. Applying a similar procedure to the overvalued options,

$$\omega_o^H \beta_o^H + \omega_o^L \beta_o^L = \beta^*, \tag{18a}$$

$$\omega_o^H + \omega_o^L = 1, \tag{18b}$$

$$\omega_o^H, \omega_o^L > 0. \tag{18c}$$

Going long one dollar in the dollar in the undervalued option portfolio and short one dollar in the overvalued option portfolio, the net investment position is

$$(\omega_u^H + \omega_u^L) - (\omega_o^H + \omega_o^L) = 1 - 1 = 0, \tag{19}$$

and the net risk position is

$$(\omega_u^H \beta_u^H + \omega_u^L \beta_u^L) - (\omega_o^H \beta_o^H + \omega_o^L \beta_o^L) = \beta^* - \beta^* = 0, \tag{20}$$

exactly the desired costless arbitrage result.

The selection of an appropriate value of β^* is somewhat arbitrary, although it should be chosen such that the allocation among the undervalued and the overvalued options is as even as possible. To accomplish this result β^* is set equal to the arithmetic average of the systematic risk coefficients of the options contained in the sample in week t. In this way there is reasonable assurance that the distribution of wealth among securities is fairly even, and the constraints (17c) and (18c) are satisfied.

Implementing the concept of systematic risk coefficients for call options presents interrelated problems. Since the stock return generating process is stationary through time [Assumption (A.2)], the call option return process is non-stationary — the mean and the standard deviation decrease as the option approaches maturity. With respect to the test methodology, this non-stationarity poses two problems: (1) how to estimate the option's beta, and (2) how to compensate for the systematic risk change in the 'riskless' hedge position.

The estimation problem is resolved by employing an estimate of the stock beta and an estimate of the elasticity of the call price with respect to the stock price. Each week the most recent 100 weeks of historical stock returns ($R_{P\tau}$) are regressed on the historical observed returns of a market index ($R_{m\tau}$),[22] that is,

$$R_{P\tau} = \alpha_P + \beta_P R_{m\tau} + v_{P\tau}, \qquad \tau = t-99, \ldots, t. \tag{21}$$

[22]The log-linear investment relative form of the market model regression may have been more appropriate in this instance since the theoretical arguments were expressed in terms of instantaneous rates of return. Weekly returns, however, were used so as to provide a consistency in units with the weekly option portfolio rebalancing activity.

to estimate the stock's systematic risk coefficient. The estimate, $\hat{\beta}_P$, is then multiplied by the elasticity of call price with respect to stock price to obtain an estimate of the call option's systematic risk, that is,

$$\hat{\beta}_C = \eta \cdot \hat{\beta}_P,^{23} \tag{22}$$

where $\eta(\equiv(\partial C/\partial P)(P/C))$ denotes the elasticity.

The remaining concern about non-stationarity has to do with the discretely-adjusted option trading activity. Since an option's risk decreases deterministically as it approaches maturity, all other things remaining constant, the risks of the undervalued and the overvalued portfolios decrease over the investment interval Δt. However, if the overvalued options are shorted, the decrease in the risk of the undervalued option position should be approximately offset by the increase in the risk of the overvalued option position, as long as the composite characteristics of the two option portfolios are nearly the same. In other words, although non-stationarity is a problem when one considers only a long or only a short position in an option or a portfolio of options, hedging long options against short options should compensate for its effect.

7.2. Results

Descriptive summary statistics of the stock betas, elasticities and call option betas are reported in table 5. On average, the stock beta was close to one, with the

Table 5

Distribution of the 15,582 call option systematic risk estimates based upon the market model regressions of stock returns and the elasticities of the call option price with respect to stock price.

		Stock's systematic risk estimate $\hat{\beta}_P$	Elasticity of call price with respect to stock price η	Call option's systematic risk estimate $\hat{\beta}_C$
Mean		1.18	8.66	10.03
Standard deviation		0.35	4.06	5.00
Mean absolute deviation		0.27	3.04	3.85
Deciles	0.10	0.80	4.44	4.58
	0.20	0.91	5.40	5.90
	0.30	1.00	6.19	7.04
	0.40	1.10	6.99	8.10
	0.50	1.17	7.86	9.22
	0.60	1.27	8.77	10.47
	0.70	1.36	9.92	11.85
	0.80	1.45	11.39	13.74
	0.90	1.62	13.48	16.58

[23]By definition $\beta_C \equiv \mathrm{cov}(\tilde{r}_C, \tilde{r}_m)/\mathrm{var}(\tilde{r}_m)$. If the call option and stock instantaneous rates of return are perfectly correlated, that is, $\tilde{r}_C = \eta \tilde{r}_P$, $\beta_C = \mathrm{cov}(\eta \tilde{r}_P, \tilde{r}_m)/\mathrm{var}(\tilde{r}_m) = \eta \beta_P$.

distribution appearing slightly skewed to the left. The average elasticity was about 8.66, indicating that option prices are very sensitive to shifts in the stock price. The options were extremely risky, with their average beta exceeding 10.

The portfolio formation procedure involved creating two equal-weighted portfolios of undervalued and of overvalued options. The two undervalued option portfolios were weighted such that constraints (17a) and (17b) were satisfied, and the two overvalued option portfolios were weighted such that constraints (18a) and (18b) were satisfied. Portfolio rebalancing occurred weekly so that 160 sets of weights were created. A summary of the portfolio compositions using the American call option valuation equation to identify undervalued and overvalued options is included in table 6.[24] The technique worked well, with

Table 6

Distributions of the allocations among the high-risk and low-risk undervalued and high-risk and low-risk overvalued option portfolios used in forming the costless arbitrage position in each of the 160 weeks during the period January 17, 1975 through February 3, 1978.

		Undervalued			Overvalued		
	Portfolio[a] risk β^*	Number of options n_u	High-[b] risk portfolio weight ω_u^H	Low-[b] risk portfolio weight ω_u^L	Number of options n_o	High-[b] risk portfolio weight ω_o^H	Low-[b] risk portfolio weight ω_o^L
Mean	9.9939	48.78	0.6192	0.3808	48.61	0.3198	0.6802
Standard deviation	2.4274	18.27	0.0959	0.0959	19.14	0.1478	0.1478
Mean absolute deviation	2.0261	14.51	0.0745	0.0745	15.47	0.1148	0.1148
Deciles 0.10	6.5801	28.00	0.4993	0.2572	24.00	0.1384	0.5123
0.20	7.7246	35.00	0.5467	0.3008	32.00	0.2176	0.5536
0.30	8.4664	38.00	0.5748	0.3363	38.00	0.2619	0.5958
0.40	9.2773	41.00	0.6030	0.3598	42.00	0.2995	0.6304
0.50	9.9085	47.00	0.6247	0.3761	46.00	0.3280	0.6730
0.60	10.8531	52.00	0.6403	0.3985	52.00	0.3749	0.7010
0.70	11.6174	56.00	0.6669	0.4270	57.00	0.4115	0.7459
0.80	12.2248	63.00	0.7037	0.4553	66.00	0.4483	0.7860
0.90	13.1151	75.00	0.7459	0.5021	75.00	0.5003	0.8621

[a]The value of β^* is the arithmetic average of the systematic risk coefficients of all options contained in the sample in each week.

[b]To form the costless arbitrage position each week, four portfolios are initially created by equal-weighted investments in high-risk and low-risk undervalued and high-risk and low-risk overvalued options. The four weights listed in these columns are the weights applied to the equal-weighted portfolios in order to match the systematic risk characteristics of the undervalued and overvalued option positions at β^*. By going long one dollar in the undervalued option position and short one dollar in the overvalued option position, a costless (i.e., $\$1-1=0$), riskless (i.e., $\beta^*-\beta^*=0$) arbitrage portfolio is formed.

[24]The 'picking of outliers' using the American call formula may systematically identify bid prices as undervalued options and ask prices as overvalued options, and, hence, the rate of return on the hedge portfolio may be slightly overstated. For an explanation of this selection bias, see Phillips and Smith (1980, pp. 186–187).

more than 80 percent of the allocations falling within the 0.2–0.8 range and with the constraints (17c) and (18c) satisfied in all of the cross-sections. Not surprisingly, the significantly negative relationship between prediction error and standard deviation of stock return influenced the structure of the costless arbitrage portfolio. The high-risk undervalued position and the low-risk overvalued portfolio were, in general, weighted more heavily in matching the systematic risks of the undervalued and the overvalued positions.

With the weekly portfolio compositions computed, generating the hedge portfolio return series became a matter of weighting and averaging the weekly option returns. To test whether the hedge portfolio had zero systematic risk during the sample period, the hedge returns were regressed on the market returns ($\tau = 1, \ldots, 160$). The OLS regression results were

$$\underset{(4.59)}{R_{h\tau} = 0.0258} - \underset{(-1.43)}{0.4469 R_{m\tau}} + e_{h\tau}, \qquad R^2 = 0.0128, \quad \hat{\rho} = -0.007,$$

$$s(R_h)/s(R_m) = 2.95,$$

where the values in parentheses are t-ratios, $\hat{\rho}$ is the estimated first-order serial correlation in the disturbance term and $s(\cdot)$ denotes the estimated standard deviation.[25] Although the absolute magnitude of the slope coefficient appears large, its size relative to the average option risk level reported in table 6, 9.9939, is small, and at the 15 percent significance level the null hypothesis that the portfolio has zero systematic risk cannot be rejected.

The null hypothesis of a zero expected return on the costless arbitrage portfolio was tested under two distributional assumptions. First, assuming the hedge returns were drawn from a normal distribution, a Student t test,

$$t = \sqrt{n}\,\bar{x}/s,$$

was performed. The mean ($\bar{x}$) and the standard deviation (s) of the 160 sample returns were 0.0246 and 0.0707, respectively, so that the Student t ratio was 4.41. If expected returns were, on average, realized during the sample period, the null hypothesis is rejected at the 0.0019 percent significance level.

If the hedging of the undervalued option portfolio against the overvalued portfolio did not adequately control for the nonstationarity of the option return process, the hedge portfolio's return distribution might be asymmetric, with the direction of the asymmetry contingent upon whether the deterministic decrease in the value of the undervalued options outweighed the deterministic increase in

[25]Using the CRSP equal-weighted market index the regression results were:

$$\underset{(4.75)}{R_{h\tau} = 0.0280} - \underset{(-1.70)}{0.4851 R_{m\tau}} + e_{h\tau}, \quad \text{with a coefficient of determination of 0.0180.}$$

the value of the overvalued options or vice versa. Assuming that the hedge returns were drawn from a non-normal distribution, a Johnson (1978) modified t test,

$$t_{\mathrm{JOHN}} = \sqrt{n}\,[\bar{x} + \hat{\mu}_3/6s^2n + \hat{\mu}_3\bar{x}^2/3s^4]/s,$$

was performed, where $\hat{\mu}_3$ denotes the third central sample moment. The Johnson t ratio was 4.08, and, at the 0.0115 percent significance level, the null hypothesis was rejected. The difference between the Student t and the Johnson t ratios, however, indicates that the hedge return distribution was slightly negatively skewed.

With the null hypothesis of a zero hedge return rejected, the question arose whether investors could realize positive trading profits after transaction costs by enacting the costless arbitrage trading strategy. To answer this question, the effect of a proportional transaction cost rate, F, was simulated. At the beginning of each week each dollar of after-transaction-cost proceeds from the overvalued option portfolio sold short, \$1$(1-F)$, was assumed to be invested in the undervalued option portfolio, \$1$(1-F)/(1+F)$. At the end of the week, the long position was closed yielding proceeds \$$(1-F)(1-F)(1+r_u)/(1+F)$, and the short position was covered costing \$1$(1+F)(1+r_o)$. The after-transaction-cost return on the hedge portfolio (r_h^t) was, therefore,

$$r_h^t = (1-F)(1-F)(1+r_u)/(1+F) - (1+F)(1+r_o).$$

The mean after-transaction-cost hedge return was assumed to be equal to zero, the mean realized returns of the undervalued and the overvalued option portfolios, 1.2515 percent and -1.2137 percent, were substituted for r_u and r_o, respectively, and the value of F^* was computed.[26] A proportional transaction cost rate of 0.616 percent was sufficient to eliminate all of the trading profits that could be realized by implementing the costless arbitrage activity.

In Phillips and Smith (1980, p. 184) the average bid–ask spread for CBOE call options priced at more than \$0.50 was reported as 4.5 percent of the average of the bid and ask prices. Assuming that the option prices used in the present test are halfway between the bid and ask prices,[27] the value of F^*, 0.616 percent, can be thought of as one-half of the bid–ask spread as a percentage of price, and can be compared with one-half of the value of the Phillips and Smith estimate, 2.25 percent. In sum, option market efficiency is soundly supported. The profits from

[26]The solution for the maximum transaction cost rate was computed using the formula, $F^* = (1+k-2\sqrt{k})/(1-k)$, where $k = (1+\bar{R}_o)/(1+\bar{R}_u)$ and $\bar{R}_u$ and $\bar{R}_o$ are the realized returns on the undervalued and overvalued portfolios. The remaining root of the quadratic equation did not yield a solution for F^* between 0 and 1.

[27]Again, it should be reminded that there may be a selection bias in the beginning of week prices since the model may have systematically picked out bid and ask prices as undervalued and overvalued options, respectively. See footnote 23.

buying undervalued options and selling overvalued options are insufficient to cover the market makers spread, let alone the remaining trading costs such as commissions and transfer fees.

8. Summary and conclusions

This study examines the pricing performance of three methods for valuing American call options on dividend-paying stocks. The methods include: (1) the simple approximation obtained by substituting the stock price net of the present value of the escrowed dividend into the Black–Scholes formula, (2) the Black approximation obtained by taking the higher of the Black–Scholes formula value using stock price net of the present value of the escrowed dividend and the Black–Scholes formula value using the time to ex-dividend as the time to expiration variable, and (3) the correctly specified equation for the American call option on a stock with a known dividend. Previous empirical investigations have focused on the simple approximation method, and have found several disturbing relationships between the degree of under- and over-pricing and the determinants of call value, for example, the standard deviation of stock return, the degree to which the option is in-the-money or out-of-the-money, and the option's time to expiration. This study shows that these correlations are, for the most part, spurious in nature, induced by the approximation's failure to account for the American call option holder's early exercise privilege.

The American call formula is shown to alleviate all of the dependencies except the relationship between prediction error and the standard deviation of stock return. Even when the American call formula is used, there remains a tendency of the model to underprice options on low-risk stocks and to overprice options on high-risk stocks. This phenomenon may be attributable to several sources: (1) non-stationarity of stock return standard deviation parameter, (2) the assumption of a known dividend, and (3) the assumption of zero tax rate differential between dividend and capital gain income.

The American call formula also better describes the observed structure of call option prices than either of the approximations. Both the mean and the standard deviation of the relative prediction errors are lower. Since the cost of implementing the correct pricing equation is only slightly higher than the approximations, it should be used for pricing American calls and for computing implied standard deviations. If an approximation is desired, however, the results of this study indicate that the Black approximation is more accurate than the simpler method.

Previous studies have also documented that weighted implied standard deviations are better predictors of future return volatility than historical estimates. In this study standard deviations based upon minimizing the sum of squared deviations of the observed call prices from the model's prices are used. In a preliminary investigation the minimum sum of squares estimates were compared with three weighted implied standard deviation measures, where the weights were: (1) equal, (2) the partial derivative of the model price with respect to stock return standard deviation, and (3) the elasticity of model price with respect to stock return standard deviation. The test results, not reported here, indicate that the minimum sum of squares method yields more accurate estimates of subsequent realized volatility than the weighted average methods.

The volatility estimation methodology employed here also differs from previous studies in two other respects. First, contemporaneous volatility estimation and option valuation is avoided. The standard deviation parameter used in option valuation process is obtained by computing the implied standard deviation from the option prices in the previous week. In this way, the presumption that the model prices and, yet, does not price options correctly at a given instant is unnecessary, and single options written on a stock in a given week are retained in the sample. Second, an implied standard deviation is computed for each option maturity. Aggregating all options written on a stock presupposes that the market's assessment of a stock's volatility is independent of the length of time into the future for which it is estimated. Available empirical evidence suggests otherwise.

This study also investigates Chicago Board Options Exchange efficiency using the American call option formula and a costless arbitrage hedging strategy. At the beginning of each week during the sample period January 17, 1975 through February 3, 1978, underpriced and overpriced options are hedged against one another in proportions such that a zero-risk, zero-investment option portfolio is formed. At the end of each week the position is closed, and the gain or loss is realized. The average weekly return using this trading strategy is 2.63 percent, and the null hypothesis of a zero expected return on the hedge portfolio is rejected. A proportional transaction cost rate of less than 1 percent, however, is sufficient to eliminate the trading profits, and the Chicago Board Options Exchange must be deemed to be an efficient market.

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