Dividends and S&P 100 Index Option Valuation

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INTRODUCTION

It is commonplace in research and practice to see S&P 100 index options valued using European-style formulas or American-style approximation methods that assume the index pays dividends at a constant proportional rate. Day and Lewis (1988, in press), Franks and Schwartz (1988), and Schwert (1990), for example, use a dividend-adjusted, European-style approximation to estimate market volatilities from S&P 100 index option prices. Sheikh (1989) uses the European-style model in transaction tests of S&P 100 call options. Stein (1989), on the other hand, relies on an American-style binomial method that assumes a constant proportional dividend yield on the index.

This study shows that both of these ad hoc valuation procedures can produce large pricing errors. The pricing errors arise largely from the distinctly discrete and seasonal pattern of dividends on the S&P 100 index portfolio. By properly accounting for these dividends, it is shown that the right to early exercise has significant value for both the call and put option holders. Moreover, the frequency with which S&P 100 call and put options are exercised early attests to the importance of this conclusion.

S&P 100 INDEX DIVIDEND SERIES

A key to valuing S&P 100 index options correctly is knowing the amount and the timing of the dividends that will be paid on the underlying index stocks during the options' life. Until recently, the cash dividend series of the S&P 100 index was not published.[1] Since the purpose of this study is to demonstrate the important effect

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[1]Beginning with the month of June 1988, Standard and Poor's reports the daily cash dividends of the S&P 100 index. However, prior to June 1988, no published series is available. The S&P 100 cash dividend series created here for the pre-June 1988 period is available on diskette from the authors.

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that dividends have on the valuation of S&P 100 index options and how the results of past studies may have been affected by unrealistic assumptions regarding the nature of S&P 100 index dividends, an S&P 100 dividend series is constructed from the cash dividends, ex-dividend dates, and number of shares outstanding of each of the component stocks. The construction of the cash dividend series for the S&P 100 index is no small task considering that the index consists of 100 stocks, is market value-weighted, and has changed in composition many times during its relatively short history.

Data Requirements

The construction of a daily dividend series for the S&P 100 requires knowledge of the stocks included in the index, the number of shares outstanding for each firm in each day, and the dividends paid by each firm. The daily value of the divisor used to compute the S&P 100 index is required also. The divisor is the mechanism that accounts for stock splits, stock dividends, and changes in the index composition.

The stocks included in the index are drawn from the *S&P 100 Information Bulletin,* which is published by Standard and Poor's. The bulletin is published monthly and includes a list of the stocks included in the index on the last trading day of the month. The bulletin also notes any changes in the composition from the previous month, including the date when Standard and Poor's made a change.

Data on the number of shares outstanding on the last trading day of the month is published in the bulletin. The number of shares outstanding for the component stocks is also available from the Center for Research in Security Prices (CRSP) daily master file. Where there are discrepancies between the CRSP figures and data in the *S&P 100 Information Bulletin,* the S&P data are used.

The number of shares for each stock changes in three situations. Once a quarter, Standard and Poor's does an update. This update reflects changes that may have occurred in the number of shares outstanding for the stocks. A change occurs when a firm issues new shares or buys back existing shares. These changes are reported in the *Bulletin.* Finally, changes occur if a stock splits or does a reverse split. S&P does not report the split. However, the split ratio can be inferred from the end-of-month number of shares outstanding. The date of the split is drawn from the CRSP master files.

Changes in the composition of the S&P 100 index are made only once a week—on Wednesday evening. If a stock drops out of the index on Thursday, for example, it is carried in the index for four trading days at its last close. On Wednesday evening, a new stock is included in the index and the divisor is adjusted so this switch of stocks does not affect the level of the index.

The S&P 100 index options started trading on March 11, 1983. The dividend information on the S&P 100 stocks for the period 1983 through 1987 is drawn from CRSP. The 1988 dividends are from *Moody's Annual Dividend Record.* The stock prices for the period 1983 through 1987 are drawn from CRSP. Stock prices for 1988 are from Trade-line. The daily time-series of the S&P 100 index divisor is obtained from the *S&P 100 Bulletin.*

Construction

Given these data, a daily series of cash dividends for the index is constructed. For each day, t, the total dividend figure for the S&P 100 stocks is calculated by multiplying the dividend of stock i by the number of shares outstanding of stock i and summing across all 100 stocks. The total dividend of the S&P 100 index portfolio is then deflated by the S&P 100 divisor, that is,

$$D_{\text{S\&P}\,100,t} = \frac{\sum_{i=1}^{100} \text{number of share outstanding}_{i,t} \times D_{i,t}}{\text{S\&P 100 index divisor}_{i,t}} \tag{1}$$

where D denotes the cash dividend.

To assess the accuracy of the computations, two checks are made. First, the S&P 100 index level is constructed from individual stock prices and compared to make sure that it matches the published index level. Second, a dividend series is created through June, 1988. S&P began to calculate and report the daily cash dividends in that month. The average difference between the S&P 100 index dividend series and that reported in the *S&P 100 Information Bulletin* in June 1988 is $.0001.

Time-Series Behavior

It is well known that firms pay dividends in quarterly cycles and that firms generally use a December 31 fiscal year-end. For this reason, monthly seasonality in index dividend payments is expected. Table I shows that the most popular ex-dividend months are February, May, August, and November. The average daily payouts are

Table I
SUMMARY OF DAILY CASH DIVIDENDS PAID ON THE S&P 100 INDEX PORTFOLIO DURING THE 1721 TRADING-DAY PERIOD FROM S&P 100 INDEX OPTION MARKET INCEPTION ON MARCH 11, 1983 TRHOUGH THE END OF DECEMBER 1989

Day/Month	Average Daily Dividend ($)	Maximum Dividend ($)	Maximum Dividend Date	Total Days	Number of Zero-Dividend Days
Monday	0.0494	0.3925	Sept. 26, 1983	328	58
Tuesday	0.0331	0.3529	Feb. 2, 1986	352	111
Wednesday	0.0176	0.4322	Nov. 5, 1986	352	193
Thursday	0.0279	0.5513	Nov. 5, 1987	345	185
Friday	0.0358	0.4683	Dec. 23, 1983	344	152
January	0.0123	0.1272	Jan. 25, 1988	127	68
February	0.0609	0.4153	Feb. 5, 1988	116	37
March	0.0245	0.3448	Mar. 25, 1988	145	38
April	0.0120	0.1534	Apr. 26, 1988	144	81
May	0.0570	0.4610	May 7, 1987	149	38
June	0.0288	0.3840	June 24, 1983	150	48
July	0.0121	0.1661	July 29, 1986	147	82
August	0.0548	0.4079	Aug. 7, 1986	156	54
September	0.0287	0.3925	Sept. 26, 1983	142	50
October	0.0160	0.2846	Oct. 31, 1984	155	89
November	0.0553	0.5513	Nov. 5, 1987	143	58
December	0.0304	0.4683	Dec. 23, 1983	147	56
All days	0.0325	0.5513	Nov. 5, 1987	1721	699

higher in these months, and the frequency of zero-dividend days is lower. Of these four months, the average daily dividend during the month of February is highest, 6.09¢, probably because extra dividend payments are often declared in the last quarter of the fiscal year. Of the total number of trading days in February in this sample, less than 32% have zero dividends. The least popular months are January, April, July, and October. During these months, the average daily dividend is less than half of the average dividend across all days. In addition, the frequency of zero-dividend days during these months exceeds 55% of the total trading days.

More insight into the monthly seasonal pattern of S&P 100 dividends is garnered from Figure 1, which plots the average daily dividends by month and year. The size of the February, May, August, and October dividends relative to other months is clearly displayed in this figure. Also clearly displayed is the fact that average dividends generally have increased over the seven-year sample period. In August 1983, for example, the average daily payout is 4.45¢ and, in August 1989, the average payout is 6.70¢. The growth in the payouts also occurred in nonpeak months. In April 1983, the average daily dividend is 0.43¢ and, in April 1989, the average dividend is 2.15¢.

Table I also presents average dividends by day of the week. Monday has the largest average payment, 4.94¢. In addition, of the total number of Mondays in the sample period, only 18% are zero-dividend days. At the other extreme, Wednesday appears to be the least popular day to pay dividends. In more than 55% of the total number of Wednesdays during the sample period, no dividends are paid. The average dividend payment across all Wednesdays is 1.76¢.

These day-of-week patterns are confirmed in Figure 2, which plots the average daily dividends by day-of-week and year. The graph shows that Wednesday is consistently the lowest payout day. Moreover, its popularity appears to be declining over the sample period. On the other hand, Monday is fairly consistent in being the highest payout day, and its popularity as an ex-dividend day appears to be increas-

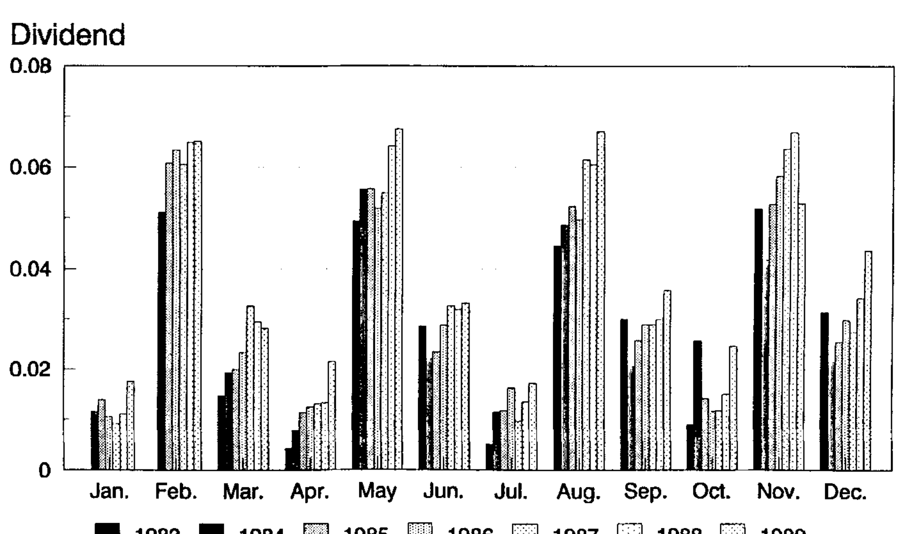


Figure 1
Average daily dividend on S&P 100 index by month and year: March 1983 through December 1989.

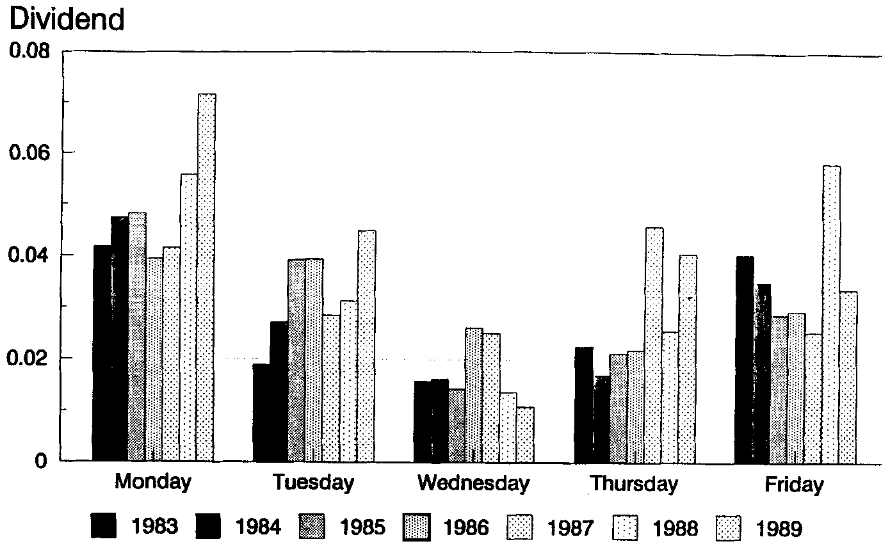


Figure 2
Average daily dividend on S&P 100 index by day and year: March 1983 through December 1989.

ing. In some part, however, the increase in dividends on Mondays must be attributable also to the fact that larger dividends are paid on S&P 100 index stocks in recent years.

Regarding the remaining days of the week, Friday is more popular than Tuesday, which, in turn, is more popular than Thursday. For Tuesdays and Thursdays, there are more zero-dividend days than nonzero-dividend days.

In Table I, the average daily dividend payment across all days is 3.25¢. This average includes 699 days when no dividend is paid. If the zero-dividend days are excluded, the average dividend payment on the S&P 100 index is 5.47¢.

The above dividend descriptions are written in terms of average dividend amounts. The importance of dividends in S&P 100 index option valuation has to do with not only the average level of dividends but also the magnitudes of individual daily dividend payments. In fact, the next section demonstrates it is the failure to recognize the large cash dividend payments on the S&P 100 index that causes the commonly applied option valuation procedures to generate large pricing errors. Many cash dividends on the S&P 100 index are quite large, as Table I shows. The largest cash dividend on the index during the March 11, 1983 to December 29, 1989 sample period is 55.13¢ on Thursday, November 5, 1987. On Friday, December 23, 1983, the cash dividend on the S&P 100 index is 46.83¢, and, on Thursday, May 7, 1987, it is 46.10¢. Cash dividends this large can have an important effect on the valuation of the S&P 100 index options. In particular, the holder of an S&P 100 index call may find it optimal to exercise his option in the day immediately before a large dividend is to be paid, and the holder of an S&P 100 index put may find it optimal to exercise his option just after. The effect of discrete cash dividends on the valuation of the S&P 100 index options is now examined.

PRICING ERRORS INDUCED BY VALUATION SIMPLIFICATIONS

Two methods are commonly applied to value S&P 100 index options: (a) a dividend-adjusted European-style valuation formula; and (b) an American-style option pricing approximation with a constant proportional dividend yield rate assumption.[2] Both of these procedures are applied for the sake of computational convenience. The Black–Scholes equation used under (a) is very easy to compute, but the effect of early exercise of the S&P 100 index options is ignored. Under (b), computational time is increased and early exercise is considered, however, the transformation of discrete cash dividends to a constant proportional dividend yield rate attentuates the effect that cash dividend payments have on the early exercise premium of the index option and, hence, on the S&P 100 index option value. This section explains how S&P 100 options should be valued.[3] Following that, the magnitudes of the pricing errors that can be generated by using the two ad hoc valuation methods described above are assessed.

Index Option Valuation

The S&P 100 index option is an American-style option on a stock portfolio that pays multiple known discrete dividends during the option's life. In general, analytical formulas for such options either have not been found or are computationally expensive, so numerical methods are used. A dividend-adjusted binomial method that may be used to quickly and accurately price S&P 100 index options is outlined here.

The key to designing an efficient procedure for valuing S&P 100 index options is to define the stock index grid in terms of the index level net of the present value of the promised dividends.[4] More specifically, first compute the current index level net of the present value of the promised dividends, that is,

$$S_0^x = S_0 - \sum_{i=1}^{n} D_i e^{-rt_i} \tag{2}$$

where $D_i(t_i)$ is the amount of (time to) the ith dividend paid during the option's life and S_0 is the current index level. Next, set up the binomial tree, beginning with S_0^x rather than S_0. The upstep and downstep factors are computed in the usual way, as are the transition probabilities.[5]

With the stock index level lattice (net of dividends) computed, the approximation method starts at the end of the option's life and works back to the present. At the end of the option's life, the option value at each index level node is given by the intrinsic value of the option. The option values one step back in time (at time $n - 1$) are computed by taking the present value of the expected future value of the

[2]The two most commonly applied algorithms for pricing American-style index options under the assumption of a constant proportional dividend yield rate are the binomial method of Cox, Ross, and Rubinstein (1979) and the quadratic approximation method of Barone-Adesi and Whaley (1987).

[3]The S&P 100 index option is the only *cash index option* that remains American-style. All others are European-style. *Index futures options* such as those written on the S&P 500 futures contract are American-style also. The valuation of American-style index futures options is distinctly different from American-style cash index options. The interested reader is referred to Ramaswamy and Sundaresan (1985) or Whaley (1986).

[4]This assumption was first introduced by Roll (1977) in his derivation of the American call option pricing formula, where the stock pays a single known dividend during the option's life. In the context of pricing an index option with multiple known dividends paid during the option life, this assumption circumvents the bifurcation that takes place at each ex-dividend instant if the *cum* dividend index level is used to define the binomial lattice.

[5]If the length of each time step is denoted Δt, the upstep and downstep factors u and d are usually defined as $u = e^{\sigma\sqrt{\Delta t}}$ and $d = \frac{1}{u}$, where σ is the standard deviation of the instantaneous returns on the index. The transition probabilities of up and down movements are $p = \frac{r^* - d}{u - d}$ and $1 - p$, respectively, where $r^* = e^{r\Delta t}$.

option. Before taking a second step back in time, it is necessary to see if any of the option values are below their early exercise value. Here is where dividends may enter the picture again. If no dividends are paid at time $n-1$, then the early exercise value is simply the lattice index level less the exercise price. However, if a dividend is paid at time $n-1$, then the early exercise proceeds equal the lattice index level plus the dividend less the exercise price. If any of the computed option values are below the exercise proceeds, they are replaced with the value of the exercise proceeds.

As the process is repeated, one step back in time, one must keep track of the sum of the present values of the dividends paid during the option's remaining life. At time $n-1$, there is only one dividend and it is paid at time $n-1$, so the sum equals the value of the single dividend paid at time $n-1$. However, if at time $n-2$, and there is a dividend paid at time $n-2$ as well as a dividend paid at time $n-1$, the sum equals the dividend payment at $n-2$ plus the present value of the dividend at $n-1$. By the time this iterative procedure finds its way back to time 0, the early exercise boundary includes the present value of all promised dividends, just as is stated in eq. (2).[6]

Approximations Using European-Style Option Formulas

With a correct method for valuing S&P 100 index options in hand, consider now the pricing errors incurred by researchers and practitioners who have applied and continue to apply ad hoc valuation procedures. The first ad hoc procedure addressed is the application of the European-style formula. Within the set of European-style applications, the most common method is to substitute the observed index level less the present value of the dividends promised during the option's life [see eq. (2)] for the stock price parameter in the Black–Scholes (1973) model. The usual rationale for doing so is that the early exercise premium for index options, particularly for index calls, is negligible. It is demonstrated here that this is simply not the case.

The simulation performed is as if one priced all S&P 100 index options during the 253 trading-day period August 1, 1988 through July 31, 1989. An entire year of trading days is used in the simulation to ensure the discovery of the full range of option pricing errors that may result given the distinctly discrete and seasonal pattern S&P 100 dividends take throughout the year. In addition, actual market parameters for the index level, the riskless rate, and the standard deviation of index return are used to make sure that pricing errors are not scaled up or down as a result of using unrealistic parameter assumptions.

Each day during the simulation period, one-, two-, and three-month options are created. For each maturity, a range of exercise prices is then generated. These exercise prices are created in five-point increments, just as they appear at the CBOE, and are centered on the closing S&P 100 index level on that day. All exercise prices that fall within the range from 12.5% out-of-the-money to 12.5% in-the-money are used. Figure 3 shows that the S&P 100 index is at the level of 260 at the beginning of the simulation period and at a level of 320 by the end. The simulation results, therefore, cover a wide range of underlying index levels.

The 30-day T-bill rate is used as a proxy for the riskless rate of interest, and the expected dividends paid during the option's remaining life are the actual dividends paid. The simulation also requires an estimate of volatility each day. The daily

[6]This S&P 100 index option valuation procedure is used in Harvey and Whaley (in press) to estimate implied volatilities.

volatility rate used here is the implied volatility from at-the-money call and put option transaction prices.[7] Figures 4 and 5 show the wide range of volatility rates used in the simulation. Volatility declines from a level exceeding 24% in August 1988 to below 16% by the end of July 1989.

Under the above assumptions regarding the underlying option pricing parameters, option values using the dividend-adjusted, European-style option procedure and the dividend-adjusted, American-style binomial method are computed each day during the 253 trading-day simulation period. The results reported in Table II are averages of (a) the difference between the American option price and the European option price; and (b) the American option price across days. In addition, the maximum deviation between the American option price and European option price is reported.

A number of significant results appear in Table II. First, the conjecture that early exercise premiums on index call options is small appears to be true on average. The average dollar pricing error for at-the-money call options (±2.5%) is 1–2¢ for one-month maturities. The maximum errors tell a somewhat different story, however. The early exercise premiums for at-the-money index call options are as high as 19¢ for one-month maturities and 26¢ for two- and three-month maturities. These mispricings are large and important.

The results for the deep in-the-money call options demonstrate more clearly the problem with using the European option pricing formula. The call options in the 10–12.5% moneyness category can produce pricing errors as high as 50¢ or more. Recall that this is approximately the amount of the largest dividends on the S&P 100 index. The use of the European formula does not recognize that the fact

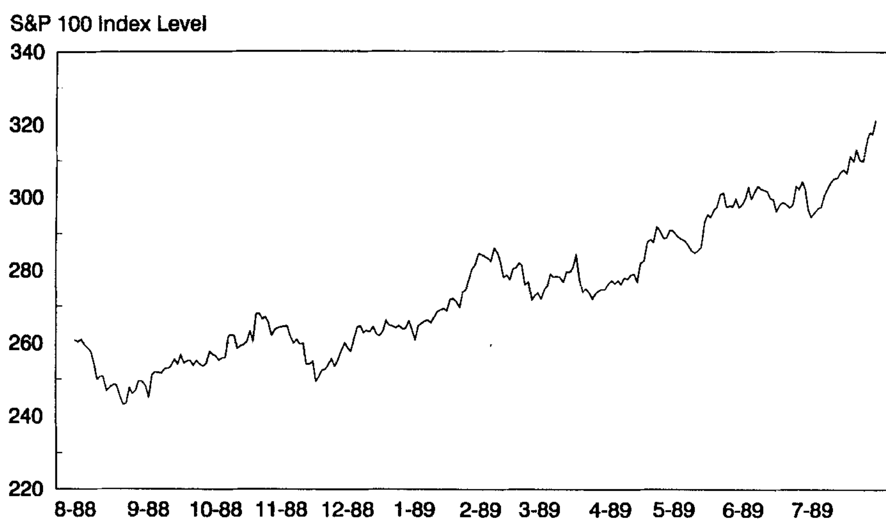


Figure 3
S&P 100 index level during simulation period August 1, 1988 through July 31, 1989.

[7]The daily volatility estimates are obtained from a nonlinear regression of at-the-money call and put option transaction prices on model prices over a ten-minute window from 2:55 to 3:05 PM CST. Separate estimates are made for calls and puts. The sample contains options of a single maturity—the shortest maturity with at least 15 days remaining to expiration.

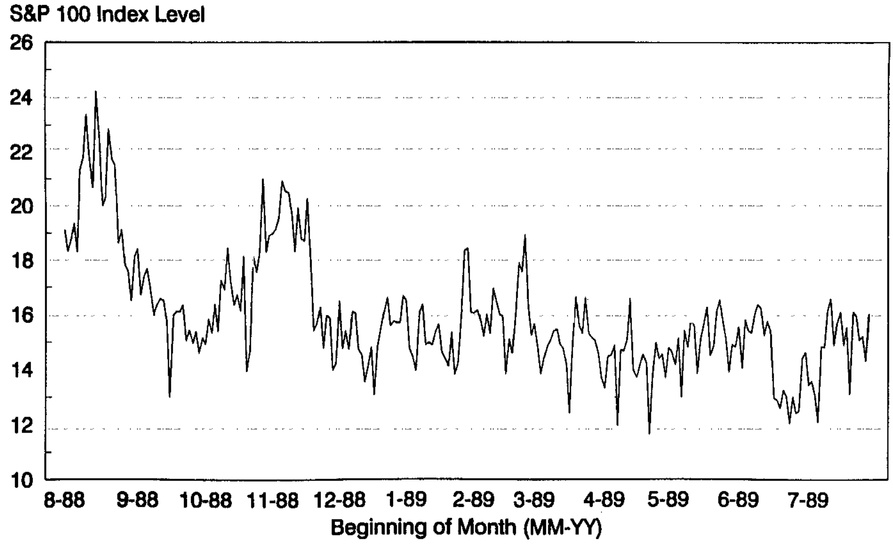


Figure 4
Implied volatility of at-the-money S&P 100 calls August 1, 1988 through July 31, 1989.

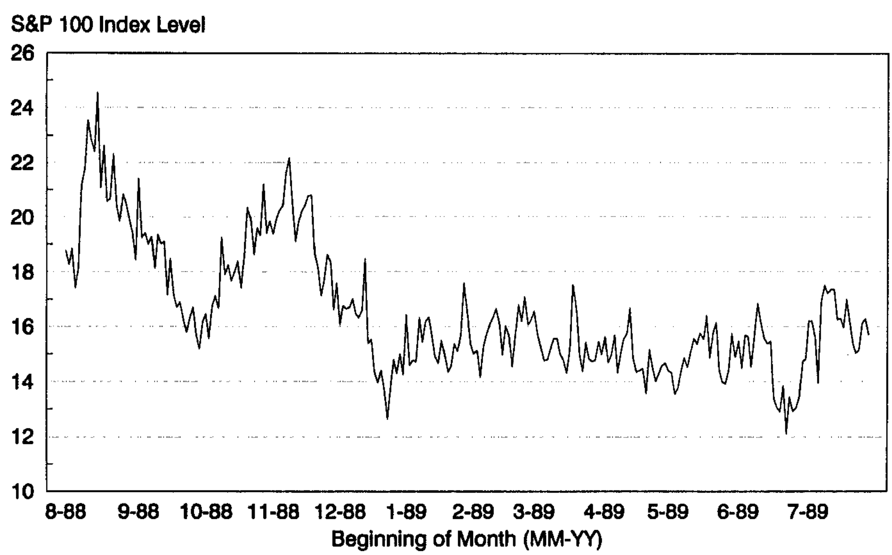


Figure 5
Implied volatility of at-the-money S&P 100 puts August 1, 1988 through July 31, 1989.

that the S&P 100 call option holder may want to capture a large dividend by exercising the option just prior to the dividend payment.[8]

The simulations show that the early exercise premium of the S&P 100 index call options may have significant value. If such is the case, there must be instances when exercising the call early is optimal. Figure 6 shows the number of call option contracts exercised prior to expiration day during the period March 12, 1983 through

[8]By exercising the option prior to the dividend payment, the dividend amount is included within cash settlement proceeds of the call.

Table II

PRICING ERRORS INDUCED BY USING A EUROPEAN-STYLE OPTION VALUATION APPROXIMATION TO PRICE AMERICAN-STYLE S&P 100 INDEX OPTIONS

Moneyness Range		One-Month Maturity			Two-Month Maturity			Three-Month Maturity		
$(S/X - 1) \times 100$[a]		Mean Error	Max. Error	Mean Price	Mean Error	Max. Error	Mean Price	Mean Error	Max. Error	Mean Price
Call options										
−12.50	−10.00	0.00	0.00	0.06	0.00	0.01	0.44	0.00	0.04	1.08
−10.00	−7.50	0.00	0.00	0.19	0.00	0.02	0.93	0.00	0.05	1.91
−7.50	−5.00	0.00	0.03	0.65	0.00	0.06	2.00	0.01	0.10	3.43
−5.00	−2.50	0.00	0.06	1.80	0.01	0.10	3.81	0.01	0.17	5.59
−2.50	0.00	0.01	0.11	4.04	0.01	0.20	6.47	0.02	0.21	8.49
0.00	2.50	0.02	0.19	7.70	0.02	0.26	10.20	0.03	0.26	12.25
2.50	5.00	0.03	0.37	12.33	0.03	0.33	14.51	0.03	0.37	16.42
5.00	7.50	0.04	0.44	17.70	0.04	0.45	19.46	0.04	0.41	21.13
7.50	10.00	0.05	0.49	23.29	0.05	0.50	24.64	0.05	0.46	26.05
10.00	12.50	0.06	0.51	28.86	0.06	0.53	29.92	0.06	0.50	31.09
Put Options										
−12.50	−10.00	1.20	2.08	33.63	1.93	3.55	33.66	2.41	4.13	33.74
−10.00	−7.50	1.09	2.03	26.58	1.57	3.28	26.73	1.90	3.75	26.97
−7.50	−5.00	0.73	1.78	18.72	0.96	2.28	19.28	1.19	2.49	19.89
−5.00	−2.50	0.37	1.10	12.00	0.54	1.24	13.20	0.70	1.50	14.17
−2.50	0.00	0.18	0.56	6.87	0.29	0.71	8.49	0.42	0.93	9.66
0.00	2.50	0.07	0.20	3.48	0.15	0.34	5.17	0.23	0.53	6.40
2.50	5.00	0.03	0.07	1.52	0.08	0.16	2.90	0.13	0.29	3.99
5.00	7.50	0.01	0.03	0.61	0.04	0.08	1.58	0.07	0.17	2.46
7.50	10.00	0.00	0.01	0.22	0.02	0.04	0.80	0.04	0.08	1.45
10.00	12.50	0.00	0.00	0.07	0.01	0.03	0.39	0.02	0.04	0.82

[a]The notation is this column is as follows: S is the index level and X is the options's exercise price. The option pricing parameters correspond to the period August 1, 1988 through July 31, 1989. The option pricing parameters are the closing index level, the 30-day T-bill rate, and the implied volatility of the at-the-money options with the nearest maturity exceeding 15 days. The option prices simulated are one-, two-, and three-month times to expiration respectively. The pricing errors reported are defined as the correct S&P 100 index option value less the value of the European-style option valuation approximation. The option price reported is the correct American-style option value. All prices and pricing errors are in dollars.

November 30, 1989. Considerable early exercise activity occurs in the last week before expiration, however, exercise activity extends out to as much as 134 days prior to expiration. It is doubtful that the early exercise activity 30 days or more before expiration is rational; however, the number of exercises in the weeks prior to expiration indicate that early exercise of index call options is not uncommon. Indeed, it may be likely if a large dividend is paid during the option's remaining life.

The put option results (at the bottom panel of Table II) show that failure to recognize the early exercise premium on the index put options can lead to even more serious pricing errors than is the case for the call options. The at-the-money options with a one-month maturity have an average early exercise premium of 7–18¢ and a maximum pricing error of 56¢. The premium is substantial. The deeper the put

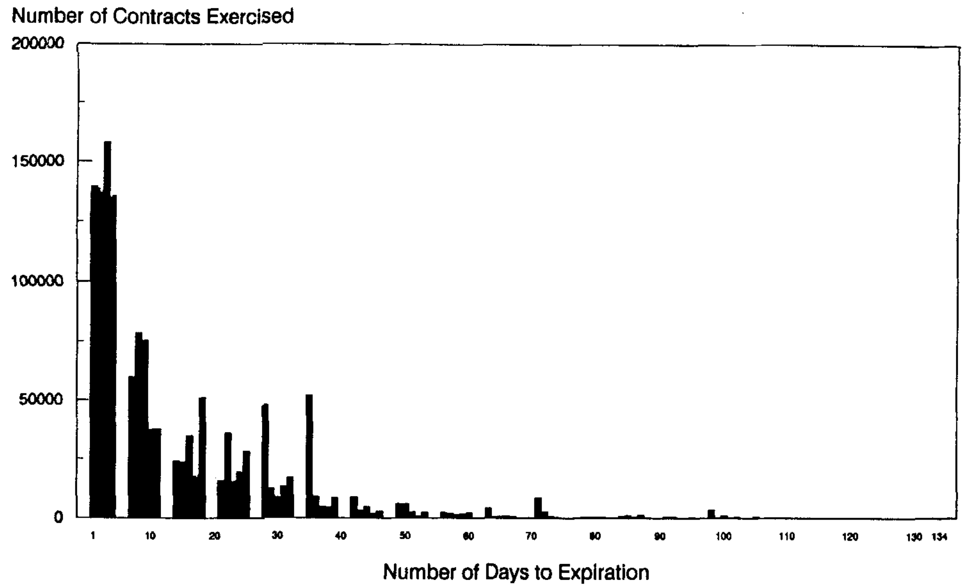
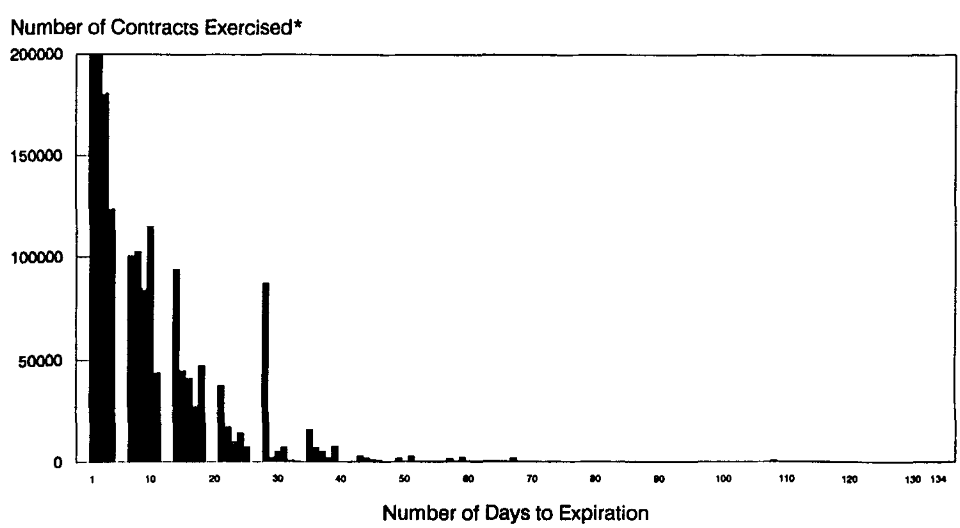


Figure 6
Exercise history of S&P 100 index call options March 12, 1983 through November 30, 1989.

option is in the money and the longer is the time to expiration, the greater is the probability of early exercise and, hence, the larger is the pricing error of the European-style valuation approach.

Figure 7 shows the number of S&P 100 index put contracts exercised prior to expiration during the period March 12, 1983 through November 1989. Early exercise of index puts is more frequent than calls. This result is expected, since the magnitudes of the early exercise premiums shown in the simulation results are larger for



*The numbers of index put option contracts exercised one and two days prior to expiration are 422,604 and 208,981, respectively.

Figure 7
Exercise history of S&P 100 index put options March 12, 1983 through November 30, 1989.

puts than for calls. The early exercise activity for puts in the weeks prior to expiration appears commonplace. Early exercise activity as far out as 109 days is observed.

In summary, the use of the European-style option pricing formulas to price S&P 100 index options can lead to serious mispricing. The discrete and seasonal nature of the dividends on the S&P 100 index can induce substantial early exercise premiums for S&P 100 index calls and puts, and these premiums are not recognized within the European-style approach.

Approximations Using Constant Dividend Yield Rate

Some studies attempt to recognize the early exercise feature of the S&P 100 index options by using an American option pricing approximation method. Unfortunately, however, they usually assume that index dividends are paid at a constant proportional rate. This approximation, like the European-style option approximation, fails to recognize the effect that large discrete cash dividend payments can have on the early exercise premium of the S&P 100 index options and, hence, on S&P 100 index option prices, and thereby can generate equally large and spurious pricing errors.

To illustrate plausible magnitudes of the pricing errors that can be introdued by using the constant dividend yield rate assumption, a second simulation is performed. Hypothetical one-, two-, and three-month index options are priced each day during the period August 1, 1988 through July 31, 1989 using the binomial method with a constant dividend yield and the corrrect dividend-adjusted binomial method described at the beginning of this section. The option pricing parameters are the same as those in the previous simulation, with the exception of the dividend yield. The constant proportional dividend yield rate for the S&P 100 index is computed as

$$d = \ell n\left(\frac{S + \Sigma_{i=1}^{n} D_i e^{r(T-t_i)}}{S}\right)/T \tag{3}$$

where S is the current index level, D_i and t_i are the amount and the timing of the ith dividend on the index, and T is the time to expiration of the option.

Prior to examining the simulation results, it should be noted that the cards are stacked in favor of finding accurate pricing with the constant dividend yield model. The dividend yield rate computed is from the actual dividends paid during the option's life and, in this sense, accounts for the monthly seasonality in dividend payments. In practice, it is much more common to see the dividend yield rate computed by taking the sum of dividends over the next *quarter* and dividing by the current index level. Such a procedure is not sensitive to monthly seasonality. Different index options should have different dividend yield rates depending upon what part of the dividend cycle their times to expiration include. All of this is to say that the pricing errors generated in this simulation are likely to understate the pricing errors that are observed in practice when the constant dividend yield rate assumption is used.

The simulation results reported in Table III are averages of (a) the difference between the American option price using actual cash dividends and the American option price using the actual dividend yield; and (b) the American option price using actual cash dividends across days. The maximum pricing errors in each moneyness category are reported also. On average, the results indicate, that there is only a small difference between the option pricing methods; i.e., the constant dividend

Table III
PRICING ERRORS INDUCED BY USING AN AMERICAN-STYLE OPTION VALUATION APPROXIMATION WITH A CONSTANT DIVIDEND YIELD RATE TO PRICE AMERICAN-STYLE S&P 100 INDEX OPTIONS

Moneyness Range $(S/X - 1) \times 100$[a]		One-Month Maturity Mean Error	One-Month Maturity Max. Error	One-Month Maturity Mean Price	Two-Month Maturity Mean Error	Two-Month Maturity Max. Error	Two-Month Maturity Mean Price	Three-Month Maturity Mean Error	Three-Month Maturity Max. Error	Three-Month Maturity Mean Price
Call options										
−12.50	−10.00	0.00	0.00	0.06	0.00	0.01	0.44	0.00	0.04	1.08
−10.00	−7.50	0.00	0.00	0.19	0.00	0.02	0.93	0.01	0.06	1.91
−7.50	−5.00	0.00	0.03	0.65	0.00	0.06	2.00	0.01	0.10	3.43
−5.00	−2.50	0.00	0.06	1.80	0.01	0.10	3.81	0.02	0.18	5.59
−2.50	0.00	0.01	0.11	4.04	0.02	0.21	6.47	0.03	0.23	8.49
0.00	2.50	0.02	0.19	7.70	0.03	0.27	10.20	0.04	0.27	12.25
2.50	5.00	0.03	0.37	12.33	0.04	0.34	14.51	0.05	0.39	16.42
5.00	7.50	0.05	0.44	17.70	0.05	0.46	19.46	0.07	0.44	21.13
7.50	10.00	0.05	0.49	23.29	0.06	0.51	24.64	0.08	0.49	26.05
10.00	12.50	0.06	0.52	28.86	0.07	0.55	29.92	0.09	0.53	31.09
Put Options										
−12.50	−10.00	0.04	0.51	33.63	0.05	0.55	33.66	0.05	0.60	33.74
−10.00	−7.50	0.05	0.51	26.58	0.08	0.51	26.73	0.08	0.54	26.97
−7.50	−5.00	0.08	0.56	18.72	0.12	0.58	19.28	0.12	0.58	19.89
−5.00	−2.50	0.09	0.44	12.00	0.10	0.43	13.20	0.08	0.46	14.17
−2.50	0.00	0.06	0.26	6.87	0.05	0.27	8.49	0.04	0.30	9.66
0.00	2.50	0.02	0.10	3.48	0.02	0.13	5.17	0.00	0.16	6.40
2.50	5.00	0.00	0.05	1.52	0.00	0.06	2.90	0.01	0.08	3.99
5.00	7.50	0.00	0.02	0.62	0.01	0.04	1.58	0.02	0.05	2.46
7.50	10.00	0.00	0.01	0.22	0.01	0.01	0.80	0.02	0.01	1.45
10.00	12.50	0.00	0.00	0.07	0.01	0.00	0.39	0.02	0.00	0.82

[a]The notation is this column is as follows: S is the index level and X is the opion's exercise price. The option pricing parameters correspond to the period August 1, 1988 through July 31, 1989. The option pricing parameters are the closing index level, the thirty-day T-bill rate, and the implied volatility of the at-the-money options with the nearest maturity exceeding fifteen days. The dividend yield rate is computed by taking the future value of all dividends paid during the option's life and converting the future value to a constant proportional dividend yield rate. The future value of the dividends paid during the option's life is $FVD = \Sigma_{i=1}^{n} D_i e^{r(T-t_i)}$. The constant dividend yield rate is then computed as $d = \ln[(S + FVD/S)]/T$, where S is the index level and T is the time to expiration of the option. The option prices simulated are one-month, two-month, and three-month times to expiration respectively. The pricing errors reported are defined as the correct S&P 100 index option value less the value of American-style option valuation approximation with a constant dividend yield rate. The option price reported is the correct American-style option value. All prices and pricing errors are in dollars.

yield method tends to understate the option value. The maximum pricing errors, however, tell a different story. The maximum pricing errors are every bit as large as for the European-style valuation procedure for both calls and puts—on order of 50¢ and more.

Both the average pricing error and maximum pricing error results are driven by the inability of the constant dividend yield rate model to properly account for the early exercise premium of the S&P 100 index options. The transformation of discrete cash dividends into a constant dividend yield rate [such as that described by eq. (3)] smooths the dividend series over the life of the option, thereby reducing the probability of early exercise. If a large cash dividend, e.g., on order of 50¢ is about to be paid, the S&P 100 call (put) option holder may want to implicitly capture this amount by exercising his option immediately before (after) ex-dividend. In the constant dividend yield formulation, this 50¢ dividend is spread out over the remaining life of the option, in which case its ability to provide an incentive for early exercise is attenuated.

In summary, the use of an American-style option pricing method where the index is assumed to pay dividends at a constant proportional rate can also lead to serious S&P 100 index option mispricing. Using a constant dividend yield rate option valuation approach fails to recognize the size of the early exercise premiums attributable to large, discrete dividend payments on the S&P 100 index.

CONCLUSIONS

This study shows that ad hoc valuation methods for S&P 100 index options can produce very large pricing errors. To value S&P 100 index options accurately, the option valuation method must account for the discrete and seasonal dividend payments of the S&P 100 index portfolio. To this end, this study (a) constructs an exact cash dividend series for the S&P 100 index over its history; and (b) provides a binomial method for pricing American-style options where the underlying index pays multiple known discrete dividends. Past studies of the S&P 100 index option market use European-style option valuation formulas or American-style option valuation approximations with a constant dividend yield rate assumption to either price index options or infer market volatility from observed option prices. Since both of these ad hoc valuation methods ignore the discrete and seasonal nature of index dividend payments, the results of these studies may be undermined. From the practical standpoint of pricing (or trading) S&P 100 index options, knowing the amount and timing of S&P 100 index cash dividends appears to be critical.

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