PERFORMANCE MEASUREMENT: RISK-ADJUSTED PERFORMANCE – SINGLE-FACTOR

Performance evaluation is one of the most critical areas of applied investment management. Investors use it to assess how well funds perform relative to benchmarks and find reasons for the under/overperformance. Performance measures fall into two categories—tracking performance and risk-adjusted performance. Within the risk-adjusted performance category are single market price risk factor (or traditional) measures and multiple market price risk factor (multi-factor) measures. This section focuses on the origins of the conventional single-factor, risk-adjusted performance measures and the statistical analysis necessary to ensure correct and meaningful measurement. The following section addresses performance measurement when the fund has multiple sources of market risk.

The traditional or single-factor performance measures emanate from the Sharpe (1964)/Lintner (1965) capital asset pricing model (CAPM).[1] The model has three main results: (a) the capital market line (CML), (b) the composition of the market portfolio, and (c) the security market line (SML). Each of these results has a bearing on performance measurement. While the CML and SML serve as the foundation of the single-factor measures, the composition of the market portfolio guides the design of performance benchmark indexes. Since this section focuses on single-factor performance, we focus on the implications of the CML and SML.

CAPM - Capital market line

The *capital market line* (CML) represents the relation between expected return and risk for efficient portfolios (i.e., portfolios with the highest expected return for a given risk tolerance). All individuals in the economy hold the same risky asset tangency portfolio M in combination with the risk-free asset. The CML is

$$E_P = r + \left(\frac{E_M - r}{\sigma_M}\right)\sigma_P, \tag{1}$$

where E_P and E_M are the expected returns on the individual's and market portfolios, σ_P and σ_M are their volatilities, and r is the risk-free return. An individual's allocation between the market portfolio and the risk-free asset depends on his degree of risk aversion. Suppose an individual's risk tolerance is below σ_M. In that case, his *optimal* portfolio will be a *lending* portfolio—some wealth invested in M and some in the risk-free asset. If the individual's risk

[1] Whaley (2023) reviews the assumptions and key results of the CAPM.

tolerance is above σ_M it is tangent to the right of σ_M, the optimal portfolio will be a *borrowing* portfolio—not only is all wealth invested in *M*, but also additional funds are borrowed and invested in *M*.

CAPM - Security market line

The *security market line* (SML) is risky assets' equilibrium expected return/risk relation. The relation is

$$E_i = r + \left(E_M - r\right)\frac{\sigma_{iM}}{\sigma_M^2} = r + \left(E_M - r\right)\beta_i . \qquad (2)$$

The SML is the equilibrium expected return/risk relation for all risky securities in the marketplace. And, if (2) holds for all risky securities, it has for portfolios of securities like ETFs.

<u>Illustration 1</u>: Estimate the total risk and relative systematic risk of ESGU.

Thematic index ETFs have become popular in recent years. The reason is simple. ETF issuers earn revenue as a percent (i.e., the management fee included in the expense ratio) of the total value of the assets under management ($AUM). They will do so if they can attract new investors by devising a new product with a particular investment theme (e.g., high dividend yield stocks). The marginal costs of creating a new stock index product for issuers like BlackRock are trivial. Currently, a popular theme is environmental, social, and corporate governance (ESG) stocks, and a strong proponent of this product structure is iShares (which is owned by BlackRock). Their largest ESG fund is ESGU, an ETF benchmarked to the ESG Aware MSCI USA index. All of this is explained in the ***ESGU fact sheet.pdf.*** *The last three years of daily data are in* ***ESGU analysis.xlsx****. Estimate its risk parameters and compare them to the stock market indexes and cash equivalents.*

The table below has summary statistics for the daily ln returns of the ESGU ETF, cash equivalents (i.e., Fed funds return (EFFR)), and three different market indexes. The MSCI USA index is MSCI's index offering intended to compete with the S&P 500. It has 625 stocks from the US stock market's large- and mid-cap segments, accounting for about 85% of the total market cap. The S&P 500 is included because BlackRock reports ESGU's beta relative to the S&P 500 index, and the S&P TMI is included because it represents the entire US stock market (as the Sharpe/Lintner model specifies).

Realized total risk, $\hat{\sigma}$, is reported in the StDev (daily) and StDev (annual) rows. By convention, StDev (annual) is called "volatility." With daily return data, volatility is computed as $\hat{\sigma}_{daily} \times \sqrt{252}$.[2] The volatility of the Fed funds returns is 0.06%. While the level is small relative to ESGU and the different market indexes, it is not risk-free. Cash equivalents should be treated as any other risky security. The total risk of ESGU is 25.85%, about the same as its underlying benchmark, MSCH USA. It is slightly higher than the S&P 500 index volatility (due to the exclusion of high market-cap, low-risk stocks such as Exxon Mobil) and lower than the S&P total market index volatility (due to the exclusion of small-cap, high-risk stocks).

Return summary statistics					
Description	ESGU	MSCI USA	S&P 500	S&P TMI	EFFR
n	756	756	756	756	756
Mean (daily)	0.00028	0.00029	0.00029	0.00026	0.00003
StDev (daily)	0.01629	0.01629	0.01611	0.01648	0.00006
Skewness	-0.72638	-0.76986	-0.74355	-0.83805	3.44605
Autocorrelation	-0.20295	-0.20701	-0.21618	-0.19959	0.57987
Minimum	-0.12786	-0.12917	-0.12761	-0.13165	0.00000
Median	0.00074	0.00074	0.00094	0.00088	0.00000
Maximum	0.09252	0.08992	0.08977	0.09054	0.00048
Mean (annual)	7.16%	7.20%	7.36%	6.65%	0.72%
StDev (annual)	25.85%	25.86%	25.57%	26.16%	0.09%
CAGR	7.43%	7.47%	7.64%	6.87%	0.72%
HPR	23.98%	24.11%	24.72%	22.07%	2.18%

Before turning to the relative systematic risk estimation, examining correlations between return series to develop intuition about the return series is helpful. Below is a summary. This matrix says that the returns of ESGU and the three market indexes are virtually perfectly correlated. The correlations of the market indexes and the Fed funds returns are not different from 0 from a statistical viewpoint. However, the negative sign suggests that more investment is made in the stock market when borrowing rates are cheap.

Correlation matrix					
	ESGU	MSCI USA	S&P 500	S&P TMI	EFFR
ESGU	1	0.999	0.998	0.997	-0.047
MSCI USA	0.999	1	0.999	0.998	-0.047
S&P 500	0.998	0.999	1	1	-0.046
S&P TMI	0.997	0.998	0.997	1.000	-0.049
EFFR	-0.047	-0.047	-0.046	-0.049	1

[2] The number of trading days in a calendar year is 252.

Excess return regressions are performed to estimate the relative systematic risk (i.e., beta). *Excess returns* are defined as $R_{i,t} - R_{F,t}$, the realized return of security *i* less the realized return on cash equivalents. In contrast, the SML is based on return expectations. The *(expected) risk premium* is security $E_i - r$, i.e., the expected return over the holding period less the risk-free rate over the holding period.[3] Now, consider the excess return regression,

$$R_{i,t} - R_{F,t} = \alpha + \beta\left(R_{M,t} - R_{F,t}\right) + \varepsilon_{i,t},$$

where $R_{i,t}$, $R_{F,t}$, and $R_{M,t}$ are the daily returns of ESGU, Fed funds, and three market indexes. If, over the sample period (i.e., in this case, three years of daily data), expectations are, on average, realized, the expected value of the intercept term is 0 since

$$E_i - r = \left(E_M - r\right)\beta_i = \overline{R}_i - \overline{R}_F = \left(\overline{R}_M - \overline{R}_F\right)\hat{\beta}_i = 0.$$

The excess return regression results table below offers at least two critical insights. First, note that while the betas are close to one, they are different from one in a statistical sense. For the null hypothesis that beta equals 1, the coefficient using the MSCI USA index is not different from one. It is significantly greater than one using the S&P 500 index and significantly less than one using the S&P TMI. Unlike total risk (i.e., volatility), measuring systematic risk depends on the choice of the proxy for the market. Indeed, BlackRock uses the S&P 500 when it reports the beta of ESGU, 1.02, on the second page of the Fact Sheet. It is interesting (and contradictory) that BlackRock publishes a beta relative to the S&P 500 when benchmarking the MSCI USA index. It seems a reflection that the S&P 500 is "the" standard. The BlackRock estimate of beta is 1.02, while ours is 1.01. The difference is negligible and is attributable to factors such as daily vs monthly data.

[3] The CAPM is a single-period model, and a risk-free rate is conceptually possible. In practice, however, estimating expected returns and risks requires multiple periods of return data.

Excess return regression results			
Benchmark index	MSCI USA	S&P 500	S&P TMI
n	756	756	756
α	0.00000	-0.00001	0.00002
s(α)	0.00003	0.00004	0.00005
β	0.99834	1.00878	0.98513
s(β)	0.00197	0.00239	0.00290
R-squared	0.9971	0.9958	0.9935
Adj. R-squared	0.9971	0.9958	0.9935
Std error of estimate	0.00088	0.00106	0.00131
t-ratio (H_0: β=1)	-0.84	3.67	-5.12

Second, the adjusted R-squared is highest for the MSCI USA index (0.9971), second highest for the S&P 500 (0.9958), and lowest for the S&P TMI. These empirical results make sense. Recall that

$$\overline{R}^2 = 1 - \frac{Var\left(\varepsilon_i\right)}{Var\left(R_{i,t} - R_{F,t}\right)} = 1 - \frac{\text{Idiosyncratic risk}}{\text{Total risk}}.$$

The 308 stocks in ESGU are chosen from the 625 stocks (see fact sheet) in the MSCI USA index. Hence, the index returns explain more of the total return volatility of ESGU. At the other extreme, the adjusted R-squared for the S&P TMI is 0.9935. While the S&P TMI explains a good deal of the excess return variance of ESGU, it does not explain as much as MSCI USA.

Total risk-adjusted performance measures

The four traditional measures of single-factor risk-adjusted performance are the Sharpe (1964) ratio, the Modigliani and Modigliani (1997) M^2, the Treynor (1965) ratio, and Jensen's (1968) alpha. The first two are based on the portfolio's total risk under consideration. Under the CAPM, all individuals hold efficient portfolios from the CML (1), which means that the risk-adjusted risk premium of each efficient portfolio equals the risk-adjusted risk premium of the market, that is, the ex-ante (before the fact) Sharpe ratio is

$$\text{Ex-ante Sharpe ratio} = \frac{E_P - r}{\sigma_p} = \frac{E_M - r}{\sigma_M}. \tag{11}$$

The Sharpe ratio operationalizes this concept using a history of portfolio return data. The ex-post (after the fact) Sharpe ratio is the mean realized excess return of the portfolio divided by its realized volatility. If expectations are realized over the evaluation period, the Sharpe ratio of the portfolio should equal the realized market risk premium divided by its volatility,

$$\text{Ex-post Sharpe ratio} = \frac{\overline{R}_P - \overline{R}_F}{\hat{\sigma}_P} = \frac{\overline{R}_M - \overline{R}_F}{\hat{\sigma}_M}. \qquad (12)$$

If the portfolio's ex-post Sharpe ratio exceeds that of the market, the portfolio "outperformed the market on a risk-adjusted basis."[4]

The M^2 measure was also developed from the CML but was rearranged to produce a <u>standalone</u> metric. From (1), we know

$$\text{Ex-ante } M^2 = \left(E_P - r\right)\left(\frac{\sigma_M}{\sigma_P}\right) - \left(E_M - r\right) = 0. \qquad (13)$$

The term $\left(E_M - r\right)$ is the expected market risk premium. The preceding term is the expected risk premium of the portfolio $\left(E_P - r\right)$, levered up or down by the factor σ_M / σ_P to match the total risk of the market σ_M. If the portfolio's total risk σ_P is below the market's σ_M, the factor exceeds one. This implies that an individual not only invests all his wealth in portfolio *P*, but also borrows $1 - \sigma_M / \sigma_P$ at the risk-free rate and invests it in *P*. If the factor is below one, the individual invests σ_M / σ_P in the risky portfolio *P* and $1 - \sigma_M / \sigma_P$ in the risk-free security. Substituting the values of the realized parameters over the evaluation period into (13), abnormal performance is

$$\text{Ex-post } M^2 = \left(\overline{R}_P - \overline{R}_F\right)\left(\frac{\hat{\sigma}_M}{\hat{\sigma}_P}\right) - \left(\overline{R}_M - \overline{R}_F\right). \qquad (14)$$

With equal risk levels, the levered portfolio's return can be compared to the market return directly. Where $M^2 > 0$, portfolio *P* outperformed the market on a risk-adjusted basis, and vice versa.

Systematic risk performance measures

The remaining two performance measures—the Treynor ratio and the Jensen alpha—are based on systematic risk (and are the counterparts to the Sharpe ratio and M^2, respectively). The measures depend on the SML (10), which applies to all risky securities in the marketplace. Since a portfolio is nothing more than a

---

[4] Sharpe ratios are often misapplied. The Sharpe ratio of a portfolio means nothing without the Sharpe ratio of the benchmark. Suppose that the ex-post Sharpe ratio of the portfolio using daily return data is *x*. The Sharpe ratio using monthly return data over the same period will be $x \times \frac{21}{\sqrt{21}} = 4.48x$.

weighted combination of securities, it is also the case that portfolios lie along the SML, that is,

$$E_P = r + \left(E_M - r\right)\beta_P . \tag{15}$$

Hence, the expected (or ex-ante) risk premium of the portfolio adjusted for its systematic risk should equal the expected risk premium of the market,

$$\text{Ex-ante Treynor ratio} = \frac{E_P - r}{\beta_P} = E_M - r . \tag{16}$$

The Treynor ratio is the realized (ex-post) version of (16) in which realized returns and risk replace expected returns and risks, that is,

$$\text{Ex-post Treynor ratio} = \frac{\overline{R}_P - \overline{R}_F}{\hat{\beta}_P} = \overline{R}_M - \overline{R}_F . \tag{17}$$

The portfolio has outperformed the market if the Treynor ratio exceeds the realized market risk premium over the evaluation period. The portfolio's realized systematic risk, or beta, $\hat{\beta}_P$ is estimated by a time-series OLS regression of the excess returns of the portfolio on the excess returns of the market, that is,

$$R_{P,t} - R_{F,t} = \alpha_P + \beta_P\left(R_{M,t} - R_{F,t}\right) + \varepsilon_{P,t} . \tag{18}$$

Suppose a portfolio outperforms the market on a risk-adjusted basis. In that case, its Treynor ratio will exceed the market's realized excess return.

Jensen's alpha, like the M^2, is a rearrangement of a relation in a manner that the expected portfolio performance is 0. Here, the SML may be written.

$$\text{Ex-ante Jensen's alpha} = E_P - r - \beta_P\left(E_M - r\right) = 0 . \tag{19}$$

Like the other performance measures, Jensen's alpha depends on the realized returns over the evaluation period,

$$\text{Ex post Jensen's alpha} = \hat{\alpha}_P = \overline{R}_P - \overline{R}_F - \hat{\beta}_P\left(\overline{R}_M - \overline{R}_F\right). \tag{20}$$

If the estimated value of Jensen's alpha $\hat{\alpha}_P$, is greater than zero, the portfolio outperformed the market on a risk-adjusted basis.

Treynor and Black (1973) offer an important extension of Jensen's alpha called the *appraisal ratio*. The need for this measure is called for by the fact that individuals or managers often choose not to hold the market portfolio passively. Instead, they actively select subsets of securities based on their investment skills. Their skills may be in tactical allocation (e.g., overweighting/underweighting different sectors of the economy or types of stocks such as small-cap vs. large-cap) or stock-picking (e.g., finding securities that are under/over-priced). Whatever their style, however, they are not holding the market portfolio and are, therefore, incurring diversifiable or idiosyncratic risk. The appraisal ratio is Jensen's alpha penalized by the idiosyncratic risk that the individual incurs from choosing not to hold a fully diversified portfolio. The standard deviation of the residual term (or the standard error of the estimate) in the excess return regression,

$$R_{P,t} - R_{F,t} = \alpha_P + \beta_P \left(R_{M,t} - R_{F,t} \right) + \varepsilon_{P,t}, \tag{21}$$

measures idiosyncratic risk. The appraisal ratio (or the Treynor-Black ratio) is.

$$\text{Appraisal ratio} = \frac{\hat{\alpha}_P}{\hat{\sigma}_\varepsilon}.$$

For portfolios with the same level of positive $\hat{\alpha}_P$, the one with the highest appraisal ratio is preferred.

Alternative risk measures

The five performance measures discussed thus far assume that investors measure total portfolio risk by the standard deviation of returns and that the standard deviation of portfolio returns (i.e., total risk) can be segmented into two components (i.e., market risk and idiosyncratic risk). As perplexing as it might seem, the use of return standard deviation implies that investors find an unexpectedly sizeable positive return equally as distasteful as an unexpectedly large negative return. Common sense dictates otherwise. Investors are willing to pay for the chance of a significant positive return (i.e., positive skewness), holding other factors constant, but will want to be paid for taking on negative skewness. Since the standard performance measures do not recognize these premiums/discounts, portfolios with positive skewness will appear to underperform the market on a risk-adjusted basis, and portfolios with negative skewness will appear to over-perform.

Ironically, while the Sharpe/Linter CAPM is based on the mean/variance portfolio theory of Markowitz (1952), it was Markowitz (1959) who first noted that using standard deviation to measure risk is too conservative since it regards all extreme returns, positive or negative, as undesirable. Markowitz (1959, Ch. 9) advocates the use of semi-variance or semi-standard deviation as a total risk measure.[5] To understand the relation between standard deviation and semi-standard deviation, begin with total risk as measured traditionally using the standard deviation of excess returns, that is,

$$\text{Standard deviation}_i = \sqrt{\frac{\sum_{t=1}^{T} \left(R_{i,t} - R_{F,t} \right)^2}{T}}. \tag{22}$$

[5] Indeed, in his Nobel Prize acceptance speech, Markowitz (1991) continues to argue that semi-variance seems more plausible than variance as a measure of risk.

In (22), the mean realized excess return is assumed to be equal to 0.[6] Without loss of generality, equation (22) may be rewritten as

$$\text{Standard deviation}_i = \sqrt{\frac{\sum_{t=1}^{T} \min\left(R_{i,t} - R_{F,t}, 0\right)^2}{T} + \frac{\sum_{t=1}^{T} \max\left(R_{i,t} - R_{F,t}, 0\right)^2}{T}}. \qquad (23)$$

Under the square root sign are two terms. The first is the sum of the squared deviations where the excess return is negative, and the second is the sum of the squared deviations where the excess return is positive. If individuals care only about risky asset returns when they are below the return of cash equivalents, semi-standard deviation (i.e., an alternative total risk measure) can be defined as

$$\text{Semi-standard deviation}_i = \sqrt{\frac{\sum_{t=1}^{T} \min\left(R_{i,t} - R_{F,t}, 0\right)^2}{T}}, \qquad (24)$$

where $i = M, P$. The ratio of realized excess return relative to the semi-standard deviation of return is the Sortino ratio.[7]

<u>Illustration 2</u>: Estimate performance of buy-write index portfolio (PBP) and its benchmark index (BXM).

*The Excel file, **PBP analysis.xlsx**, holds ten years of daily returns for Invesco's S&P 500 BuyWrite ETF (PBP), the CBOE's S&P 500 BuyWrite Index (BXM), the S&P 500 index, and the overnight Fed funds return (EFFR). The **PBP fact sheet.pdf** explains that PBP holds the S&P 500 stocks and sells a one-month at-the-money call each month and holds it until expiration. The investment strategy is called a "buy-write" because it "buys" the stock and "writes" a call option on the stock. It is also often called an "income enhancement strategy" because selling calls generates cash. The label is misleading, however. The money is gained at the expense of giving up part (if not all) of the upside on the performance of the S&P 500 stocks. In theory, buy-writes reduce expected risk and, therefore, reduce expected return. The truncation of the upside of the return distribution also means that it is skewed to the left. Recall that traditional risk-adjusted performance measures depend on symmetric return distributions. Evaluate PBP's tracking error relative to its benchmark, BXM, and then examine the risk-adjusted performance of PBP and BXM relative to the S&P 500.*

Recall tracking error does not depend on the shape of the ETF return distribution per se. The ETF and its benchmark have the same return distribution. The tracking error results reported in the table below are

[6] The denominator is T rather than $T-1$ since we are not using up a degree of freedom to estimate the mean.
[7] See Sortino and Van der Meer (1991).

reasonable. The first column is daily holding period returns, while the second column is daily ln returns. The tracking difference (TD) for daily holding period returns is -0.000023 a day or 57 basis points a year. The 57 basis points are in line with the fund's stated management fee of 49 basis points reported in the fact sheet. The *TD* is not significantly different from 0 (i.e., the RATD is -0.3747). Signs of inferior performance appear, however. First, the minimum and maximum tracking errors are -3.9% and 2.9%, respectively. It seems highly unlikely that these errors are possible. The primary driver of the daily returns of PBP and BXM is the daily return of the S&P 500 index. Outliers are often generated from misreporting or recording errors. Other symptoms of outliers are the extremely high negative autocorrelation of tracking error, -0.4480, and the low contemporaneous correlation between return, 0.934 (i.e., it should be close to one). The results for daily ln returns are qualitatively similar. From a practical perspective, daily holding period returns, and daily ln returns are similar in size.

Tracking performance		
Description	HP returns	ln returns
No. of daily returns (*n*)	2,518	2,518
Tracking difference (*TD*)	-0.000023	-0.000026
Standard deviation of *TE*	0.003041	0.003041
Standard error of *TD*	0.000061	0.000061
Risk-adjusted tracking difference (*RATD*)	-0.3747	-0.4209
Prob *TD* = 0	0.7079	0.6738
Autocorrelation	-0.4880	-0.4883
Minimum	-0.039261	-0.040705
Median	0.000019	0.000019
Maximum	0.028639	0.026812
Root mean square tracking error (*RMSE*)	0.003040	0.003040
Annualized *TD*	-0.57%	-0.64%
Correlation	0.934	0.935

Before turning to the risk-adjusted measures, examining the summary statistics of the ln returns is worthwhile. Two observations are directly relevant to our analysis. First, note the extreme negative skewness of PBP and BXM relative to the S&P 500 and S&P TMI. The buy-write strategy induces this skewness. Second, the annualized return volatility of PBP is about ¾ of the return volatilities of the S&P 500 and S&P TMI. This is a reflection that the buy-write strategies are risk-reducing. Unfortunately, it is the upside risk that is being truncated.

Return summary statistics					
Description	PBP	BXM	S&P 500	S&P TMI	EFFR
n	2,518	2,518	2,518	2,518	2,518
Mean (daily)	0.000195	0.000221	0.000469	0.000451	0.000032
StDev (daily)	0.008581	0.008250	0.011125	0.011334	0.000047
Skewness	-1.97146	-2.43469	-0.83790	-0.93479	2.86335
Autocorrelation	-0.22928	-0.23248	-0.14576	-0.13425	0.52738
Minimum	-0.12010	-0.12956	-0.12761	-0.13165	0.00000
Median	0.00048	0.00049	0.00072	0.00071	0.00001
Maximum	0.08411	0.08984	0.08977	0.09054	0.00048
Mean (annual)	4.92%	5.56%	11.83%	11.36%	0.80%
StDev (annual)	13.62%	13.10%	17.66%	17.99%	0.08%
CAGR	5.04%	5.72%	12.56%	12.03%	0.80%
HPR	63.44%	74.28%	226.12%	211.10%	8.27%

The risk reduction can also be seen through excess return regression results. The excess returns of PBP are regressed on the S&P 500 and the S&P TMI. In both instances, the beta coefficient is about 0.65, showing that PBP is 35% less risky than the market indexes. The adjusted R-squared values are about 72%, meaning that about 28% of the buy-write strategy risk is driven by the influence of call option writing.

Excess return regression results		
	S&P 500	S&P TMI
n	2,518	2,518
α	-0.00012	-0.00011
s(α)	0.00009	0.00009
β	0.65836	0.64268
s(β)	0.00801	0.00798
R-squared	0.7285	0.7205
Adj. R-squared	0.7284	0.7204
Std error of estimate	0.00447	0.00454

The single-factor risk-adjusted measures displayed in the table show a range of interesting results. Daily ln returns are used. If the S&P 500 index is used as the market proxy, all traditional performance measures show that the PBP underperformed relative to the market. The Sharpe ratio is less than that of the market, the M-squared is negative, the Treynor ratio is below the excess market return, and Jensen's alpha is negative. The skewness of the return distribution does not influence the conclusion. The Sortino ratio for the PBP is 0.0247 and 0.0539 for the S&P 500.

The CAGRs of the M-squared and Jensen measures are displayed at the bottom of the table. They are computed using $e^{r_{daily}*252}-1$. The difference between the CAGRs using the S&P 500 and the S&P TMI indexes primarily reflects that the S&P 500 return, 12.56%, was higher than the S&P TMI return, 12.03%, during the sample period. The choice of benchmark is a crucial decision in assessing performance. If the question is “What has been the abnormal performance of PBP?” the S&P TMI is the correct choice. But, if the question is “How well did PBP perform relative to the S&P 500 portfolio, the excess return regression PBP on the S&P 500 supplies valuable information. The excess return regression results tell us that PBP underperformed a portfolio with a weight of 0.658 in the S&P 500 portfolio and 0.342 in cash equivalents by about 3.09%.

Risk-adjusted performance		
Description	S&P 500	S&P TMI
n	2,518	2,518
Sharpe ratio - portfolio	0.019053	0.019053
Sharpe ratio - market	0.039353	0.036978
M-squared	-0.000174	-0.000154
Treynor ratio - portfolio	0.000248	0.000254
Treynor ratio - market	0.000438	0.000419
Jensen's alpha	-0.000125	-0.000106
Appraisal ratio	-0.027891	-0.023325
Sortino ratio - portfolio	0.024709	0.024709
Sortino ratio - market	0.053872	0.050330
M-squared - CAGR	-4.30%	-3.80%
Jensen's alpha - CAGR	-3.09%	-2.63%

References and suggested readings

Lintner, John, 1965, The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets, *Review of Economics and Statistics* 47, 13-37.

Modigliani, Franco, and Leah Modigliani, 1997, Risk-adjusted performance, *Journal of Portfolio Management* (Winter), 45-54.

Sharpe, William F., 1964, Capital asset prices: A theory of market equilibrium under conditions of risk, *Journal of Finance* 19, 425-442.

Sortino, Frank A. and van der Meer, Robert, 1991, Downside risk, *Journal of Portfolio Management* (Summer), 27-31.

Treynor, Jack L., and Fischer Black, 1973, How to use security analysis to improve portfolio selection. *Journal of Business* 46, 66-86.

Whaley, Robert E., 2023, CAPM review. *Applied Investment Management*.