The Investor Fear Gauge

Explication of the CBOE VIX.

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The Chicago Board Options Exchange's market volatility index (the VIX) is called the "investor fear gauge." The name fits. The index is set by *investors* and expresses their consensus view about *expected future stock market volatility.* The higher the VIX, the greater the fear.

To see why, we need to know how the VIX is constructed. To see how VIX works, we need to examine its history and its relation to stock market movements.

CONSTRUCTION OF THE VIX

VIX is an *implied* volatility in the parlance of the securities industry. It is similar in spirit to a bond's yield to maturity. To compute a bond's yield to maturity, we search numerically for the discount rate that equates the present value of the bond's promised coupon payments and principal repayment to the bond's current price. What permits the computation is the fact that, although the bond valuation model includes a number of terms (or parameters), only one is unknown: yield to maturity. The amount and the timing of the coupon payments and principal repayment are known.

Equating the market price of a bond to its model value and solving for yield, therefore, means that the computed yield to maturity is an *implied* (by the bond price) yield to maturity. It is the market's "best" assessment of the *expected* rate of return over the remaining life of the bond.

Like a bond, a stock index option has a valuation

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model. And, like the bond valuation model, the stock index option valuation model has a number of parameters, all but one of them known or able to be estimated with a high degree of accuracy. The unknown parameter is the index's expected future volatility.

By equating the market price of an index option to its model value and solving for volatility, we identify the *implied* (by the option price) volatility. This implied volatility is the market's "best" assessment of the *expected* volatility of the underlying stock index over the remaining life of the option. Since VIX is based on S&P 100 index (or OEX) option prices, VIX represents a market consensus view of the expected volatility of the S&P 100 index.

To be more precise, VIX is computed on a minute-by-minute basis from the implied volatilities of the eight near-the-money, nearby, and second-nearby OEX option series. These implied volatilities are then weighted in such a manner that the VIX represents the implied volatility of a thirty-calendar day (twenty-two-trading day) at-the-money OEX option.

Computing OEX Implied Volatilities

Computing an implied volatility requires three things:

- An option valuation model.
- Values of the model parameters (except for volatility).
- An observed option price.

The option valuation model underlying the computation of VIX is based on the Nobel Prize-winning work of Black and Scholes [1973] and Merton [1973]. A cash dividend-adjusted binomial method is used to account for the facts that OEX index options are American-style and that the underlying index portfolio pays discrete cash dividends.

Aside from volatility, the option valuation model parameters are the current index level, the option's exercise price and time expiration, the risk-free rate of interest, and the amount and timing of the anticipated cash dividends paid during the option's life. The anticipated daily cash dividends of the S&P 100 index portfolio must be forecast. These forecasts are generally very accurate, considering that the VIX has a thirty-calendar day horizon and firms declare cash dividend payments well before they are paid. The risk-free interest rate is the continuous yield on a T-bill whose maturity most closely matches the option expiration, except when the option time to expiration is shorter than thirty days, in which case the T-bill with thirty days to maturity is used. The exercise price and time to expiration are terms of the option contract. The OEX index level is reported continuously throughout the trading day.

The option prices used to compute implied volatilities are midpoints of the bid and ask prices quoted at the time the VIX is computed. Midmarket quotes are used for two reasons. First, bid-ask price quotes reflect current market conditions. Second, using the midpoint of the bid-ask price quotes eliminates the bouncing between bid-ask price levels (and, hence, implied volatilities) observed in the sequence of trade prices.

Finally, it should be noted that the VIX is based on trading days. If the time to expiration of the option is measured in calendar days, the implied volatility is a volatility rate per calendar day. This means, among other things, that the return variance of the OEX index over the weekend (from Friday close to Monday close) should be three times higher than it is over any other pair of adjacent trading days during the week (say, Monday close to Tuesday close). Empirically, this is not true. Stock return volatility over the weekend is approximately the same as it is for other trading days. Consequently, each (calendar-day) implied volatility rate is transformed to a trading-day basis. If the number of calendar days to expiration is N_c, the number of trading days, N_t, is computed as

$$N_t = N_c - 2 \times \text{int}(N_c/7)$$

An option with eight calendar days to expiration, for example, has six trading days to expiration. The implied volatility rate is multiplied by the ratio of the square root of the number of calendar days to the square root of number of trading days, that is:

$$\sigma_t = \sigma_c\left(\frac{\sqrt{N_c}}{\sqrt{N_t}}\right)$$

where $\sigma_t(\sigma_c)$ is the trading-day (calendar-day) implied volatility rate.[1]

Weighting the Implied Volatilities

The CBOE market volatility index is constructed from the implied volatilities of the eight near-the-money, nearby, and second-nearby OEX option series. The nearby OEX series is defined as the series with the shortest time to expiration but with at least eight calendar days to expiration. The second-nearby OEX series is the series of the next adjacent contract month.

To explain index construction, we first need to identify the eight OEX option series that underlie the VIX computation at any particular time. Denote the exercise price just below the current index level, S, as X_l (i.e., the lower exercise price), and the exercise price just above the current index level as X_u (i.e., the upper exercise price). The implied volatilities of the eight options (four calls and four puts) used to compute the VIX are therefore defined as in Exhibit 1.

Now, the averaging process begins. The first step involves averaging the call and put option-implied volatilities in each of the four categories of options, that is:

$$\sigma_1^{X_l} = (\sigma_{c,1}^{X_l} + \sigma_{p,1}^{X_l})/2$$

$$\sigma_1^{X_u} = (\sigma_{c,1}^{X_u} + \sigma_{p,1}^{X_u})/2$$

$$\sigma_2^{X_l} = (\sigma_{c,2}^{X_l} + \sigma_{p,2}^{X_l})/2$$

and

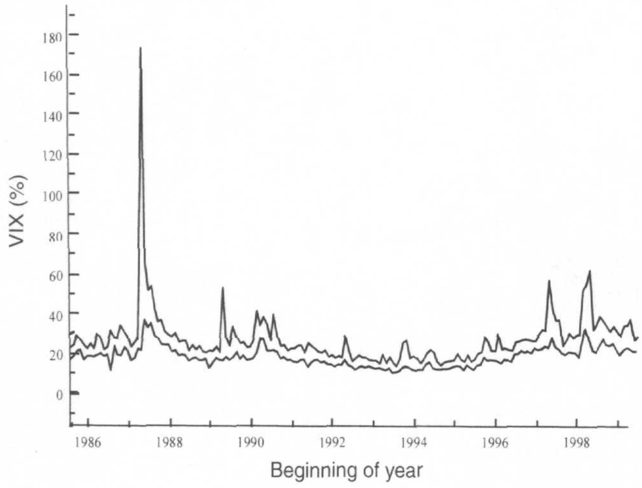
$$\sigma_2^{X_u} = (\sigma_{c,2}^{X_u} + \sigma_{p,2}^{X_u})/2$$

Four volatilities now remain. The next step is to interpolate between the nearby implied volatilities and the second-nearby implied volatilities to get an at-the-money implied volatility for each maturity. The at-the-money nearby and second-nearby average volatilities (σ_1 and σ_2, respectively) are

EXHIBIT 1
IMPLIED VOLATILITIES OF EIGHT OPTIONS

Exercise Price	Nearby Contract (1) Call	Nearby Contract (1) Put	Second-Nearby Contract (2) Call	Second-Nearby Contract (2) Put
X_l (< S)	$\sigma_{c,1}^{X_l}$	$\sigma_{p,1}^{X_l}$	$\sigma_{c,2}^{X_l}$	$\sigma_{p,2}^{X_l}$
X_u (≥ S)	$\sigma_{c,1}^{X_u}$	$\sigma_{p,1}^{X_u}$	$\sigma_{c,2}^{X_u}$	$\sigma_{p,2}^{X_u}$

EXHIBIT 2
HIGH AND LOW OF CBOE MARKET VOLATILITY INDEX (VIX)



$$\sigma_1 = \sigma_1^{X_l}\left(\frac{X_u - S}{X_u - X_l}\right) + \sigma_1^{X_u}\left(\frac{S - X_l}{X_u - X_l}\right)$$

and

$$\sigma_2 = \sigma_2^{X_l}\left(\frac{X_u - S}{X_u - X_l}\right) + \sigma_2^{X_u}\left(\frac{S - X_l}{X_u - X_l}\right)$$

The final step is to interpolate between (or, occasionally, extrapolate from) the nearby and second-nearby implied volatilities to create a thirty-calendar day (or $N_t = 30 - 2 \times \text{int}(30/7) = 22$-trading day) implied volatility. If N_{t_1} is the number of trading days to expiration of the nearby contract, and N_{t_2} is the number of trading days of the second-nearby contract, the CBOE market volatility index is

$$VIX = \sigma_1\left(\frac{N_{t_2} - 22}{N_{t_2} - N_{t_1}}\right) + \sigma_2\left(\frac{22 - N_{t_1}}{N_{t_2} - N_{t_1}}\right)$$

HISTORY OF THE VIX

The CBOE computes the VIX on a real-time basis throughout each trading day, from 9:00 a.m. until 3:00 p.m. CST. It has done so since January 1993. The history of the VIX, however, extends back to January 1986.[2]

Exhibit 2 shows the monthly high and low values of the VIX from January 1986 though December 1999. Perhaps the most interesting phenomenon shown in the graph is that the monthly high level of VIX has had periodic spikes. At the time of the market crash in October 1987, for example, VIX reached its recorded high, 172.79%. The jump in October 1989 is the "mini-crash" resulting from the UAL restructuring failure. The jump in mid-1990 occurred when Iraq invaded Kuwait, and the jump in early 1991 corresponds to the attack on Iraq by United Nations forces. From then forward, the monthly high levels for the VIX remained below 30%.

Then two sharp spikes occurred — one in October 1997, and one in October 1998. The October 1997 spike (55.48%) occurred following a stock market sell-off in which the DJIA fell 555 points. The October 1998 spike (66.63%) occurred in a period of general nervousness in the stock market. The VIX then returned to more normal levels in 1999.

Normal Range

The normal ranges of VIX are reported in Exhibit 3 and illustrated in Exhibit 4. Over its entire history, the median daily closing level of VIX is 18.77%. 50% of the time VIX closed between 15.36% and 23.27% (a range of 7.91 percentage points), and 90% of the time VIX closed between 11.70% and 31.46% (a range of 19.76 percentage points). Since the CBOE began reporting the VIX in 1993, the 50% and 90% ranges are 12.97 to 23.04 percentage points and 11.22 to 30.28 percentage points, respectively.

Exhibits 3 and 4 also show that there is a great deal of variation in what is considered normal from year to year. In 1986, for example, the median daily closing level of the VIX was about 19.79%. During that same year, the closing VIX levels were between 18.58% and 21.66% (a 308-basis point range) about half the time and between 17.40% and 24.92% (a 752-basis point range) about 90% of the time. The widest range occurred during 1987 — the year of the market crash. The 5% and 95% percentiles indicate that the range of daily VIX levels was from 17.19% to 55.64%, or 3,845 basis points.

Following the crash, both the median level and the range of VIX fell in general (except for 1990). Between 1992 and 1996, the VIX level was low not only by historical standards but also in terms of its trading range. In

EXHIBIT 3
MEDIAN AND PERCENTILE RANGES FOR DAILY CLOSING VIX LEVELS

				Percentile			Normal Range	
Period		5%	25%	50%	75%	95%	50%	90%
1986–1999	3,535	11.70	15.36	18.77	23.27	31.46	7.91	19.76
1993–1999	1,765	11.22	12.97	17.30	23.04	30.28	10.07	19.06
1986	253	17.40	18.58	19.79	21.66	24.92	3.08	7.52
1987	253	17.19	21.46	23.33	27.59	55.64	6.13	38.45
1988	253	17.97	20.97	24.75	28.01	37.19	7.04	19.22
1989	252	15.99	16.94	17.79	18.73	23.22	1.79	7.23
1990	253	17.03	18.84	21.82	26.87	31.35	8.03	14.32
1991	252	15.43	16.49	17.77	19.67	25.14	3.18	9.71
1992	254	12.58	13.73	15.17	16.43	18.46	2.70	5.88
1993	251	10.74	11.72	12.61	13.40	14.78	1.68	4.04
1994	252	10.55	11.63	13.17	14.89	16.53	3.26	5.98
1995	252	11.05	11.85	12.63	13.55	14.55	1.70	3.50
1996	253	13.85	16.16	17.25	18.66	20.87	2.50	7.02
1997	253	20.49	21.72	22.87	25.34	31.21	3.62	10.72
1998	251	18.57	21.03	23.24	28.45	42.74	7.42	24.17
1999	253	20.28	23.02	24.95	27.34	31.19	4.32	10.91

1998, VIX experienced its second-widest range ever, with 90% of the VIX levels between 18.57% and 42.74%. Using 1999 levels as a guide, the expected normal range for VIX during the year 2000 is between 20% and 31%.

Relation of Stock Market

The fact that the VIX spikes during periods of market turmoil is why it has become known as the "investor fear gauge." Naturally, such fears are usually reflected in stock prices also. This stands to reason. If expected market volatility increases, investors demand higher rates of return on stocks, so stock prices fall. The relation is not perfect, however.

Exhibit 5 shows the weekly closing levels of the VIX and the S&P 100 index back to January 1995. As the graph shows, some spikes in the VIX are coincident

EXHIBIT 4
MEDIAN AND 25-75 AND 5-95 PERCENTILE RANGES FOR DAILY CLOSING VIX LEVELS

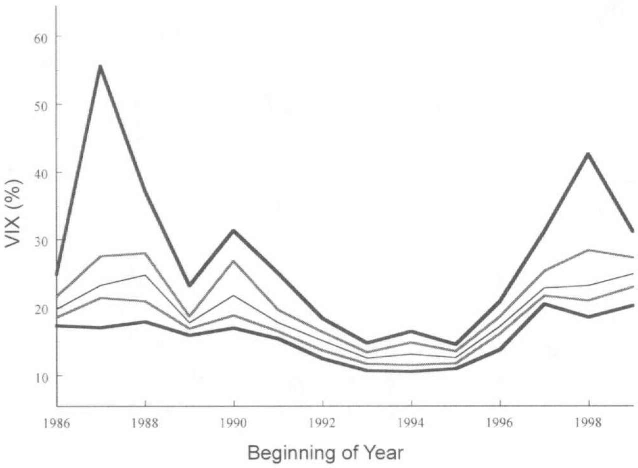
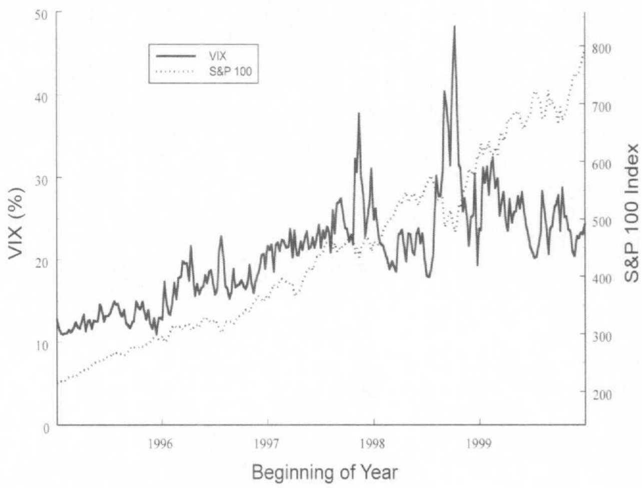


EXHIBIT 5
WEEKLY LEVELS OF VIX AND S&P 100 INDEX



with spikes in the opposite direction for the S&P 100 index. In October 1998, for example, VIX spiked upward, and the S&P 100 spiked downward. Similar effects are seen in months such as July 1996, October 1997, and July 1999.

At other times, however, there can be a run-up in stock prices as well as volatility. In January 1999, the VIX was rising (i.e., investors were becoming more nervous) while the level of the S&P 100 index was rising. At even other times, there can be a run-up in stock prices with little movement in volatility. See, for example, the first two months of 1995, June and July of 1997, and December 1999.

To assess more precisely the relation between the returns of the S&P 100 index and changes in the VIX, we use regression analysis. More specifically, using the weekly data shown in Exhibit 5, we regress the Wednesday-to-Wednesday return of the S&P 100 index (denoted R_t) on the weekly change in the VIX (denoted ΔVIX_t) and the weekly change in the VIX when the change is positive (i.e., ΔVIX_t^+ where $\Delta VIX_t^+ = \Delta VIX_t$ if $\Delta VIX_t > 0$ and $\Delta VIX_t^+ = 0$, otherwise).[3]

The regression results are:

$$\text{Predicted } R_t = 0.775 - 0.469\Delta VIXt - 0.238\Delta VIX_t^+$$

where the number of observations is 260 and the regression R^2 is 58.6%. All regression coefficients are significantly different from zero at less than the 1% level. The estimated intercept of the regression is 0.775%. This means that if the VIX does not change over the week, the S&P 100 index is expected to rise by 0.775%. This is reasonable, considering that the S&P 100 rose by in excess of 270% over the five-year sample period used to fit the regression, as is shown in Exhibit 5.

If we want to understand the relation between changes in the VIX and stock market returns, however, the two slope coefficients rather than the intercept term tell the story. What they say is that if the VIX falls by 100 basis points, the S&P 100 index will rise by

$$\text{Incremental } R_t = -0.469(-1.00) = 0.469\%$$

while, if the VIX rises by 100 basis points, the S&P 100 index will fall by

$$\begin{aligned}\text{Incremental } R_t &= -0.469(-1.00) - 0.238(1.00) \\ &= -0.707\%\end{aligned}$$

Interestingly, the relation between stock market returns and changes in the VIX is asymmetric. The stock market reacts more negatively to an increase in the VIX than it reacts positively when the VIX falls. Put differently, VIX is more a barometer of investors' fear of the downside than it is a barometer of investors' excitement (or greed) in a market rally.[4]

SUMMARY

VIX is said to be the "investor fear gauge." "Gauge" simply means a measure. We say "fear" because investors are averse to risk. Since the VIX is constructed from the implied volatilities of S&P 100 index options, it is, by definition, a measure of expected stock market risk. We say "investor" because investors set the level of the VIX, albeit indirectly. Investor demands for S&P 100 call and put options set prices, and these prices, in turn, are used to imply the level of the VIX.

Over its fourteen-year history, VIX has acted reliably as a fear gauge. High levels of VIX are coincident with high degrees of market turmoil, whether the turmoil is attributable to stock market decline, the threat of war, unexpected change in interest rates, or any number of other newsworthy events. The higher the VIX, the greater the fear.

ENDNOTES

[1]For the logic underlying this transformation as well as other details concerning the VIX construction, see Whaley [1993, endnote 23].

[2]Details on construction of the backdated VIX are provided in Whaley [1993, appendix].

[3]We use Wednesday-to-Wednesday returns because there are fewer holidays on Wednesdays than any other day of the week.

[4]One possible explanation for this result is that in times of market turmoil, investors buy S&P 100 index put options as portfolio insurance. The excess demand for index puts drives prices (and hence the VIX) upward. The converse is not true, however.

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