THE JOURNAL OF FINANCE • VOL. XLI, NO. 1 • MARCH 1986

Valuation of American Futures Options: Theory and Empirical Tests

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ABSTRACT

This paper reviews the theory of futures option pricing and tests the valuation principles on transaction prices from the S&P 500 equity futures option market. The American futures option valuation equations are shown to generate mispricing errors which are systematically related to the degree the option is in-the-money and to the option's time to expiration. The models are also shown to generate abnormal risk-adjusted rates of return after transaction costs. The joint hypothesis that the American futures option pricing models are correctly specified and that the S&P 500 futures option market is efficient is refuted, at least for the sample period January 28, 1983 through December 30, 1983.

FUTURES OPTION CONTRACTS NOW trade on every major futures exchange and on a wide variety of underlying futures contracts. The Chicago Mercantile Exchange, the Chicago Board of Trade, the New York Futures Exchange, and the Commodity Exchange now collectively have more than twenty options written on futures contracts, where the underlying spot commodities are financial assets such as stock portfolios, bonds, notes and Eurodollars, foreign currencies such as West German marks, Swiss francs and British pounds, precious metals such as gold and silver, livestock commodities such as cattle and hogs, and agricultural commodities such as corn and soybeans. Moreover, new contract applications are before the Commodity Futures Trading Commission and should be actively trading in the near future.

With the markets for these new contingent claims becoming increasingly active, it is appropriate that the fundamentals of futures option valuation be reviewed and tested. Black [5] provides a framework for the analysis of commodity futures options. Although his work is explicitly directed at pricing European options on forward contracts, it applies to European futures contracts as well if the riskless rate of interest is constant during the futures option life.[1] The options currently

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[1] Cox, Ingersoll, and Ross [11, p. 324] demonstrate that the price of a futures contract is equal to the price of a forward contract when interest rates are nonstochastic.

trading, however, are American options, and only recently has theoretical work begun to focus on the American futures option pricing problem.[2]

The purpose of this paper is to review the theory underlying American futures option valuation and to test it on transaction prices from the S&P 500 equity futures option market. In the first section of the paper, the theory of futures option pricing is reviewed. The partial differential equation of Black ([5]) is presented, and the boundary conditions of the American and European futures option pricing problems are shown to imply different valuation equations. For the American futures options, efficient analytic approximations of the values of the call and put are presented, and the magnitude of the early exercise premium is simulated.

In the second section of the paper, the American futures option valuation principles are tested on S&P 500 futures option contract data for the period January 28, 1983 through December 30, 1983. Included are an examination of the systematic biases in the mispricing errors of the option pricing models, a test of the stationarity of the volatility of the futures price change relatives, and a test of the joint hypothesis that the American futures option models are correctly specified and that the S&P market is efficient. The paper concludes with a summary of the major results of the study.

I. Theory of Futures Option Valuation

An option on a futures contract is like an option on a common stock in the sense that it provides its holder with the right to buy or sell the underlying security at the exercise price of the option. Unlike a stock option, however, a cash exchange in the amount of the exercise price does not occur when the futures option is exercised. Upon exercise, a futures option holder merely acquires a long or short futures position with a futures price equal to the exercise price of the option. When the futures contract is marked-to-market at the close of the day's trading, the option holder is free to withdraw in cash an amount equal to the futures price less the exercise price in the case of a call and the exercise price less the futures price in the case of a put. Thus, exercising a futures option is like receiving in cash the exercisable value of the option.

A. Assumptions and Notation

Black [5] provides the groundwork for futures option valuation. Although his work is directed at pricing a European call option, it is general in the sense that the partial differential equation describing the dynamics of the call option price through time applies to put options as well as call options and to American options as well as European options. The assumptions necessary to develop Black's partial differential equation are as follows:

[2] Following Black's [5] seminal article, Moriarity, Phillips, and Tosini [18], Asay [1], Wolf [24], and others discussed the European futures option pricing problem. Other than the studies by Whaley [22] and Stoll and Whaley [21], the theoretical work on American futures options is unpublished and includes studies by Ramaswamy and Sundaresan [19] and Brenner, Courtadon, and Subrahmanyam [9].

(A1) There are no transaction costs in the option, futures, and bond markets. These include direct costs such as commissions and implicit costs such as the bid-ask spread and penalties on short sales.
(A2) Markets are free of costless arbitrage opportunities. If two assets or portfolios of assets have identical terminal values, they have the same price, and/or, if an asset or portfolio of assets has a future value which is certain to be positive, the initial value (cost) of the asset or portfolio is certain to be negative (positive).
(A3) The short-term riskless rate of interest is constant through time.
(A4) The instantaneous futures price change relative is described by the stochastic differential equation,

$$dF/F = \mu \, dt + \sigma \, dz,$$

where μ is the expected instantaneous price change relative of the futures contract, σ is the instantaneous standard deviation, and z is a Wiener process.

Assumptions (A1) and (A2) are fairly innocuous. Transaction costs are trivial for those making the market in the various financial assets, and available empirical evidence suggests investors behave rationally. Assumption (A3) may appear contradictory, since some futures options are written on long-term debt instrument futures contracts[3] where the driving force behind the volatility of the futures price change relatives is interest rate uncertainty. The two interest rates are, to some degree, separable, however. Assumption (A3) describes the behavior of the short-term interest rate on, say, Treasury bills, while the volatility of T-bond futures prices, for example, is related to the volatility of the long-term U.S. Treasury bond forward rate.[4] Assumption (A4) describes the dynamics of the futures price movements through time. It is important to note that no assumption about the relationship between the futures price and the price of the underlying spot commodity has been invoked.[5] The valuation results presented in this section, therefore, apply to any futures option contract, independent of the nature of the underlying spot commodity.

[3] The Chicago Board of Trade, for example, lists options on U.S. T-bond and T-note futures contracts.

[4] A priori, the assumption of constant short-term interest rate is untenable for all option pricing models. A constant short-term rate implies a constant, flat term structure, with interest rate uncertainty having no bearing on the volatility of the underlying asset prices. Such is hardly the case. The validity of such option pricing models, however, need not be evaluated on the basis of their assumptions and can be judged on the merits of their predictions.

[5] Note that Assumption (A4) defines the dynamics of the futures price movements with no reference to the relationship between the futures price and the price of the underlying spot commodity. Whether such an assumption is more appropriate for the futures price dynamics or the underlying spot commodity dynamics is an open empirical question.

Assumption (A4) is consistent with the assumption that the underlying spot price, S, follows the stochastic differential equation.

$$dS/S = \alpha \, dt + \sigma \, dz,$$

where α is the expected relative spot price change, and σ is the instantaneous standard deviation if there is (a) a constant, continuous riskless rate of interest, r, and (b) a constant, continuous

For expositional purposes, the following notation is adopted in this study to describe futures options and their related parameters:

F = current futures price
F_T = random futures price at expiration
$C(F, T; X)[c(F, T; X)]$ = American [European] call option price
$P(F, T; X)[p(F, T; X)]$ = American [European] put option price
$\varepsilon_C(F, T; X)[\varepsilon_p(F, T; X)]$ = early exercise premium of American call [put] option
r = riskless rate of interest
T = time to expiration of futures options
X = exercise price of futures options.

B. Solution to Futures Option Pricing Problem

Under the above-stated assumptions, Black demonstrates that, if a riskless hedge can be formed between the futures option and its underlying futures contract, the partial differential equation governing the movements of the futures option price (V) through time is

$$\tfrac{1}{2}\sigma^2 F^2 V_{FF} - rV + V_t = 0. \tag{1}$$

This equation applies to American call ($C = V$) and put ($P = V$) options, as well as European call ($c = V$) and put ($p = V$) options. What distinguishes the four valuation problems is the set of boundary conditions applied to each problem.

C. European Futures Options

The boundary condition necessary to develop an analytic formula for the European call option is that the terminal call price is equal to the maximum value of 0 or the in-the-money amount of the option, that is, $\max(0, F_T - X)$. Black shows that, when this terminal boundary condition is applied to Equation

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proportional rate of receipt (payment), d, for holding the underlying spot commodity. To show this result, apply Ito's lemma to the cost-of-carry relationship, $S_t = F_t e^{-(r-d)(T-t)}$, where F_t is defined in (A4). The expected futures price change relative, μ, is equal to the expected spot price change relative less the difference between the riskless rate of interest and the continuous rate of receipt, $\alpha - (r - d)$, and the standard deviation, σ, is the same for both the underlying spot commodity and futures price changes.

The interpretation of d depends on the nature of the underlying spot commodity. For example, in the foreign currency futures market, d represents the foreign interest rate earned on the investment in the foreign currency. For agricultural commodity futures, d is less than zero and represents the rate of cost for holding the spot commodity (i.e., storage costs, insurance costs, etc.), and for stock index futures, d represents the continuous proportional dividend yield on the underlying stock portfolio.

A continuous proportional dividend yield assumption may not be appropriate for a stock index since dividend payments are discrete and have a tendency to cluster according to the day of the week and the month of the year. With uncertain discrete dividend payments during the futures' life, the cost-of-carry relationship between the prices of the stock index and stock index futures is unclear, however, as long as (A4) holds for the futures price dynamics, the option pricing relationships contained in the paper will hold.

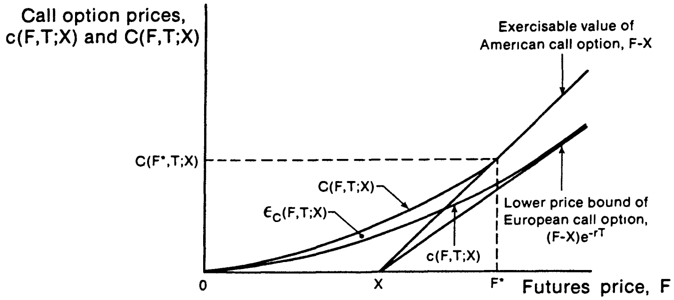

Figure 1. European and American Call Option Prices As a Function of the Underlying Futures Contract Price.

(1) where $c = V$, the value of a European call option on a futures contract is

$$c(F,\ T;\ X) = e^{-rT}[FN(d_1) - XN(d_2)], \tag{2}$$

where $d_1 = [\ln(F/X) + 0.5\sigma^2T]/\sigma\sqrt{T}$, and $d_2 = d_1 - \sigma\sqrt{T}$, and where $N(\ \ )$ is the cumulative univariate normal distribution. When the lower boundary condition for the European put, $\max(0,\ X - F_T)$, is applied to the partial differential Equation (1), the analytic solution is

$$p(F,\ T;\ X) = e^{-rT}[XN(-d_2) - FN(-d_1)], \tag{3}$$

where all notation is as it was defined above.

D. American Futures Options

The European call formula (2) provides a convenient way of demonstrating that the American call option may be exercised early. As the futures price becomes extremely large relative to the exercise of the option, the values of $N(d_1)$ and $N(d_2)$ approach one, and the European call value approaches $(F - X)e^{-rT}$. But, the American option may be exercised immediately for $F - X$, which is higher than the European option value. Thus, the American call option may be worth more "dead" than "alive"[6] and will command a higher price that the European call option.

Figure 1 illustrates the value of the American call option's early exercise privilege. In the figure, F^* represents the critical current futures price level where the American call option holder is indifferent about exercising his option immediately or continuing to hold it. Below F^*, the value of the early exercise premium, $\varepsilon_C(F,\ T;\ X)$, is equal to the difference between the American and European call functions, $C(F,\ T;\ X) - c(F,\ T;\ X)$. Above F^*, $\varepsilon_C(F,\ T;\ X)$ is equal to $(F - X) - c(F,\ T;\ X)$. Note that as the futures price becomes large relative to the exercise price, the European call option value approaches $(F - X)e^{-rT}$, and the early exercise premium approaches $(F - X)(1 - e^{-rT})$. In other words, the maximum

[6] Merton [17] demonstrates that, because the exercisable value of an American call option on a nondividend-paying stock, $S - X$, is always below the lower price bound of the corresponding European option, $S - Xe^{-rT}$, the American call option is always worth more alive than dead, and, therefore, will not be exercised early.

value the early exercise premium may attain is the present value of the interest income which can be earned if the call option is exercised immediately.

Unlike the European option case, there are no known analytic solutions to the partial differential Equation (1), subject to the American call option on a futures contract boundary condition, $C(F, t; X) \geq \max(0, F_t - X)$ for all $0 \leq t \leq T$, and, subject to the American put option on a futures contract boundary condition, $P(F, t; X) \geq \max(0, X - F_t)$ for all $0 \leq t \leq T$. Usually, the valuation of American futures options has resorted to finite difference approximation methods.[7] Ramaswamy and Sundaresan [19] and Brenner, Courtadon, and Subrahmanyam [9], for example, use such techniques. Unfortunately, finite difference methods are computationally expensive because they involve enumerating every possible path the futures option price could travel during its remaining time to expiration.

Whaley [23] adapts the Geske-Johnson [13] compound option analytic approximation method to price American futures options. In addition to being computationally less expensive than numerical methods, the compound option approach offers the advantages of being intuitively appealing and easily amenable to comparative statics analysis. Unfortunately, even though the compound option approach is about twenty times faster than numerical methods, it is still not inexpensive because it requires the evaluation of cumulative bivariate and cumulative trivariate normal density functions.

The analytic approximation of American futures option values used in this study is that derived by Barone-Adesi and Whaley [3]. The method is based on MacMillan's [16] quadratic approximation of the American put option on a stock valuation problem and is considerably faster than either the finite difference or the compound option approximation methods.

The quadratic approximation of the American call option on a futures contract, as provided in Barone-Adesi and Whaley [3], is

$$C(F, T; X) = c(F, T; X) + A_2(F/F^*)^{q_2}, \quad \text{where} \quad F < F^*, \quad \text{and}$$

$$C(F, T; X) = F - X, \quad \text{where} \quad F \geq F^*, \tag{4}$$

and where $A_2 = (F^*/q_2)\{1 - e^{-rT}N[d_1(F^*)]\}$, $d_1(F^*) = [\ln(F^*/X) + 0.5\sigma^2 T]/\sigma\sqrt{T}$, $q_2 = (1 + \sqrt{1 + 4k})/2$, *and* $k = 2r/[\sigma^2(1 - e^{-rT})]$. F^* is the critical futures price above which the American futures option should be exercised immediately (see Figure 1) and is determined iteratively by solving

$$F^* - X = c(F^*, T; X) + \{1 - e^{-rT}N[d_1(F^*)]\}F^*/q_2. \tag{4a}$$

Although the valuation equation may appear ominous, its intuition is simple. For a current futures price below the critical stock price, F^*, the American call value is equal to the European value plus the early exercise premium, as approximated by the term, $A_2(F/F^*)^{q_2}$. Above F^*, the worth of the American call is its exercisable proceeds.

[7] The first applications of finite difference methods to option pricing problems were by Schwartz [20] who valued warrants written on dividend-paying stocks and by Brennan and Schwartz [7] who priced American put options on nondividend-paying stocks. These techniques are reviewed in Brennan and Schwartz [8] and Geske and Shastri [15].

The only parameter to the American option formula (4) which requires computational sophistication beyond that required for the European formula (2) is the determination of the critical futures price F^*. To this end, Barone-Adesi and Whaley [3] provide an algorithm for solving (4a) in five iterations or less.

The quadratic approximation of the American put option on a futures contract is

$$P(F, T; X) = p(F, T; X) + A_1(F/F^{**})^{q_1}, \quad \text{where} \quad F > F^{**}, \quad \text{and}$$
$$P(F, T; X) = X - F, \quad \text{where} \quad F \le F^{**}, \tag{5}$$

and where $A_1 = -(F^{**}/q_1)\{1 - e^{-rT}N[-d_1(F^{**})]\}$, $q_1 = (1 - \sqrt{1 + 4k})/2$, and where all other notation is as it was defined for the American call, F^{**} is the critical futures price below which the American futures option should be exercised immediately and is determined iteratively by solving

$$X - F^{**} = p(F^{**}, T; X) - \{1 - e^{-rT}N[-d_1(F^{**})]\}F^{**}/q_1. \tag{5a}$$

E. Simulation of Early Exercise Premium Values

To demonstrate plausible magnitudes of the early exercise premium on American futures options, the European and American models prices were computed for a range of option pricing parameters. The results are reported in Table I. It is interesting to note that out-of-the-money futures options have negligible early exercise premiums. For example, when the futures price (F) is 90, the riskless rate of interest (r) is 8 percent, and the standard deviation of the futures price relatives (σ) is 0.15, an out-of-the-money call option with an exercise price (X) of 100 and a time to expiration (T) of 0.5 years has an early exercise premium of 0.0106, only slightly more than 1 percent of the American option price. Even at-the-money options have small early exercise premiums which account for only a small percentage of the option price. Only when the option is considerably in-the-money does the early exercise premium account for a significant proportion of the price of the option.

In summary, the theory of futures option valuation suggests that the early exercise privilege of American futures options contributes meaningfully to the futures option value. The simulation results, based on option pricing parameters that are typical for S&P 500 futures option contracts, suggest that this is true, but only for in-the-money options.

II. Empirical Tests

In this section, the performance of the American futures option pricing models is analyzed using transaction information for S&P 500 equity futures options. After the description of the data in the first subsection, the implied standard deviation methodology is discussed. Volatility estimates are made using nonlinear regression of observed futures option prices on model prices. The third subsection presents an examination of the systematic patterns in the models prediction errors. This analysis is motivated by the evidence reported in the stock option

Table I

Theoretical European and American Futures Option Values: Exercise Price (X) = 100

Futures Option Parameters[a]	Futures Price (F)	Call Options: European $c(F, T; X)$[b]	Call Options: American $C(F, T; X)$[c]	Call Options: Early Exercise Premium $\varepsilon_C(F, T; X)$	Put Options: European $p(F, T; X)$[b]	Put Options: American $P(F, T; X)$[c]	Put Options: Early Exercise Premium $\varepsilon_P(F, T; X)$
$r = 0.08$	80	0.0027	0.0029	0.0002	19.6067	20.0000	0.3933
$\sigma = 0.15$	90	0.2529	0.2547	0.0018	10.0549	10.1506	0.0957
$T = 0.25$	100	2.9321	2.9458	0.0137	2.9321	2.9458	0.0137
	110	10.1752	10.2627	0.0875	0.3732	0.3756	0.0024
	120	19.6239	20.0000	0.3761	0.0199	0.0204	0.0005
$r = 0.12$	80	0.0027	0.0030	0.0003	19.4116	20.0000	0.5884
$\sigma = 0.15$	90	0.2504	0.2533	0.0029	9.9549	10.1153	0.1605
$T = 0.25$	100	2.9029	2.9257	0.0228	2.9029	2.9257	0.0228
	110	10.0740	10.2205	0.1465	0.3695	0.3734	0.0039
	120	19.4286	20.0000	0.5714	0.0197	0.0205	0.0008
$r = 0.08$	80	0.3956	0.3986	0.0030	19.9996	20.2032	0.2036
$\sigma = 0.30$	90	1.9817	1.9913	0.0096	11.7837	11.8543	0.0707
$T = 0.25$	100	5.8604	5.8878	0.0274	5.8604	5.8878	0.0274
	110	12.2527	12.3237	0.0710	2.4507	2.4624	0.0116
	120	20.4776	20.6470	0.1694	0.8737	0.8790	0.0053
$r = 0.08$	80	0.0583	0.0603	0.0020	19.2740	20.0000	0.7260
$\sigma = 0.15$	90	0.8150	0.8256	0.0106	10.4229	10.6044	0.1815
$T = 0.50$	100	4.0637	4.1099	0.0463	4.0637	4.1099	0.0463
	110	10.6831	10.8584	0.1753	1.0752	1.0887	0.0134
	120	19.4105	20.0018	0.5913	0.1947	0.1991	0.0043

[a] The notation used in this column is as follows: r = riskless rate of interest; σ = standard deviation of the futures price change relative; and T = time to expiration.

[b] The European futures option values are computed using the Black [5] pricing equations.

[c] The American futures option values are computed using the Barone-Adesi and Whaley [3] analytic approximations.

pricing tests. In the fourth subsection, the hypothesis that the standard deviation of futures price change relatives is the same across call and put options is tested. The final subsection presents the results of a joint test of the hypothesis that the American futures option pricing models are correctly specified and that the S&P 500 futures option market is efficient.

A. Data

The data used in this study consist of transaction information for the S&P 500 equity futures and futures option contracts traded on the Chicago Mercantile Exchange (CME) from the first day of trading of the S&P futures options, January 28, 1983, through the last business day of the year, December 30, 1983. The data were provided by the CME and are referred to as "Quote Capture" information. Essentially, the data set contains the time and the price of every transaction in which the price changed from the previously recorded transaction. Bid and ask prices are also recorded if the bid price exceeds or the ask price is below the price at the last transaction. The volume of each transaction and the number of transactions at a particular price are not recorded.

Two exclusionary criteria were applied to the Quote Capture information. First, bid and ask price quotes were eliminated because they do not represent prices at which there were both a buyer and seller available to transact. Both sides of the market transaction were necessary within the market efficiency test design. Second, futures options with times to expiration in excess of 26 weeks were excluded. The trading activity in these options and their underlying futures contracts was too sparse to warrant consideration with the market efficiency test. What remained was a sample of 28,736 transactions, 21,613 in the nearest contract month, and 7,123 in the second nearest contract month.

The futures option pricing models require the futures price at the instant at which the option is traded. To represent the contemporaneous futures price, the futures price at the trade most closely preceding the futures option trade is used. Because the S&P 500 futures market was so active during the investigation period, the average time between the futures and the subsequent futures option transactions was only 21 seconds.

Table II offers a summary of the characteristics of the transactions contained in the 232-day sample period. Of the 28,736 transactions, 15,063 were call option transactions and 13,763 were puts. The at-the-money options appear to have been the most active, with 55 percent of the call option trades and 50 percent of the put option trades being at futures prices ± 2 percent of the exercise price. Out-of-the-money options were more active than in-the-money options: 25 percent of total trades to 20 percent of total trades for calls and 42 percent to 8 percent for puts, respectively. Over 64 percent of the transactions were on options with maturities of less than 8 weeks, verifying that most of the trading activity was in the nearest contract month.

The yield on the U.S. Treasury bill maturing on the contract month expiration day[8] was used to proxy for the riskless rate on interest. The yields were computed

[8] S&P 500 futures option contracts expired the third Thursday of the contract month until the June 1984 contract. Beginning with the June 1984 contract, the third Friday of the month is the expiration day.

Table II

Summary of S&P 500 Futures Option Transactions during the Period January 28, 1983 through December 30, 1983

Futures Price/ Exercise Price (F/X)	No. of Transactions Call	Put	Both	Time to Expiration (in weeks) (T)	No. of Transactions Call	Put	Both
$F/X < 0.90$	11	2	13	$T < 2$	2,307	2,234	4,541
$0.90 \leq F/X < 0.92$	77	9	86	$2 \leq T < 4$	2,375	2,190	4,565
$0.92 \leq F/X < 0.94$	339	42	381	$4 \leq T < 6$	2,567	2,211	4,778
$0.94 \leq F/X < 0.96$	1,014	191	1,205	$6 \leq T < 8$	2,480	2,064	4,544
$0.96 \leq F/X < 0.98$	2,281	773	3,054	$8 < T < 10$	1,708	1,623	3,331
$0.98 \leq F/X < 1.00$	4,091	2,615	6,706	$10 \leq T < 12$	1,479	1,371	2,850
$1.00 \leq F/X < 1.02$	4,260	4,252	8,512	$12 \leq T < 14$	1,255	1,164	2,419
$1.02 \leq F/X < 1.04$	1,783	2,559	4,342	$14 \leq T < 16$	337	445	782
$1.04 \leq F/X < 1.06$	830	1,524	2,354	$16 \leq T < 18$	222	173	395
$1.06 \leq F/X < 1.08$	241	875	1,116	$18 \leq T < 20$	175	90	265
$1.08 \leq F/X < 1.10$	78	453	531	$20 \leq T$	158	108	266
$1.10 \leq F/X$	58	378	436				
All	15,063	13,673	28,736	All	15,063	13,673	28,736

daily on the basis of the average of the T-bill's bid and ask discounts reported in the *Wall Street Journal.*

B. Implied Standard Deviation Methodology

The American futures option pricing models have five parameters: F, X, T, r, and σ. Of these, four are known or are easily estimated. The exercise price, X, and the time to expiration, T, are terms of the futures option contract, and the futures price, F, and the riskless rate of interest, r, are easily accessible market values. The troublesome parameter to estimate is the standard deviation of the futures price change relatives.

The methodology used to estimate the standard deviation of the futures price change relative is described in Whaley [22, pp. 39–40]. Observed futures option prices, V_j, were regressed on their respective model prices, $V_j(\sigma)$, that is,

$$V_j = V_j(\sigma) + \varepsilon_j. \tag{6}$$

where ε_j is a random disturbance term,[9] each day during the sample period. All transaction prices for the day were used in each regression. The number of transactions used to estimate σ in a given day ranged from 30 to 300, with the average number being 124. The estimates of σ ranged from 0.1009 to 0.2176, with the average being approximately 0.1555.

The time series of standard deviation estimates indicates that the volatility of the S&P 500 futures price relatives declined during 1983. During the first 116 trading days of the sample period, the average estimate of σ using the American model was 0.1711, while, during the last 116 days of the period, it was 0.1399. It is interesting to note that, during the same two subperiods, the S&P 500 Index rose by 15.07 percent and −0.65 percent, respectively.[10]

C. Tests for Systematic Biases

One way in which the performance of an option pricing model may be evaluated is by examining its mispricing errors for systematic tendencies. Whaley [22] demonstrates that, when the early exercise premium of the American call option on a dividend-paying stock is accounted for in the valuation model, the exercise price and time to expiration biases which had been documented for the European model disappear. Geske and Roll [14] later verify this result and also attempt to explain the variance bias. Here, the variance bias is not of concern since there is only one underlying commodity. The ability of the American futures option models to eliminate the first two biases, however, should be examined.

The tests for systematic biases in the futures option pricing models involved clustering and then averaging the price deviations by the degree the option is in-

[9] The relationship between observed and model prices is not exact and is affected by: (a) model misspecification; (b) nonsimultaneity of futures and futures option price quotations; and (c) the bid-ask spread in the futures and futures option markets. If the residuals in the nonlinear regression (6) are independent and normally distributed, the resulting value of σ is the maximum likelihood estimate.

[10] This evidence is consistent with the notion that the variance rate depends on the price of the underlying asset.

Table III

Summary of Average Mispricing Errors of American Futures Option Pricing Models by the Option's Moneyness (F/X) and by the Option's Time to Expiration in Weeks (T) for S&P 500 Futures Option Transactions during the Period January 28, 1983 through December 30, 1983

	$C - C(F, T; X)$				$P - P(F, T; X)$			
	$T < 6$	$6 \leq T < 12$	$T \geq 12$	All T	$T < 6$	$6 \leq T < 12$	$T \geq 12$	All T
$F/X < 0.98$	−0.0630[a]	−0.1372	−0.0872	−0.1028	−0.1064	−0.0914	−0.1056	−0.1014
	(1,221)	(1,760)	(741)	(3,722)	(593)	(335)	(89)	(1,017)
$0.98 \leq F/X < 1.02$	−0.1228	−0.0775	0.0073	−0.0924	−0.0816	−0.0196	0.1336	−0.0406
	(4,452)	(2,858)	(951)	(8,351)	(3,999)	(2,193)	(675)	(6,867)
$F/X \geq 1.02$	0.0577	0.1175	0.0702	0.0806	0.1286	0.1906	30.3060	0.1929
	(1,486)	(1,049)	(455)	(2,990)	(2,043)	(2,530)	(1,216)	(5,789)
All F/X	−0.0757	−0.0599	−0.0120	−0.0606	−0.0191	0.0808	0.2287	0.0537
	(7,249)	(5,667)	(2,147)	(15,063)	(6,635)	(5,058)	(1,980)	(13,673)

[a] The average deviation of the observed option price from the model price for the 1,221 call option transaction prices with in-the-moneyness (F/X) less than 0.98 and time to expiration (T) less than 6 weeks is −0.0630.

the-money of the option and by the option's time to expiration. Table III contains a summary of the results for the 15,063 call option and the 13,673 put option transactions in the sample.

Both a "moneyness" bias and a "maturity" bias appear for the call option transaction prices of the sample. The moneyness bias is just the opposite of that reported for stock options.[11] The further the call option is in-the-money, the lower is the model price relative to the observed price (i.e., out-of-the-money calls are overpriced by the model and the in-the-money calls are underpriced). This is true for the American models when all maturities are clustered together and when the intermediate-term and long-term options are considered separately. For the short-term options, the greatest mispricing occurs for the at-the-money calls, which appear dramatically underpriced relative to the model [e.g., for the American call option pricing model, the average value of $C - C(F, T; X)$ is -0.1228].

The maturity bias for the calls is also just the opposite of that reported for call options on stocks. Here, the model prices are higher than the observed prices for short-term options and are lower than observed for long-term options. The relationship is not consistent across the moneyness groupings, however. For out-of-the-money calls, the mispricing is greatest for the intermediate term options with the model considerably overstating observed values [e.g., the average $C - C(F, T; X)$ is -0.1372], and, for in-the-money options, the mispricing is still greatest for the intermediate term options, but with the models understating observed values [e.g., the average $C - C(F, T; X)$ is 0.1175]. Overall, however, the maturity bias does not appear to be as serious as the moneyness bias for the sample of call option transaction prices.

The average price deviations for the put options appear to have a more orderly pattern, with the relationships between average price deviation and the moneyness and maturity of the options monotonic. Like the call option results, the maturity bias takes the form of short-term options being underpriced relative to the model and long-term options being overpriced. Unlike the call option results, however, the maturity bias is almost as serious as the moneyness bias, and the moneyness bias takes the form of out-of-the-money options being overpriced relative to the model and in-the-money options underpriced. (Recall the put option is in-the-money where $F/X < 1$.) A possible explanation of this latter result is that floor traders engage in conversion/reversal arbitrage using the European put-call parity relationship,[12]

$$c(F, T; X) - p(F, T; X) = (F - X)e^{-rT}. \tag{7}$$

[11] See, e.g., Black [4] or Whaley [22].

[12] The European put-call parity relationship can be found in a variety of papers, including Black [5], Moriarity, Phillips, and Tosini [18], Asay [1], and Wolf [24]. In all of these studies, the futures contract underlying the option contract is treated like a forward, but no problems arise because the European option can be exercised only at expiration.

For American futures options, the assumption of equivalence between forward and futures contract positions can lead to erroneous statements about futures option pricing. Some of these results are outlined in Ramaswamy and Sundaresan [19]. Stoll and Whaley [21] derive the put-call parity relationship for American futures options.

If the put-call parity relationship (7) is actively arbitraged, overpricing of in-the-money call options should result in overpricing of out-of-the-money put options, and underpricing of out-of-the-money call options should result in underpricing of in-the-money put options, or vice versa.

One final note about the results in Table III is worthwhile. During the period examined, put options were overpriced on average while call options were underpriced. Obviously, this result is sensitive to the volatility estimate used to price the options, but, nonetheless, the difference between the average mispricing errors of the put and call option formulas would be approximately the same even if a different estimate of σ were used. This peculiarity indicates that the market's assessment of the volatility of the relative futures price changes may be greater for puts than for calls and provides the motivation for the tests in the next subsection.

D. Stationarity of Volatility Estimates Across Options

To test the hypothesis that the standard deviation of futures price change relatives is the same in the pricing of call and put options on the S&P 500 futures contracts, the ratio,

$$R = [SSE_C(\sigma_C) + SSE_p(\sigma_p)]/SSE(\sigma), \tag{8}$$

was computed each day during the sample period. In (8), $SSE_C(\sigma_C)$ is the sum of squared errors realized by estimating the nonlinear regression (6) using only the call option transaction prices during the day, and $SSE_p(\sigma_p)$ is the sum of squared errors using only the put option prices. $SSE(\sigma)$ is the sum of squared errors using both the call and put option prices. If the residuals of the regressions are independent and normally distributed, Gallant [12] shows that the test statistic,

$$F = (n - 2)(1 - R), \tag{9}$$

is approximately distributed, $F_{1,n-2}$.[13] The results of these tests are reported in Table IV.

The test results indicate that the null hypothesis that the volatility estimates are equal for calls and puts is rejected in 75 percent of the cases for the American model. The standard deviation of futures price relatives implied by call option prices is lower, on average, than that implied by put option prices. The cause of this anomaly is difficult to determine. One possible explanation is that the stochastic process governing the futures price movements is ill-defined, so the option pricing models are misspecified. Another is that perhaps two separate clienteles trade in call options and in put options. But, this latter explanation fails to account for the floor traders who could costlessly benefit from such a clientele arrangement.

Regardless of the explanation, the anomaly may be only transitory. The only fact established so far is that the futures option pricing models do not adequately explain the observed structure of option prices. It may well be the case that the

[13] Barone-Adesi [2] uses a similar maximum likelihood test to compare the structural forms of competing option pricing models.

Table IV

Frequency Distribution of Non-Rejection/Rejection of the Null Hypothesis that the Standard Deviations Implied by Option Prices Are Equal for Call-and-Put Options Using S&P 500 Futures Option Transaction Prices during the Period January 28, 1983 through December 30, 1983

Hypothesis[a, b]	Frequency
H_O: The standard deviation of the futures price relatives for call options is equal to the standard deviation for put options.	59
H_A: The standard deviation of the futures price relatives for call options is *not* equal to the standard deviation for put options.	173
Total	232

[a] The probability level used in the evaluation of the test statistics is 5 percent.

[b] The test statistic for the hypothesis test is $F = (n - 2)(1 - R)$, where n is the number of option transactions and $R = [SSE_C(\sigma_C) + SSE_p(\sigma_p)]/SSE(\sigma)$. Assuming the residuals are independent and normally distributed, the ratio F is approximately distributed as $F_{1,n-2}$.

market is mispricing S&P 500 futures options and that abnormal risk-adjusted rates of return may be earned by trading on the basis of the models' prices.

E. Market Efficiency Test

The systematic biases reported in Table III and the σ-anomaly reported in Table IV may result because the futures option pricing models are misspecified or because the S&P 500 futures option market is inefficient or both. One way of attempting to isolate the two effects is to test whether abnormal rates of return after transaction costs may be earned by trading futures options on the basis of the models' prices. If abnormal returns after transaction costs can be earned, it is likely to be the case that the market is inefficient. The price deviations, systematic or not, signal profit opportunities. If abnormal profits cannot be earned, there are no grounds for rejecting the null hypothesis that the model is correctly specified and that the S&P 500 futures option market is efficient.

The market efficiency test design involved hedging mispriced futures options against the underlying futures contract. Each day options were priced using the American futures option pricing models and the standard deviations estimated from *all* of the previous day's transaction prices.[14] Because no estimate of σ was available for the transactions of the first day of the sample period, January 28, 1983, the first day's transactions were eliminated, and only 231 days and 28,493 options remained in the sample.

[14] Because both call and put option transaction prices are used in the daily regression to estimate the σ, the estimate is, in essence, an average of the estimates implied by call and puts separately.

Each of the 28,493 option transactions was examined to see whether the option was undervalued or overvalued relative to the futures option pricing models. The hedge formed at that instant in time[15] depended on the nature of the transaction price:

Nature of Transaction Price	Futures Option Position	Futures Position
Undervalued call	Long 1 contract	Short $\delta C/\delta F$ contracts
Overvalued call	Short 1 contract	Long $\delta C/\delta F$ contracts
Undervalued put	Long 1 contract	Long $-\delta P/\delta F$ contracts
Overvalued put	Short 1 contract	Short $-\delta P/\delta F$ contracts

where the partial derivatives of the call and put option prices were computed using valuation Equations (4) and (5).

Two types of hedge portfolios were considered in the analysis. The first was a "buy-and-hold" hedge portfolio. Each hedge was formed according to the weights described above and was held until the futures option/futures expiration or until the end of the sample period, whichever came first. At such time, the futures option/futures positions were closed, and the hedge profit was computed. The second was the "rebalanced" hedge portfolio. Here, the initial hedge composition was the same as the buy-and-hold strategy, but at the end of each day, the futures position was altered to account for the change in the futures option's hedge ratio. The difference between the profits of these two hedge portfolio strategies was, therefore, the net gain or loss on the intermediate futures position adjustments within the rebalanced portfolio.[16]

Note that the hedge portfolios are assumed to be held until the option's

[15] The hedge portfolio strategy assumed that the hedge is formed at the prices which signalled the profit opportunity. This was done for two reasons. First, floor traders have the opportunity to transact at these prices. If a sell order at a price below the model price enters the pit, the floor trader can buy the options and then hedge his position within seconds using the futures. Second, the transaction price for retail customers may be handled by simply adding the bid-ask spread to the price which triggered a buy and subtracting the bid-ask spread from the price which triggered a sell.

[16] To illustrate the mechanics of the buy-and-hold and rebalanced hedge portfolio strategies, consider the following example. A call option with an exercise price of $100 and with two days to expiration is priced at $1, where its theoretical price is $1.50 and its hedge ratio is 0.8. The current futures price is $100. Because the call is underpriced relative to the model, it is purchased, and 0.8 futures contracts are sold. The net investment of both the buy-and-hold and rebalanced hedge portfolios is, therefore, $1 (i.e., one option contract times $1 per contract).

By the end of the day before expiration, the futures price rises to say, $102. At the new futures price, the model price is $3.00 and the hedge ratio is 0.9. Since the hedge ratio has changed, 0.1 more futures contracts must be sold in order to maintain the riskless hedge of the rebalanced portfolio. The additional futures contracts are assumed to be bought or sold at the day's closing price, in this case $102.

Now, suppose that on the following day, the futures expires at $106, and the futures option at $6.00 (i.e., the futures price $106 less the exercise price $100). The buy-and-hold hedge portfolio profit would be computed as the option position profit, $6 − 1 = $5, plus the futures position profit, −0.8 × ($106 − 100) = −$4.80, or $0.20 in total. The rebalanced hedge portfolio profit is computed as the $0.20 buy-and-hold profit plus the net gain (loss) on the intermediate futures position change, −0.1 × ($106 − 102) = −$0.40, or −$0.20 in total.

expiration. This is unlike the empirical procedures used in the stock option market efficiency tests which assume that an option position is opened at one price and then closed at the next available price. If the option pricing models have systematic mispricing tendencies, an option which is undervalued on one day is likely to be undervalued on the next. By holding the option position open until expiration, at which time the observed and model prices converge to the same value, there is some assurance that the prospective option mispricing profits are being captured.

In Table V, the average cost, profit, and rate of return of the hedge portfolios formed on the basis of the American futures option prices are presented. When no minimum size restriction was placed on the absolute price deviation, 28,493 hedge portfolios were formed. On an average, the number of futures contracts in each hedge at formation was 0.442 (1.442 less one futures option contract). The average investment cost of each hedge was −\$46.75 (−0.0935 × \$500),[17] indicating that, on an average, money was collected when the hedge portfolios were formed.

The average profit for the buy-and-hold hedge portfolio was \$88 (0.1760 × \$500), and the average rebalanced hedge portfolio profit was \$77.85. The daily rebalancing of the futures position lowered overall hedge profits. On the other hand, the standard deviation of the buy-and-hold profit was 1.9302 compared with 0.8574 for the rebalanced portfolio profits.[18] The daily rebalancing of the futures position decreased the volatility of the hedge profits portfolio by more than 55 percent.

Immediately to the right of the rebalanced portfolio profit column is a column with break-even transaction cost rates. These numbers represent the average of the transaction cost rate per contract sufficient to eliminate rebalanced portfolio profit. In other words, if the transaction cost rate was less than \$57.60 (0.1152 × \$500) per contract, the average portfolio profit was greater than zero. Note that the transaction costs were assumed to be paid only on the contracts bought or sold when the portfolio was formed. The overall net effect of the incremental transaction costs on the intermediate daily rebalancing of the futures position of the hedge portfolios was assumed to be equal to zero.[19]

The rebalanced portfolio excess rate of return column contains the average rate of return and the net of any interest carrying charge. If the option in the hedge portfolio was purchased, the excess rate of return of the hedge was equal to the rate of return on the hedge less the riskless rate of interest. If the option was sold, interest was assumed to be earned on the proceeds from the sale, so the excess rate of return on the hedge was equal to the rate of return on the hedge plus the riskless rate of interest. The excess rate of return for the rebalanced

[17] The value for the S&P 500 futures and futures options are index values. The dollar worth of the contract is obtained by multiplying the index value by \$500.

[18] The standard deviations are not reported, but they can be inferred from the reported numbers of observations and the *t*-ratios.

[19] To account for the transaction costs of the daily readjustment of the futures position within each portfolio separately would dramatically overstate the role of transaction costs within the hedge portfolio because, at the end of the day, some hedges will require that futures contracts be purchased and some that futures be sold. The net overall daily adjustment in the futures position would likely be near zero, so no intermediate transaction costs were imposed.

Table V

Average Cost, Profit, and Rate of Return of Hedge Portfolios by Size of Absolute Price Deviation from the American Futures Option Pricing Models for S&P 500 Futures Option Transaction Prices during the Period January 31, 1983 through December 30, 1983

Minimum Absolute Price Deviation	No. of Observations	Average Investment[a]	Average No. of Contracts[b]	Buy-and-Hold Portfolio Profit[c]	Rebalanced Portfolio Profit[d]	Break-Even Transaction Cost Rate[e]	Rebalanced Portfolio Excess Rate of Return[f]	Rebalanced Portfolio Excess Return after Transaction Costs[g]	Relative Systematic Risk[h]
All $\|\Delta\|$	28,493	−0.0935	1.442	0.1760	0.1557	0.1152	0.0905	0.0696	0.1193
				(15.39)[i]	(30.64)		(35.77)	(27.78)	(2.11)
$\|\Delta\| \geq 0.05$	22,850	−0.1035	1.441	0.2054	0.1854	0.1372	0.1026	0.0850	0.0745
				(15.83)	(31.41)		(38.48)	(32.21)	(1.27)
$\|\Delta\| \geq 0.10$	17,596	−0.1160	1.437	0.2444	0.2181	0.1615	0.1164	0.1006	0.0375
				(16.24)	(30.70)		(39.91)	(34.81)	(0.59)
$\|\Delta\| \geq 0.15$	13,116	−0.1370	1.430	0.2507	0.2424	0.1802	0.1247	0.1099	0.0924
				(14.07)	(27.53)		(37.69)	(33.48)	(1.30)
$\|\Delta\| \geq 0.20$	9,521	−0.1200	1.425	0.2607	0.2696	0.2006	0.1309	0.1168	0.1632
				(12.18)	(23.82)		(33.64)	(30.20)	(1.98)

[a] The cost of the hedge portfolio is equal to the option price if the option is purchased and minus the option price if the option is sold. The futures position involves no net investment.

[b] The average absolute number of option and futures contracts in the hedge.

[c] The buy-and-hold portfolio profit assumes the hedge is formed and held until the expiration of the contracts or the end of the sample period.

[d] The rebalanced portfolio profit is equal to the buy-and-hold profit plus (less) the net gains (losses) from the futures position adjustments made during the option's life.

[e] The break-even transaction cost per contract sufficient to eliminate the rebalanced portfolio profit.

[f] The rate of return of the rebalanced hedge portfolio less the riskless rate of interest.

[g] The excess rate of return of the rebalanced hedge portfolio after a $10 per contract transaction cost.

[h] The relative systematic risk is estimated by regressing the excess rate of return of the hedge on the relative futures price changes over the same period.

[i] The values in parentheses are t-ratios for the null hypothesis that the parameter is equal to 0.

portfolio using all of the transactions was 9.05 percent and is significantly greater than zero.

Before proceeding with a description of the remaining two columns, it is worthwhile to point out three facts about the excess rates of return for the rebalanced hedge portfolio. First, the excess return did not fall very much if the proceeds from the futures option sales were assumed to earn no interest. In this case, the average excess rate of return was 8.41 percent, with a t-ratio of 33.49. Second, the excess rate of return for the American model was only slightly higher than it was for the European model. For the latter model, the average return was 8.91 percent, with a t-ratio of 35.03. This evidence is consistent with the simulation results in the last section. Finally, the use of Student t-ratios to evaluate the significance of the excess rates of return is appropriate since the return distributions were symmetric and only slightly leptokurtic.

The column labelled excess rate of return after transaction costs incorporated a $10 per contract transaction cost assumption. Such a fee is probably appropriate for a floor trader.[20] The average excess rate of return after transaction costs was 6.96 percent, again significantly greater than zero.

The final column contains estimated slope coefficients from the regression of rebalanced portfolio excess rates of return on the futures price change relatives over the corresponding period. In essence, this regression is intended to evaluate the effectiveness of the portfolio rebalancing at maintaining a riskless hedge. For the entire sample of hedge portfolio, the relative systematic risk is significantly positive at the 5 percent level, however its magnitude, 0.1193, is very small.

Table V also contains the hedge portfolio profit characteristics when minimum absolute option price deviations of 0.05, 0.10, 0.15, and 0.20 were imposed. Naturally, the higher was the demanded absolute price deviation, the fewer were the option transactions to qualify as hedge portfolio candidates. In the case where the minimum absolute deviation was set equal to 0.10, for example, only 17,596 hedges were formed.

With all of the price deviation strategies reported in Table V, the average excess rates of return are significantly greater than zero. For floor traders,

[20] Actually, the assumed $10 per contract overstates the transaction costs a floor trader might face. The only transaction cost paid by floor traders is a clearing fee, which is on order of $1.50 per contract. The $10 per contract assumption, therefore, presents a conservative view of the floor trader's hedge portfolio profits after transaction costs.

Two other institutional considerations are worthy of note. The transaction cost rates in this market are quoted on a "round-turn" basis. That is, a $50 per contract commission charge covers the cost of entering the market at the time of purchase or sale and the cost of closing the position at a subsequent date. For futures contract positions, the broker charges all of the commission when the position is closed, and, for futures option positions, half the commission is charged when the position is opened and half when it is closed.

Since commission rates are negotiated between each customer and his or her broker, it is difficult to assess what are representative commission charges for the various futures/futures option customers. Large institutional customers such as mutual funds typically pay commissions at a rate of $20 to $30 per contract and are allowed to post margin requirement in the form of interest-bearing T-bills. Smaller customers likely pay commissions of $50 or more, and are also allowed to the T-bill margin-posting privilege. Some brokers quote lower rates for small customers, but demand margin money in the form of cash.

demanding a minimum price deviation of 0.05 is reasonable since they face only the cost of clearing their transactions, which is considerably less than $25 per contract. When such a minimum price deviation was imposed, the average hedge portfolio excess rate of return was 10.26 percent before clearing costs and 8.50 percent after a $10 per contract clearing cost was applied to both the futures option and futures transactions. Retail customers, however, not only face the commission rates imposed by their broker, but also the bid-ask spread imposed by the market maker. Assuming a commission rate of $50 per contract and a bid-ask spread of $50 per option contract, demanding a minimum price deviation of 0.20 is reasonable. However, in this case, the average break-even transaction cost rate was 0.2006, so the retail customer would have earned about $0.30 per hedge after transaction costs.

In the previous section, systematic mispricing errors related to the moneyness of the option were documented. For this reason, the option transactions were categorized by the type of option and by the degree to which the option is in-the-money. The results are reported in Table VI. Most of the abnormal profits associated with the trading strategy appear to be concentrated in out-of-the-money put options. The average excess rate of return after the floor trader's clearing costs was 16.88 percent. In comparison, none of the other option categories had an average return greater than 3 percent after clearing costs.

One plausible explanation for this result is that more than 72 percent out-of-the-money put options were overpriced (see Table III) and thus sold within the trading strategy. Over the period January 31, 1983 through December 30, 1983, the S&P 500 Index rose from 145.30 to 164.93, indicating that writing out-of-the-money puts would have been profitable indeed. But, the put options sold within the hedge strategy were balanced against short positions in the futures, so what was gained on the put transactions should have been lost on the futures transactions. Moreover, the estimated systematic risk for the hedge portfolios in this category was significantly negative, indicating that, if anything, not enough put options were sold to immunize the portfolio against movements in the underlying futures price. The overall upward market movement in the equity market during the examination period must, therefore, be discounted as a potential explanation of the market inefficiency.

Although the results of Table VI indicate that floor traders could profit by writing out-of-the-money puts, it is doubtful whether retail customers could profit by such a strategy. As was noted in Table II, at-the-money options enjoyed the greatest volume of activity and, therefore, probably experienced the lowest bid-ask spread. Out-of-the-money S&P 500 futures options have less liquidity, and it is not uncommon to find the bid-ask spread as high as 0.15 or 0.20. Assuming a commission rate of $50 per contract and a bid-ask spread of $50 per contract takes the average profit from $159.70 per hedge to an average gain after transaction costs of $45.40.

Overall, the results reported in Tables V and VI provide evidence that the joint hypothesis that the American futures option valuation models are correctly specified and that the S&P 500 futures option market is efficient is refuted for the period January 31, 1983 through December 30, 1983, at least from the

Table VI

Average Cost, Profit, and Rate of Return of Hedge Portfolios by the Moneyness of the Option for S&P 500 Futures Option Transaction Prices during the Period January 31, 1983 through December 30, 1983

Futures Option Category	No. of Observations	Average Investment[a]	Average No. of Contracts[b]	Buy-and-Hold Portfolio Profit[c]	Rebalanced Portfolio Profit[d]	Break-Even Transaction Cost Rate[e]	Rebalanced Portfolio Excess Rate of Return[f]	Rebalanced Portfolio Excess Return after Transaction Costs[g]	Relative Systematic Risk[h]
Calls	7,736	−1.0521	1.339	−0.0077	0.0204	0.0160	0.0432	0.0159	0.7339
$F/X < 1$				(−0.34)[i]	(2.34)		(7.00)	(2.60)	(5.25)
Calls	7,150	0.5963	1.670	0.1052	0.1284	0.0763	0.0295	0.0206	0.4858
$F/X \geq 1$				(4.60)	(16.81)		(12.58)	(8.84)	(9.02)
Puts	3,620	−1.9300	1.646	0.0975	0.0497	0.0273	0.0186	0.0074	0.4150
$F/X < 1$				(2.98)	(1.95)		(3.99)	(1.61)	(3.66)
Puts	9,987	0.8208	1.286	0.3979	0.3194	0.2518	0.1968	0.1688	−0.7379
$F/X \geq 1$				(21.53)	(37.46)		(42.10)	(36.42)	(−7.56)

[a] The cost of the hedge portfolio is equal to the option price if the option is purchased and minus the option price if the option is sold. The futures position involves no net investment.

[b] The average absolute number of option and futures contracts in the hedge.

[c] The buy-and-hold portfolio profit assumes the hedge is formed and held until the expiration of the contracts or the end of the sample period.

[d] The rebalanced portfolio profit is equal to the buy-and-hold profit plus (less) the net gains (losses) from the futures position adjustments made during the option's life.

[e] The break-even transaction cost per contract sufficient to eliminate the rebalanced portfolio profit.

[f] The rate of return of the rebalanced hedge portfolio less the riskless rate of interest.

[g] The excess rate of return of the rebalanced hedge portfolio after a $10 per contract transaction cost.

[h] The relative systematic risk is estimated by regressing the excess rate of return of the hedge on the relative futures price changes over the same period.

[i] The value in parentheses are t-ratios for the null hypothesis that the parameter is equal to 0.

Table VII

Average Cost, Profit, and Rate of Return of Hedge Portfolios by Subperiod for S&P 500 Futures Option Transaction Prices during the Period January 31, 1983 through December 30, 1983

Subperiod	No. of Observations	Average Investment[a]	Average No. of Contracts[b]	Buy-and-Hold Portfolio Profit[c]	Rebalanced Portfolio Profit[d]	Break-Even Transaction Cost Rate[e]	Rebalanced Portfolio Excess Rate of Return[f]	Rebalanced Portfolio Excess Return after Transaction Costs[g]	Relative Systematic Risk[h]
1/31/83–4/21/83	9,846	−0.0509	1.454	−0.1758	0.0308	0.0271	0.0047	−0.1024	0.8848
				(−8.73)[i]	(7.01)		(1.56)	(−4.13)	(15.40)
4/22/83–7/14/83	8,237	−0.1623	1.450	0.5118	0.3884	0.2682	0.2067	0.1876	0.8641
				(22.84)	(55.08)		(50.06)	(39.82)	(5.66)
7/15/83–10/6/83	6,001	−0.1902	1.423	0.2323	0.0953	0.0780	0.0737	0.0515	−0.2587
				(8.86)	(7.25)		(10.97)	(7.72)	(−0.90)
10/7/83–12/30/83	4,409	0.0710	1.430	0.2588	0.0968	0.0740	0.0879	0.0567	−1.769
				(14.55)	(4.49)		(12.48)	(8.14)	(−4.52)

[a] The cost of the hedge portfolio is equal to the option price if the option is purchased and minus the option price if the option is sold. The futures position involves no net investment.

[b] The average absolute number of option and futures contracts in the hedge.

[c] The buy-and-hold portfolio profit assumes the hedge is formed and held until the expiration of the contracts or the end of the sample period.

[d] The rebalanced portfolio profit is equal to the buy-and-hold profit plus (less) the net gains (losses) from the futures position adjustments made during the option's life.

[e] The break-even transaction cost per contract sufficient to eliminate the rebalanced portfolio profit.

[f] The rate of return of the rebalanced hedge portfolio less the riskless rate of interest.

[g] The excess rate of return of the rebalanced hedge portfolio after a $10 per contract transaction cost.

[h] The relative systematic risk is estimated by regressing the excess rate of return of the hedge on the relative futures price changes over the same period.

[i] The values in parentheses are t-ratios for the null hypothesis that the parameter is equal to 0.

standpoint of floor traders who stood ready to transact based on model prices. From a retail customer's standpoint, however, it is doubtful whether abnormal profits after transaction costs could have been earned.

In Table VII, the option transactions in four separate subperiods are considered. In the first subperiod, the average excess rate of return on the hedge portfolio was 0.47 percent, insignificantly different from zero. In the remaining three subperiods, the excess rate of return was significantly greater than zero, with the return highest in the second subperiod and second highest in the final subperiod. In other words, there does not appear to be any indication that the market became more efficient during 1983. Whether floor traders can continue to earn abnormal rates of return after clearing costs by buying undervalued and selling overvalued S&P 500 futures options must await further empirical investigation.

III. Summary and Conclusions

The purpose of this paper is to review the theory underlying American futures option valuation and to test the theory in one of the recently developed futures option markets. The theoretical work begins by focusing on the partial differential equation of Black [5] and by discussing how the boundary conditions to the equation imply different structural forms to the pricing equations. Although no analytic solutions to the American futures option pricing problems are provided, efficient analytic approximations are presented. Simulations of futures option prices using the European and American models and plausible option pricing parameters show that the early exercise premium of the American futures option has a significant impact on pricing if the option is in-the-money.

The empirical work focuses on transaction prices for S&P 500 equity futures options during the first 232 trading days of the market's existence, the period from January 28, 1983 through December 30, 1983. The major empirical results are as follows:

1. A moneyness bias and a maturity bias appear for the American futures option pricing models. For calls, the moneyness bias is the opposite of that reported for stock options—out-of-the-money options are underpriced relative to the model and in-the-money options are overpriced. For puts, just the reverse is true—out-of-the-money puts are overpriced relative to the model and in-the-money puts are underpriced. The maturity bias is the same for both the calls and the puts—short time-to-expiration options are underpriced relative to the model and long time-to-expiration are overpriced, but the bias appears more serious for put options than for call options.
2. The standard deviation implied by call option transaction prices is lower, on average, than that implied by put option prices.
3. A riskless hedging strategy using the American futures option pricing models (as well as the European futures option pricing models) generates abnormal risk-adjusted rates of return after the transaction costs paid by floor traders or large institutional customers. If a retail customer was to try to capture the profits implied by the futures option mispricing, however, transaction costs will likely eliminate the hedge portfolio profit opportunities.

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